Kinematical correlations: from RHIC to LHC

Antoni Szczurek

INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES
IFJ PAN
AND
UNIVERSITY OF RZESZOW
Our recent works on correlations

- Jet-jet correlations (with A. Rybarska and G. Ślipek)
- Photon-jet correlations (with T. Pietrycki)
- Charm-anticharm correlations (with M. Łuszczak)
- Correlations of leptons from semileptonic decays of heavy mesons
- Drell-Yan pair production (with G. Ślipek)
- $J/\psi - gluon$ correlations (with S. Baranov)
Plan of the first part

- Introduction/Motivation
- Theoretical approach(es)
- Matrix elements
- Unintegrated gluon distributions
- Results
- Conclusions

based on:
A. Szczurek, A. Rybarska and G. Slipek,
Introduction/Motivation

Experimental motivation:
New RHIC data for hadron-hadron correlations – indication of jet structure down to small transverse momenta
(→ Jan Rak)
New PHENIX data

Theoretical motivation:
Dynamics of gluon/parton ladders – a theoretical challenge.
The QCD dynamics (collinear, $k_t$-factorization) is usually investigated for inclusive reactions:

- $\gamma^*$-proton total cross section (or $F_2$)
- Inclusive production of jets
- Inclusive production of mesons (pions)
- Inclusive production of open charm, bottom, top
- Inclusive production of direct photons
- Inclusive production of quarkonia
Very interesting are:

- Dijet correlations (Leonidov-Ostrovsky, Bartels et al.)
- $Q\bar{Q}$ correlations (→ Marta Luszczak)
- $\gamma^* –$ jet correlations (→ Tomasz Pietrycki)
- jet – $J/\psi$ correlations (Baranov-Szczurek)
- Exclusive reactions: $pp \rightarrow pXp$ where $X = J/\psi, \chi_c, \chi_b, \eta', \eta_c, \eta_b$
  (Matrin-Khoze-Ryskin, Szczurek-Pasechnik-Teryaev)

They contain much more information about QCD ladders.
QCD motivation

HERA $\gamma^* p$ total cross section ($F_2(x, Q^2)$)
Collinear approach to dijet correlations

In LO:

\[ \frac{d\sigma}{d\phi} = f(W) \delta (\phi - \pi) \]  

(1)

In NLO:

\[ h_1 \]

\[ X_1 \]

\[ x_1 \]

\[ (y_1, p_{1t}) \]

\[ (y_3, p_{3t}) \]

\[ h_2 \]

\[ X_2 \]

\[ x_2 \]

\[ (y_2, p_{2t}) \]

Figure 1: A typical diagram for $2 \rightarrow 3$ contributions.
Figure 2: Typical diagrams for $k_t$-factorization approach.
Pair of partons in $\kappa_t$-factorization approach

$$
\frac{d\sigma(h_1 h_2 \rightarrow jj)}{d^2 p_{1,t} d^2 p_{2,t}} = \int dy_1 dy_2 \frac{d^2 \kappa_{1t}}{\pi} \frac{d^2 \kappa_{2t}}{\pi} \frac{1}{16\pi^2(x_1 x_2 s)^2} |\mathcal{M}(gg \rightarrow jj)|^2 \\
\cdot \delta^2(\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}) f(x_1, \kappa_{1,t}^2) f(x_2, \kappa_{2,t}^2)
$$

(2)

where

$$x_1 = \frac{m_{1t}}{\sqrt{s}} e^{+y_1} + \frac{m_{2t}}{\sqrt{s}} e^{+y_2}, \quad (3)$$

$$x_2 = \frac{m_{1t}}{\sqrt{s}} e^{-y_1} + \frac{m_{2t}}{\sqrt{s}} e^{-y_2}. \quad (4)$$

The final partonic state is $jj = gg, q\bar{q}$.

There are other (quark/antiquark initiated) processes ($\rightarrow$ see soon)
Pair of partons in $\kappa_t$-factorization approach

\[ f_1(x_1, \kappa_{1,t}^2) \to x_1 g_1(x_1) \delta(\kappa_{1,t}^2) \]  
\[ f_2(x_2, \kappa_{2,t}^2) \to x_2 g_2(x_2) \delta(\kappa_{2,t}^2) \]

then one recovers the standard collinear formula.

Inclusive cross sections:

\[ \frac{d\sigma(h_1 h_2 \to j)}{dy_1 d^2 p_{1,t}} = 2 \int dy_2 \frac{d^2 \kappa_{1,t}}{\pi} \frac{d^2 \kappa_{2,t}}{\pi} \left( \ldots \right) | \bar{p}_{2,t} = \bar{\kappa}_{1,t} + \bar{\kappa}_{2,t} - \bar{p}_{1,t} \]  
\[ \frac{d\sigma(h_1 h_2 \to j)}{dy_2 d^2 p_{2,t}} = 2 \int dy_1 \frac{d^2 \kappa_{1,t}}{\pi} \frac{d^2 \kappa_{2,t}}{\pi} \left( \ldots \right) | \bar{p}_{1,t} = \bar{\kappa}_{1,t} + \bar{\kappa}_{2,t} - \bar{p}_{2,t} . \]
The integration with the Dirac delta function in (2)

\[ \int dy_1 dy_2 \frac{d^2 \kappa_{1t}}{\pi} \frac{d^2 \kappa_{2t}}{\pi} (\ldots) \delta^2(\ldots) . \]  

