Integrálhatóság az AdS/CFT-ben

Bajnok Zoltán: AdS/CFT, mint integrálható modell,YM oldal

Jevicki Antal: Húrelmélet $AdS_5 \times S^5$-on

Hegedűs Árpád: Bethe Ansatz AdS/CFT-re

Bevezetés az AdS/CFT-be

SYM: elemi terek, hatás, szimmetria, anomális dimenziók

Koszet hatás, szimmetriák, töltések

Redukció: kapcsolat a sine-Gordon modellel

Planáris limesz: spin lánc megfeleltetés, integrálhatóság

Megmaradó szimmetriák: S-mátrix

Konkrét megoldások: forgó es pörgő húr

BA: $su(2)$ szektor

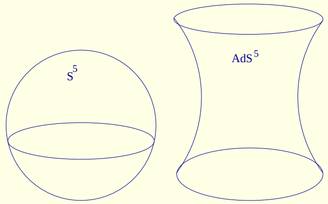
BA: általános $psu(2|2) \otimes psu(2|2)$ eset

S-mátrix húrelméletből

Végesméret effektusok

09.03.14^{00} V. Kazakov: TBA: from usual to string sigma models

CFT, mint integrálható modell

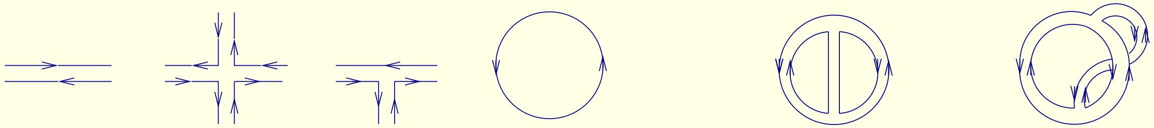
Az AdS/CFT szótár 

IIB $\leftrightarrow$ D=4 SYM

Húrelmélet AdS-en bozonikus rész

N=4 SUSY YM: elemi terek $A_\mu, \Psi_i, \overline{\Psi}_i, \Phi_a$, hatás

Szimmetria: Szuperkonform nem anomális $\leftrightarrow$ QCD

Planáris limesz 

Anomális dimenziók, $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{1}{x^{2\Delta}}$ Konishi $\mathcal{O} = \mathrm{Tr}\Phi^2$

AdS/CFT megint, kibúvó az erős-gyenge csatolás alól

AdS/CFT megfeleltetés: szótár

II_B szuperhúr $AdS_5 \times S^5$-ön	$\equiv$	$\mathcal{N}=4$ $SU(N)$ Szuper Yang-Mills	Maldacena
$\sqrt{\lambda} = \frac{R^2}{\alpha'}$ effektív húrfeszültség	$\equiv$	$\lambda = g_{YM}^2 N$ t'Hooft csatolás	

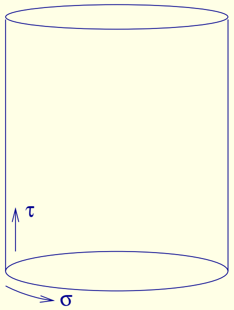
Erős-gyenge csatolás

$g_s = \frac{\lambda}{N}$ húrcsatolás	$\equiv$	N színek száma

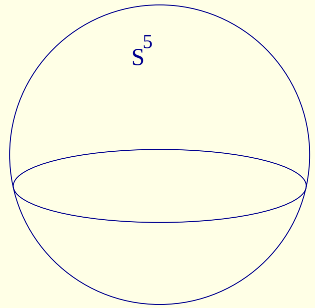
$N \to \infty$ Planáris limesz

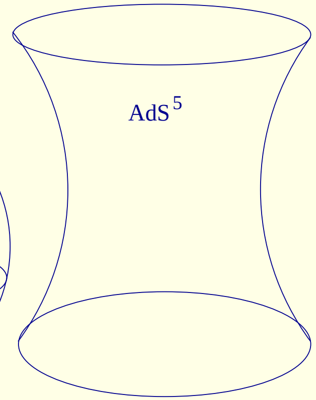
Húrspektrum $E(\lambda)$	$\equiv$	$\Delta(\lambda)$ anomális dimenziók spektruma	(exp)

AdS/CFT: húrelmélet



Húr világlepedő $(\sigma,\tau) \longrightarrow$targettér $X, Y(\sigma,\tau)$





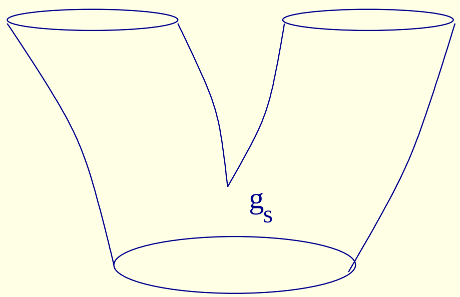
$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$

$$\boxed{S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + fermionok}$$

$S^5 \times AdS_5 = \frac{SO(6)}{SO(5)} \times \frac{SO(2,4)}{SO(1,4)}$ Szuper változat: $Szuper[S^5 \times AdS_5] = \frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

