Unitarity cutting rules for hard processes on nuclear targets

Wolfgang Schäfer

1Institute of Nuclear Physics, PAN, Kraków

High-pT Physics at LHC, Tokaj’08, 16-19 March 2008, Tokaj, Hungary
Outline

Overview: nonlinear $k_{\bot}$–factorization

Nuclear collective glue and its unitarity cut interpretation

Standard Abramovskii-Gribov-Kancheli (AGK) vs. QCD

Summary

N.N. Nikolaev & W.S.
Unitarity cutting rules and topological cross sections in hard production off nuclei from nonlinear $k_{\bot}$–factorization.
For $y_{a,b,c} \gtrsim \log(2R_A m_N) = \log(1/x_A)$, $x_A \sim 0.01$, the breakup $a \rightarrow bc$ is coherent over the whole nucleus.
Production as excitation of beam partons $a \rightarrow bc$

- For $y_{a,b,c} \gtrsim \log(2R_A m_N) = \log(1/x_A)$, $x_A \sim 0.01$, the breakup $a \rightarrow bc$ is coherent over the whole nucleus.
- Partons move along straight-line trajectories. Impact parameters $b_i$ conserved in the interaction.
Production as excitation of beam partons $a \to bc$

For $y_{a,b,c} \gtrsim \log(2R_A m_N) = \log(1/x_A)$, $x_A \sim 0.01$, the breakup $a \to bc$ is coherent over the whole nucleus.

Partons move along straight–line trajectories. Impact parameters $b_i$ conserved in the interaction.

Interactions with the nucleus *before* and *after* the virtual decay interfere destructively.
Production as excitation of beam partons $a \rightarrow bc$

- For $y_{a,b,c} \gtrsim \log(2R_A m_N) = \log(1/x_A)$, $x_A \sim 0.01$, the breakup $a \rightarrow bc$ is coherent over the whole nucleus.

- Partons move along straight-line trajectories. Impact parameters $b_i$ conserved in the interaction.

- Interactions with the nucleus before and after the virtual decay interfere destructively.

- Nuclear target as a testing ground for unitarity/rescattering effects in hadronic hard interactions.
Production as excitation of beam partons $a \rightarrow bc$

- On the beam side: $bc$–system will evolve in all possible color multiplets.
Production as excitation of beam partons $a \rightarrow bc$

- On the **beam side**: $bc$–system will evolve in all possible color multiplets.
- On the **target side**: nucleus will be left in a state with multiple color excited nucleons / _wounded nucleons_.

\[ y_{a,b,c} \quad y_A \quad A \]
Production as excitation of beam partons $a \rightarrow bc$

- On the **beam side**: $bc$–system will evolve in all possible color multiplets.
- On the **target side**: nucleus will be left in a state with multiple color excited nucleons / wounded nucleons.
- more excited nucleons $\rightarrow$ higher multiplicity in target hemisphere $\rightarrow$ a nonperturbative source of energy loss for the $bc$–state.
Production as excitation of beam partons $a \rightarrow bc$

- On the **beam side**: $bc$–system will evolve in all possible color multiplets.
- On the **target side**: nucleus will be left in a state with multiple color excited nucleons / *wounded nucleons*.
- More excited nucleons $\rightarrow$ higher multiplicity in target hemisphere $\rightarrow$ a nonperturbative source of energy loss for the $bc$–state.
- Predictions for: cut pomeron/topological cross sections; forward-backward correlations; centrality/multiplicity dependence of hard interactions.
We need to separate:

- Elastic, color–diagonal (≡ uncut Pomeron) and color–rotation/excitation (≡ cut Pomeron) scatterings.
Production as excitation of beam partons $a \rightarrow bc$

- We need to separate:
  - Elastic, color–diagonal (≡ uncut Pomeron) and color–rotation/excitation (≡ cut Pomeron) scatterings.
  - *first* square, average over target states, and apply closure in summing over the nucleon and nucleus excitation.
We need to separate:

- Elastic, color–diagonal (≡ uncut Pomeron) and color–rotation/excitation (≡ cut Pomeron) scatterings.

- First square, average over target states, and apply closure in summing over the nucleon and nucleus excitation.

