UPDATES ON DARK ENERGY

Norbert Straumann

LOR2006, Budapest, June (2006)

Program

- Hubble diagram for Type Ia SNe

- Three-year WMAP results

- Alternatives to Dark Energy?

NS: Mod. Phys. Lett. A 21, 1083-1097 (2006)

Luminosity-Redshift Relation

Definition of **luminosity distance**:

$$D_L = (\mathcal{L}/4\pi\mathcal{F})^{1/2}.$$

distance modulus $m - M$:

$$m - M = 5 \log \left(\frac{D_L}{1 \ Mpc} \right) + 25$$

Friedmann equations $\rightarrow$

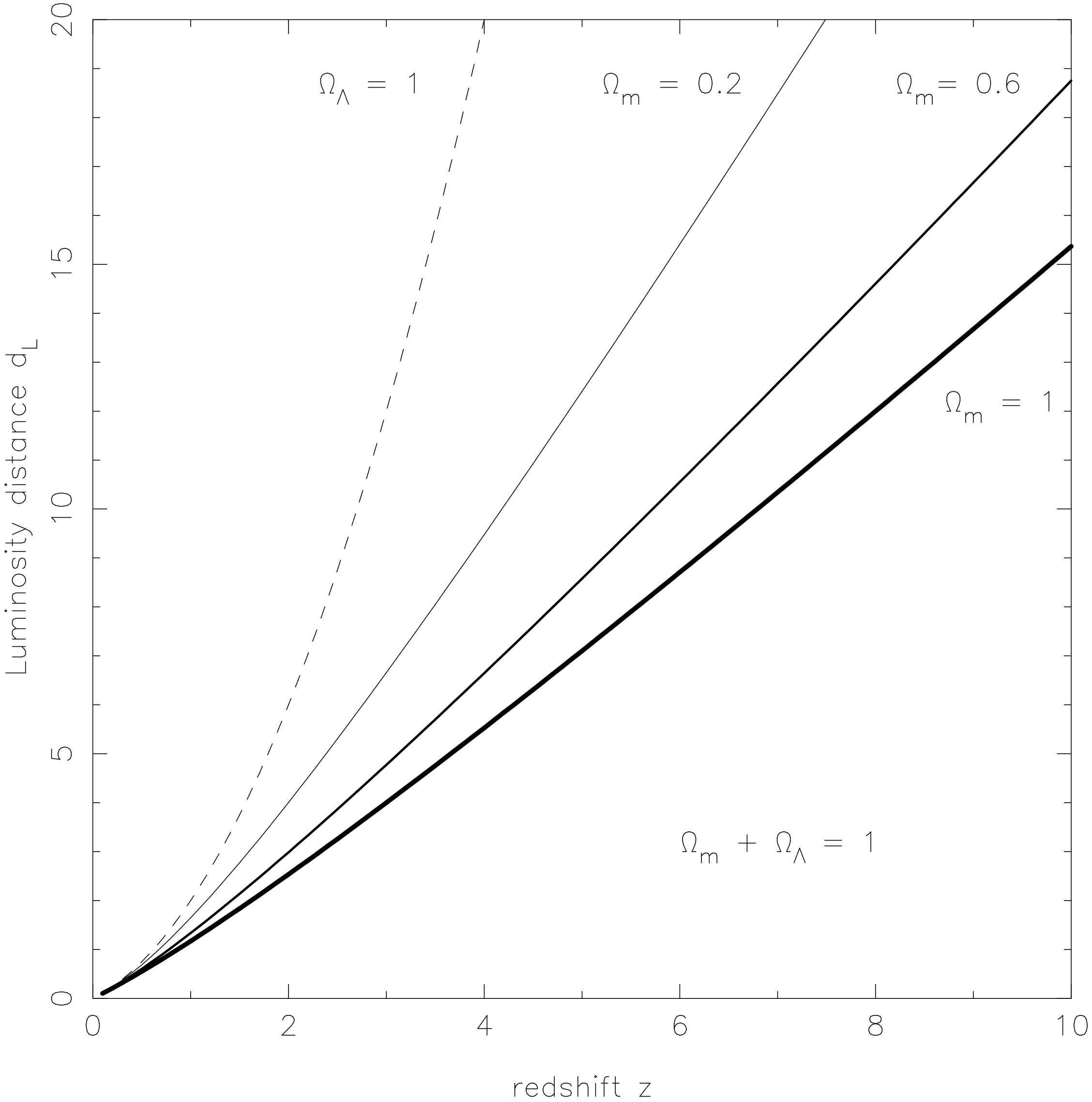
$$D_L(z) = \frac{1}{H_0} \mathcal{D}_L(z; \Omega_K, \Omega_X), \quad \Omega_X = \frac{(\rho_X)_0}{\rho_{crit}}; \quad \Omega_K + \sum_X \Omega_X = 1.$$

Hubble-free luminosity distance in Friedmann models:

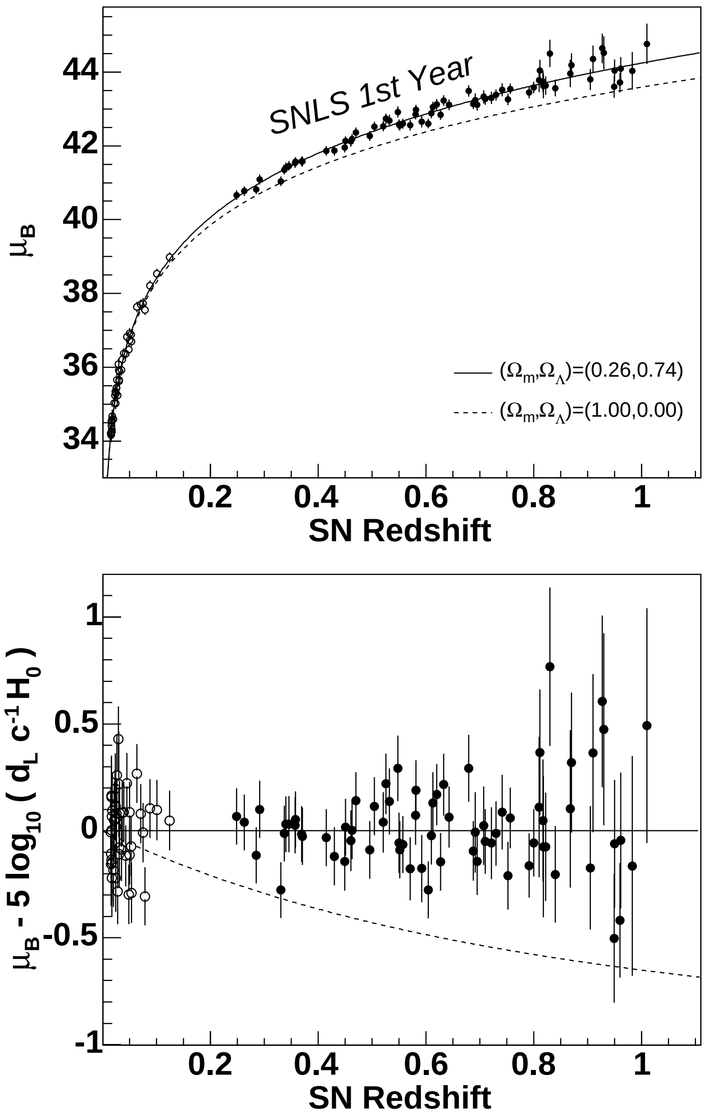
$$\mathcal{D}_L(z) = (1+z)\frac{1}{|\Omega_K|^{1/2}}\mathcal{S}\left(|\Omega_K|^{1/2}\int_0^z \frac{dz'}{E(z')}\right) \quad ;$$

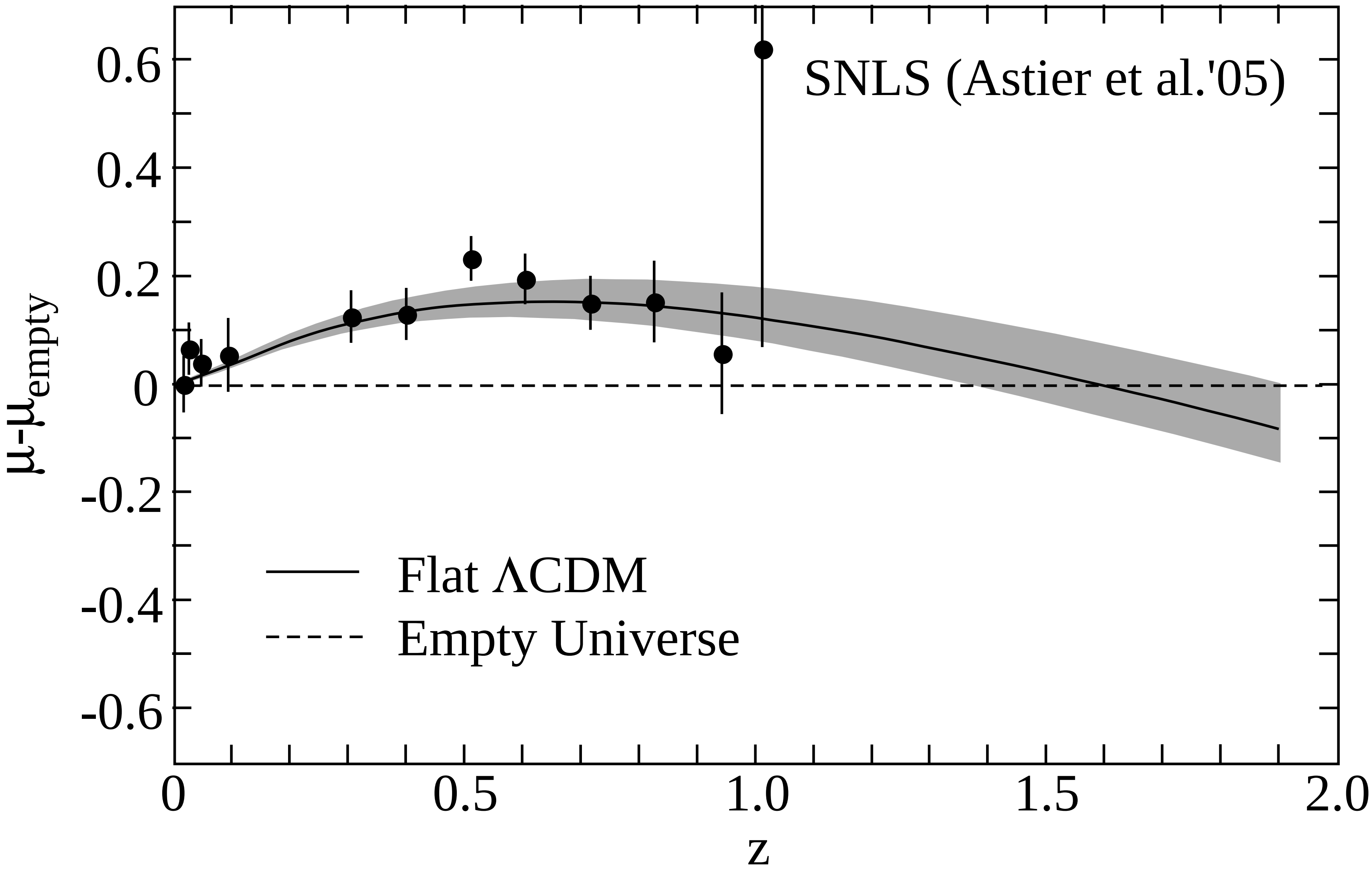
$$E^2(z;\Omega_K,\Omega_X) = \Omega_K(1+z)^2 + \sum_X \Omega_X(1+z)^{3(1+w_X)}.$$