Húrcsatolás $g_s \to 0$ csak húrkvantummechanika



Keressük: Spektrum $E(\lambda, g_s) =?$ $\sqrt{\lambda} = \frac{R^2}{\alpha'}$

$\mathcal{N}=4$ SU(N) szuper Yang-Mills: hatás

Minden tér az adjungált ábrázolásban: $N \times N$ spúrtalan matrixok

Mértéktér A_0, A_1, A_2, A_3 mátrix alak A_μ^{ij} szf: 2

Fermionok $\Psi_1^\alpha, \Psi_2^\alpha, \Psi_3^\alpha, \Psi_4^\alpha, \Psi_1^{\dot\alpha}, \Psi_2^{\dot\alpha}, \Psi_3^{\dot\alpha}, \Psi_4^{\dot\alpha}$ $\alpha = 1,2,\ \dot\alpha = 3,4$ szf: 8

Skalárok $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6$ mátrix alak Φ_a^{ij} szf: 6

Térerősség: $D_\mu = \partial_\mu - i[A_\mu, .]$ $F_{\mu\nu} = i[D_\mu, D_\nu]$

Hatás $\boxed{S = \frac{2}{g_{YM}} \int d^4x \mathrm{Tr} \left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi] \right]}$

Szimmetriák rögzítik: $\mathcal{N}=4$ SUSY $\supset$Poincare, R-szimmetria

$\mathcal{N} = 4$ SU(N) szuper Yang-Mills: szimmetriák

Lorentz szimmetria: $su(2) \times su(2)$ 3 forgatás J_i, 3 boost, B_i

Poincare: 4 eltolás is P_μ

R-szimmetria: $SO(6)$ a bozonokon $SU(4)$ fermionokon

Nincs skála: $\mathcal{D}$ dilatáció $[\mathcal{D}, \mathcal{O}] = \Delta_{\mathcal{O}} \mathcal{O}$ megmarad

$\Delta_F = 2$, $\Delta_\Phi = 1$, $\Delta_{\overline{\Psi}} = \Delta_{\overline{\Psi}} = \frac{3}{2}$, $\Delta_D = 1$,

Speciális konform transzformációk K_μ(inverzió,eltolás, inverzió)

Konform szimmetria $SO(4,2) = SU(2,2)$ (konform családok)

Szupertranszformációk nemlineárisan ábrázolódnak (on-shell)

$$\begin{pmatrix} SU(2,2) & Q, \bar{S} \\ \bar{Q}, S & SU(4) \end{pmatrix} = \begin{pmatrix} B^{\alpha\beta} & P^{\alpha\dot{\beta}} & Q_a^\alpha & \bar{S}_{\dot{a}}^\alpha \\ K & J & Q & \bar{S} \\ \bar{Q} & S & R & R \\ \bar{Q} & S & R & R \end{pmatrix} = PSU(2,2|4)$$

s Δ $\bar{s}$ q_1 p q_2

Szimmetriák nem anomálisak: Beta függvény egzaktul eltűnik

Csatolás nem fut, λ fizikai paraméter

Minden tér tömegtelen, nincs bezárás

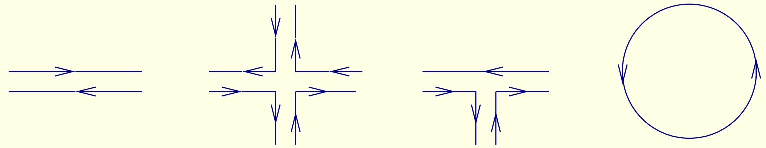
$\mathcal{N} = 4$ SU(N) szuper Yang-Mills: korrelációs függvények

Mértékinvarians operátorok: tracek $\mathcal{O} = \mathrm{Tr}\left[\Psi F \Phi\Phi \ldots D^k \Phi\right]$ determinánsok ...

Primérek $[K_\mu, \mathcal{O}_n] = 0$ két pontfüggvényei: $\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$

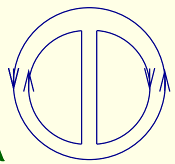
dimenzió $\Delta(\lambda) = \Delta_0 + \lambda\Delta_1 + \ldots$

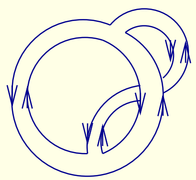
Perturbációszámítás, gráfszabályok



g_{YM}^2 $\quad g_{YM}^{-2}$ $\quad g_{YM}^{-2}$ $\quad N$

Planáris diagramok vezetnek $g_{YM}^2 N^3 = N^2\lambda$





$\lambda = g_{YM}^2 N$

particiós függvény $Z(\lambda, \frac{1}{N}) = N^2 \sum_{g=0}^{\infty} (\frac{1}{N})^{2g} \sum_{n=0}^{\infty} \alpha(g,n)\lambda^n$

t'Hooft: Feynmann gráfok $\leftrightarrow$világlepedő triangulációja

$\mathcal{N}=4$ SU(N) szuper Yang-Mills: Konishi

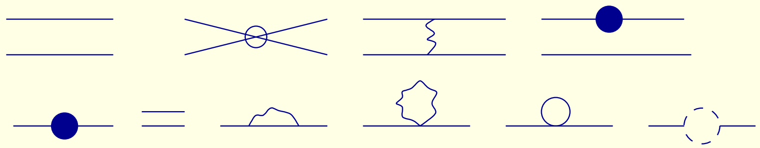
Konishi: $\mathcal{O}_1(x) = \mathrm{Tr}\left(\Phi_1(x)\Phi_1(x)\right)$ legegyszerűbb primér $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i(\lambda)}}$