(9)

can be performed by introducing the following new auxiliary variables:

\[ \vec{Q}_t = \vec{\kappa}_{1t} + \vec{\kappa}_{2t} , \]
\[ \vec{q}_t = \vec{\kappa}_{1t} - \vec{\kappa}_{2t} . \]  

(10)

The jacobian of this transformation is:

\[ \frac{\partial (\vec{Q}_t, \vec{q}_t)}{\partial (\vec{\kappa}_{1t}, \vec{\kappa}_{2t})} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = 2 \cdot 2 = 4 . \]  

(11)
Pair of partons in $k_t$-factorization approach

Then:

$$
\frac{d\sigma(h_1 h_2 \rightarrow Q \bar{Q})}{d^2 p_{1,t} d^2 p_{2,t}} = \frac{1}{4} \int dy_1 dy_2 d^2 Q_t d^2 q_t (\ldots) \delta^2(\vec{Q}_t - \vec{p}_{1,t} - \vec{p}_{2,t})
$$

(12)

$$
= \frac{1}{4} \int dy_1 dy_2 d^2 q_t (\ldots) \bigg|_{\vec{Q}_t = \vec{P}_t} =
$$

(13)

$$
= \frac{1}{4} \int dy_1 dy_2 \left( q_t dq_t \right) d\varphi (\ldots) \bigg|_{\vec{Q}_t = \vec{P}_t} =
$$

(14)

$$
= \frac{1}{4} \int dy_1 dy_2 \left( \frac{1}{2} dq_t^2 \right) d\varphi (\ldots) \bigg|_{\vec{Q}_t = \vec{P}_t}.
$$

(15)

Above $\vec{P}_t = \vec{p}_{1,t} + \vec{p}_{2,t}$. 
If one is interested in the distribution of the sum of transverse momenta of the outgoing quarks, then it is convenient to write

\[
\frac{1}{4} d^2 P_t d^2 p_t = \frac{1}{4} d\phi + P_t dP_t \ d\phi - p_t dp_t
\]

(16)

If one is interested in studying a two-dimensional map \( p_{1,t} \times p_{2,t} \) then

\[
\frac{1}{4} 2\pi P_t dP_t \ d\phi - p_t dp_t .
\]

(16)

Then

\[
\frac{d\sigma(p_{1,t}, p_{2,t})}{dp_{1,t} dp_{2,t}} = \int d\phi_1 d\phi_2 p_{1,t} p_{2,t} \int dy_1 dy_2 \ \frac{1}{4} q_t dq_t d\phi_{q_t} (\ldots) .
\]

(18)
Pair of partons in $k_t$-factorization approach

It is convenient to make the following transformation of variables

$$(\phi_1, \phi_2) \rightarrow (\phi_{\text{sum}} = \phi_1 + \phi_2, \phi_{\text{dif}} = \phi_1 - \phi_2), \quad (19)$$

where $\phi_{\text{sum}} \in (0, 4\pi)$ and $\phi_{\text{dif}} \in (-2\pi, 2\pi)$. Now the new domain $(\phi_{\text{sum}}, \phi_{\text{dif}})$ is twice bigger than the original one $(\phi_1, \phi_2)$.

$$d\phi_1 d\phi_2 = \left( \frac{\partial \phi_1 \partial \phi_2}{\partial \phi_{\text{sum}} \partial \phi_{\text{dif}}} \right) d\phi_{\text{sum}} d\phi_{\text{dif}}. \quad (20)$$

The transformation jacobian is:

$$\left( \frac{\partial \phi_1 \partial \phi_2}{\partial \phi_{\text{sum}} \partial \phi_{\text{dif}}} \right) = \frac{1}{2}. \quad (21)$$
Pair of partons in $k_t$-factorization approach

\[
d^2 p_{1,t} \ d^2 p_{2,t} = p_{1,t} d p_{1,t} \ p_{2,t} d p_{2,t} \frac{d \phi_{\text{sum}} d \phi_{\text{dif}}}{2} \\
= p_{1,t} d p_{1,t} \ p_{2,t} d p_{2,t} 2 \pi d \phi_{\text{dif}} . \quad (22)
\]

The integrals in Eq.(18) can be written equivalently as

\[
\frac{d \sigma (p_{1,t}, p_{2,t})}{dp_{1,t} dp_{2,t}} = \frac{1}{2} \cdot \frac{1}{2} \ \int d \phi_{\text{sum}} d \phi_{\text{dif}} \ p_{1,t} p_{2,t} \ \int dy_1 dy_2 \ \frac{1}{4} q_t dq_t d \phi_{q_t} (\ldots) . \quad (23)
\]

First $\frac{1}{2}$ – jacobian, second $\frac{1}{2}$ – extra extension of the domain.