- Then the problem reduces to the calculation of few–particle $S$–matrices in a color–coupled channel Glauber–Gribov multiple scattering theory.
Four parton dipole cross section operator:

- A matrix in the space of possible $SU(N)$–color singlets.
Four parton dipole cross section operator:

\[ \hat{\Sigma}^{(4)}(b_1, b_2, b_3, b_4) = \hat{\Sigma}^{(4)}_{\text{ex}} + \hat{\Sigma}^{(4)}_{\text{el}} \]

- A matrix in the space of possible $SU(N)$–color singlets.

-

\[ \hat{\Sigma}^{(4)}_{\text{ex}} \]
  color rotation/excitation

\[ \hat{\Sigma}^{(4)}_{\text{el}} \]
  color diagonal
Four parton dipole cross section operator:

$$\hat{R}_i \hat{R}_j \hat{R}_i \hat{R}_j$$

- A matrix in the space of possible $SU(N)$–color singlets.

$$\hat{\Sigma}^{(4)}(b_1, b_2, b_3, b_4) = \hat{\Sigma}_{\text{ex}}^{(4)} + \hat{\Sigma}_{\text{el}}^{(4)}$$

- Color rotation/excitation
- Color diagonal

Nuclear multiparton $S$–matrix: $S^{(4)}_A(\{b_i\}) = \exp[-\frac{1}{2} T_A (\hat{\Sigma}_{\text{ex}}^{(4)} + \hat{\Sigma}_{\text{el}}^{(4)})]$
Four parton dipole cross section operator:

A matrix in the space of possible $SU(N)$–color singlets.

\[
\hat{\Sigma}^{(4)}(b_1, b_2, b_3, b_4) = \hat{\Sigma}^{(4)}_{\text{ex}} + \hat{\Sigma}^{(4)}_{\text{el}}
\]

- color rotation/excitation
- color diagonal

Nuclear multiparton $S$–matrix:

\[
S_A^{(4)}(\{b_i\}) = \exp\left[-\frac{1}{2} T_A (\hat{\Sigma}^{(4)}_{\text{ex}} + \hat{\Sigma}^{(4)}_{\text{el}})\right]
\]

Expansion in powers of $\hat{\Sigma}^{(4)}_{\text{ex}}$ is an expansion in *cut pomerons* (wounded nucleons).
Four parton dipole cross section operator:

- A matrix in the space of possible SU($N$)–color singlets.

\[ \hat{\Sigma}^{(4)}(b_1, b_2, b_3, b_4) = \hat{\Sigma}_{\text{ex}}^{(4)} + \hat{\Sigma}_{\text{el}}^{(4)} \]

- color rotation/excitation
- color diagonal

Nuclear multiparton S–matrix: \[ S_A^{(4)}(\{b_i\}) = \exp[-\frac{1}{2} T_A (\hat{\Sigma}_{\text{ex}}^{(4)} + \hat{\Sigma}_{\text{el}}^{(4)})] \]

Expansion in powers of \( \hat{\Sigma}_{\text{ex}}^{(4)} \) is an expansion in cut pomerons (wounded nucleons).

Expansion in powers of \( \hat{\Sigma}_{\text{el}}^{(4)} \) gives the multipomeron absorptive corrections.
Four parton dipole cross section operator:

A matrix in the space of possible $SU(N)$–color singlets.

\[
\hat{\Sigma}^{(4)}(b_1, b_2, b_3, b_4) = \hat{\Sigma}_{\text{ex}}^{(4)} + \hat{\Sigma}_{\text{el}}^{(4)}
\]

color rotation/excitation color diagonal

Nuclear multiparton $S$–matrix: $S_A^{(4)}(\{b_i\}) = \exp[-\frac{1}{2} T_A (\hat{\Sigma}_{\text{ex}}^{(4)} + \hat{\Sigma}_{\text{el}}^{(4)})]$

Expansion in powers of $\hat{\Sigma}_{\text{ex}}^{(4)}$ is an expansion in *cut pomerons* (*wounded nucleons*).

Expansion in powers of $\hat{\Sigma}_{\text{el}}^{(4)}$ gives the multipomeron absorptive corrections.