Anomális dimenzió: $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \langle \mathrm{Tr}\left(\Phi_i(x)\Phi_i(x)\right)\mathrm{Tr}\left(\Phi_j(0)\Phi_j(0)\right)e^{-S_I}\rangle_0$

Perturbatív számolás: $\langle \Phi_i^a(x)\Phi_j^b(0)\rangle_0 = \frac{g_{YM}^2}{8\pi^2}\frac{\delta^{ab}\delta_{ij}}{x^2}$ $\qquad \Phi_i = \Phi_i^a t_a$

Fagráf $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = (\frac{g_{YM}^2}{8\pi^2})^2\frac{\delta^{ab}\delta^{ba}\delta_{ij}\delta_{ji}}{x^4}2 = (\frac{\lambda}{8\pi^2})^2\frac{12}{x^4} \quad \Delta_0 = 2$

Egy hurok $d = 4-2\epsilon$ dimregben:



$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = (\frac{\lambda}{8\pi^2})^2\frac{12}{x^4}\left[1-\frac{3\lambda}{4\pi^2}(\frac{1}{\epsilon}+\gamma-\log\frac{1}{x^2})\right]$$

Hullámfüggvény renormálás $\mathcal{O}^{ren} = Z_{\mathcal{O}}\mathcal{O} = (1 + \frac{3\lambda}{8\pi^2}(\frac{1}{\epsilon} + \gamma))\mathcal{O}$

Anomális dim$\langle \mathcal{O}_i^{ren}(x)\mathcal{O}_j^{ren}(0)\rangle = (\frac{\lambda}{8\pi^2})^2 \frac{12}{x^4}\left[1 + \frac{3\lambda}{4\pi^2}\log\frac{1}{x^2}\right] = (\frac{\lambda}{8\pi^2})^2\frac{12}{x^4}\left[\frac{1}{x^{2\frac{3\lambda}{4\pi^2}}}\right]$

$\Delta(\lambda) = 2 + 2\lambda\frac{3}{4\pi^2}$

Magasabb rendekben:

$E(\lambda) = 2+6\frac{\lambda}{4\pi^2}-24(\frac{\lambda}{4\pi^2})^2+168(\frac{\lambda}{4\pi^2})^3-(1410+144\zeta(3)+\Delta E/2)(\frac{\lambda}{4\pi^2})^4+\dots$

$\Delta E_{wrapping} = 324 + 864\zeta(3) - 1440\zeta(5)$

AdS/CFT megfeleltetés

II_B szuperhúr $AdS_5 \times S^5$-ön	$\equiv$	$\mathcal{N}=4$ $SU(N)$ Szuper Yang-Mills
$\sqrt{\lambda}=\frac{R^2}{\alpha'}$ effektív húrfeszültség	$\equiv$	$\lambda = g_{YM}^2 N$ t'Hooft csatolás
$g_s = \frac{\lambda}{N}$ húrcsatolás	$\equiv$	N színek száma
Húrspektrum $E(\lambda)$	$\equiv$	$\Delta(\lambda)$ anomális dimenziók spektruma

Erős-gyenge csatolás

$$E(\lambda)=E(\infty)+\frac{E_1}{\sqrt{\lambda}}+\frac{E_2}{\lambda}+\ldots \leftrightarrow \Delta(\lambda)=\Delta(0)+\lambda\Delta_1+\lambda^2\Delta_2+\ldots$$

Szemiklasszikus limesz nagy R-töltés J or $\mathcal{J}=\frac{J}{\sqrt{\lambda}}$ BMN-FT skálázás

$$E(J,\lambda)=J\left[\overbrace{1+\frac{1}{\mathcal{J}^2}\left(c_0^1+\frac{c_1^1}{\mathcal{J}\sqrt{\lambda}}+\ldots\right)}^{klasszikus}+\overbrace{\frac{1}{\mathcal{J}^4}(c_0^2+\frac{c_1^2}{\mathcal{J}\sqrt{\lambda}}+\ldots)}^{kvantum}\right]$$

$$\Delta(J,\lambda)=J\left[1+\frac{\lambda}{J^2}\left(a_0^1+\frac{a_1^1}{J}+\ldots\right)+\frac{\lambda^2}{J^4}(a_0^2+\frac{c_1^2}{J}+\ldots)\right]$$

Konform osztályozás

Lokális terek: $F, \Psi, \Phi, \overline{\Psi}, \overline{F}$, és D^k végtelen torony (de hol a teteje)