By symmetry, there is no dependence on $\phi_{\text{sum}}$

\[
\frac{d \sigma (p_{1,t}, p_{2,t})}{dp_{1,t} dp_{2,t}} = \frac{1}{2} \cdot \frac{1}{2} \cdot 4 \pi \ \int d \phi_{\text{dif}} \ p_{1,t} p_{2,t} \ \int dy_1 dy_2 \ \frac{1}{4} q_t dq_t d \phi_{q_t} (\ldots) . \quad (24)
\]
The matrix elements for on-shell initial gluons/partons

\[ |\mathcal{M}_{gg\to gg}|^2 = \frac{9}{2} g_s^4 \left( 3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right), \]

\[ |\mathcal{M}_{gg\to q\bar{q}}|^2 = \frac{1}{8} g_s^4 \left( 6 \frac{\hat{t}\hat{u}}{\hat{s}^2} + \frac{4}{3} \frac{\hat{u}}{\hat{t}} + \frac{4}{3} \frac{\hat{t}}{\hat{s}} + 3 \frac{\hat{u}}{\hat{s}} \right), \]

\[ |\mathcal{M}_{gg\to gq}|^2 = g_s^4 \left( -\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2} \right), \]

\[ |\mathcal{M}_{qq\to gg}|^2 = g_s^4 \left( -\frac{4}{9} \frac{\hat{s}^2 + \hat{t}^2}{\hat{s}\hat{t}} + \frac{\hat{t}^2 + \hat{s}^2}{\hat{u}^2} \right). \quad (25) \]

The matrix elements for off-shell initial gluons – the same formulae but with \( \hat{s}, \hat{t}, \hat{u} \) from off-shell kinematics. In this case \( \hat{s} + \hat{t} + \hat{u} = k_1^2 + k_2^2 \), where \( k_1^2, k_2^2 < 0 \). Our prescription – a smooth analytic continuation of the on-shell formula off mass shell.
processes in collinear approach

Standard parton model formula:

\[
d\sigma(h_1h_2 \rightarrow ggg) = \int dx_1 dx_2 \, g_1(x_1, \mu^2) g_2(x_2, \mu^2) \, d\hat{\sigma}(gg \rightarrow ggg)
\]

(26)

The elementary cross section can be written as

\[
d\hat{\sigma}(gg \rightarrow ggg) = \frac{1}{2\hat{s}} |\mathcal{M}_{gg \rightarrow ggg}|^2 dR_3 .
\]

(27)

The three-body phase space element is:

\[
dR_3 = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} (2\pi)^4 \delta^4 (p_a + p_b - p_1 - p_2 - p_3) ,
\]

(28)
It can be written in an equivalent way as:

\[
dR_3 = \frac{dy_1 d^2 p_{1t}}{(4\pi)(2\pi)^2} \frac{dy_2 d^2 p_{2t}}{(4\pi)(2\pi)^2} \frac{dy_3 d^2 p_{3t}}{(4\pi)(2\pi)^2} (2\pi)^4 \delta^4 (p_a + p_b - p_1 - p_2 - p_3) ,
\]

(29)

The last formula is useful for practical purposes. Now

\[
d\sigma = dy_1 d^2 p_{1t} dy_2 d^2 p_{2t} dy_3 \cdot \frac{1}{(4\pi)^3(2\pi)^2} \frac{1}{\hat{s}^2} x_1 f_1(x_1, \mu_f^2)x_2 f_2(x_2, \mu_f^2) |\mathcal{M}_{2\to3}|^2
\]

(30)

where

\[
x_1 = \frac{p_{1t}}{\sqrt{s}} \exp(+y_1) + \frac{p_{2t}}{\sqrt{s}} \exp(+y_2) + \frac{p_{3t}}{\sqrt{s}} \exp(+y_3) ,
\]

\[
x_2 = \frac{p_{1t}}{\sqrt{s}} \exp(-y_1) + \frac{p_{2t}}{\sqrt{s}} \exp(-y_2) + \frac{p_{3t}}{\sqrt{s}} \exp(-y_3) .
\]

(31)
Repeating similar steps as for $2 \rightarrow 2$:

$$d\sigma = \frac{1}{64\pi^4 \hat{s}^2} \ x_1 f_1(x_1, \mu_f^2) x_2 f_2(x_2, \mu_f^2) \ |\mathcal{M}_{2\rightarrow 3}|^2$$

$$p_1 t dp_1 t p_2 t dp_2 t d\Phi_- dy_1 dy_2 dy_3,$$

where $\Phi_-\) is restricted to the interval $(0, \pi)$.
Matrix elements for $2 \rightarrow 3$ processes

