Gravitational Waves

Volume 1

Theory and Experiments

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Preface

The physics of gravitational waves is in a very special period. At the time of writing (2007) various gravitational-wave detectors, after decades of developments, have reached a sensitivity where there are significant chances of detection, and future improvements are expected to lead, in a few years, to advanced detectors with even better sensitivities. As a result of these experimental efforts, there are good reasons to hope that the next decade will witness the direct detection of gravitational waves and the opening of the field of gravitational-wave astronomy and, possibly, cosmology. Stimulated by this intense experimental activity, there has also been in parallel a vigorous theoretical effort. We now understand much better many potentially interesting mechanisms for the production of gravitational waves, both in astrophysics and in cosmology, while long-standing conceptual and technical problems, for instance related to the production of gravitational waves by self-gravitating systems (such as coalescing binaries) have been solved. For these reasons, it is now appropriate to attempt a summary of the knowledge that has accumulated over the last few decades.

The theory of gravitational waves is a rich subject that brings together different domains such as general relativity, field theory, astrophysics, and cosmology. The experimental side is as rich, with extraordinary techniques that nowadays allow us to obtain sensitivities that, intuitively, might seem totally out of reach. For instance, one can now monitor the length L of the two arms of an interferometer (with $L \sim$ a few kms), detecting a relative displacement ΔL many orders of magnitude smaller than the size of a nucleus; or one can detect vibrations corresponding to just a few tens of phonons, in a resonant-mass detector which weights several tons. The aim of this book is to bring the reader to the forefront of present-day research, both theoretical and experimental, assuming no previous knowledge of gravitational-wave physics.

Part I of this volume is devoted to the theory of gravitational waves (GWs). Here we assume an elementary knowledge of general relativity. Typically, we recall the most important notions when we use them; nevertheless, it should be borne in mind that this is *not* a textbook on general relativity. In some sections, we also require some knowledge of field theory; in some cases, e.g. for the Noether theorem, we recall them in some detail.

We have attempted to rederive afresh and in a coherent way all the results that we present, trying to clarify or streamline the existing derivations whenever possible. Throughout this book, we try to go into sufficient detail, and we do our best to avoid standard sentences like "it can be shown that..." or, even worse, "it is easy to show that...", unless what is left to the reader is really only straightforward algebra.[1] In order not to burden the main text too much with details, some more technical issues are collected into a "Solved problems" section at the end of some chapters, where we present the relevant calculations in all details.

[1] The exception to this rule will be Chapter 5, on the post-Newtonian generation of GWs. Here the computations are so long that they sometimes required years of work by highly specialized teams. In this case, we explain in detail the basic principles, we perform explicitly some lowest-order computation, and then we quote the final high-order results.

The theory of gravitational waves is a domain where two different traditions meet: one more geometrical, where one uses the language of general relativity, and one more field-theoretical, where one uses the language of classical and even quantum field theory. This is due to the fact that, at the fundamental level, linearized gravity is just the field theory of a massless particle, the graviton. At this level, the most appropriate language is that of field theory. However, at the macroscopic level the collective excitations of the gravitational field are described in terms of a metric, and here the geometric language of general relativity becomes the most appropriate. Between these two description there is no real conceptual tension and, in fact, they complement each other very well. The field-theoretical point of view often gives a better understanding of some issues of principle; for example, the problem of what is the energy carried by GWs, which in the past has been surrounded by some confusion, can be answered using the Noether theorem, a typical tool of classical field theory, and is further illuminated looking at it from the point of view of quantum field theory. On the other hand, for example, the interaction of GWs with detectors is much more easily understood using the geometric language of general relativity, making use of tools such as the equation of the geodesic deviation. We will therefore make use of both languages, depending on the situation, and we will often try to discuss the most important conceptual problems from both vantage points.

Part II of this volume is devoted to a description of experimental GW physics. We discuss in great detail both resonant-mass detectors and interferometric detectors. The former belong to "small-scale" science, with experimental groups sometimes as small as half a dozen people, and limited needs for funding. They have been important historically for the development of the field, and they are remarkable instruments by themselves, with their ability to detect variation in the length L of a bar at the level $\Delta L/L \sim 10^{-18}$ or better, corresponding, for a bar of length $L = 3$ m, to about 10^{-3} fm. Interferometers rather belong to "Big Science", with collaborations of hundreds of people, and costs of several hundreds millions of euros. At their present sensitivity, interferometers are by now the main actors on the experimental scene, and give us our best chances for detection, while advanced interferometers, planned for the near future, have an extraordinary potential for discoveries. We will also devote a chapter to data analysis for GWs, which is quite a crucial issue. This is also a domain where the interaction between theorists and experimentalists has been very fruitful, since in many instances (in particular for coalescing binary systems) the theoretical predictions of

the waveform are crucial for extracting a real GW signal from a noisy detector.