$\mathcal{D}$ skáladimenziót növeli: $[\mathcal{D}, P] = P$, $[\mathcal{D}, Q] = \frac{1}{2}Q$, $[\mathcal{D}, \bar{Q}] = \frac{1}{2}\bar{Q}$

$\mathcal{D}$ skáladimenziót csökkenti: $[\mathcal{D}, K] = K$, $[\mathcal{D}, S] = -\frac{1}{2}S$, $[\mathcal{D}, \bar{S}] = -\frac{1}{2}\bar{S}$

báziscsere: $\begin{pmatrix} J & P & Q & \bar{S} \\ K & B & Q & \bar{S} \\ \bar{Q} & S & R & R \\ \bar{Q} & S & R & R \end{pmatrix} \to \begin{pmatrix} J & Q & Q & P \\ S & R & R & \bar{Q} \\ S & R & R & \bar{Q} \\ K & \bar{S} & \bar{S} & B \end{pmatrix}$ nincs negatív dimenzió

primér X: $[S, X] = 0$, $[\bar{S}, X] = 0$ királis primér $[Q, Y] = 0$

$Z_1 = \Phi_1 + i\Phi_2$ BPS $[Q, Z_1] = 0 = [\bar{Q}, Z_1]$ a $Q, \bar{Q}$-k felére

$\{Q, S\} = R + L + \mathcal{D}$ $\to$nincs anomális dimenzió: $\mathcal{O} = \mathrm{Tr}\left[Z_1^J\right]$ $\Delta_{\mathcal{O}}(\lambda) = J$ (család)

Maradék szimmetria $Psu(2,2|4) \to Psu(2|2) \otimes Psu(2|2)$

Spinlánc: SU(2) szektor 1-hurok, bázis

Csere $Z_1 = \Phi_1 + i\Phi_2 \to Z_2 = \Phi_3 + i\Phi_4$ operátorok száma 2^J

$\mathcal{O} = \mathrm{Tr}\left[Z_1^{J-1} Z_2\right]$ $\mathcal{O} = \mathrm{Tr}\left[Z_1^{J-2} Z_2 Z_1\right]$ $\mathcal{O} = \mathrm{Tr}\left[Z_2 Z_1^{J-1}\right]$

$\Delta_{\mathcal{O}}(\lambda) = J + \gamma_{\mathcal{O}}(\lambda)$

$\mathcal{O}_1 = \mathrm{Tr}\left[Z_1^{J-2} Z_2^2\right]$ $\mathcal{O}_2 = \left[Z_1^{J-2} Z_2 Z_1 Z_2\right]$ $\mathcal{O} = \mathrm{Tr}\left[Z_2^2 Z_1^{J-2}\right]$

Anomális dimenzió $\langle \mathcal{O}_n^{ren}(x) \mathcal{O}_m^{ren}(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta n(\lambda)}}$

perturbatív számolása: $\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \langle \mathcal{O}_i(x) \mathcal{O}_j(0) e^{-S_I} \rangle_0$

Operátor keveredés 1-hurok szinten $\langle \mathcal{O}_1(x) \mathcal{O}_2(0) S_I \rangle_0 \neq 0$

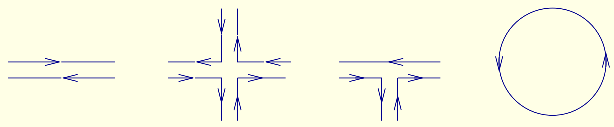
Keveredési mátrixot diagonizálni$\to$integrálható spinlánc

Spinlánc:SU(2) szektor 1-hurok, keveredés

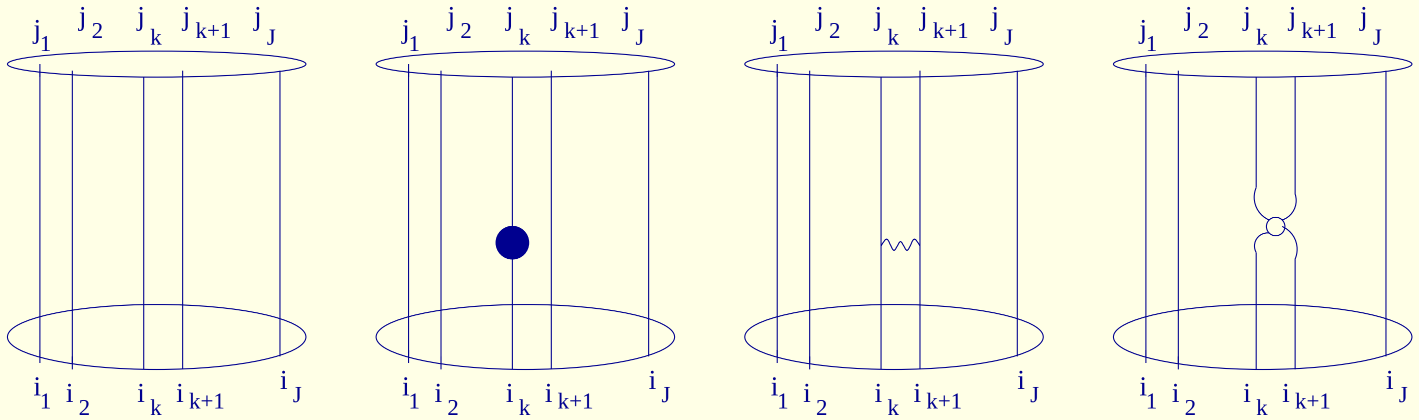
Bázis: $Z_i, \quad i = 1,2 \qquad \mathcal{O}_{i_1 \dots i_J} = \mathrm{Tr}\left[Z_{i_1} \dots Z_{i_J}\right]$