For the $gg \rightarrow ggg$ process ($k_1 + k_2 \rightarrow k_3 + k_4 + k_5$) the squared matrix element is

$$|\mathcal{M}|^2 = \frac{1}{2} g_s^6 \frac{N_c^3}{N_c^2 - 1} \left[ (12345) + (12354) + (12435) + (12453) + (12534) + (12543) + (13245) + (13254) + (13425) + (13524) + (12453) + (14325) \right]$$

$$\times \sum_{i<j} (k_i k_j) / \prod_{i<j} (k_i k_j),$$

(33)

where $(ijlmn) \equiv (k_i k_j)(k_j k_l)(k_l k_m)(k_m k_n)(k_n k_i)$. 
Matrix elements for $2 \rightarrow 3$ processes

It is useful to calculate matrix element for the process $q\bar{q} \rightarrow ggg$. The squared matrix elements for other processes can be obtained by crossing the squared matrix element for the process $q\bar{q} \rightarrow ggg (p_a + p_b \rightarrow k_1 + k_2 + k_3)$

$$|\mathcal{M}|^2 = g_s^6 \frac{N_c^2 - 1}{4N_c^4}$$

$$\sum_{i}^3 a_i b_i (a_i^2 + b_i^2) / (a_1 a_2 a_3 b_1 b_2 b_3)$$

$$\times \left[ \frac{s}{2} + N_c^2 \left( \frac{s}{2} - \frac{a_1 b_2 + a_2 b_1}{(k_1 k_2)} - \frac{a_2 b_3 + a_3 b_2}{(k_2 k_3)} - \frac{a_3 b_1 + a_1 b_3}{(k_3 k_1)} \right) \right]$$

$$+ \frac{2N_4}{\hat{s}} \left( \frac{a_3 b_3 (a_1 b_2 + a_2 b_1)}{(k_2 k_3)(k_3 k_1)} + \frac{a_1 b_1 (a_2 b_3 + a_3 b_2)}{(k_3 k_1)(k_1 k_2)} + \frac{a_2 b_2 (a_3 b_1 + a_1 b_3)}{(k_1 k_2)(k_2 k_3)} \right)$$

(34)
Matrix elements for $2 \rightarrow 3$ processes

The matrix element for the process $gg \rightarrow q\bar{q}g$ is obtained from that of $q\bar{q} \rightarrow ggg$ by appropriate crossing:

$$|M|_{gg\rightarrow q\bar{q}g}^2(k_1, k_2, k_3, k_4, k_5) = \frac{9}{64} \cdot |M|_{q\bar{q}\rightarrow ggg}^2(-k_4, -k_3, -k_1, -k_2, k_5).$$  \hspace{1cm} (36)

We sum over 3 final flavours ($f = u, d, s$).

For the $qg \rightarrow qgg$ process

$$|M|_{qg\rightarrow qgg}^2(k_1, k_2, k_3, k_4, k_5) = \left(-\frac{3}{8}\right) \cdot |M|_{q\bar{q}\rightarrow ggg}^2(k_1, -k_3, -k_2, k_4, k_5)$$  \hspace{1cm} (37)

and finally for the process $g\bar{q} \rightarrow \bar{q}gg$

$$|M|_{g\bar{q}\rightarrow \bar{q}gg}^2(k_1, k_2, k_3, k_4, k_5) = \left(-\frac{3}{8}\right) \cdot |M|_{q\bar{q}\rightarrow ggg}^2(-k_3, k_2, -k_1, k_4, k_5).$$  \hspace{1cm} (38)
Gaussian smearing

\[ \mathcal{F}_{naive}(x, \kappa^2, \mu_F^2) = x g^{coll}(x, \mu_F^2) \cdot f_{Gauss}(\kappa^2), \]  
\[ (39) \]

\[ f_{Gauss}(\kappa^2) = \frac{1}{2\pi\sigma_0^2} \exp \left( -\frac{\kappa_t^2}{2\sigma_0^2} \right) / \pi. \]  
\[ (40) \]

BFKL UGDF

\[ -x \frac{\partial f(x, q_t^2)}{\partial x} = \frac{\alpha_s N_c}{\pi} q_t^2 \int_0^\infty \frac{dq_{1t}^2}{q_{1t}^2} \left[ \frac{f(x, q_{1t}^2) - f(x, q_t^2)}{|q_t^2 - q_{1t}^2|} + \frac{f(x, q_t^2)}{\sqrt{q_t^4 + 4q_{1t}^4}} \right]. \]  
\[ (41) \]
Unintegrated gluon distributions (part 2)

Golec-Biernat-Wuesthoff saturation model from dipole-nucleon cross section to UGDF

\[ \alpha_s \mathcal{F}(x, \kappa_t^2) = \frac{3\sigma_0}{4\pi^2} R_0^2(x) \kappa_t^2 \exp(-R_0^2(x) \kappa_t^2), \quad (42) \]

\[ R_0(x) = \left( \frac{x}{x_0} \right)^{\lambda/2} \frac{1}{\text{GeV}}. \quad (43) \]

Parameters adjusted to HERA data for $F_2$.

Kharzeev-Levin gluon saturation

\[ \mathcal{F}(x, \kappa^2) = \begin{cases} f_0 & \text{if } \kappa^2 < Q_s^2, \\ f_0 \cdot \frac{Q_s^2}{\kappa^2} & \text{if } \kappa^2 > Q_s^2. \end{cases} \quad (44) \]

$f_0$ adjusted by Szczurek to HERA data for $F_2$. Describes nicely inclusive pion production at RHIC.
Kwiecinski parton distributions

QCD-most-consistent approach – CCFM.

