The form factor program – a review and new results

the nested **SU**(**N**)-off-shell Bethe ansatz

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The "Bootstrap Program"

Construct a <u>quantum</u> field theory *explicitly* in 3 steps

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1. S-matrix using
   1. general Properties: unitarity, crossing etc.
   2. "Yang-Baxter Equation"
   3. "bound state bootstrap"
   4. 'maximal analyticity'

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   2. LSZ-assumptions
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The bootstrap program classifies integrable quantum field theoretic models

Integrability

Integrability in (quantum) field theories means:
there exits ∞-many conservation laws

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Consequence in 1+1 dim.:
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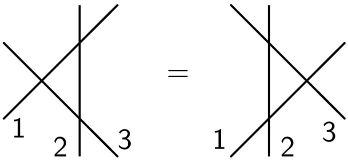
Further consequences

Integrability

"Yang-Baxter equation" $\quad \mathbf{S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12}}$



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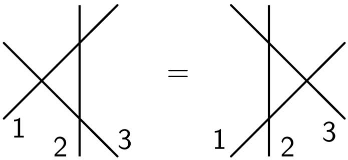
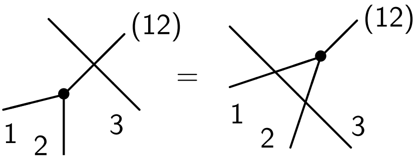


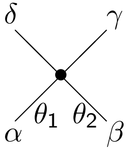
"bound state bootstrap equation" $\mathbf{S_{(12)3}\,\Gamma_{12}^{(12)} = \Gamma_{12}^{(12)}\,S_{13}S_{23}}$



$SU(N)$ S-matrix

Example

$$S^{\delta\gamma}_{\alpha\beta}(\theta) =$$



$$= \delta_{\alpha\gamma}\delta_{\beta\delta}\, b(\theta) + \delta_{\alpha\delta}\delta_{\beta\gamma}\, c(\theta)\,.$$

particles $\alpha, \beta, \gamma, \delta = 1, \ldots, N \leftrightarrow$ vector representation of $SU(N)$

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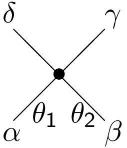

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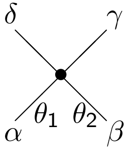

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Yang-Baxter $\Longrightarrow$ $c(\theta) = -\frac{2\pi i}{N\theta}\; b(\theta)$

$$\Longrightarrow \qquad a(\theta) = b(\theta) + c(\theta) = -\frac{\Gamma\left(1 - \frac{\theta}{2\pi i}\right)\Gamma\left(1 - \frac{1}{N} + \frac{\theta}{2\pi i}\right)}{\Gamma\left(1 + \frac{\theta}{2\pi i}\right)\Gamma\left(1 - \frac{1}{N} - \frac{\theta}{2\pi i}\right)}$$

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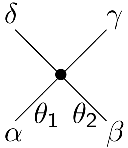
$S^{\delta\gamma}_{\alpha\beta}(\theta) =$ 

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The S-matrix has a bound state pole at $\theta = i\eta = 2\pi i/N$.

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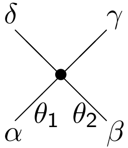
$S^{\delta\gamma}_{\alpha\beta}(\theta) =$ 

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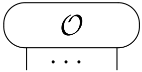
Bound state of $N-1$ particles = anti-particle [Swieca]

Definition: Let $\mathcal{O}(x)$ be a local operator

$$\langle\, 0 \,|\, \mathcal{O}(0) \,|\, p_1, \ldots, p_n \,\rangle^{in}_{\alpha_1 \ldots \alpha_n} = F^{\mathcal{O}}_{\alpha_1 \ldots \alpha_n}(\theta_1, \ldots, \theta_n)$$

$$=$$



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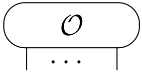
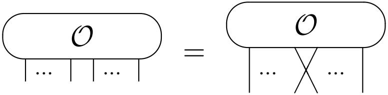


LSZ-assumptions
+ 'maximal analyticity' $\Big\} \Longrightarrow$ **Properties of form factors**

Form factor equations

(i) Watson's equation

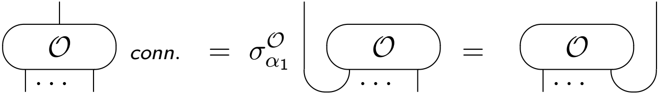
$$F^{\mathcal{O}}_{\dots ij \dots}(\dots,\theta_i,\theta_j,\dots) = F^{\mathcal{O}}_{\dots ji \dots}(\dots,\theta_j,\theta_i,\dots)\, S_{ij}(\theta_i-\theta_j)$$



(ii) Crossing

$${}_{\bar\alpha_1}\langle\, p_1 \,|\, \mathcal{O}(0) \,|\, \dots, p_n \,\rangle^{in,conn.}_{\dots\alpha_n} =$$