Fagráf: $\langle \mathcal{O}_{i_1 \dots i_J}(x) \mathcal{O}_{j_1 \dots j_J}(0) \rangle_0 = (\frac{g_{YM}^2}{8\pi^2 x^2})^J N^J (\delta_{j_1}^{i_1} \delta_{j_2}^{i_2} \dots \delta_{j_J}^{i_J} + ciklikus\, perm.)$
$\Delta_0 = J$

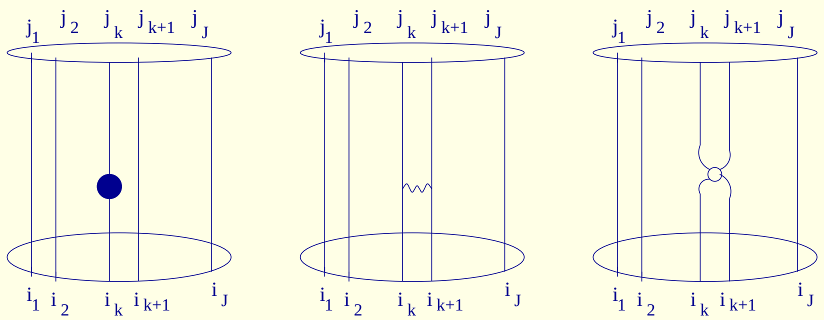
1-hurok perturbáció $-\langle \mathcal{O}_{i_1 \dots i_J}(x) \mathcal{O}_{j_1 \dots j_J}(0) S_I \rangle_0$



Feynman gauge: $\langle \Phi_i^a(x) \Phi_j^b(0) \rangle_0 = \frac{g_{YM}^2}{8\pi^2} \frac{\delta^{ab} \delta_{ij}}{x^2} \; \langle A_\mu^a(x) A_\nu^b(0) \rangle_0 = \frac{g_{YM}^2}{8\pi^2} \frac{\delta^{ab} \delta_{\mu\nu}}{x^2}$



Spinlánc:SU(2) szektor 1-hurok, Heisenberg lánc



1-hurok renormálás után: $\langle \mathcal{O}_{i_1 \ldots i_J}(x) \mathcal{O}_{j_1 \ldots j_J}(0) \rangle_0 =$

$$\left[1 + \frac{\lambda}{8\pi^2} \log x^{-2} \sum_{k=1}^{J} (1 - P_{k,k+1})\right] \mathcal{M}_{tree} (\delta_{j_1}^{i_1} \delta_{j_2}^{i_2} \ldots \delta_{j_J}^{i_J} + cycl.)$$

Diagonalizálandó keveredési matrix: $\boxed{\frac{\lambda}{8\pi^2} \sum_{k=1}^{J} (1 - P_{k,k+1})}$

$$P_{k,k+1} : \delta_{j_1}^{i_1} \ldots \delta_{j_k}^{i_k} \delta_{j_{k+1}}^{i_{k+1}} \ldots \delta_{j_J}^{i_J} \rightarrow \delta_{j_1}^{i_1} \ldots \delta_{j_k}^{i_{k+1}} \delta_{j_{k+1}}^{i_k} \ldots \delta_{j_J}^{i_J}$$

Heisenberg spinlánc $\sum_k (1 - \vec{\sigma}_k \cdot \vec{\sigma}_{k+1})$ $\qquad Z_1 = \uparrow Z_2 = \downarrow$

SO(6) szektor $Z_3 = (\Phi_5 + i\Phi_6)$ $\qquad$ planáris $\rightarrow$ integrálható

Spinlánc: általános eset

Alapállapot: $\mathrm{Tr}\left[Z^J\right] = \mathrm{Tr}\,[ZZZZZ\dots ZZZZ]$ J nagy (BMN) $Z = Z_1$

Kis gerjesztések: síkhullám $\sum_{i=1}^{J} e^{ipn}\mathrm{Tr}\left[\overbrace{ZZZZZ}^{n}XZ\dots ZZZZ\right]$

$X = Z_2, Z_3, \Psi_a^{\alpha}, \Psi_a^{\dot\alpha}, D_\mu, F_{\mu\nu} = [D_\mu D_\nu]$ multiplett $psu(2,2|4)$-re

Szórás állapotok: $\sum_{i_1 i_2 a_1 a_2} e^{ip_1 n_1 + ip_2 n_2}\mathrm{Tr}\left[\overbrace{\underbrace{ZZZZZ}_{n_1}X_{a_1}ZZZ}^{n_2}X_{a_2}Z\dots ZZZZ\right] + S(12)$