For LO (2 → 1) processes convenient to use UPDFs in a space conjugated to transverse momentum (Kwieciński et al.)

\[ \tilde{f}(x, b, \mu^2) = \frac{1}{2\pi} \int d^2\kappa \exp \left( -i\kappa \cdot \vec{b} \right) F(x, \kappa^2, \mu^2) \]

\[ F(x, \kappa^2, \mu^2) = \frac{1}{2\pi} \int d^2 b \exp \left( i\kappa \cdot \vec{b} \right) \tilde{f}(x, b, \mu^2) \]

The relation between

Kwieciński UPDF and the collinear PDF:

\[ x p_k(x, \mu^2) = \int_0^\infty d\kappa_t^2 f_k(x, \kappa_t^2, \mu^2) \]
At $b = 0$ the functions $f_j$ are related to the familiar integrated parton distributions, $p_j(x, Q)$, as follows:

$$f_j(x, 0, Q) = \frac{x}{2} p_j(x, Q).$$

$$p_{NS} = u - \bar{u}, \quad d - \bar{d},$$

$$p_S = \bar{u} + u + \bar{d} + d + \bar{s} + s + \ldots,$$

$$p_{sea} = 2\bar{d} + 2u + \bar{s} + s + \ldots,$$

$$p_G = g,$$

where $\ldots$ stand for higher flavors.
Kwiecinski equations

for a given impact parameter:

\[
\frac{\partial f_{NS}(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_0^1 dz P_{qq}(z) \left[ \Theta(z - x) J_0((1 - z)Qb) f_{NS} \left( \frac{x}{z}, b, Q \right) 
- f_{NS}(x, b, Q) \right]
\]

\[
\frac{\partial f_S(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_0^1 dz \left\{ \Theta(z - x) J_0((1 - z)Qb) \left[ P_{qq}(z) f_S \left( \frac{x}{z}, b, Q \right) 
+ P_{qg}(z) f_G \left( \frac{x}{z}, b, Q \right) \right] 
- [zP_{qq}(z) + zP_{qg}(z)] f_S(x, b, Q) \right\}
\]

\[
\frac{\partial f_G(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_0^1 dz \left\{ \Theta(z - x) J_0((1 - z)Qb) \left[ P_{qg}(z) f_S \left( \frac{x}{z}, b, Q \right) 
+ P_{gg}(z) f_G \left( \frac{x}{z}, b, Q \right) \right] 
- [zP_{qg}(z) + zP_{gg}(z)] f_G(x, b, Q) \right\}
\]
Nonperturbative effects

Transverse momenta of partons due to:

- perturbative effects
  (solution of the Kwieciński-CCFM equations),
- nonperturbative effects
  (intrinsic momentum distribution of partons)

Take factorized form in the b-space:

\[
\tilde{f}_q(x, b, \mu^2) = \tilde{f}_q^{CCFM}(x, b, \mu^2) \cdot F_{q}^{np}(b). 
\]

We use a flavour and x independent form factor

\[
F_{q}^{np}(b) = F^{np}(b) = \exp\left(\frac{-b^2}{4b_0^2}\right)
\]

May be too simplistic?
Unintegrated gluon distributions (comparison)

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Processes included in our $k_t$-factorization approach

There are 4 important contributions:

- $\text{gluon+gluon} \rightarrow \text{gluon+gluon}$ (Leonidov-Ostrovsky)
- $\text{gluon+gluon} \rightarrow \text{quark+antiquark}$ (Leonidov-Ostrovsky)
- $\text{gluon+(anti)quark} \rightarrow \text{gluon+(anti)quark}$ (new !!!)
- $\text{(anti)quark+gluon} \rightarrow \text{(anti)quark+gluon}$ (new !!!)

First two processes discussed also by: Bartels-Sabio-Vera-Schwennsen
New contributions

Figure 3:
Processes included in $k_t$-factorization

$gg \rightarrow gg$ (left upper), $gg \rightarrow q\bar{q}$ (right upper), $gq \rightarrow gq$ (left lower), $qg \rightarrow qg$ (right lower).

Kwieciński UPDFs with $b_0 = 1$ GeV$^{-1}$, $\mu^2 = 100$ GeV$^2$. Full range of parton rapidities.
Processes included in $k_t$-factorization

Fractional contributions of different subprocesses

$gg \rightarrow gg$ (left upper),
$gg \rightarrow q\bar{q}$ (right upper),
$gq \rightarrow gq$ (left lower),
$qg \rightarrow qg$ (right lower).