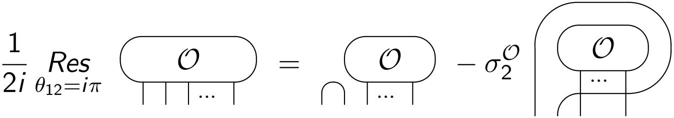
$$\sigma^{\mathcal{O}}_{\alpha_1} F^{\mathcal{O}}_{\alpha_1\dots\alpha_n}(\theta_1+i\pi,\dots,\theta_n) = F^{\mathcal{O}}_{\dots\alpha_n\alpha_1}(\dots,\theta_n,\theta_1-i\pi)$$



Form factor equations

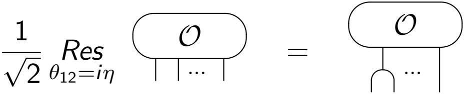
(iii) Annihilation recursion relation

$$\frac{1}{2i}\operatorname*{Res}_{\theta_{12}=i\pi} F^{\mathcal{O}}_{1\dots n}(\theta_1,..) = \mathbf{C}_{12}\; F^{\mathcal{O}}_{3\dots n}(\theta_3,..)\left(\mathbf{1} - \sigma_2^{\mathcal{O}} S_{2n}\dots S_{23}\right)$$



(iv) Bound state form factors

$$\frac{1}{\sqrt{2}}\operatorname*{Res}_{\theta_{12}=i\eta} F^{\mathcal{O}}_{123\dots n}(\underline{\theta}) \quad = \quad F^{\mathcal{O}}_{(12)3\dots n}(\theta_{(12)},\underline{\theta}')\,\Gamma^{(12)}_{12}$$



(v) Lorentz invariance

$$F^{\mathcal{O}}_{1\dots n}(\theta_1+u,\dots,\theta_n+u) = e^{su}\,F^{\mathcal{O}}_{1\dots n}(\theta_1,\dots,\theta_n)$$

Locality and field equations

(i) – (iv) and a general crossing formula $\Rightarrow$

$$^{in}\langle\,\phi\,|\;[\mathcal{O}(x),\mathcal{O}(y)]\;|\,\psi\,\rangle^{in}=0$$

for all matrix elements, if $x-y$ is space like.

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the "bare" mass $\sqrt{\alpha}$ is related to the renormalized mass m by

$$\alpha = m^2\frac{\pi\nu}{\sin\pi\nu} \quad \text{where} \quad \nu = \frac{\beta^2}{8\pi-\beta^2}$$

2-particle form factors

$$\langle\, 0\,|\,\mathcal{O}(0)\,|\,p_1,p_2\rangle^{in/out} = F\left((p_1+p_2)^2 \pm i\varepsilon\right) = \; F\left(\pm\theta_{12}\right)$$

where $p_1 p_2 = m^2 \cosh\theta_{12}$.

Watson's equations

$$\begin{aligned} F\left(\theta\right) &= F\left(-\theta\right)S\left(\theta\right) \\ F\left(i\pi-\theta\right) &= F\left(i\pi+\theta\right) \end{aligned}$$

"maximal analyticity" $\Rightarrow$ unique solution

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Example: The highest weight $SU(N)$ 2-particle form factor

$$F(\theta) = \exp \int_0^\infty \frac{dt}{t \sinh^2 t} e^{\frac{t}{N}} \sinh t \left(1 - \frac{1}{N}\right) \left(1 - \cosh t \left(1 - \frac{\theta}{i\pi}\right)\right)$$

The general formula

$$F^{\mathcal{O}}_{\alpha_1\ldots\alpha_n}(\theta_1,\ldots,\theta_n) = K^{\mathcal{O}}_{\alpha_1\ldots\alpha_n}(\underline{\theta}) \prod_{1\leq i<j\leq n} F(\theta_{ij})$$

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"Nested off-shell Bethe Ansatz"

$$\boxed{K^{\mathcal{O}}_{\alpha_1\ldots\alpha_n}(\underline{\theta}) = \int_{\mathcal{C}_{\underline{\theta}}} dz_1 \cdots \int_{\mathcal{C}_{\underline{\theta}}} dz_m\, h(\underline{\theta},\underline{z})\, p^{\mathcal{O}}(\underline{\theta},\underline{z})\, L_{\beta_1\ldots\beta_m}(\underline{z}) \Psi^{\beta_1\ldots\beta_m}_{\alpha_1\ldots\alpha_n}(\underline{\theta},\underline{z})}$$

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$$h(\underline{\theta},\underline{z}) = \prod_{i=1}^{n}\prod_{j=1}^{m} \phi(z_j-\theta_i) \prod_{1\le i<j\le m} \tau(z_i-z_j)\,,$$

$$\tau(z) = \frac{1}{\phi(z)\phi(-z)}$$

depend only on $F(\theta)$ i.e. on the S-matrix (see below),

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$p^{\mathcal{O}}(\underline{\theta},\underline{z})$ = polynomial of $e^{\pm z_i}$ depends on the operator.

