Charges in Gauge Theories

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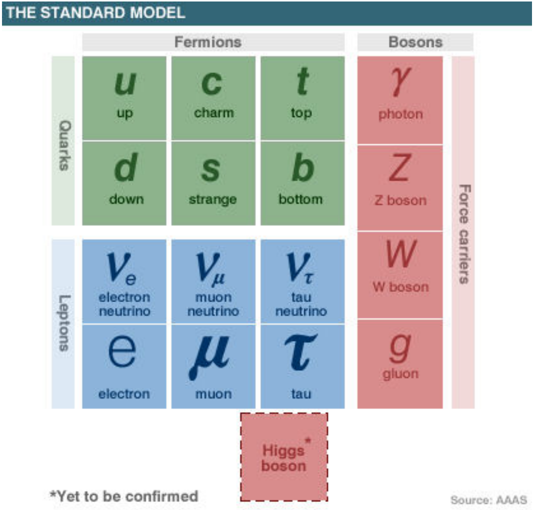
LOR 2006
Budapest
29^{th} June

Outline of talk

1. Charges in nature
2. The theoretical challenge of charges
3. Magnetic charges
4. Conclusions

The building blocks of *particle* physics



Theoretical description

The relativistic concept of a charged particle does not exist.

Kulish and Faddeev, 1970

- Massless photon
- Long range nature of force between (electric) charges
- Non-trivial asymptotic dynamics
- Soft infrared divergences in QED
- Massless charges produce additional collinear divergences.

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The party line

- The Bloch-Nordsieck (1937) method in QED:
  suitable inclusive cross-sections are finite
  - Does not work for massless charges
  - Unnatural time asymmetry
- The Lee-Nauenberg 'theorem' (1964):
  remove divergences by summing over *all* degenerate states
  - Works fine for final state degeneracies (ok for collinear structures as in LEP)
  - Does not work for initial and final state degeneracies [M Lavelle and DM JHEP 2006]
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Can we do better?

- Basic Question: Should we identify particles directly with the matter fields ψ that enters the Lagrangian?
  - Coupling does not switch off as $t \to \pm\infty$
  - Matter $\psi(x)$ is never gauge invariant $\psi(x) \to U(x)\psi(x)$
  - Matter field is never a physical field.
- Our response [M.Lavelle and DM]: Need to 'dress' matter to make a charge
  - Find a field dependent dressing $h^{-1}(x)$ that transforms as

$$h^{-1}(x) \to h^{-1}(x)U^{-1}(x)$$

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A static charge

Dirac's dressed electron

$$\psi_D(r) = \exp\left(ie\frac{\partial_i A_i}{\nabla^2}(r)\right)\psi(r)$$

Creates a charged state

$$|\psi_D(r)\rangle = \psi_D(r)\,|0\rangle$$

The state has the proper Coulombic field for a static charge

$$E_i(x)\,|\psi_D(r)\rangle = \frac{e}{4\pi}\frac{(x-r)_i}{|\underline{x}-\underline{r}|^3}\,|\psi_D(r)\rangle$$

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[E.Bagan, M.Lavelle, DM]

- Can extend Dirac's suggestion to moving and colour charges
- Find that the dressing has structure

$$h^{-1} =$$

- Structure responsible for different infrared effects.
- Structure in non-abelian theory reflects screening and anti-screening forces between charges.
- Global obstruction to construction of coloured charges.
- Direct interplay between Gribov copies and confinement.

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Monopoles

Common lore: condensation of magnetic monopoles is responsible for confinement

- Numerous lattice investigations
- Many open questions
- Analytic description lacking
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Monopole operator

A magnetic monopole operator $M(r)$ should:

- Create a one monopole state

$$|M(r)\rangle := M(r)\,|0\rangle$$

- Create a Coulombic magnetic charge

$$B_i(x)\,|M(r)\rangle = \frac{1}{g}\frac{(x-r)_i}{|\underline{x}-\underline{r}|^3}\,|M(r)\rangle$$

- Gauge invariant
- Finite energy

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Monopoles in electrodynamics

Dirac: the need for singular potentials

$$\underline{\lambda}^N := -\tfrac{1}{2}g\frac{\underline{r}\times\underline{\hat{z}}}{r(r+z)} \qquad \underline{\lambda}^S := \tfrac{1}{2}g\frac{\underline{r}\times\underline{\hat{z}}}{r(r-z)}$$

A candidate operator

$$M(r) = \exp\left(\frac{i}{g}\int d^3w\lambda_i^N(w-r)E_i(w)\right)$$

- Gauge invariant ✓
- Generates Coulombic field ✓
- Generates Dirac string X
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Monopoles on $\mathbb{R}^3 - \{0\}$

Removing the position of the monopole means we can introduce multi-valued potentials

$$\underline{\Lambda}(r) = \theta(z)\underline{\lambda}^N + \theta(-z)\underline{\lambda}^S + \frac{1}{g}\delta(z)\phi(r)\underline{\hat{z}}$$

An improved operator

$$M(r) = \exp\left(\frac{i}{g}\int_{\mathbb{R}^3-\{r\}} d^3w\Lambda_i(w-r)E_i(w)\right)$$

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Georgi-Glashow model

- SU(2) gauge field coupled to adjoint Higgs

$$L = -\frac{1}{4}F^2 + (DH)^2 - V(H^2)$$

- Can define a gauge invariant field strength

$$F_{\mu\nu} = \frac{H^a}{|H|}F^a_{\mu\nu} - \frac{1}{g}\frac{1}{|H|^3}\epsilon^{abc}H^a(D_\mu H)^b(D_\nu H)^c$$

- Define magnetic current $J^M_\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}\partial^\nu F^{\lambda\sigma}$
- Magnetic charge exists as a physical observable.

$$Q_M = \frac{1}{4\pi}\int d^3x J^M_0 = \frac{1}{8\pi g}\int d^2S_i\epsilon_{ijk}\epsilon^{abc}\hat{H}^a\partial_j\hat{H}^b\partial_k\hat{H}^c$$

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- Define magnetic current $J^M_\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}\partial^\nu F^{\lambda\sigma}$
- Magnetic charge exists as a physical observable.

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Georgi-Glashow model

- SU(2) gauge field coupled to adjoint Higgs

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Monopole creation operator

We find [A. Khvedelidze, A. Kovner, DM, JHEP 2006]

$$M(r) = D(r)M_A(r)$$

where we first create monopole and string

$$M_A(r) = \exp\left(\frac{i}{g}\int d^3w\lambda_i^N(w-r)\hat{H}^a(w)E_i^a(w)\right)$$

then we remove string contribution to magnetic field

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Perturbative tests $\langle M(r) \rangle$

- In Higgs phase monopoles are massive so expect that in a finite but large volume L,

$$\langle M \rangle \propto \exp(-\mu L)$$

- In the confining phase we expect that

$$\langle M \rangle \neq 0$$

This is a non-perturbative effect.

- In a perturbative calculation expect a milder volume dependence.

We find within path integral calculation (steepest descent method, dandelion configuration)

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- Charges can be defined in gauge theories and a relativistic description of a charged particle *is* possible.
- Charges have structure which is reflected in their infrared behaviour and forces between them.
- A promising approach to magnetic charges has been initiated.
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