Kwieciński UPDFs with $b_0 = 1 \text{ GeV}^{-1}$, $\mu^2 = 100 \text{ GeV}^2$.
$5 \text{ GeV} < p_{1t}, p_{2t} < 20 \text{ GeV}$. 
Azimuthal correlations

$W = 200 \text{ GeV}$

$p_{t1}, p_{t2} = (5, 15) \text{ GeV}$

$\frac{d\sigma}{d\varphi} (\text{mb})$

$\varphi (\text{deg})$

- $\text{gg} \rightarrow \text{gg}$
- $\text{gg} \rightarrow \text{qq\overline{q}q}$
- $\text{gg} \rightarrow \text{qq}$
- $\text{gg} \rightarrow \text{qq}$
$\mu^2 = 0.25$ (black), $10$ (blue), $100$ (red) GeV$^2$
Different UGDFs

\[ W = 200 \text{ GeV} \]
\[ p_{1t}, p_{2t} = (5, 15) \text{ GeV} \]
\[ y_1, y_2 = (-4, 4) \]

- Kwiecinski
- Kharzeev–Levin
- BFKL
- Ivanov–Nikolaev

\( \frac{d\sigma}{d\varphi} (\text{mb}) \)
2 \rightarrow 3 \text{ processes in collinear approach}

Figure 7: $gg \rightarrow ggg$ component for $W = 200$ GeV.

Singularities when $\vec{p}_1 \rightarrow 0$, $\vec{p}_2 \rightarrow 0$ and $\vec{p}_3 \rightarrow 0$. 
How to remove NLO singularities?

$k_t$-factorization – no singularities, no delta functions !!!
$gg \rightarrow gg$, different UGDFs vs $gg \rightarrow ggg$

KL (left upper), BFKL (right upper), Ivanov-Nikolaev (left lower), $gg \rightarrow ggg$ (right lower).

$-4 < y_1, y_2 < 4$. 
Dijet correlations for $gg \rightarrow ggg$, leading jets

$p_{1t}(selected) > p_{3t}$ and $p_{2t}(selected) > p_{3t}$
Dijet correlations for $gg \rightarrow ggg$, leading jets

$p_{1t}(selected) > p_{3t}$ and $p_{2t}(selected) > p_{3t}$

Figure 9:
Figure 10: Definition of windows in $p_{1t} \times p_{2t}$ plane.
Figure 11:
Extra scalar cuts

- to eliminate LO and NLO singularities (yes!)
- to enhance resummation with respect to NLO (no!)

Figure 12: $|p_{1t} - p_{2t}| > \Delta_s$. 
Extra vector cuts

- to eliminate LO and NLO singularities (yes!)
- to enhance resummation with respect to NLO (no!)

Figure 13: \(|\vec{p}_{1t} + \vec{p}_{2t}| > \Delta_v\).
Summary/Conclusions of the first part

- Dijet correlations at RHIC have been calculated in the $k_t$-factorization approach with different UGDFs (UPDFs) from the literature.

- Two new mechanisms have been included compared to the literature. They are dominant at larger rapidities (or rapidity gaps) i.e. constitute competition for Mueller-Navelet (BFKL) jets.

- Results have been compared with collinear NLO calculations.

- At $\phi < 120^0$ and/or asymmetric jet transverse momenta the $k_t$-factorization is superior over the collinear NLO.

- This calculation is a first step for hadron-hadron correlations measured at RHIC. Here internal structure of both jets enters in addition.

- The method can be used in semihard region (small $p_t$) at LHC.
Photon-jet correlations
Plan of the second part of the talk

- Introduction
- Inclusive spectra
- Photon-jet correlations
- Results
- Conclusions

based partially on:
in collaboration with T. Pietrycki
Cascade mechanism 1

\[
\begin{align*}
&h_1 \\
&X_1 \\
&(x_1, k_{1t}) \\
&p_{1t} \\
&(x_2, k_{2t}) \\
&p_{2t} \\
&h_2 \\
&X_2
\end{align*}
\]

\[
\begin{align*}
&h_1 \\
&X_1 \\
&(x_1, k_{1t}) \\
&p_{1t} \\
&(x_2, k_{2t}) \\
&p_{2t} \\
&h_2 \\
&X_2
\end{align*}
\]

\text{soft emissions}

\text{hard } \gamma, q

\text{soft emissions}

\text{soft emissions}

\text{soft emissions}

\text{soft emissions}

\text{soft emissions}

\text{soft emissions}

\text{soft emissions}
Cascade mechanism 2

\[ (x_1, k_{1t}) \rightarrow p_{1t} \rightarrow \text{hard } \gamma, g \rightarrow (x_2, k_{2t}) \rightarrow p_{2t} \rightarrow \text{soft emissions} \]

\[ (x_1, k_{1t}) \rightarrow p_{1t} \rightarrow \text{hard } \gamma, g \rightarrow (x_2, k_{2t}) \rightarrow p_{2t} \rightarrow \text{soft emissions} \]
Kimber-Martin-Ryskin for $k_t^2 > k_t^2,0$

$$f_q(x, k_t^2, \mu^2) = T_q(k_t^2, \mu^2) \frac{\alpha_s(k_t^2)}{2\pi}$$

$$\cdot \int_x^1 dz \left[ P_{qq}(z) \frac{x}{z} q(\frac{x}{z}, k_t^2) \Theta(\Delta - z) + P_{qg}(z) \frac{x}{z} g(\frac{x}{z}, k_t^2) \right]$$