Intrinsic dispersion, after corrections (using empircal correlations): $\simeq 0.15$ mag.

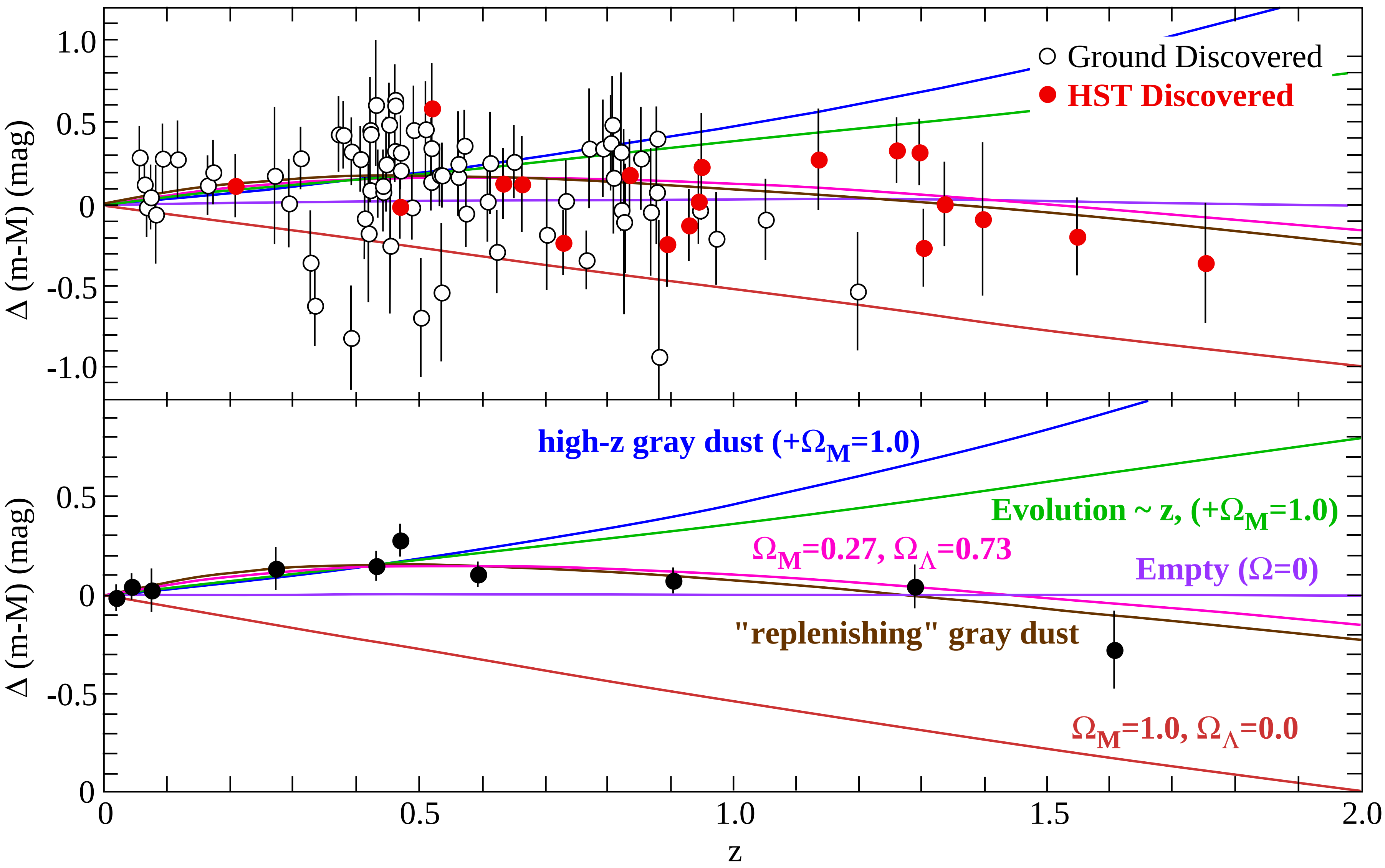


20
15
10
5
0
Luminosity distance d_L
Ω_Λ = 1
Ω_m = 0.2
Ω_m= 0.6
Ω_m = 1
Ω_m + Ω_Λ = 1
0
2
4
6
8
10
redshift z


SNLS 1st Year
μ_B
44
42
40
38
36
34
$(\Omega_m,\Omega_\Lambda)$=(0.26,0.74)
$(\Omega_m,\Omega_\Lambda)$=(1.00,0.00)
0.2
0.4
0.6
0.8
1
SN Redshift
μ_B - 5 $\log_{10}$ (d_L c^{-1} H_0)
1
0.5
0
-0.5
-1
0.2
0.4
0.6
0.8
1
SN Redshift



SNLS (Astier et al.'05)
μ-μempty
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.5
1.0
1.5
2.0
z
Flat ΛCDM
Empty Universe


Ground Discovered
HST Discovered
Δ (m-M) (mag)
high-z gray dust (+Ω_M=1.0)
Evolution ~ z, (+Ω_M=1.0)
Ω_M=0.27, Ω_Λ=0.73
Empty (Ω=0)
"replenishing" gray dust
Ω_M=1.0, Ω_Λ=0.0
z

CMB

TT- correlation function:

$$C(\vartheta) := \left\langle \frac{\Delta T(\mathbf{n})}{T} \cdot \frac{\Delta T(\mathbf{n}')}{T} \right\rangle = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos\vartheta)$$

$\Theta(\mathbf{n}) \equiv \frac{\Delta T(\mathbf{n})}{T}$ is random field,

$$\Theta(\mathbf{n}) = \sum_{lm} a_{lm} Y_{lm}(\mathbf{n})$$

$$\langle a_{lm} \rangle = 0, \quad \langle a^{\star}_{lm} a_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l(\eta)$$

$$C_l = \frac{1}{2l+1} \left\langle \sum_{m=-l}^{l} a^{\star}_{lm} a_{lm} \right\rangle .$$

For Gaussian a_{lm}:

$$\frac{\sigma(C_l)}{C_l} = \sqrt{\frac{2}{2l+1}}.$$

Polarization tensor:

$$(\mathcal{P}_{ab}) = \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix},$$

$(V = 0)$.

$$Q + iU = \sqrt{2} \sum_{l=2}^{\infty} \sum_{m} \left[a^{E}_{(lm)} + i a^{B}_{(lm)} \right] {}_2Y_l^m.$$

Evolution of fluctuations (qualitative discussion)

- recombination: decoupling of photons ($\rightarrow$ 'free' propagation)

- below $\sim 6000K$: $\gamma,\ (e^-,\ p,\ ...),\ \nu's,\ CDM$

- before $\sim 6000K$: two fluida description:
$(\gamma,\ e^-,\ p,\ He^{++},\ ...),\ CDM$;
coupled only gravitationally.