Only the sum will be infrared safe, separation into color excitations and elastic rescatterings is infrared sensitive.
Master formula for dijets

\[ \frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2 p_b d^2 p_c} = \int \frac{d^2 b_b d^2 b_c d^2 b'_b d^2 b'_c}{(2\pi)^4} e^{-i p_b(b_b - b'_b) - i p_c(b_c - b'_c)} \psi(z_b, b_b - b_c)\psi^*(z_b, b'_b - b'_c) \]

\[ \left\{ S_{b\bar{c}cb}^{(4)}(b'_b, b'_c, b_b, b_c) + S_{\bar{a}a}^{(2)}(b'_b, b) - S_{b\bar{c}a}^{(3)}(b, b'_b, b'_c) - S_{\bar{a}bc}^{(3)}(b', b_b, b_c) \right\} \]
Master formula for dijets

\[
\frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2 p_b d^2 p_c} = \int \frac{d^2 b_b d^2 b_c d^2 b'_b d^2 b'_c}{(2\pi)^4} e^{-i p_b (b_b - b'_b) - i p_c (b_c - b'_c)} \\
\psi(z_b, b_b - b_c) \psi^*(z_b, b'_b - b'_c) \\
\left\{ S_{b\bar{c}cb}^{(4)}(b'_b, b'_c, b_b, b_c) + S_{a\bar{a}}^{(2)}(b', b) - S_{b\bar{c}a}^{(3)}(b, b'_b, b'_c) - S_{\bar{a}bc}^{(3)}(b', b_b, b_c) \right\}
\]

**DIS:** \( \gamma^* \rightarrow q\bar{q} \quad \Rightarrow \quad 1 + \frac{8}{N_c^2} \)
Master formula for dijets

\[
\frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2 p_b d^2 p_c} = \int \frac{d^2 b_b d^2 b_c d^2 b'_b d^2 b'_c}{(2\pi)^4} e^{-i p_b(b_b - b'_b) - i p_c(b_c - b'_c)} \\
\psi(z_b, b_b - b_c) \psi^*(z_b, b'_b - b'_c) \\
\left\{ S_{b\bar{c}cb}^{(4)}(b'_b, b'_c, b_b, b_c) + S_{aa}^{(2)}(b', b) - S_{b\bar{c}a}^{(3)}(b, b'_b, b'_c) - S_{\bar{a}bc}^{(3)}(b', b_b, b_c) \right\}
\]

- DIS: \( \gamma^* \rightarrow q\bar{q} \quad \Rightarrow \quad 1 + \frac{8}{N_c^2} \)

- Open charm: \( g \rightarrow c\bar{c} \quad \Rightarrow \quad 1 + \frac{8}{N_c^2} \)

\( 1(\text{N}_c-\text{suppressed}) \)
Master formula for dijets

\[
\frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2p_b d^2p_c} = \int \frac{d^2b_b d^2b_c d^2b'_b d^2b'_c}{(2\pi)^4} e^{-i p_b (b_b - b'_b) - i p_c (b_c - b'_c)} \\
\psi(z_b, b_b - b_c) \psi^*(z_b, b'_b - b'_c) \\
\left\{ S^{(4)}_{b\bar{c}cb}(b'_b, b'_c, b_b, b_c) + S^{(2)}_{a\bar{a}}(b', b) - S^{(3)}_{b\bar{c}a}(b, b'_b, b'_c) - S^{(3)}_{\bar{a}bc}(b', b_b, b_c) \right\}
\]

- **DIS:** $\gamma^* \rightarrow q\bar{q} \quad \Rightarrow \quad \frac{1}{1} + \frac{8}{N_c^2}$

- **Open charm:** $g \rightarrow c\bar{c} \quad \Rightarrow \quad \frac{1}{1(N_c \text{-suppressed})} + \frac{8}{N_c^2}$

- **Forward dijets** $q \rightarrow qg \quad \Rightarrow \quad \frac{3}{N_c} + \frac{6 + 15}{N_c \times N_c}$
Master formula for dijets

\[
\frac{d\sigma(a^* \rightarrow bc)}{dz_b d^2p_b d^2p_c} = \int \frac{d^2b_b d^2b_c d^2b'_b d^2b'_c}{(2\pi)^4} e^{-i p_b(b_b - b'_b) - i p_c(b_c - b'_c)} 
\psi(z_b, b_b - b_c)\psi^*(z_b, b'_b - b'_c) 
\left\{ S^{(4)}_{\bar{b}\bar{c}cb}(b'_b, b'_c, b_b, b_c) + S^{(2)}_{\bar{a}a}(b', b) - S^{(3)}_{\bar{b}\bar{c}a}(b, b'_b, b'_c) - S^{(3)}_{\bar{a}abc}(b', b_b, b_c) \right\}
\]