$$f_g(x, k_t^2, \mu^2) = T_g(k_t^2, \mu^2) \frac{\alpha_s(k_t^2)}{2\pi}$$

$$\cdot \int_x^1 dz \left[ P_{gg}(z) \frac{x}{z} g(\frac{x}{z}, k_t^2) \Theta(\Delta - z) + \sum_q P_{qg}(z) \frac{x}{z} q(\frac{x}{z}, k_t^2) \right]$$

saturation for $k_t^2 < k_t^2,0$
\[
\frac{d\sigma(h_1h_2 \rightarrow \gamma, \text{parton})}{d^2p_{1,t}d^2p_{2,t}} = \int dy_1dy_2 \frac{d^2k_{1,t}}{\pi} \frac{d^2k_{2,t}}{\pi} \frac{1}{16\pi^2(x_1x_2s)^2} \sum_{i,j,k} |M(i,j \rightarrow \gamma k)|^2 
\]

\[\cdot \delta^2(\vec{k}_{1,t} + \vec{k}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}) f_i(x_1, k_{1,t}^2) f_j(x_2, k_{2,t}^2)\]

\[(i, j, k) = (q, \bar{q}, g), (\bar{q}, q, g), (g, \bar{q}, q), (q, g, q)\]

**standard collinear formula**

\[f_i(x_1, k_{1,t}^2) \rightarrow x_1p_i(x_1)\delta(k_{1,t}^2)\]

\[f_j(x_2, k_{2,t}^2) \rightarrow x_2p_j(x_2)\delta(k_{2,t}^2)\]
Differential cross section

$2 \rightarrow 2$ in $k_t$-factorization approach

$$d\sigma_{h_1 h_2 \rightarrow \gamma, k} = dy_1 dy_2 d^2 p_{1,t} d^2 p_{2,t} \frac{d^2 k_{1,t}}{\pi} \frac{d^2 k_{2,t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \sum_{i,j,k} |M_{ij \rightarrow \gamma k}|^2$$

$$\cdot f_i(x_1, k_{1,t}^2) f_j(x_2, k_{2,t}^2) \delta^2(\vec{k}_{1,t} + \vec{k}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t})$$

$2 \rightarrow 3$ in collinear-factorization approach

$$d\sigma_{h_1 h_2 \rightarrow \gamma kl} = dy_1 dy_2 dy_3 d^2 p_{1,t} d^2 p_{2,t} \frac{1}{(4\pi)^3 (2\pi)^2} \frac{1}{\hat{s}^2} \sum_{i,j,k,l} |M_{ij \rightarrow \gamma kl}|^2$$

$$\cdot x_1 p_i(x_1, \mu^2) x_2 p_j(x_2, \mu^2)$$

see Aurenche et al., Nucl. Phys. B286 553 (87)
Photon-jet correlations $d\sigma/d\phi_-$

2 → 2 in $k_t$-factorization approach

\[
\frac{d\sigma_{h_1h_2\to\gamma k}}{d\phi_-} = \int \frac{2\pi}{16\pi^2(x_1x_2s)^2} \frac{f_i(x_1, k_{1,t}^2)}{\pi} \frac{f_j(x_2, k_{2,t}^2)}{\pi} \sum_{i,j,k} |M_{ij\to\gamma k}|^2 \cdot p_1, t d\mathbf{p}_1, t d\mathbf{p}_2, t dy_1 dy_2 q_t dq_t d\phi_{qt}
\]

2 → 3 in collinear-factorization approach

\[
\frac{d\sigma_{h_1h_2\to\gamma kl}}{d\phi_-} = \int \frac{1}{64\pi^4 S^2} x_1 p_i(x_1, \mu^2) x_2 p_j(x_2, \mu^2) \sum_{i,j,k,l} |M_{ij\to\gamma kl}|^2 \cdot p_1, t d\mathbf{p}_1, t d\mathbf{p}_2, t dy_1 dy_2 dy_3
\]
Decorrelations in \((p_{1,t}, p_{2,t})\) space
Scale dependence in Kwieciński UPDFs
Photon-jet correlations $d\sigma/d\phi_-$

NLO collinear vs $k_t$-factorization approach

$\sqrt{s} = 1960$ GeV

$p_{1,t}, p_{2,t} \in (5, 20)$ GeV

$y_1, y_2, y_3 \in (-4, 4)$

NLO collinear

Gauss $\sigma_0 = 1$ GeV

KMR $k_{t0}^2 = 1$ GeV$^2$

Kwieciński $b_0 = 1/\text{GeV}$
Scalar cuts

\[ |p_{1,t} - p_{2,t}| > \Delta S \]

\[ \sqrt{s} = 1960 \text{ GeV} \]

\[ p_{1,t}, p_{2,t} \in (5, 20) \text{ GeV} \]

\[ y_1, y_2, y_3 \in (-4, 4) \]

**NLO collinear Gauss**

**Gauss**

\[ \sigma_0 = 1 \text{ GeV} \]