- need: *initial conditions* (primordial power spectrum on super-horizon scales deep in the plasma era)

Basic eqs. for evolution

linearize about FL behavior:

▷ fluida eqs. for various components;

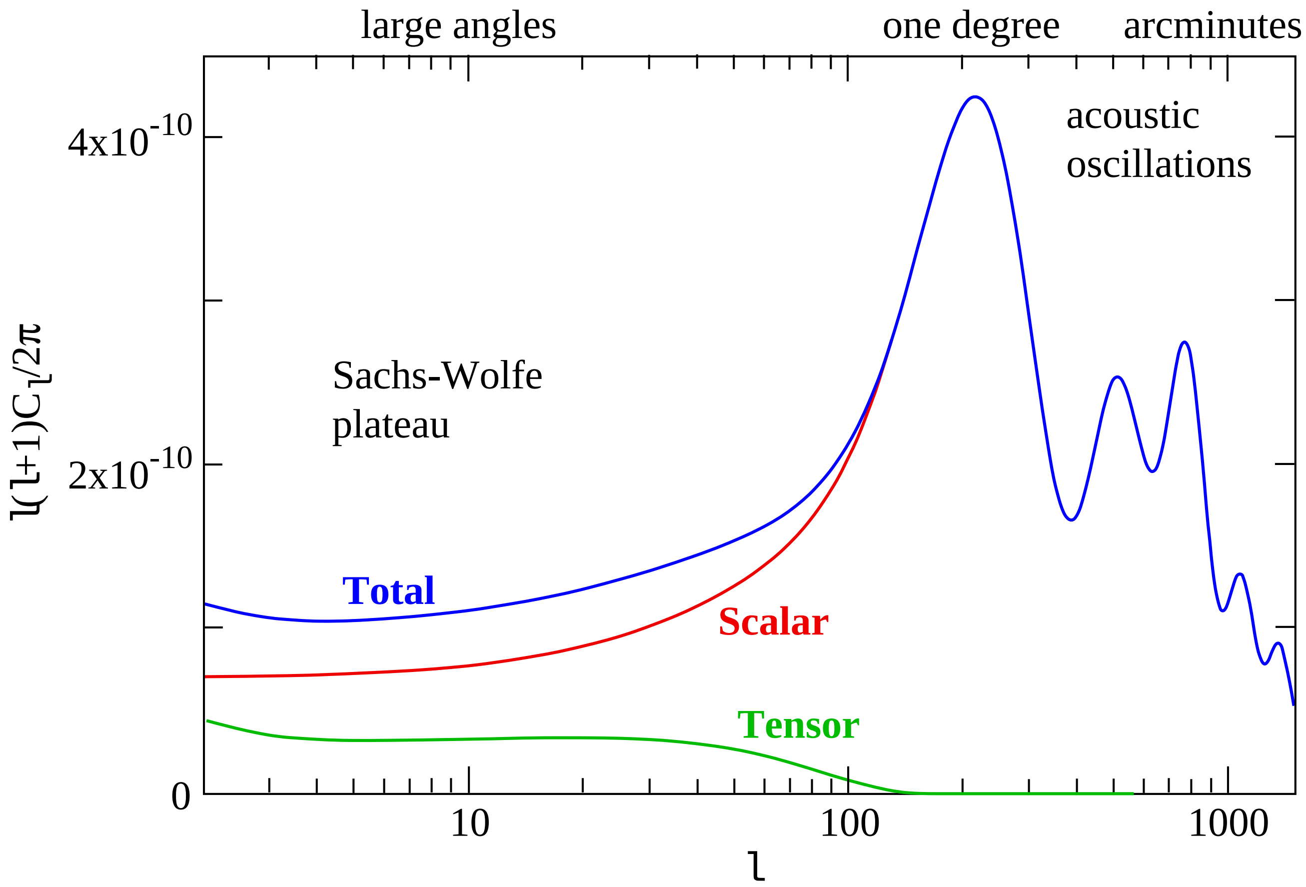
▷ Boltzmann eqs. for photons and neutrinos;

▷ Einstein's field eqs.

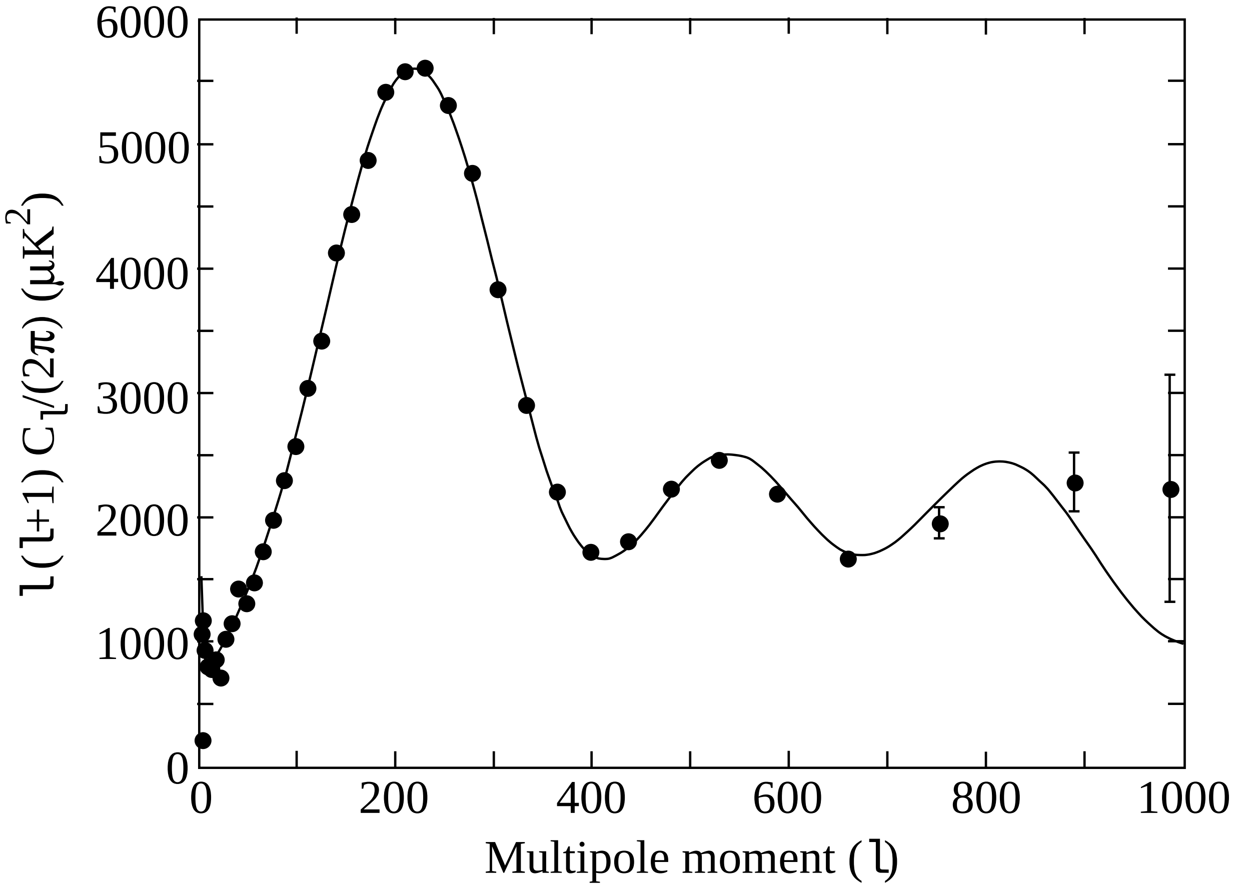
Through initial conditions: Window to the **very early** Universe.

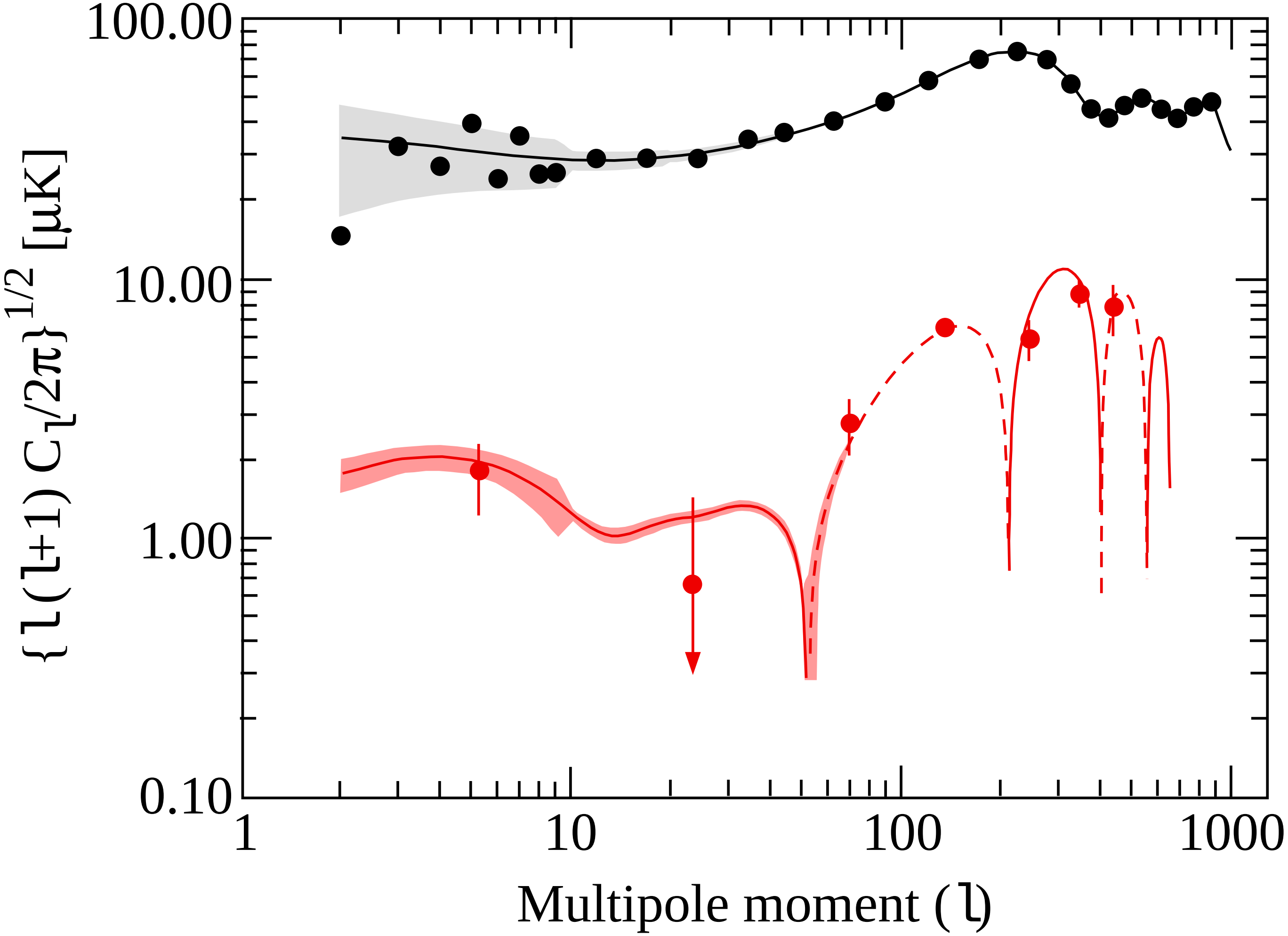
NS: Combo-Lectures, *Ann. Phys. (Leipzig)*, (2006); [hep-ph/0505249].