- **DIS:** \( \gamma^* \rightarrow q\bar{q} \implies \frac{1}{1} + \frac{8}{N_c^2} \)

- **Open charm:** \( g \rightarrow c\bar{c} \implies \frac{1}{1(N_c-\text{suppressed})} + \frac{8}{N_c^2} \)

- **Forward dijets** \( q \rightarrow qg \implies \frac{3}{N_c} + 6 + 15 \frac{N_c \times N_c^2}{N_c} \)

- **Central dijets**
  \( g \rightarrow gg \implies \frac{1}{1(N_c-\text{suppressed})} + \frac{8_A + 8_S}{N_c^2} + \frac{10 + 10 + 27 + R_7}{N_c^2 \times N_c^2} \)
Diffractive dijets define nuclear unintegrated glue

Diffractive hard dijets from pions:
\[ \pi A \rightarrow Jet_1 + Jet_2 \]
Diffractive dijets define nuclear unintegrated glue

Diffractive hard dijets from pions: 
\[ \pi A \rightarrow Jet_1 + Jet_2 \]

Hard jets acquire \( p_\perp \) from gluons
Diffractive dijets define nuclear unintegrated glue

- Diffractive hard dijets from pions: \( \pi A \rightarrow Jet_1 + Jet_2 \)
- Hard jets acquire \( p_\perp \) from gluons
- Collective glue is a physical observable: \( M_A(p) \propto \phi(b, p) \).
nuclear coherent glue per unit area in impact parameter space:

\[
\phi(b, \kappa) = \sum w_j(b) f^{(j)}(\kappa)
\]

collective glue of \( j \) overlapping nucleons:

\[
f^{(j)}(\kappa) = \int \left[ \prod d^2 \kappa_i f(\kappa_i) \right] \delta(\kappa - \sum \kappa_i)
\]

probab. to find \( j \) overlapping nucleons

\[
w_j(b) = \frac{\nu_j^j(b)}{j!} \exp[-\nu_A(b)], \quad \nu_A(b) = \frac{1}{2} \sigma_0 T(b), \quad \sigma_0 = \sigma(r \to \infty)
\]

Nuclear S–matrix for the dipole

\[
\int \frac{d^2 r}{(2\pi)^2} S_A(b, r) \exp(-i \kappa r) = w_0(b) \delta(k) + \phi(b, \kappa)
\]
Nuclear unintegrated glue

- nuclear coherent glue per unit area in impact parameter space:
  \[ \phi(b, \kappa) = \sum w_j(b)f^{(j)}(\kappa) \]

- collective glue of \( j \) overlapping nucleons:
  \[ f^{(j)}(\kappa) = \int \left[ \prod d^2 \kappa_i f(\kappa_i) \right] \delta(\kappa - \sum \kappa_i) \]

- probab. to find \( j \) overlapping nucleons
  \[ w_j(b) = \frac{\nu^j_A(b)}{j!} \exp[-\nu_A(b)], \nu_A(b) = \frac{1}{2}\sigma_0 T(b), \sigma_0 = \sigma(r \to \infty) \]

- Nuclear \( S \)-matrix for the dipole \( S_A(b, r) = \exp[-\frac{1}{2}\sigma(r) T_A(b)] \)

\[ \int \frac{d^2 r}{(2\pi)^2} S_A(b, r) \exp(-i\kappa r) = w_0(b)\delta(k) + \phi(b, \kappa) \]
Nuclear unintegrated glue

- Nuclear coherent glue per unit area in impact parameter space:
  \[ \phi(b, \kappa) = \sum w_j(b) f^{(j)}(\kappa) \]

- Collective glue of \( j \) overlapping nucleons:
  \[ f^{(j)}(\kappa) = \int \left[ \prod d^2 \kappa_i f(\kappa_i) \right] \delta(\kappa - \sum \kappa_i) \]

- Probability to find \( j \) overlapping nucleons
  \[ w_j(b) = \frac{\nu_j^A(b)}{j!} \exp[-\nu_A(b)] , \quad \nu_A(b) = \frac{1}{2} \sigma_0 T(b) , \quad \sigma_0 = \sigma(r \to \infty) \]

