

# The form factor program – a review and new results the nested $SU(N)$ -off-shell Bethe ansatz

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  - Construct a quantum field theory explicitly
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  - $SU(N)$  Form factors
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Construct a quantum field theory **explicitly** in 3 steps

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  - 4 ‘maximal analyticity’

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- 2 LSZ-assumptions
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The bootstrap program classifies integrable quantum field theoretic models

**Integrability** in (quantum) field theories means:  
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**Consequence** in 1+1 dim.:

The  $n$ -particle S-matrix is a product of 2-particle S-matrices

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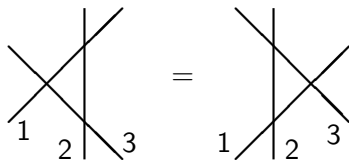
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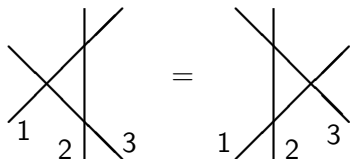
$$S^{(n)}(p_1, \dots, p_n) = \prod_{i < j} S_{ij}(p_i, p_j)$$

**Further consequences**

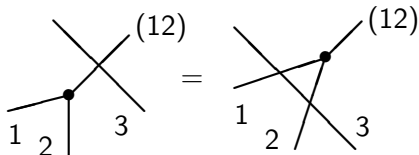
**“Yang-Baxter equation”**  $S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12}$



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**“bound state bootstrap equation”**  $S_{(12)3} \Gamma_{12}^{(12)} = \Gamma_{12}^{(12)} S_{13}S_{23}$



# $SU(N)$ S-matrix

## Example

$$S_{\alpha\beta}^{\delta\gamma}(\theta) = \begin{array}{c} \delta \quad \gamma \\ \diagdown \quad \diagup \\ \bullet \\ \diagup \quad \diagdown \\ \alpha \quad \theta_1 \quad \theta_2 \quad \beta \end{array} = \delta_{\alpha\gamma} \delta_{\beta\delta} b(\theta) + \delta_{\alpha\delta} \delta_{\beta\gamma} c(\theta).$$

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$$\implies \quad a(\theta) = b(\theta) + c(\theta) = -\frac{\Gamma\left(1 - \frac{\theta}{2\pi i}\right) \Gamma\left(1 - \frac{1}{N} + \frac{\theta}{2\pi i}\right)}{\Gamma\left(1 + \frac{\theta}{2\pi i}\right) \Gamma\left(1 - \frac{1}{N} - \frac{\theta}{2\pi i}\right)}$$

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Bound state of  $N - 1$  particles = anti-particle [\[Swieca\]](#)

**Definition:** Let  $\mathcal{O}(x)$  be a local operator

$$\begin{aligned} \langle 0 | \mathcal{O}(0) | p_1, \dots, p_n \rangle_{\alpha_1 \dots \alpha_n}^{in} &= F_{\alpha_1 \dots \alpha_n}^{\mathcal{O}}(\theta_1, \dots, \theta_n) \\ &= \text{Diagram} \end{aligned}$$

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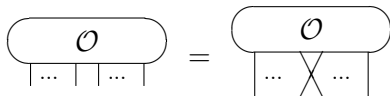
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LSZ-assumptions  
+ 'maximal analyticity' }  $\implies$  Properties of form factors

# Form factor equations

## (i) Watson's equation

$$F_{\dots ij \dots}^{\mathcal{O}}(\dots, \theta_i, \theta_j, \dots) = F_{\dots jj \dots}^{\mathcal{O}}(\dots, \theta_j, \theta_i, \dots) S_{ij}(\theta_i - \theta_j)$$



## (ii) Crossing

$$\bar{\alpha}_1 \langle p_1 | \mathcal{O}(0) | \dots, p_n \rangle_{\dots \alpha_n}^{in, conn.} =$$

$$\sigma_{\alpha_1}^{\mathcal{O}} F_{\alpha_1 \dots \alpha_n}^{\mathcal{O}}(\theta_1 + i\pi, \dots, \theta_n) = F_{\dots \alpha_n \alpha_1}^{\mathcal{O}}(\dots, \theta_n, \theta_1 - i\pi)$$



# Form factor equations

## (iii) Annihilation recursion relation

$$\frac{1}{2i} \operatorname{Res}_{\theta_{12}=i\pi} F_{1\dots n}^{\mathcal{O}}(\theta_1, \dots) = \mathbf{C}_{12} F_{3\dots n}^{\mathcal{O}}(\theta_3, \dots) (\mathbf{1} - \sigma_2^{\mathcal{O}} S_{2n} \dots S_{23})$$

$$\frac{1}{2i} \operatorname{Res}_{\theta_{12}=i\pi} \text{Diagram} = \text{Diagram} - \sigma_2^{\mathcal{O}} \text{Diagram}$$