Theoretical TT Power Spectrum

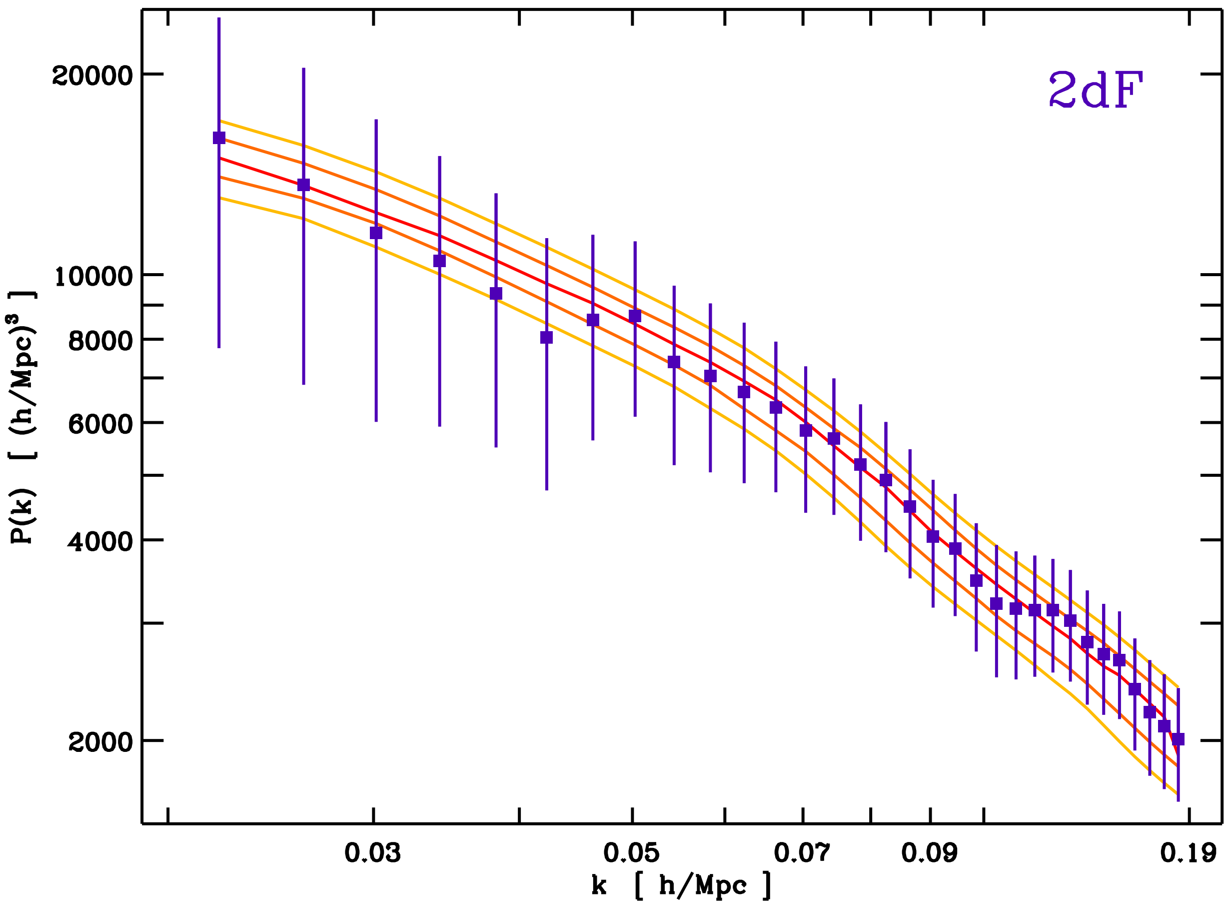


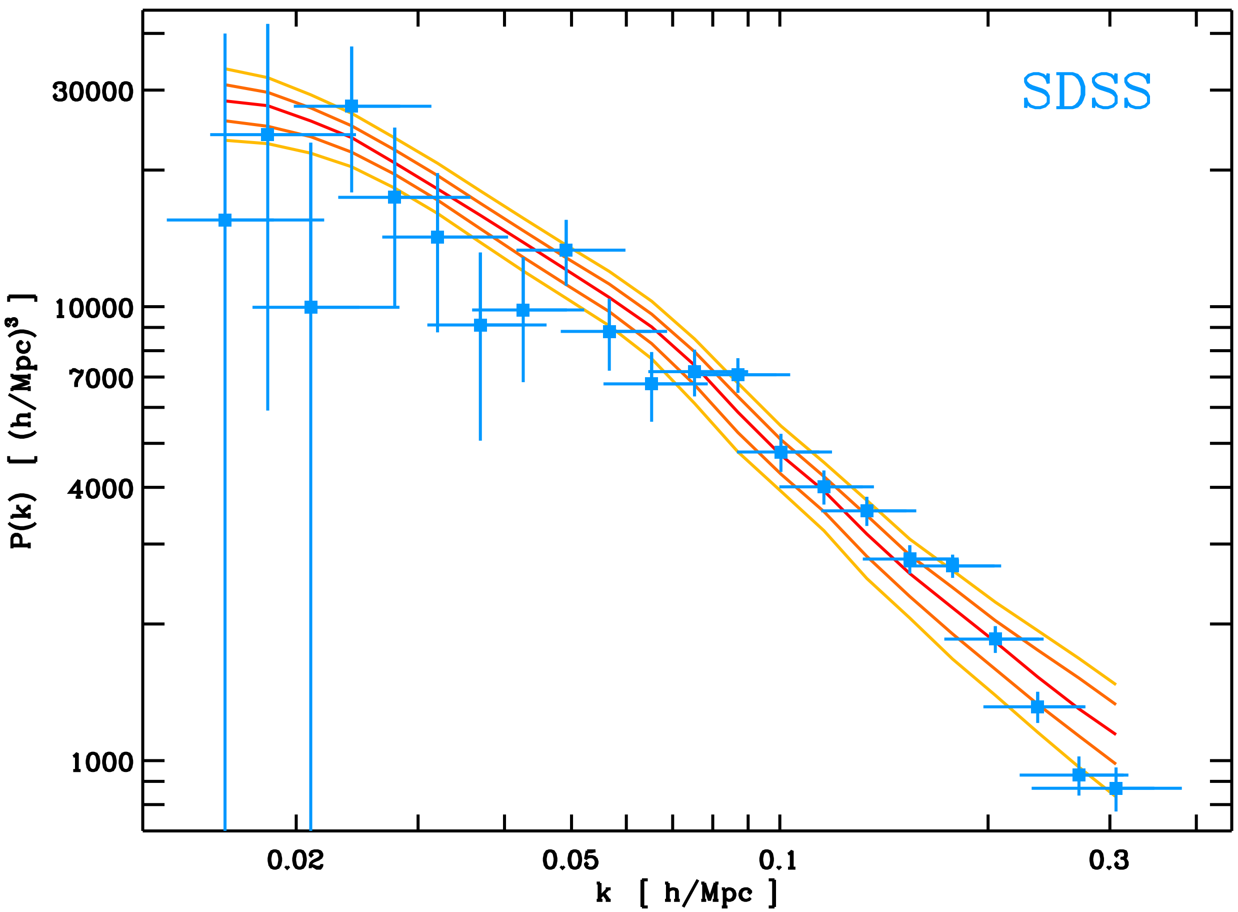
Three-year WMAP data



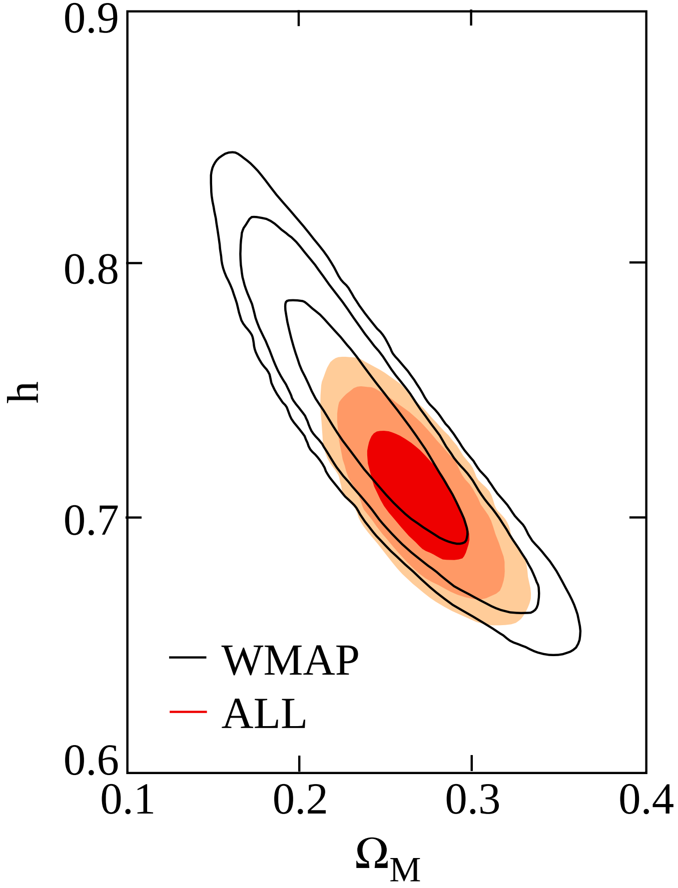

100.00
10.00
1.00
0.10
1
10
100
1000
Multipole moment (l)
{l(l+1) C_l/2π}^1/2 [μK]

Matter power spectrum





SDSS
30000
10000
7000
4000
1000
P(k) [(h/Mpc)³]
0.02
0.05
0.1
0.3
k [h/Mpc]