- Nuclear S–matrix for the dipole \( S_A(b, r) = \exp[-\frac{1}{2} \sigma(r) T_A(b)] \)
  \[ \int \frac{d^2 r}{(2\pi)^2} S_A(b, r) \exp(-i\kappa r) = w_0(b)\delta(k) + \phi(b, \kappa) \]
Nuclear unintegrated glue

- nuclear coherent glue per unit area in impact parameter space:
  \[ \phi(b, \kappa) = \sum w_j(b) f(j)(\kappa) \]

- collective glue of \( j \) overlapping nucleons:
  \[ f(j)(\kappa) = \int \left[ \prod d^2 \kappa_i f(\kappa_i) \right] \delta(\kappa - \sum \kappa_i) \]

- probab. to find \( j \) overlapping nucleons
  \[ w_j(b) = \frac{\nu^j_A(b)}{j!} \exp[-\nu_A(b)], \quad \nu_A(b) = \frac{1}{2} \sigma_0 T(b), \quad \sigma_0 = \sigma(r \to \infty) \]

- Nuclear \( S \)-matrix for the dipole \( S_A(b, r) = \exp[-\frac{1}{2} \sigma(r) T_A(b)] \)
  \[ \int \frac{d^2 r}{(2\pi)^2} S_A(b, r) \exp(-i\kappa r) = w_0(b) \delta(k) + \phi(b, \kappa) \]
Nuclear unintegrated glue: salient features

collective glue $f^{(j)}(\kappa)$

- nuclear coherent glue per unit area in impact parameter space:

$$\phi(b, \kappa) = \sum w_j(b) f^{(j)}(\kappa)$$
Nuclear unintegrated glue: salient features

- collective glue $f^{(j)}(\kappa)$

- nuclear coherent glue per unit area in impact parameter space:
  \[ \phi(b, \kappa) = \sum w_j(b) f^{(j)}(\kappa) \]

- typical scale: the saturation scale $Q_A^2 \sim 0.8 \div 1$ GeV$^2$ for realistic glue and heavy nuclei.
Nuclear unintegrated glue: salient features

Collective glue $f^{(j)}(\kappa)$

- Nuclear coherent glue per unit area in impact parameter space:
  $$\phi(b, \kappa) = \sum w_j(b) f^{(j)}(\kappa)$$

- Typical scale: the saturation scale $Q_A^2 \sim 0.8 \div 1 \text{ GeV}^2$ for realistic glue and heavy nuclei.

- Large-$\kappa^2$ Cronin–type antishadowing enhancement
Nuclear unintegrated glue: salient features

- Nuclear coherent glue per unit area in impact parameter space:
  \[ \phi(b, \kappa) = \sum w_j(b) f^{(j)}(\kappa) \]

- Typical scale: the saturation scale \( Q_A^2 \sim 0.8 \div 1 \text{ GeV}^2 \) for realistic glue and heavy nuclei.

- Large–\( \kappa^2 \) Cronin–type antishadowing enhancement

- Furnishes linear \( k_\perp \)–factorization of inclusive deep inelastic, forward jets in DIS, and diffractive dijets.

Collective glue \( f^{(j)}(\kappa) \)

![Graph showing collective glue](image-url)
Nuclear unintegrated glue: small-\(x\) evolution

- unintegrated glue:
\[
\Phi(b, x, p) \equiv \int \frac{d^2 r}{(2\pi)^2} \exp[-ipr] S_{q\bar{q}}(b, x, r) = w_0 \delta^{(2)}(p) + \phi(b, x, p)
\]

- small-\(x\) evolution \(\text{Nikolaev}, \text{Zakharov}, \text{Zoller} / \text{Mueller}'94\):
\[
S_{q\bar{q}}(b, x_0, r) \rightarrow S_{q\bar{q}}(b, x_0, r) + \log(x_0/x)\delta S_{q\bar{q}}(b, x, r)
\]
\[
\delta S_{q\bar{q}}(b, x, r) \propto \int |\psi_{q\bar{q}g}|^2 \left(S_{q\bar{q}g} - S_{q\bar{q}}\right)
\]

