# Twisted Superconducting Strings in Extended Abelian Higgs Models

with Sébastien Reuillon, Mikhail Volkov

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- Extended Abelian Higgs model: introducing several (complex) scalars with a global symmetry acting on the scalars $\rightarrow$ semilocal models
- The case of 2 complex scalars with an SU(2) symmetry: $\rightarrow \sin^2\theta_w \rightarrow 1$ limit of the bosonic sector of the standard electroweak model (decoupling of the SU(2) gauge fields).

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where

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- global SU(2) symmetry acting on the scalars $(\phi_1, \phi_2)$ *and* local U(1) gauge symmetry.
- mass spectrum: a massive vector particle of mass $m_{\rm v} = g\eta$, ($\eta$ is the vev of the scalar field)
  one scalar particle of mass $m_{\rm s} = \sqrt{\beta}\eta$, (i.e. $\sqrt{\beta} = m_{\rm s}/m_{\rm v}$)
  and two Nambu-Goldstone bosons.

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- In the case of two complex scalars the vacuum manifold

$$\mathcal{V} = \{\Phi^\dagger\Phi = 1\} \cong S^3 \;\Rightarrow\; \pi_1(\mathcal{V}) \equiv 0$$

$\longrightarrow$ $\nexists$ topological vortex solutions in the plane

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- Their magnetic field, $B$, does not decrease exponentially as for the ANO vortices: $B \sim |w|^2/r^4$.

# Twisted semilocal vortices.

- Main point: In the case for $\beta > 1$ new vortices/strings exist when one allows for a $z$-dependent relative phase (twist) between the two complex scalar field; $\Rightarrow$ a current is induced flowing along the $z$-direction.

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- For $\mathcal{I}_3 \to 0$ ($\omega \to \omega_{\rm bif}(\beta, n, m)$) the twisted vortices *bifurcate* with the embedded ANO vortex.
- Their energy per unit length is smaller than that of the embedded ANO vortices!

# The general stationary Ansatz.

The most general $z$-translationally symmetric and stationary Ansatz:

$$A_\mu = (A_\alpha(x_1, x_2)\,, A_i(x_1, x_2)), \quad \alpha = 0, 3\,, \; i = 1, 2,$$
$$\phi_1 = f_1(x_1, x_2), \quad \phi_2 = f_2(x_1, x_2)e^{i(\omega_0 t + \omega_3 z)}\,,$$

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- the Ansatz breaks the global SU(2) symmetry to U(1).
- The Noether current corresponding to the remaining U(1) global symmetry:

$$J_\mu = 2i(\bar{\phi}_2 D_\mu \phi_2 - \phi_2 \overline{D_\mu \phi_2})$$

- $\exists$ conserved Noether charge per unit length, $\mathcal{Q}$

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- translational symmetry of the Ansatz $\rightarrow$ conserved momentum, $P$:

$$P = \int d^2x \; T^0{}_z = 2\omega_0\mathcal{I}_3 \,,$$

  and for configurations with rotational symmetry in the plane a conserved angular momentum, $J$:

$$J = \int d^2x \; T^0{}_\varphi \propto \mathcal{I}_0 \,. \tag{1}$$

# Lorentz symmetry.

- Lorentz symmetry of the Ansatz : boosts in the $(t, z)$-plane:

$$
\begin{aligned}
t &= t' \cosh\gamma + z' \sinh\gamma & \qquad z &= z' \cosh\gamma + t' \sinh\gamma \\
A'_0 &= A_0 \cosh\gamma + A_3 \sinh\gamma & \qquad A'_3 &= A_0 \sinh\gamma + A_3 \cosh\gamma \\
\omega'_0 &= \omega_0 \cosh\gamma + \omega_3 \sinh\gamma & \qquad \omega'_3 &= \omega_3 \cosh\gamma + \omega_0 \sinh\gamma
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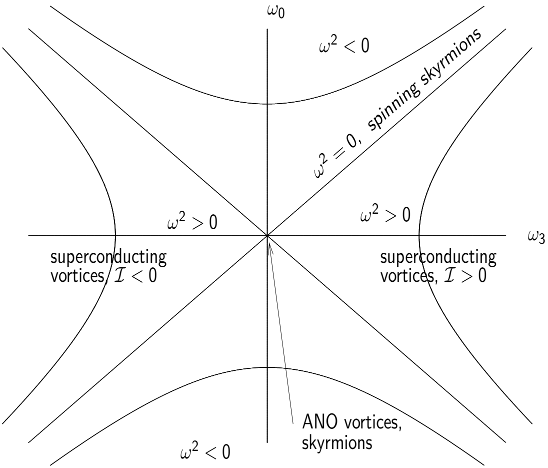
- $\Rightarrow$ only the Lorentz invariant combination $\omega^2 = \omega_3^2 - \omega_0^2$, appears in the eqs. of motion.
- Therefore the space of solutions decomposes into three classes labelled by the possible Lorentz types of the length of $\omega^2$ (Carter):