Equation for $\phi(z)$

bound state $(\underbrace{1,\ldots,1}_{N-1}) \rightarrow (N-1) = \bar{1} =$ anti-particle of 1

and the form factor recursion relations (iii) + (iv) $\Longrightarrow$

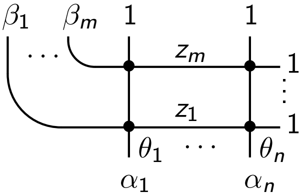
$$\prod_{k=0}^{N-2} \phi\left(\theta + ki\eta\right) \prod_{k=0}^{N-1} F\left(\theta + ki\eta\right) = 1 \ , \qquad \eta = \frac{2\pi}{N}$$

Solution:

$$\boxed{\phi(\theta) = \Gamma\left(\frac{\theta}{2\pi i}\right)\Gamma\left(1 - \frac{1}{N} - \frac{\theta}{2\pi i}\right)}$$

"Bethe ansatz" state

$$\Psi^{\underline{\beta}}_{\underline{\alpha}}(\underline{\theta}, \underline{z}) =$$



$$2 \leq \beta_i \leq N$$
$$1 \leq \alpha_i \leq N$$

The two-point function

$$w(x) = \langle\, 0 \,|\, \mathcal{O}(x)\, \mathcal{O}'(0) \,|\, 0 \,\rangle$$

Summation over all intermediate states

$$w(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\theta_1 \ldots \int d\theta_n e^{-ix(p_1+\cdots+p_n)} g_n(\underline{\theta})$$

where $$g_n(\underline{\theta}) = \frac{1}{(4\pi)^n} \langle\, 0 \,|\, \mathcal{O}(0) \,| p_1, \ldots, p_n \,\rangle^{in\; in}\langle p_n, \ldots, p_1 \,|\, \mathcal{O}'(0) \,|\, 0 \,\rangle$$

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The Log of the two-point function

$$\ln w(x) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d\theta_1 \dots \int d\theta_n e^{-ix(p_1+\cdots+p_n)} h_n(\underline{\theta})$$

Wightman functions

The functions g_n and h_n are obviously related by the cummulant formula

$$g_I = \sum_{I_1 \cup \cdots \cup I_k = I} h_{I_1} \dots h_{I_k}$$

For example

$$\begin{aligned} g_1 &= h_1 \\ g_{12} &= h_{12} + h_1 h_2 \\ g_{123} &= h_{123} + h_{12} h_3 + h_{13} h_2 + h_{23} h_1 + h_1 h_2 h_3 \\ &\dots \end{aligned}$$

Short distance behavior

The two-point Wightman function for short distances

$$\langle 0 \,|\, \mathcal{O}(x)\, \mathcal{O}(0) \,|\, 0 \,\rangle \sim \left(\sqrt{-x^2}\right)^{-4\Delta} \quad \text{for } x \to 0$$

with the 'dimension'

$$\Delta = \frac{1}{8\pi} \sum_{n=1}^{\infty} \frac{1}{n!} \int d\theta_1 \ldots \int d\theta_{n-1}\, h_n(\theta_1, \ldots, \theta_{n-1}, 0)$$

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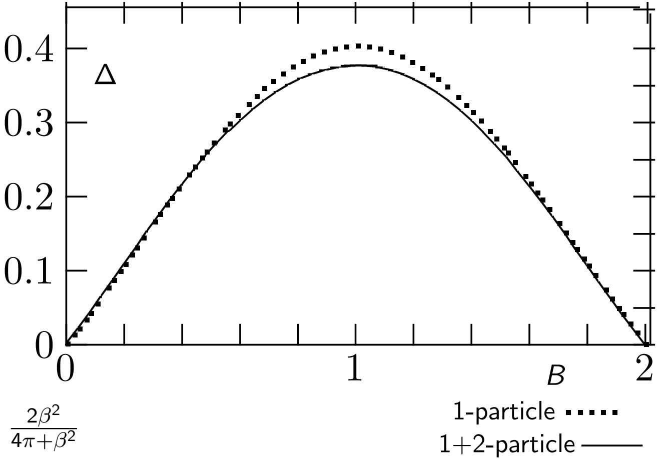
Example: The sinh-Gordon model $\quad \Box\varphi + \frac{\alpha}{\beta} \sinh \beta\varphi = 0$

Operator: $\quad \mathcal{O}(x) = :\exp \beta\varphi:(x)$

Short distance behavior

sinh-Gordon

1- and 1+2-particle intermediate state contributions



where $B = \frac{2\beta^2}{4\pi+\beta^2}$

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