S-mátrix szimmetriái: BPS alap $psu(2,2|4) \to psu(2|2) \otimes psu(2|2)$

$$\mathrm{Tr}\,[ZZZZZ\dots ZZZZ]\ \text{BPS}\ \begin{pmatrix} J & Q & Q & P \\ S & R & R & \bar{Q} \\ S & R & R & \bar{Q} \\ K & \bar{S} & \bar{S} & B \end{pmatrix} \to \begin{pmatrix} J & Q \\ S & R \end{pmatrix} \otimes \begin{pmatrix} R & \bar{Q} \\ \bar{S} & B \end{pmatrix}$$

Scatterings for AdS/CFT: $Psu(2,2|4) \overset{Tr(ZZZZ)}{\longrightarrow} su(2|2)^2 \supset su(2) \otimes su(2)$

$$[J_a^b, G_c] = \delta_c^b G_a - \frac{1}{2}\delta_a^b G_c \qquad [J_a^b, G^c] = -\delta_a^c G^b + \frac{1}{2}\delta_a^b G^c$$

$$[R_\alpha^\beta, G_\gamma] = \delta_\gamma^\beta G_\alpha - \frac{1}{2}\delta_\alpha^\beta G_\gamma \qquad [R_\alpha^\beta, G^\gamma] = -\delta_\alpha^\gamma G^\beta + \frac{1}{2}\delta_\alpha^\beta G^\gamma$$

$$\{Q_\alpha^a, Q_\beta^b\} = \epsilon_{\alpha\beta}\epsilon^{ab} C \qquad \{S_a^\alpha, S_b^\beta\} = \epsilon_{ab}\epsilon^{\alpha\beta} C^+$$

$$\{Q_\alpha^a, S_b^\beta\} = \delta_b^a R_\alpha^\beta + \delta_\alpha^\beta J_b^a + \frac{1}{2}\delta_b^a \delta_\alpha^\beta H \qquad C = i\frac{\sqrt{\lambda}}{2\pi}(e^{iP} - 1)e^{2i\xi}$$

Fundamental representation $\mathcal{V}_\xi^1 = \frac{1}{2} \otimes 0 + 0 \otimes \frac{1}{2} = (\phi^a, \psi^\alpha),\ a = 1,2;\ \alpha = 3,4$

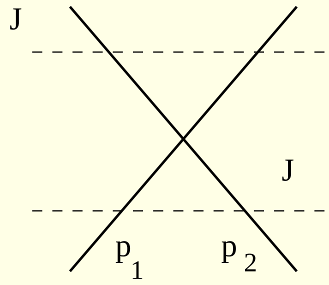
BPS rövidülési feltétel $\longrightarrow$Diszperziós reláció: $\boxed{H(p)^2 = 1 + \frac{4\lambda}{\pi^2}\sin^2\frac{p}{2}}$

Erős csatolás limesz: $\lambda \to \infty \qquad H(p) \to \frac{4\lambda}{\pi^2}\sin^2\frac{p}{2}$

Elemi terek: $\Phi_a = \phi_a \phi_{\dot{a}}, D_\mu = \psi_\alpha \psi_{\dot{\alpha}}, \Psi_{\dot{a}}^\alpha = \phi_{\dot{a}}\psi_\alpha, \bar{\Psi}_a^{\dot{\alpha}} = \phi_a \psi_{\dot{\alpha}}$

S-mátrix

S-mátrix a szimmetriából: tenzorszorzat irreducibilis $\mathcal{V}^1 \otimes \mathcal{V}^1 = \mathcal{W}^2$



$$S(p_1,p_2)\left[G(p_1,e^{ip_2})+G(p_2,1)\right]=$$

$$\left[S(p_1,1)+G(p_2,e^{ip_1})\right]S(p_1,p_2)$$

YB: $S(p_1,p_2)S(p_1,p_2)S(p_2,p_3) = S(p_1,p_2)S(p_1,p_2)S(p_2,p_3)$

Unitaritás: $S(p_1,p_2)S(p_2,p_1) = 1$

Következmény: Integrálható spin-lánc, koordinátatér Bethe Ansatz

Skalár faktor?

S-mátrix expliciten

használjunk $psu(2|2) \supset su(2) \otimes su(2)$ invariánsokat: $\Lambda^i_j = v^i D_j$

$(\frac{1}{2} \otimes 0 + 0 \otimes \frac{1}{2}) \quad (\frac{1}{2} \otimes 0 + 0 \otimes \frac{1}{2}) = 1 \otimes 0 + 2\, 0 \otimes 0 + 2\, \frac{1}{2} \otimes \frac{1}{2} + 0 \otimes 1$

$= a^1_1 \Lambda^1_1 + \sum_{i,j=2}^3 a^j_i \Lambda^i_j + + \sum_{i,j=4}^5 a^j_i \Lambda^i_j + a^6_6 \Lambda^6_6$

$$v_1 D^1 = \frac{1}{2}(w^1_a w^2_{a_1} + w^2_a w^1_{a_1}) \frac{\partial^2}{\partial w^1_a \partial w^2_{a_1}}$$