$$
\omega^2 \begin{cases} = 0 & \text{null or chiral case} \rightarrow \text{ANO, Hindmarsh, Abraham} \\ < 0 & \text{time-like or electric case} \\ > 0 & \text{space-like or magnetic case} \rightarrow \text{new twisted vortices} \end{cases} \tag{2}
$$

# Decomposition of the phase space



If $\omega^2 > 0$ (magnetic case) by a Lorentz boost one can always achieve $\omega_0 = 0\,, A_0 = 0$, i.e. it is sufficient to consider the *static* case.
The two "Gauss-law" eqs. for $A_\alpha = (A_0, A_3)$:

$$\left.\begin{array}{l} \triangle A_0 - 2A_0|\Phi|^2 + 2\omega_0\bar{\phi}_2\phi_2 = 0 \\ \triangle A_3 - 2A_3|\Phi|^2 + 2\omega_3\bar{\phi}_2\phi_2 = 0 \end{array}\right\} \Rightarrow A_0 = \frac{\omega_0}{\omega_3}A_3\,.$$

$\triangle = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$

- We shall consider solutions with cylindrical symmetry: the most general such Ansatz in polar coordinates can be written as:

$$A_0 = \omega_0 a_0(\rho)\,, A_\rho = 0\,, A_\varphi = n a(\rho)\,, A_3 = \omega_3 a_3(\rho)\,,$$
$$\phi_1 = f_1(\rho) e^{in\varphi}, \quad \phi_2 = f_2(\rho) e^{im\varphi} e^{i(\omega_0 t + \omega_3 z)}\,,$$

where the integer $n \in \mathbb{Z}_+$ determines the magnetic flux, $m = 0, \ldots n-1$.

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- Note that the electric potential is given either by

$$A_0 = A_3$$

chiral case ($|\omega_0| = |\omega_3|$), or by

$$A_0 = \omega_0 A_3 / \omega_3$$

magnetic case, i.e. in both cases one can take

$$a_0(\rho) = a_3(\rho)$$

.

- A Bogomoln'y-type rearrangement of the energy yields:

$$E = 2\pi n + (\omega_0^2 + \omega_3^2)Q + \pi(\beta - 1)\int_0^\infty \rho d\rho \left(1 - |f|^2\right)^2 + \dots \quad (3)$$

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$$Q = 2\pi \int_0^\infty \rho d\rho \, (1 - a_3) f_2^2 = 2\pi \int_0^\infty \rho d\rho \, a_3 f_1^2 \,,$$

determines the vortex worldsheet current,

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- the momentum and the angular momentum can be expressed as

$$\begin{aligned} P &= 2\omega_0 \omega_3 Q \,, \\ J &= -2\omega_0 \nu Q \,, \quad \text{where } \nu := n - m = 1, \dots, n \,. \end{aligned}$$

# Field Equations

- the cylindrically symmetric field equations can be written as:

$$\frac{1}{\rho}(\rho a_3')' = 2a_3|f|^2 - 2f_2^2\,, \quad \text{where } ' = d/d\rho\,.$$

$$\rho\left(\frac{a'}{\rho}\right)' = 2f_1^2(a-1) + 2f_2^2(a - \frac{m}{n})\,,$$

$$\frac{1}{\rho}(\rho f_1')' = f_1\left[n^2\frac{(1-a)^2}{\rho^2} + \omega^2 a_3^2 - \beta(1-|f|^2)\right]\,,$$

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- These equations depend only on the Lorentz-invariant combination $\omega^2 = \omega_3^2 - \omega_0^2$, $\to$ any solution determines a whole class, i.e. its Lorentz orbit corresponding to boosts.
- Finite energy implies that $\omega^2 \geq 0$ (space-like or null classes).