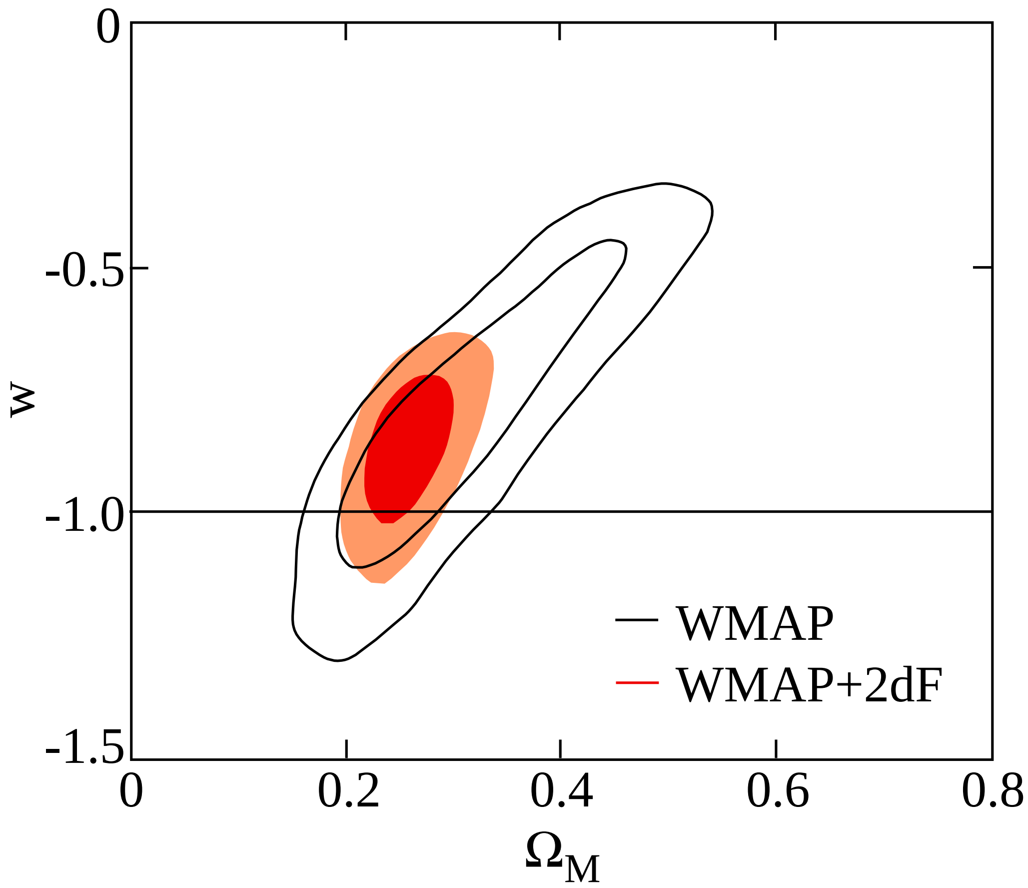

0.9
0.8
0.7
0.6
h
0.1
0.2
0.3
0.4
Ω_M
WMAP
ALL

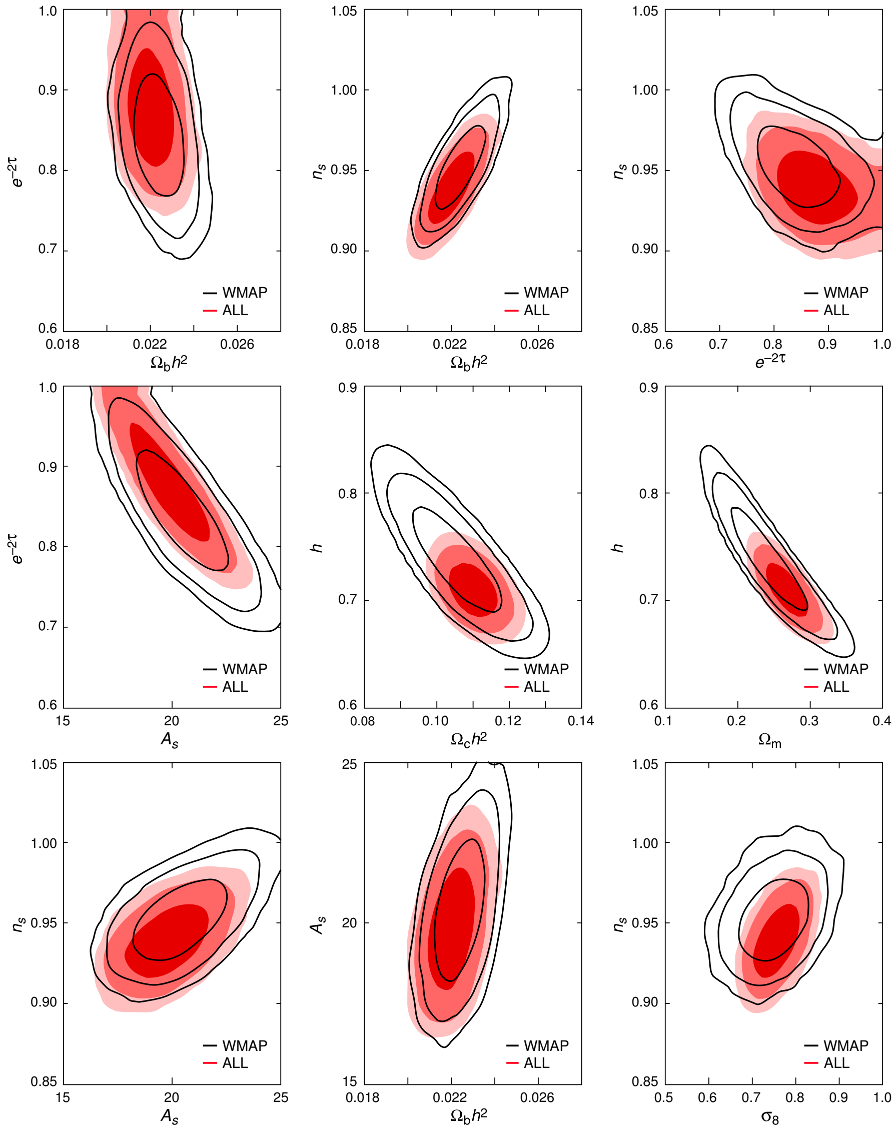
Table

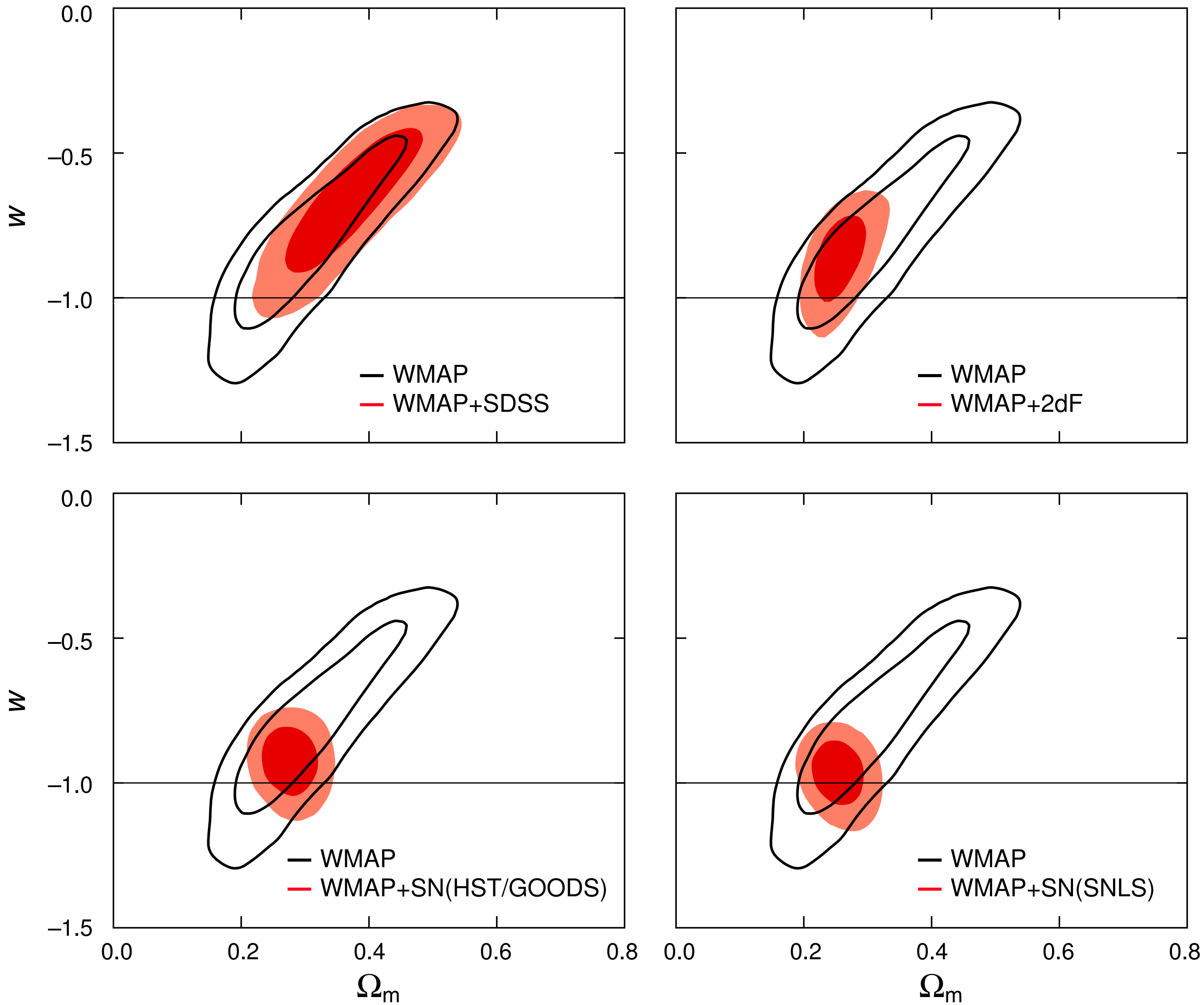
Parameter	WMAP alone	WMAP + 2dFGRS
$100\Omega_b h^2$	$2.233^{+0.072}_{-0.091}$	$2.223^{+0.066}_{-0.083}$
$\Omega_M h^2$	$0.1268^{+0.0072}_{-0.0095}$	$0.1262^{+0.0045}_{-0.0062}$
h	$0.734^{+0.028}_{-0.038}$	$0.732^{+0.018}_{-0.025}$
Ω_M	$0.238^{+0.030}_{-0.041}$	$0.236^{+0.016}_{-0.029}$
σ_8	$0.744^{+0.050}_{-0.060}$	$0.737^{+0.033}_{-0.045}$
τ	$0.088^{+0.028}_{-0.034}$	$0.083^{+0.027}_{-0.031}$
n_s	$0.951^{+0.015}_{-0.019}$	$0.948^{+0.014}_{-0.018}$