The diagram equation shows a form factor with two external legs (1 and 2) and other legs (3, ..., n). The left side is the residue at  $\theta_{12}=i\pi$ . The right side is the sum of two diagrams: the first is the form factor with legs 1 and 2 connected by a cap, and the second is the form factor with legs 1 and 2 connected by a cap and a loop, multiplied by  $-\sigma_2^{\mathcal{O}}$ .

## (iv) Bound state form factors

$$\frac{1}{\sqrt{2}} \operatorname{Res}_{\theta_{12}=i\eta} F_{123\dots n}^{\mathcal{O}}(\underline{\theta}) = F_{(12)3\dots n}^{\mathcal{O}}(\theta_{(12)}, \underline{\theta}') \Gamma_{12}^{(12)}$$

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## (v) Lorentz invariance

$$F_{1\dots n}^{\mathcal{O}}(\theta_1 + u, \dots, \theta_n + u) = e^{su} F_{1\dots n}^{\mathcal{O}}(\theta_1, \dots, \theta_n)$$

# Locality and field equations

(i) – (iv) and a general crossing formula  $\Rightarrow$

$${}^{in}\langle \phi | [\mathcal{O}(x), \mathcal{O}(y)] | \psi \rangle^{in} = 0$$

for all matrix elements, if  $x - y$  is space like.

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E.g. sine-Gordon equation:

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the “bare” mass  $\sqrt{\alpha}$  is related to the renormalized mass  $m$  by

$$\alpha = m^2 \frac{\pi\nu}{\sin \pi\nu} \quad \text{where} \quad \nu = \frac{\beta^2}{8\pi - \beta^2}$$

## 2-particle form factors

$$\langle 0 | \mathcal{O}(0) | p_1, p_2 \rangle^{in/out} = F((p_1 + p_2)^2 \pm i\varepsilon) = F(\pm\theta_{12})$$

where  $p_1 p_2 = m^2 \cosh \theta_{12}$ .

Watson's equations

$$\begin{aligned} F(\theta) &= F(-\theta) S(\theta) \\ F(i\pi - \theta) &= F(i\pi + \theta) \end{aligned}$$

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**Example:** The highest weight  $SU(N)$  2-particle form factor

$$F(\theta) = \exp \int_0^\infty \frac{dt}{t \sinh^2 t} e^{\frac{t}{N}} \sinh t \left(1 - \frac{1}{N}\right) \left(1 - \cosh t \left(1 - \frac{\theta}{i\pi}\right)\right)$$

# The general formula

$$F_{\alpha_1 \dots \alpha_n}^{\mathcal{O}}(\theta_1, \dots, \theta_n) = K_{\alpha_1 \dots \alpha_n}^{\mathcal{O}}(\underline{\theta}) \prod_{1 \leq i < j \leq n} F(\theta_{ij})$$

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"Nested off-shell Bethe Ansatz"

$$K_{\alpha_1 \dots \alpha_n}^{\mathcal{O}}(\underline{\theta}) = \int_{\mathcal{C}_{\underline{\theta}}} dz_1 \cdots \int_{\mathcal{C}_{\underline{\theta}}} dz_m h(\underline{\theta}, \underline{z}) p^{\mathcal{O}}(\underline{\theta}, \underline{z}) L_{\beta_1 \dots \beta_m}(\underline{z}) \Psi_{\alpha_1 \dots \alpha_n}^{\beta_1 \dots \beta_m}(\underline{\theta}, \underline{z})$$

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$$h(\underline{\theta}, \underline{z}) = \prod_{i=1}^n \prod_{j=1}^m \phi(z_j - \theta_i) \prod_{1 \leq i < j \leq m} \tau(z_i - z_j),$$

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depend only on  $F(\theta)$  i.e. on the S-matrix (see below),

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$p^{\mathcal{O}}(\underline{\theta}, \underline{z}) =$  polynomial of  $e^{\pm z_i}$  depends on the operator.