$a^1_1 = 1$, $a^2_2 = 2\frac{(x^+_2 - x^-_1)}{(x^+_1 - x^-_2)} \frac{(x^-_1 x^+_2 - 1)}{(x^-_1 x^-_2 - 1)} - 1$,stb. DE $S(p_1, p_2) \neq S(f(p_1) - f(p_2))$

ahol $\frac{x^+}{x^-} = e^{ip}$, $x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g} \qquad g^2 = \frac{\lambda}{4\pi}$

Teljes $su(2|2) \otimes su(2|2)$ S-mátrix $S_0(p_1, p_2) S(p_1, p_2) \otimes \dot{S}(p_1, p_2)$

Skalár faktor $S_0(p_1, p_2) = \frac{u_1 - u_2 + \frac{i}{g}}{u_1 - u_2 - \frac{i}{g}} \sigma(p_1, p_2)$ ahol $u = \frac{1}{2}(x^+ + \frac{1}{x^+} + x^- + \frac{1}{x^-})$

Spinlánc: megjegyzések

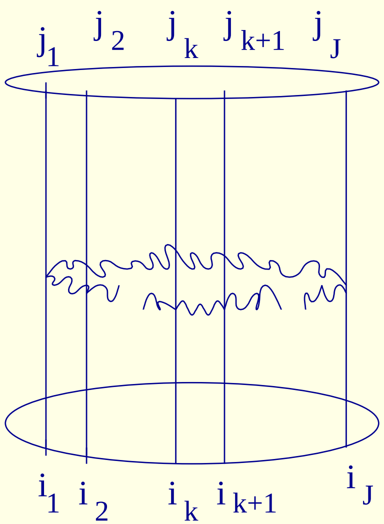
1-hurok, planáris : elsőszomszéd kölcsönhatás, BA

Nem planáris: nem integrálható

2-hurok, integrálható, de másodszomszéd, változó hossz (dinamikai), BA

De BA $\leftrightarrow$ S-mátrix általánosítsuk a BA-ot!

Milyen húrgerjesztések S-mátrixa?



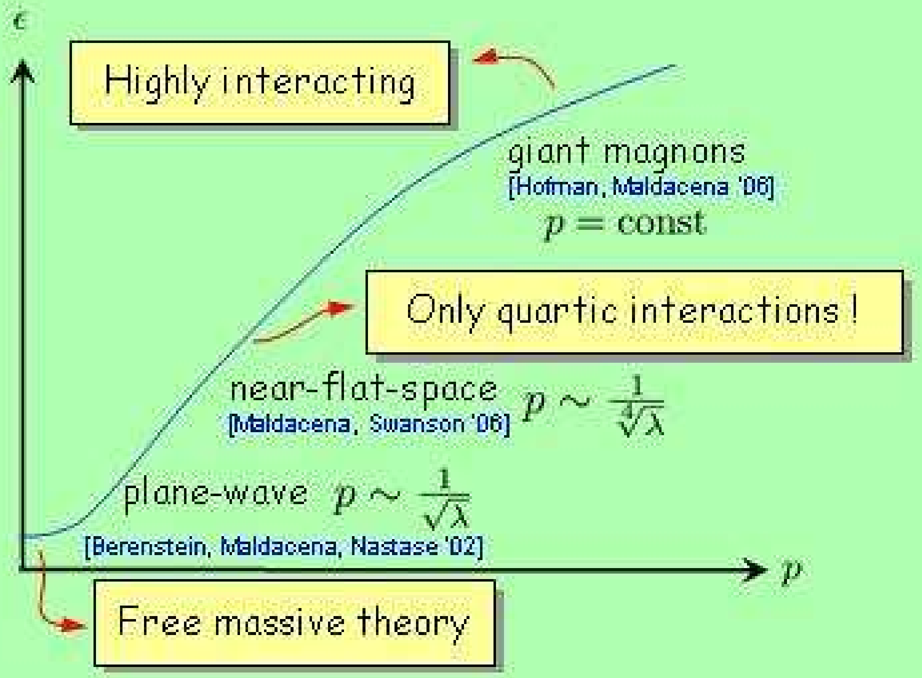
Wrapping probléma

Near-flat-space limit

$$X = (Y, Z) \qquad \psi = (\Psi, \Upsilon)$$

$$\mathcal{L} = \tfrac{1}{2}(\partial \vec{X})^2 - \tfrac{1}{2}\vec{X}^2 + 2i\psi_-\partial_+\psi_- + 2i\psi_+\partial_-\psi_+ + 2i\psi_-\Pi\psi_+$$
$$+ \gamma\Big[(\vec{Y}^2 - \vec{Z}^2)(\partial_-\vec{X})^2 + i(\vec{Y}^2 - \vec{Z}^2)\psi_-\partial_-\psi_- + i\psi_-\partial_-\vec{X}(\vec{Y} - \vec{Z})\psi_-$$
$$+ \tfrac{1}{24}(\psi_-\Gamma^{ij}\psi_-\,\psi_-\Gamma^{ij}\psi_- - \psi_-\Gamma^{i'j'}\psi_-\,\psi_-\Gamma^{i'j'}\psi_-)\Big]$$