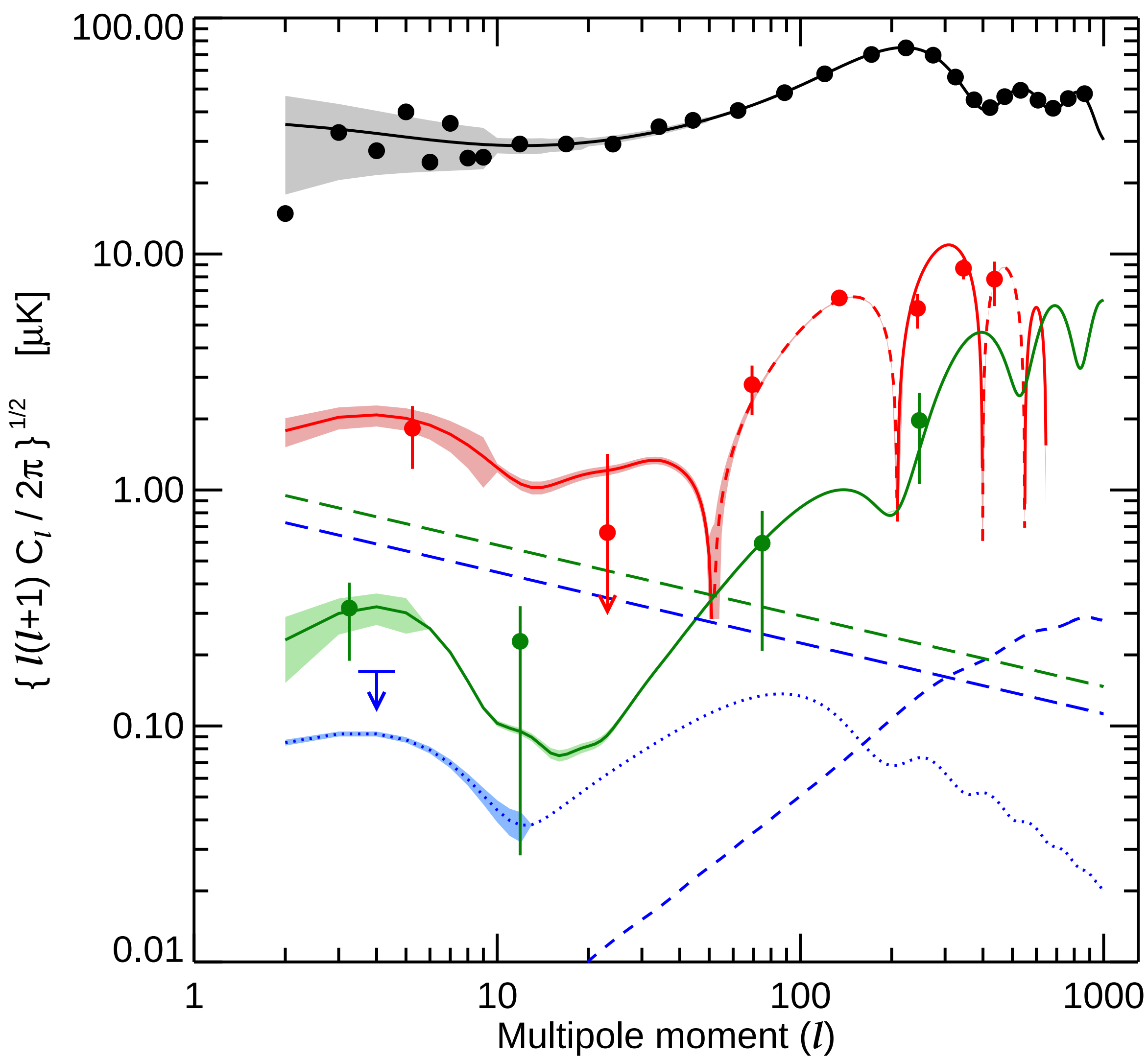
(Six-parameter flat ΛCDM model)

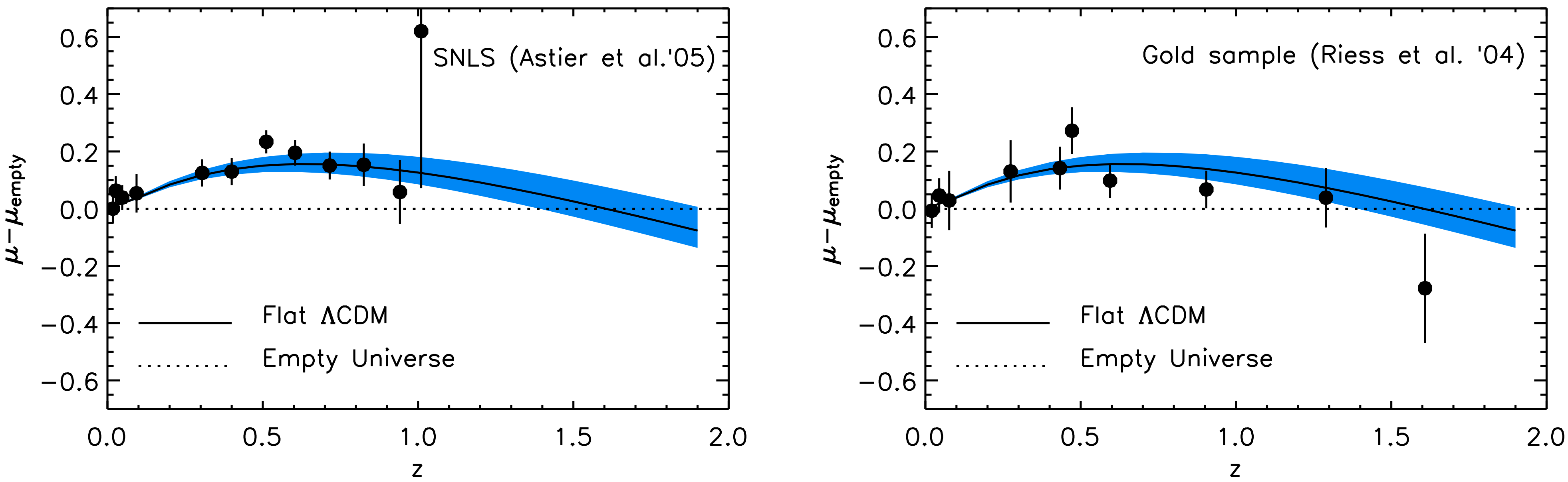
Equation of state $w = p/\rho$ for Dark Energy