- evolution of unintegrated glue \text{first step of Balitsky–Kovchegov}'96–'98:
\[
\phi(b, x, p) \rightarrow \phi(b, x, p) + \log(x_0/x)\delta \phi(b, x, p)
\]
\[
\delta \phi(b, x, p) = \mathcal{K}_{BFKL} \otimes \phi(b, x, p) + \mathcal{Q}[\phi](b, x, p)
\]
\[
\mathcal{Q}[\phi](b, x, p) = \int d^2 q d^2 \kappa \phi(b, x, q) \left\{ \left[ K(p+\kappa, p+q) - K(p, \kappa+p) - K(p, q+p) \right] \phi(b, x, p) \right. \\
- \left. \phi(b, x, p) \left[ K(\kappa, \kappa+q+p) - K(\kappa, \kappa+p) \right] \right\} ; \ K(p, q) = \frac{(p-q)^2}{p^2 q^2}
\]
Nuclear unintegrated glue: small-$x$ evolution

- **unintegrated glue:**
  \[
  \Phi(b, x, p) \equiv \int \frac{d^2r}{(2\pi)^2} \exp[-ipr] S_{q\bar{q}}(b, x, r) = w_0 \delta^{(2)}(p) + \phi(b, x, p)
  \]

- **small-$x$ evolution** \cite{NikolaevZakharovZollerMueller94}:
  \[
  S_{q\bar{q}}(b, x_0, r) \rightarrow S_{q\bar{q}}(b, x_0, r) + \log(x_0/x)\delta S_{q\bar{q}}(b, x, r)
  \]
  \[
  \delta S_{q\bar{q}}(b, x, r) \propto \int |\psi_{q\bar{q}g}|^2 \left( S_{q\bar{q}g} - S_{q\bar{q}} \right)
  \]

- **evolution of unintegrated glue** \textit{first step of Balitsky–Kovchegov'96–'98}:
  \[
  \phi(b, x, p) \rightarrow \phi(b, x, p) + \log(x_0/x)\delta\phi(b, x, p)
  \]
  \[
  \delta\phi(b, x, p) = K_{BFKL} \otimes \phi(b, x, p) + Q[\phi](b, x, p)
  \]
  \[
  Q[\phi](b,x,p) = \int d^2q d^2\kappa \phi(b,x,q) \left\{ \left[ K(p+\kappa,p+q) - K(p,\kappa+p) - K(p,q+p) \right] \phi(b,x) \right\}
  \]
  \[
  -\phi(b,x,p) \left[ K(\kappa,\kappa+q+p) - K(\kappa,\kappa+p) \right] \right\} ;
  K(p,q) = \frac{(p-q)^2}{p^2q^2}
  \]
Nuclear unintegrated glue: small-\(x\) evolution

**unintegrated glue:**

\[
\Phi(b, x, p) \equiv \int \frac{d^2 r}{(2\pi)^2} \exp[-ip r] S_{q\bar{q}}(b, x, r) = w_0 \delta^{(2)}(p) + \phi(b, x, p)
\]

**small-\(x\) evolution** Nikolaev, Zakharov, Zoller / Mueller'94:

\[
S_{q\bar{q}}(b, x_0, r) \rightarrow S_{q\bar{q}}(b, x_0, r) + \log(x_0/x)\delta S_{q\bar{q}}(b, x, r)
\]

\[
\delta S_{q\bar{q}}(b, x, r) \propto \int |\psi_{q\bar{g}}|^2 (S_{q\bar{q}} - S_{q\bar{q}})
\]

**evolution of unintegrated glue** first step of Balitsky–Kovchegov'96–'98:

\[
\phi(b, x, p) \rightarrow \phi(b, x, p) + \log(x_0/x)\delta \phi(b, x, p)
\]

\[
\delta \phi(b, x, p) = K_{BFKL} \otimes \phi(b, x, p) + Q[\phi](b, x, p)
\]

\[
Q[\phi](b, x, p) = \int d^2 q d^2 \kappa \phi(b, x, q) \left\{ \left[ K(p+\kappa, p+q) - K(p, \kappa+p) - K(p, q+p) \right] \phi(b, x) \right\}
\]

\[
-\phi(b, x, p) \left[ K(\kappa, \kappa+q+p) - K(\kappa, \kappa+p) \right] \} ;
\]

\[
K(p, q) = \frac{(p-q)^2}{p^2 q^2}
\]
Nuclear unintegrated glue: small-\(x\) evolution