**KMR**

\[ k_{t0}^2 = 1 \text{ GeV}^2 \]

**Kwieciński**

\[ b_0 = 1/ \text{ GeV} \]
\[ |\vec{p}_{1,t} + \vec{p}_{2,t}| > \Delta V \]

- \[ \sqrt{s} = 1960 \text{ GeV} \]
- \( p_{1,t}, p_{2,t} \in (5, 20) \text{ GeV} \)
- \( y_{1}, y_{2}, y_{3} \in (-4, 4) \)

**NLO collinear Gauss**
- \( \sigma_0 = 1 \text{ GeV} \)
- \( KMR \)
- \( k_{t0}^2 = 1 \text{ GeV}^2 \)
- \( b_0 = 1/\text{ GeV} \)
Leading photon/jet

NLO collinear

\[ \sqrt{s} = 1960 \text{ GeV} \]

\[ p_{1,t}, p_{2,t} \in (5, 20) \text{ GeV} \]

\[ y_1, y_2, y_3 \in (-4, 4) \]

\( p_{1,t} \) - photon

\( p_{2,t} \) - observed parton

\( p_{3,t} \) - unobs. parton

(dashed) no limits on \( p_{3,t} \)

(solid) \( p_{3,t} < p_{2,t} \)

(dotted) \( p_{3,t} < p_{1,t} \)

\[ p_{3,t} < p_{2,t} \]
Leading photon/jet

NLO collinear versus $k_t$-factorization

(solid) $p_{3,t} < p_{2,t}$
(dotted) $p_{3,t} < p_{1,t}$

$\sqrt{s} = 1960$ GeV

$p_{1,t}, p_{2,t} \in (5, 20)$ GeV

$y_1, y_2, y_3 \in (-4, 4)$

$p_{1,t}$ - photon
$p_{2,t}$ - observed parton
$p_{3,t}$ - unobs. parton

$\sigma/d\varphi_-$ (nb)
Leading photon/jet in \((p_1,t, p_2,t)\) space

- No limits on \(p_3,t\)
- \(p_3,t < p_2,t\)
- \(p_3,t < p_1,t\)
- \(p_3,t < p_2,t\)
Windows in \((p_1,t, p_2,t)\)
Windows in \((p_{1,t}, p_{2,t})\) - RHIC
Photon hadron correlations
Good agreement with exp. data using Kwiecinski UPDFs
(carefull treatment of the evolution of the QCD ladder)

Predictions made for LHC based on several UPDFs

The $k_t$-factorization approach is also better tool

for $\phi_- < \pi/2$ if leading parton/photon condition is imposed

for $\phi_- = \pi$ (no singularities)

RHIC measures $\gamma$-hadron, next step inclusion of jet hadronization
Drell-Yan with $k_t$ smearing

Lowest order process:

Initial quarks and antiquarks
Kwieciński UPDFs a good tool to include initial transverse momenta
Nonzero transverse momenta of the lepton pair.
Examples of higher order subprocesses:

Initial $k_t$ included
No singularities
Drell-Yan versus semileptonic decays

Drell-Yan, $O(\alpha_s^1)$   Semileptonic decays of $D$ and $\bar{D}$

W=200 GeV, Kwieciński UPDFs for both
Summary of SLD and DY

We have calculated \((p_{1t}, p_{2t})\) and azimuthal \(e^+e^-\) correlations including:
(a) \(gg \rightarrow c\bar{c} \rightarrow D\bar{D} \rightarrow e^+e^-\),
(b) \(gg \rightarrow b\bar{b} \rightarrow B\bar{B} \rightarrow e^+e^-\),
(c) \(O(\alpha_s^0)\) and \(O(\alpha_s^1)\) Drell-Yan within \(k_t\)-factorization approach.

For SLD decorrelation in azimuthal angle \(\phi_{ee}\) is due to:
(a) decay \(D \rightarrow e^+ (\bar{D} \rightarrow e^-)\)
(b) initial \(k_t\)-smearing of gluons (UGDFs)

For Drell-Yan decorrelation in \(\phi_{ee}\) is due to:
(a) initial \(k_t\)-smearing of quarks and antiquarks.

At RHIC (\(W=200\) GeV) dominance of SLD over DY in the large part of the phase-space. At LHC it may be even worse!
$J/\psi$ - gluon correlations

a)

b)

c)

d)

e)
$J/\psi$ - gluon correlations

Kwieciński UGDF

$\mu^2 = 10 \text{ GeV}^2$ (left), $\mu^2 = 100 \text{ GeV}^2$ (right)
$J/\psi$ - gluon correlations, gluons from the ladder
Our future: LHC

Calculations must be done, more difficulties, small $x$, saturation effects?

Large rapidities will be accessible (very small $x$).

Small $p_t$ with ALICE (saturation effects).

Really large $p_t$ will be available (domain of NLO, NNLO).

Good lack for LHC and the correlation program for proton-proton collisions.

Nuclear correlation program is very interesting at RHIC, It will be the same at LHC.