# Equation for $\phi(z)$

bound state  $(\underbrace{1, \dots, 1}_{N-1}) \rightarrow (N-1) = \bar{1} = \text{anti-particle of } 1$

and the form factor recursion relations (iii) + (iv)  $\implies$

$$\prod_{k=0}^{N-2} \phi(\theta + k\eta) \prod_{k=0}^{N-1} F(\theta + k\eta) = 1, \quad \eta = \frac{2\pi}{N}$$

Solution:

$$\phi(\theta) = \Gamma\left(\frac{\theta}{2\pi i}\right) \Gamma\left(1 - \frac{1}{N} - \frac{\theta}{2\pi i}\right)$$



# “Bethe ansatz” state

$$\Psi_{\underline{\alpha}}^{\underline{\beta}}(\underline{\theta}, \underline{z}) =$$

Diagram illustrating the Bethe ansatz state  $\Psi_{\underline{\alpha}}^{\underline{\beta}}(\underline{\theta}, \underline{z})$ . The diagram shows a network of paths connecting nodes. The top row of nodes is labeled with parameters  $\beta_1, \beta_m, 1, 1$ . The bottom row of nodes is labeled with parameters  $\alpha_1, \dots, \alpha_n$ . The leftmost node is connected to the bottom row nodes by a vertical line. From the bottom row nodes, horizontal lines extend to the right, labeled  $z_1$  and  $z_m$ . Vertical lines connect these horizontal lines to the top row nodes. The rightmost node is connected to the top row nodes by a vertical line. Ellipses indicate intermediate nodes and lines.

$$\begin{aligned} 2 &\leq \beta_i \leq N \\ 1 &\leq \alpha_i \leq N \end{aligned}$$

# Wightman functions

The two-point function

$$w(x) = \langle 0 | \mathcal{O}(x) \mathcal{O}'(0) | 0 \rangle$$

Summation over all intermediate states

$$w(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\theta_1 \dots \int d\theta_n e^{-ix(p_1 + \dots + p_n)} g_n(\underline{\theta})$$

where

$$g_n(\underline{\theta}) = \frac{1}{(4\pi)^n} \langle 0 | \mathcal{O}(0) | p_1, \dots, p_n \rangle^{in} \langle p_n, \dots, p_1 | \mathcal{O}'(0) | 0 \rangle$$

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The Log of the two-point function

$$\ln w(x) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d\theta_1 \dots \int d\theta_n e^{-ix(p_1 + \dots + p_n)} h_n(\underline{\theta})$$

The functions  $g_n$  and  $h_n$  are obviously related by the cumulant formula

$$g_I = \sum_{I_1 \cup \dots \cup I_k = I} h_{I_1} \dots h_{I_k}$$

For example

$$\begin{aligned} g_1 &= h_1 \\ g_{12} &= h_{12} + h_1 h_2 \\ g_{123} &= h_{123} + h_{12} h_3 + h_{13} h_2 + h_{23} h_1 + h_1 h_2 h_3 \\ &\dots \end{aligned}$$

# Short distance behavior

The two-point Wightman function for short distances

$$\langle 0 | \mathcal{O}(x) \mathcal{O}(0) | 0 \rangle \sim \left( \sqrt{-x^2} \right)^{-4\Delta} \quad \text{for } x \rightarrow 0$$

with the 'dimension'

$$\Delta = \frac{1}{8\pi} \sum_{n=1}^{\infty} \frac{1}{n!} \int d\theta_1 \dots \int d\theta_{n-1} h_n(\theta_1, \dots, \theta_{n-1}, 0)$$

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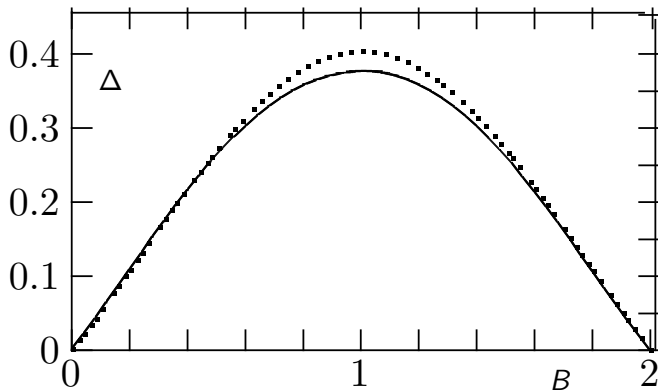
**Example:** The sinh-Gordon model  $\square\varphi + \frac{\alpha}{\beta} \sinh \beta\varphi = 0$

Operator:  $\mathcal{O}(x) = : \exp \beta\varphi : (x)$

# Short distance behavior

## sinh-Gordon

1- and 1+2-particle intermediate state contributions



where  $B = \frac{2\beta^2}{4\pi + \beta^2}$

1-particle .....  
1+2-particle —

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