- unintegrated glue:
  \[
  \Phi(b, x, p) \equiv \int \frac{d^2 r}{(2\pi)^2} \exp[-ipr] S_{q\bar{q}}(b, x, r) = w_0 \delta^{(2)}(p) + \phi(b, x, p)
  \]

- small-\(x\) evolution \cite{Nikolaev,Zakharov,Zoller/Mueller'94}:
  \[
  S_{q\bar{q}}(b, x_0, r) \rightarrow S_{q\bar{q}}(b, x_0, r) + \log(x_0/x)\delta S_{q\bar{q}}(b, x, r)
  \]
  \[
  \delta S_{q\bar{q}}(b, x, r) \propto \int |\psi_{q\bar{q}g}|^2 \left( S_{q\bar{q}g} - S_{q\bar{q}} \right)
  \]

- evolution of unintegrated glue first step of Balitsky–Kovchegov'96–'98:
  \[
  \phi(b, x, p) \rightarrow \phi(b, x, p) + \log(x_0/x)\delta\phi(b, x, p)
  \]
  \[
  \delta\phi(b, x, p) = K_{BFKL} \otimes \phi(b, x, p) + Q[\phi](b, x, p)
  \]
  \[
  Q[\phi](b, x, p) = \int d^2 q d^2 \kappa \phi(b, x, q) \left\{ \left[ K(p+\kappa, p+q) - K(p, \kappa+p) - K(p, q+p) \right] \phi(b, x, p) \right. \\
  \left. - \phi(b, x, p) \left[ K(\kappa, \kappa+q+p) - K(\kappa, \kappa+p) \right] \right\} ; K(p, q) = \frac{(p-q)^2}{p^2 q^2}
  \]
Unitarity cut interpretation of the nuclear glue

\[ \phi(b, x, \kappa) = \sum_{j \geq 1} w_j(\nu_A(b)) f^{(j)}(x, \kappa) \]
Unitarity cut interpretation of the nuclear glue

\[ \phi(b, x, \kappa) = \sum_{j \geq 1} w_j (\nu_A(b)) f^{(j)}(x, \kappa) \]

- Expansion of the cut nuclear pomeron in the cut free-nucleon pomeron
Unitarity cut interpretation of the nuclear glue

\[ \phi(b, x, \kappa) = \sum_{j \geq 1} w_j(\nu_A(b)) f^{(j)}(x, \kappa) \]

- Expansion of the cut nuclear pomeron in the cut free-nucleon pomeron
- Screening by uncut pomeron in the expansion coefficients
Uncut, and two types of cut Pomerons

- coherent distortion of lightcone
  $WF \rightarrow$ uncut Pomeron exchanges.
Uncut, and two types of cut Pomerons

- Coherent distortion of lightcone $\text{WF} \rightarrow$ uncut Pomeron exchanges.
- Color coupled channel property $\rightarrow$ two types of cut pomeron:

![Diagram showing cut Pomerons and their interactions]
Uncut, and two types of cut Pomerons

- coherent distortion of lightcone $WF \rightarrow$ uncut Pomeron exchanges.
- Color coupled channel property $\rightarrow$ two types of cut pomerons:
  - transitions between two multiplets of different dimensionality (here $1 \rightarrow 8$):
    necessarily leaves a color excited nucleon, coupling $\propto T_A(b)f(x, \kappa)$, couples to both constituents.
Uncut, and two types of cut Pomerons

- coherent distortion of lightcone $WF \rightarrow$ uncut Pomeron exchanges.
- Color coupled channel property $\rightarrow$ two types of cut pomerons:
  - transitions between two multiplets of different dimensionality (here $1 \rightarrow 8$): necessarily leaves a color excited nucleon, coupling $\propto T_A(b) f(x, \kappa)$, couples to both constituents.
  - rotations within the same color multiplet: summed up in the nuclear Reggeons. Coupling $\propto \Phi(b, x, \kappa_i)$. 
Uncut, and two types of cut Pomerons

- A color rotation Pomeron contributes \( j \) color excited nucleons with a weight
  \[ \propto w_j(b) f^{(j)}(x, \kappa_i). \]
Uncut, and two types of cut Pomerons

- A color rotation Pomeron contributes $j$ color excited nucleons with a weight $\propto w_j(b) f^{(j)}(x, \kappa_i)$.
- Even for single–particle spectra, in topological cross sections, *spectator interactions leave a trace*
QCD vs. standard AGK: two-Pomeron cuts