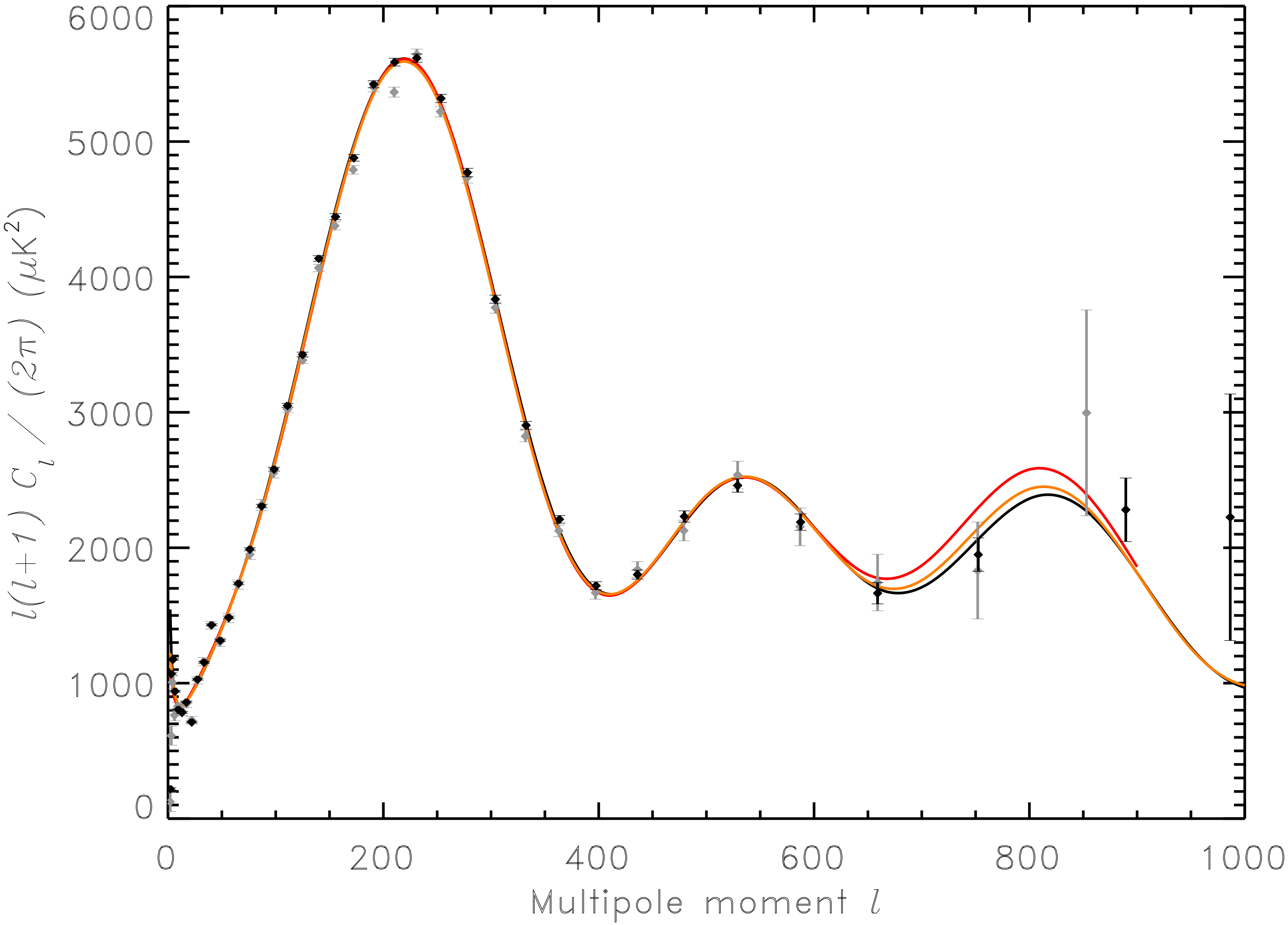



WMAP
ALL
$e^{-2\tau}$
$\Omega_b h^2$
n_s
A_s
h
$\Omega_c h^2$
Ω_m
σ_8


0.0
–0.5
–1.0
–1.5
w
WMAP
WMAP+SDSS
WMAP
WMAP+2dF
WMAP
WMAP+SN(HST/GOODS)
WMAP
WMAP+SN(SNLS)
0.0
0.2
0.4
0.6
0.8
Ωm


100.00
10.00
1.00
0.10
0.01
1
10
100
1000
Multipole moment (l)
{ l(l+1) C_l / 2π }^{1/2} [μK]


SNLS (Astier et al.'05)
Flat ΛCDM
Empty Universe
μ−μempty
z
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.5
1.0
1.5
2.0
Gold sample (Riess et al. '04)
Flat ΛCDM
Empty Universe
μ−μempty
z



6000
5000
4000
3000
2000
1000
0
0
200
400
600
800
1000
Multipole moment l
$l(l+1)\ C_l\ /\ (2\pi)\ (\mu K^2)$

Alternatives to Dark Energy

- Changes in the initial conditions?
- Inhomogeneous models?
- Modifications of gravity?
  - ▷ Generalizations of the Einstein-Hilbert action
  - ▷ First-order modifications of GR
  - ▷ Brane-world models

Generalizations of the Einstein-Hilbert action

$R \rightarrow f(R)$; equivalent to a *scalar-tensor theory*:
define a new metric $\tilde{g}_{\mu\nu} = \exp\left[\sqrt{\frac{2}{3}}\kappa\varphi\right] g_{\mu\nu}$,
$\kappa^2 = 8\pi G$, then action becomes

$$S = \int \left[\frac{1}{2\kappa^2} R[\tilde{g}] - \frac{1}{2}\tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - V(\varphi) + L_{matter}\right] dv_{\tilde{g}}$$

$V =$ Legendre transform of f.

Solar System tests, etc. ; interpretation ? Even the Newtonian limit is not a trivial issue; post-Newtonian expansion? $\rightarrow$ T.P.Sotirou; gr-qc/0507027.

Arbitrary evolution of the scale factor $a(t)$ can be obtained with appropriate choice of $f(R)$.

Inclusion of other curvature invariants

$$R_{\mu\nu}R^{\mu\nu}, \;\; R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$$

dangerous, except for functions of R and the Gauss-Bonnet invariant $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$.

M. Ostrogradski (1850): generic instability of Lagrangian systems in mechanics with higher derivatives $L(q, \dot{q}, \ddot{q}, ..., q^{(N)})$.

$$\sum_{i=0}^{N}\left(-\frac{d}{dt}\right)^{i}\frac{\partial L}{\partial q^{(i)}} = 0.$$

canonical variables:

$$Q_i = q^{(i-1)}, \quad P_i = \sum_{j=i}^{N}\left(-\frac{d}{dt}\right)^{j-i}\frac{\partial L}{\partial q^{(j)}};$$

assume that $P_N = \frac{\partial L}{\partial q^{(N)}}$ is invertible, $q^{(N)} = \mathcal{A}(Q_1, ..., Q_N, P_N)$, then

$$\begin{aligned} H &= \sum_{i=1}^{N} P_i q^{(i)} - L = P_1 Q_2 + \cdots + P_{N-1} Q_N \\ &+ P_N \mathcal{A}(Q_1, ..., Q_N, P_N) \\ &- L(Q_1, ..., Q_N, \mathcal{A}(Q_1, ..., Q_N, P_N)). \end{aligned}$$

unstable!!!

Consistency problems of $f(R, G)$- models?

There are models for which there exist accelerated late-time power law attractors and which satisfy the Solar System constraints. In Friedmann background there are **no ghosts**, however, superluminal propagation for a wide range of parameter space.

A. De Felice, M. Hindmarsh & M. Trodden,
astro-ph/0604154

First-order (affine) modifications of GR

Consider metric and affine symmetric connection Γ ($\Gamma^{\alpha}{}_{\mu\nu}$) as independent fields (for GR equivalent formulations: Palatini (1919)):

$$S = \int \left[\frac{1}{2\kappa} f(R) + L_{matter} \right] \sqrt{-g} d^4 x,$$

$R[g, \Gamma] = g^{\alpha\beta} R_{\alpha\beta}[\Gamma]$, $R_{\alpha\beta}[\Gamma]$ = Ricci tensor of independent torsionless connection Γ. Equations of motion:

$$f'(R) R_{(\mu\nu)}[\Gamma] - \frac{1}{2} f(R) g_{\mu\nu} \ = \ \kappa T_{\mu\nu},$$
$$\nabla^{\Gamma}_{\alpha} \left(\sqrt{-g} f'(R) g^{\mu\nu} \right) = 0.$$

For second equation one has to assume that L_{matter} is functionally independent of Γ. (It may, however, contain metric covariant derivatives.)

$\Rightarrow$

$$\nabla^{\Gamma}_{\alpha}\left[\sqrt{-\widehat{g}}\widehat{g}^{\mu\nu}\right] = 0$$

for conformally equivalent metric $\widehat{g}_{\mu\nu} = f'(R)g_{\mu\nu}$. Hence, $\Gamma^{\alpha}{}_{\mu\nu} =$ Christoffel symbols for the metric $\widehat{g}_{\mu\nu}$. First eq. $\Rightarrow$

$$Rf'(R) - 2f(R) = \kappa^2 T;$$

algebraic for matter-free case $\rightarrow$ Einstein's vacuum eq. with cosmological constant.

• In general, one can rewrite the field equations in the form of Einstein gravity with nonstandard matter couplings. Because of this it is, for instance, straightforward to develop cosmological perturbation theory.

• It can be shown that if the matter action is independent of Γ, the theory is dynamically equivalent to a Brans-Dicke theory with Brans-Dicke parameter $-3/2$, plus a potentiel term.

• T. Koivisto (astro-ph/0602031): confronts model $f(R) = R - \alpha R^{\beta}$ with matter power spectrum, etc; f(R) must be close to $R - 2\Lambda$.

Brane-world models

interesting example: DGP

$$S = \frac{1}{2}\tilde{M}_{Pl}^3 \int_{M^5} \tilde{R}[\tilde{g}] dv_{\tilde{g}} + \frac{1}{2} M_{Pl}^2 \int_{M^4} R[g] dv_g$$

$$+ \int_{M^4} \mathcal{L}_{SM}[\psi, g] dv_g.$$

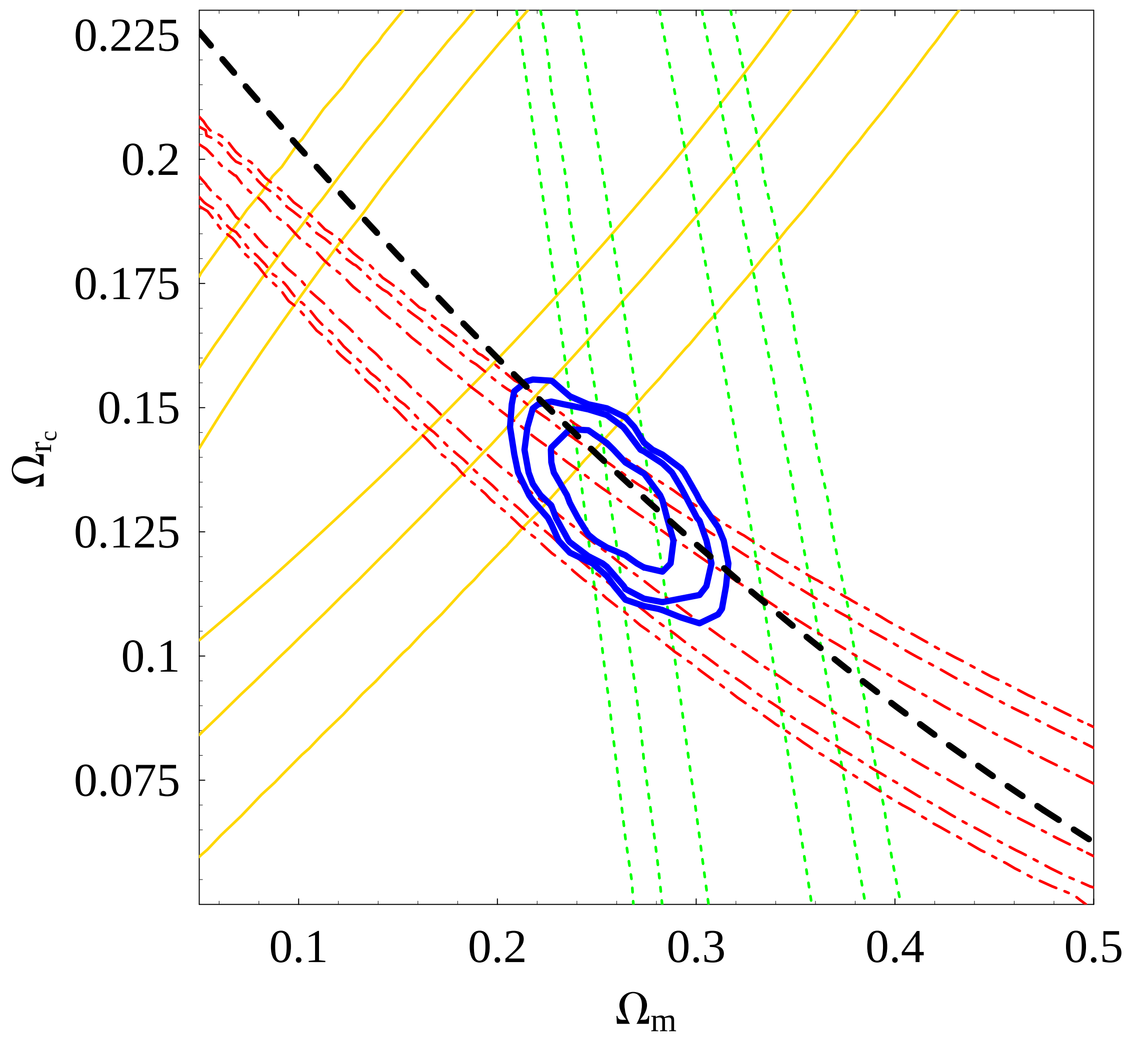
properties:

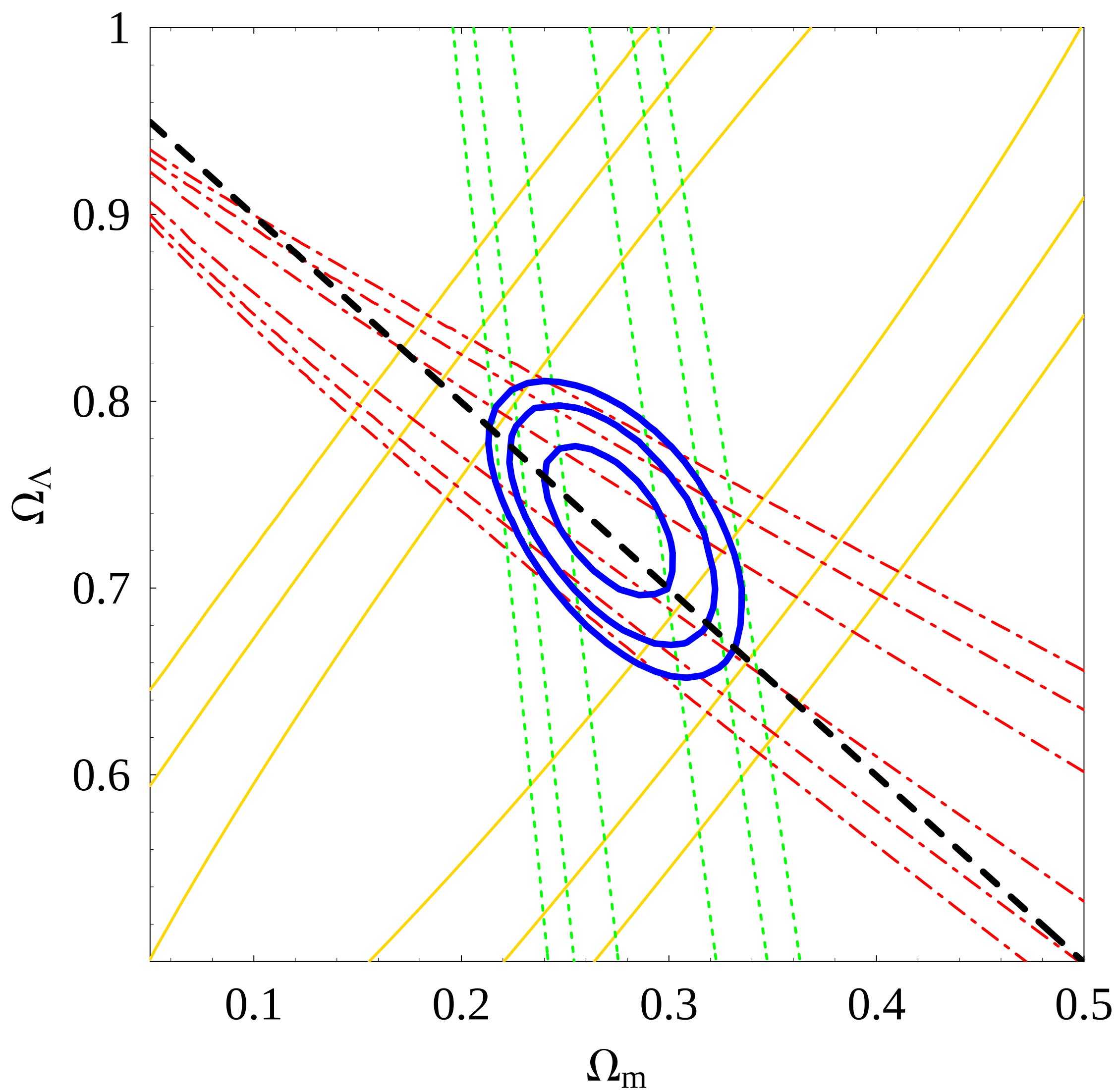
• remains 4-dim. at 'short' distances ;

• crosses over to higher dimensional behavior of gravity at very large distances $> r_c$;

• modified Friedmann equations $\rightarrow$ accelerated universe; instead of Λ there is a new scale:

$$m_c := \frac{2\tilde{M}_{Pl}^3}{M_{Pl}^2} \quad (r_c = m^{-1}).$$


0.225
0.2
0.175
0.15
0.125
0.1
0.075
Ω_{r_c}
0.1
0.2
0.3
0.4
0.5
Ω_m


1
0.9
0.8
0.7
0.6
Ω_Λ
0.1
0.2
0.3
0.4
0.5
Ω_m

Ghost on self-accelerating DGP brane

• for positive brane tension ($\sigma > 0$): perturbative ghost resides in the helicity-0 lightest tensor mode (effective action: Einstein-Hilbert + Pauli-Fierz mass term);
• for $\sigma < 0$: ghost is scalar mode:

$$h_{\mu\nu}^{(\phi)} = (\nabla_\mu \nabla_\nu + H^2)\phi;$$

• for $\sigma = 0$: ghost is an admixture of the the helicity-0 and scalar modes (become degenerate).

Ghost signals an unacceptable instability of the effective 4D perturbative description.

D. Gorbunov, K. Koyama, S. Sibiyakov, Phys.Review D **73**, 044016 (2006);

Ch. Charmousis, R. Gregory, N. Kaloper, A. Padilla, hep-th/0604086.

Has Dark Energy been discovered in the Lab?

C. Beck & M.C. Mackey: claim in several papers that dark energy can be discovered in the Lab through noise measurements of Josephson junctions.

Certainly not: Ph. Jetzer & NS: Phys. Lett B 606, 77 (2005), and astro-ph/0604522:
misunderstanding of fluctuation-dissipation theorem:

$$\langle Q^2 \rangle = \frac{2}{\pi} \int_0^\infty \left[\frac{1}{2}\omega + \frac{\omega}{e^{\beta\omega} - 1} \right] \alpha''(\omega) \frac{d\omega}{\omega}.$$

Table

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Ghosts, Acausalities for generalized GR:

$$S[g] = \int [\gamma R - F(Z)]\sqrt{-g}d^4x, \quad Z = \sum_{i=1}^{3} a_i Z_i,$$

$Z_1 = R^2, \quad Z_2 = R_{\alpha\beta}R^{\alpha\beta}, \quad Z_3 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}.$

Example. CDDETT model: $F(Z) \propto Z^{-n}$.

Introduce scalar field ϕ, and replace $F(Z)$ by $F(\phi) + (Z - \phi)F'(\phi)$; equivalent action:

$$S[g] = \int [\gamma R + F(\phi) + (Z - \phi)F'(\phi)]\sqrt{-g}d^4x;$$

leads to higher order field eqs., except for $a_2 = -4a_3$; then action is of type

$$S[g] = \int [\gamma R + bf(\phi)R^2 + f(\phi)R_{GB}^2 - U(\phi)]\sqrt{-g}d^4x.$$

Cosmological perturbations

• for gauge invariant scalar field perturbation Φ:

$$\frac{1}{a^3}\partial_t(a^3 Q\dot{\Phi} - \frac{1}{a^2}P\triangle\Phi = 0;$$

▷ **no ghosts** $(Q > 0)$: $1 + 4bfR + 8\ddot{f} > 0$;

▷ **real propagation and causality**: $0 \leq P/Q \leq 1$;
for b=0:

$$0 \leq 1 + \frac{4\dot{H}}{3H^2} - \frac{8}{3}\frac{\ddot{f} - 8\dot{f}}{1 + 8H\dot{f}} \leq 1.$$

• tensor perturbations (gravitational waves):

$\triangleright$ correct signs $\longrightarrow$

$$1 + 4bfR + 8\ddot{f} > 0, \quad 1 + 4bfR + 8H\dot{f} > 0;$$

$\triangleright$ causality:

$$\frac{1 + 4bfR + 8\ddot{f}}{1 + 4bfR + 8H\dot{f}} < 1;$$

stringent conditions !

CDDETT model: causality violations for wide range of parameter space, but *no ghosts*.