A second volume, dedicated to astrophysical and cosmological sources, is currently in preparation, and I expect to complete it in a few more years. The logic underlying this division is that Vol. 1 presents the tools, theoretical and experimental, of GW physics, while Vol. 2 will describe what can we learn about Nature, in astrophysics and in cosmology, using these tools. An Errrata web page will be maintained at http://theory.physics.unige.ch/~maggiore/home.html

Finally, a comment about the bibliography. Relevant papers are quoted (and sometimes commented on) in a Further Reading section at the end of each chapter. The principle that guided me in choosing them is *not* historical accuracy. Considering that I am summarizing developments that took place along many decades, at least from the 1960s, it is beyond my competence to give a detailed account of who did what, and who did it first. Rather, the papers that I quote are the ones that I consider interesting reading today, and which I recommend for learning more about the subject. A number of these references will however provide the reader with a more accurate guide through the historical development of the field.

Acknowledgments

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Geneva, June 2007

Notation

Constants and units. The Planck constant $\hbar$ and the speed of light c are normally written explicitly. Occasionally there are sections where we use units $\hbar = c = 1$, but in this case the use of these units is always stated at the beginning of the section. The Newton constant G is always written explicitly, and we never set $G = 1$. The solar mass is denoted by $M_{\odot}$.

Indices, metric signature, four-vectors. Greek indices, such as $\alpha, \beta, \ldots$ or $\mu, \nu, \ldots$, take the values $0, \ldots, 3$, while spatial indices are denoted by Latin letters, $i, j, \ldots = 1, 2, 3$. The totally antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ has $\epsilon^{0123} = +1$. The flat space metric is

$$\eta_{\mu\nu} = (-, +, +, +) \, .$$

This is nowadays the most common choice in general relativity, while the opposite signature is the most common choice in quantum field theory and particle physics. We also define

$$x^{\mu} = (x^0, \mathbf{x}), \qquad x^0 = ct \, ,$$
$$\partial_{\mu} = \frac{\partial}{\partial x^{\mu}} = \left(\frac{1}{c}\,\partial_t, \partial_i\right) \, ,$$
$$d^4x = dx^0 d^3x = c\, dt d^3x \, .$$

A dot denotes the time derivative, so $\dot{f}(t) = \partial_t f = c\partial_0 f$. Contrary to widespread use in the literature on general relativity, we never use commas to denote derivatives (nor semicolons to denote covariant derivatives).

The four-momentum is $p^{\mu} = (E/c, \mathbf{p})$, so $p_{\mu}x^{\mu} = -Et + \mathbf{p}{\cdot}\mathbf{x}$, and $d^4p = (1/c)dEd^3p$. Repeated upper and lower indices are summed over. When we have only spatial indices we do not need to be careful about raising and lowering of the indices since, with our choice of signature, the spatial metric is δ_{ij}. Then we will also sum over repeated lower spatial indices or over repeated upper spatial indices.

In Section 3.5.1 and in Chapter 5, where we study the multipole expansion to all orders, we use a multi-index notation where a tensor with l indices $i_1 i_2 \ldots i_l$ is labeled simply using a capital letter L, so $F_L \equiv F_{i_1 i_2 \ldots i_l}$. Various conventions related to this notation are explained on page 134. There, we also used the notation $f^{(n)}(u) \equiv d^n f/du^n$ to denote the n-th derivative with respect to retarded time.

Riemann and Ricci tensor, Einstein equations. Our conventions on the metric signature, Riemann tensor, etc. are the same as Misner,

Thorne and Wheeler (1973). We denote the curved space-time metric by $g_{\mu\nu}(x)$ and its determinant by g (so $g < 0$). The Christoffel symbol is

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_{\mu} g_{\sigma\nu} + \partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu}) \, .$$

The Riemann tensor is defined as

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\Gamma^{\mu}_{\nu\sigma} - \partial_{\sigma}\Gamma^{\mu}_{\nu\rho} + \Gamma^{\mu}_{\alpha\rho}\Gamma^{\alpha}_{\nu\sigma} - \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\nu\rho} \, .$$

The Ricci tensor is $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$, and the Ricci scalar is $R = g^{\mu\nu}R_{\mu\nu}$. The energy–momentum tensor $T^{\mu\nu}$ is defined from the variation of the matter action S_M under a change of the metric $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$, according to

$$\delta S_M = \frac{1}{2c} \int d^4x \sqrt{-g}\, T^{\mu\nu} \delta g_{\mu\nu} \, .$$

The Einstein equations read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \, .$$

Fourier transform. Our conventions on the n-dimensional Fourier transform are

$$F(x) = \int \frac{d^n k}{(2\pi)^n} \tilde{F}(k) e^{ikx} \, ,$$
$$\tilde{F}(k) = \int d^n x \, F(x) e^{-ikx} \, ,$$

With our choice of metric signature this implies that, for a function of time

$$F(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{F}(\omega)\, e^{-i\omega t} \, ,$$
$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} dt \, F(t)\, e^{+i\omega t} \, .$$

The Dirac delta satisfies

$$\int d^n x \, e^{ikx} = (2\pi)^n \delta^{(n)}(k) \, ,$$

so in particular

$$\int dt \; e^{i 2\pi f t} = \delta(f) \, .$$