# Regularity conditions

- There is a 4-parameter family of local solutions regular at the origin, $\rho = 0$:

$$a = a^{(2)}\rho^2 + O(\rho^{2m+2})\,, \qquad a_3 = a_3^{(0)} + O(\rho^{2m+2})\,,$$

$$f_1 = f_1^{(n)}\rho^n + O(\rho^{n+2})\,, \qquad f_2 = f_2^{(m)}\rho^m + O(\rho^{m+2})\,,$$

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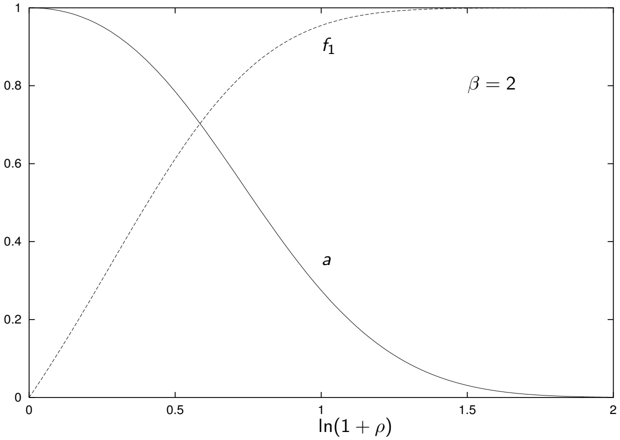
- possible asymptotic behaviours for $\rho \to \infty$ $(\omega > 0)$:

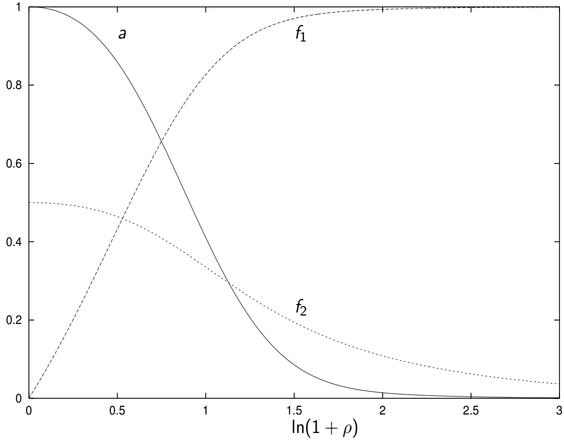
$$a = 1 + A\sqrt{\rho}\, e^{-\sqrt{2}\rho} - D^2 \left[(1 - m/n)/(1 - 2\omega^2)\right] e^{-2\omega\rho}/\rho + \dots,$$
$$a_3 = Be^{-\sqrt{2}\rho}/\sqrt{\rho} + D^2/(1 - 2\omega^2)e^{-2\omega\rho}/\rho + \dots,$$
$$f_1 = 1 + Ce^{-\sqrt{2\beta}\rho}/\sqrt{\rho} - \tilde{D}^2 e^{-2\omega\rho}/\rho + (\tilde{A}^2 + \tilde{B}^2)e^{-2\sqrt{2}\rho}/\rho + \dots,$$
$$f_2 = De^{-\omega\rho}/\sqrt{\rho} + \dots,$$

where $\{a^{(2)}, a_3^{(0)}, f_1^{(n)}, f_2^{(m)}\}$ and $\{A, B, C, D\}$ are free parameters.

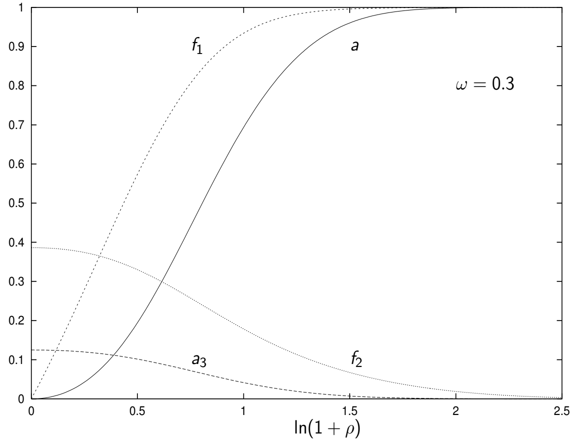
# The profile of the ANO vortex for $\beta = 2$ and $n = 1$.