AGK for DIS off a nucleus:

\[ \Delta_2 \Gamma_2^{\text{in}}(\mathbf{P} \mathbf{P}; b, r) = -[\sigma(x, r) \, T(b)]^2 \]

\[ \Delta_2 \Gamma_D(\mathbf{P} \mathbf{P}; b, r) : \Delta_2 \Gamma_1^{\text{in}}(\mathbf{P} \mathbf{P}; b, r) : \Delta_2 \Gamma_2^{\text{in}}(\mathbf{P} \mathbf{P}; b, r) = 1 : -4 : 2 \]
QCD vs. standard AGK: two-Pomeron cuts

- QCD: the cut pomerons couple differently to *singlet–to–octet excitation* and *octet–to–octet rotation*:

\[
\Delta_2 \Gamma_{1}^{in} \left( \frac{\mathbf{P}}{e} \frac{\mathbf{P}}{e} ; b, r \right) = -\frac{1}{2} \cdot \left[ \sigma_0(x) T(b) \right] \cdot \left[ \sigma(x, r) T(b) \right] - \frac{1}{2} \left[ \sigma(x, r) T(b) \right]^2
\]

\[
\Delta_2 \Gamma_{2}^{in} \left( \frac{\mathbf{P}}{e} \frac{\mathbf{P}}{e} ; b, r \right) = \frac{1}{2} \cdot \left[ \sigma_0(x) T(b) \right] \left[ \sigma(x, r) T(b) \right]
\]
Inelastic cross section of the $q\bar{q}$-dipole-Nucleus interaction:

\[
\Gamma^{inel}(r, b) = 1 - \exp[-\sigma(r)T(b)] \\
= \exp[-\sigma(r)T(b)] \sum_{\nu} \frac{1}{\nu!} [\sigma(r)T(b)]^\nu
\]
QCD vs. standard AGK: topological cross sections

Inelastic cross section of the $q\bar{q}$-dipole-Nucleus interaction:

$$\Gamma^{inel}(r, b) = 1 - \exp[-\sigma(r) T(b)]$$

$$= \exp[-\sigma(r) T(b)] \sum_{\nu} \frac{1}{\nu!} [\sigma(r) T(b)]^\nu$$

Following Capella-Kaidalov-Bertocchi-Treleani:

$$\exp[-\sigma(r) T(b)][\sigma(r) T(b)]^\nu / \nu! = \text{contribution from } \nu \text{ cut Pomerons}$$
Inelastic cross section of the $q\bar{q}$-dipole-Nucleus interaction:

$$\Gamma^{inel}(r, b) = 1 - \exp[-\sigma(r) T(b)]$$

$$= \exp[-\sigma(r) T(b)] \sum_{\nu} \frac{1}{\nu!} [\sigma(r) T(b)]^\nu$$

- Following Capella-Kaidalov-Bertocchi-Treleani:
  \[ \exp[-\sigma(r) T(b)][\sigma(r) T(b)]^\nu / \nu! = \text{contribution from } \nu \text{ cut Pomerons} \]
- Invalid in QCD. Ignores the role of spectator interactions and the two types of cut Pomerons.
Inelastic cross section of the $q\bar{q}$-dipole-Nucleus interaction:

$$\Gamma^{inel}(r, b) = 1 - \exp[-\sigma(r) T(b)]$$

$$= \exp[-\sigma(r) T(b)] \sum_{\nu} \frac{1}{\nu!} [\sigma(r) T(b)]^\nu$$

- Following Capella-Kaidalov-Bertocchi-Treleani:
  $$\exp[-\sigma(r) T(b)][\sigma(r) T(b)]^\nu / \nu! = \text{contribution from } \nu \text{ cut Pomerons}$$
- Invalid in QCD. Ignores the role of spectator interactions and the two types of cut Pomerons.
- Spectrum of forward quarks in DIS. $\nu$ cut Pomerons affect $x_F$ distribution:

$$\frac{d\sigma_\nu}{d^2b dx_F} = \int d^2r |\psi(x_F, r)|^2 \Gamma^{(\nu)inel}(r, b)$$
Summary

- Topological cross sections follow directly from nonlinear-$k_\perp$ factorization for inclusive cross sections
- Novel property of QCD unitarity cutting rules: two kinds of cut pomeron
- Comover/spectator interactions contribute to topological cross sections