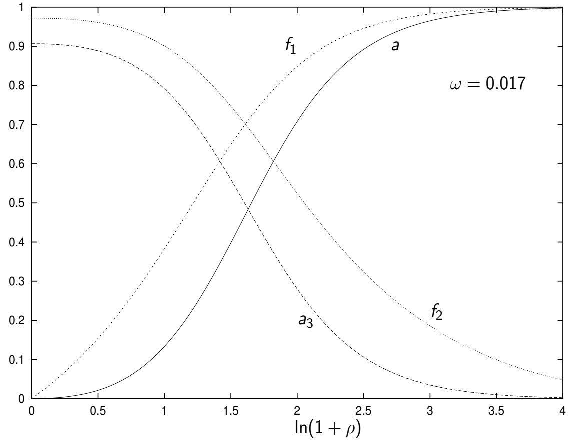
## The profile of a typical member of the $\beta = 1$ family



## Twisted semilocal vortex solutions for $n = 1$, $\beta = 2$



# Twisted semilocal vortex solutions for $n = 1$, $\beta = 2$



# $\beta = \infty$ — $\mathbf{CP}^1$-modell

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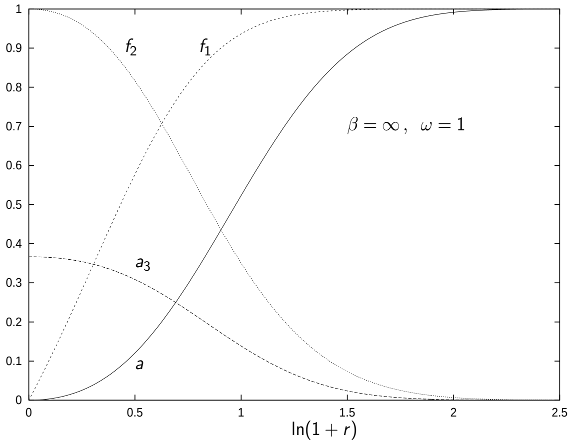
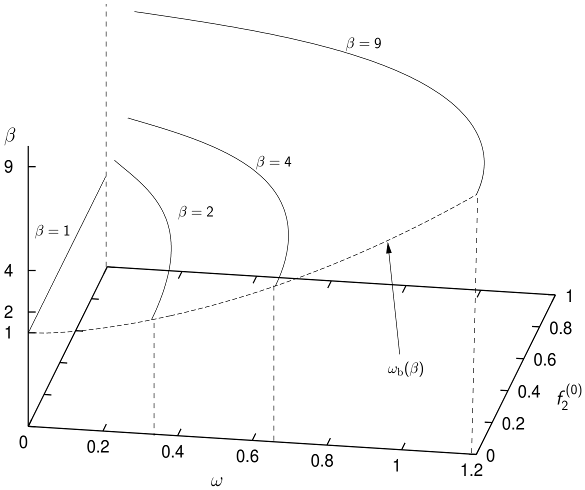
- For $\beta = \infty$ the vortices are completely different from the corresponding ANO ones, whose energy is divergent in this limit.

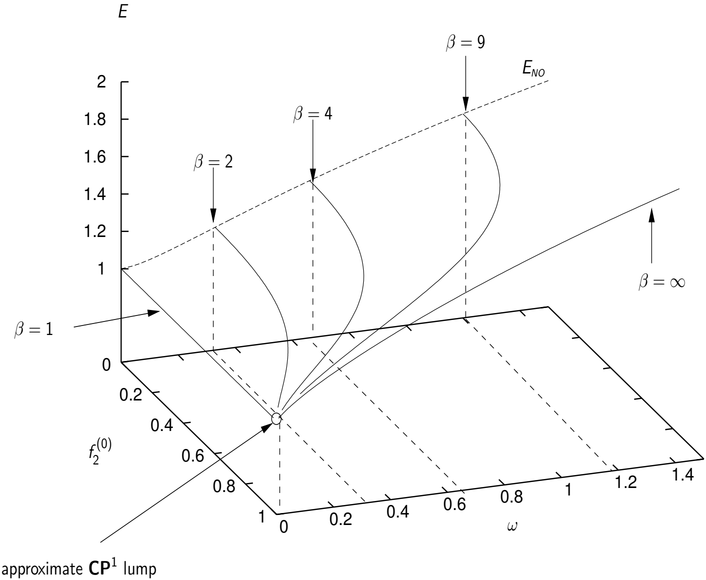
## A superconducting vortex solution for $\beta = \infty, \omega = 1$



# Phase space of the n=1 twisted vortices.



# Energy landscape of the n=1 twisted vortices.



# The current, $\tilde{\mathcal{I}}_3$ as a function of $\omega$ for $\beta = 1.5,\ 2,\ 3$.