- Decompactification limit built in
- Non-Lorentz invariant interactions
- Coupling strength dependent on particle momenta
- Decoupling of right-movers
- UV-finiteness
- quantum mechanically consistent reduction at least to two-loops



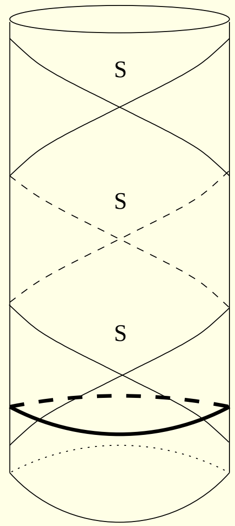
Finite size effects: AdS/CFT

Luscher correction for the Konishi = wrapping

$\mathcal{N}=4$ SYM	$g \leftrightarrow g^{-1}$	II_B on $AdS_5\times S^5$
planar on strip	spin chain	Bethe Yang

Maldacena

Konishi: $\Delta_F = 2\mathrm{Tr}(X^2Z^2+\dots) \leftrightarrow e^{i4p} = S(p,-p) \leftrightarrow qGMagnon$



$$\Delta(g) = 2E(p) = 2\sqrt{1+16g^2\sin^2\frac{p}{2}}$$

$$p = \frac{2\pi}{3} - \sqrt{3}g^2 + \frac{9\sqrt{3}}{2}g^4 - \frac{72(1+\zeta(3))}{\sqrt{3}}g^6 + \dots$$

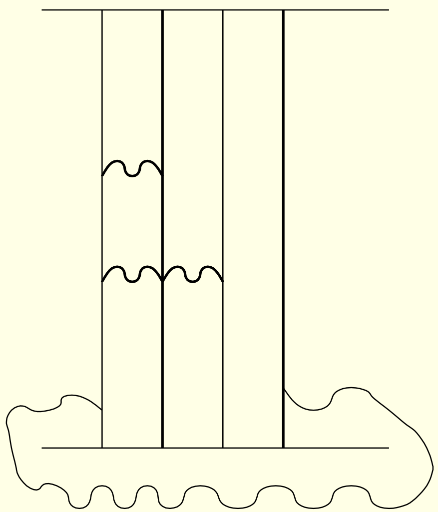
N. Mann et al $\neq$ Zanon et al

$$E = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8$$

$$\boxed{E_{wrapping} = 236 + 864\zeta(3) - 1440\zeta(5)} \qquad 324 \leftrightarrow 236$$

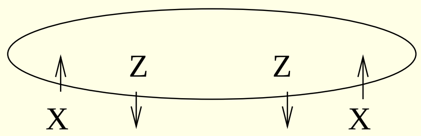
Wrapping for the anomalous dimension of Konishi $Tr(\Phi^2)$

$L = 4$ operator from the SU(2) sector: $\mathcal{O}_{su2} = \mathrm{Tr}(Z_2^2 Z^2 - Z_2 Z Z_2 Z)$
$L = 2$ operator from the SL(2) sector $\mathcal{O}_{sl2} = \mathrm{Tr}(D^2 Z^2 + \ldots)$



Planar diagrams for $\langle \mathcal{O}(r)\mathcal{O}(0)\rangle \propto r^{-2\Delta(p)}$

Spin chain description: BDS
1-loop 2-loop all-loops: BA $\not\supset$ wrapping=Luscher



$$e^{i4p} = S_{BDS}(p,-p) = \frac{\cot(\frac{p}{2})E(p) + i}{\cot(\frac{p}{2})E(p) - i} e^{2i\theta(p,-p)} \to p = \frac{2\pi}{3} + \sqrt{3}g^2 + \ldots$$

$$\Delta(g) = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8$$

wrapping contribution from Zanon et al.

$$(236 + 864\zeta(3) - 1440\zeta(5))g^8$$

Finite size correction in the sinh-Gordon from TBA Teschner

$$\epsilon(\theta) = mL\cosh\theta + \sum_{j=1}^{N} \log S(\theta-\theta_j-\frac{i\pi}{2}) - \frac{d\log S(\theta)}{id\theta}\star\log(1+e^{-\epsilon(\theta)})$$

singularity of the log: $\epsilon(\theta_j+\frac{i\pi}{2}) = i(2n_j+1)\pi$ so we have $\theta_i, \epsilon(\theta)$

$$E_{\{n_j\}}(L) = m\sum_{j=1}^{N}\cosh\theta_j - m\int\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})$$

Expand for large volume:

$$\epsilon(\theta) = mL\cosh\theta + \log S(\theta-\theta_0-\tfrac{i\pi}{2}) - \int\tfrac{d\theta'}{2\pi}\phi(\theta-\theta')S(\tfrac{i\pi}{2}+\theta_0-\theta')e^{-mL\cosh\theta'}$$

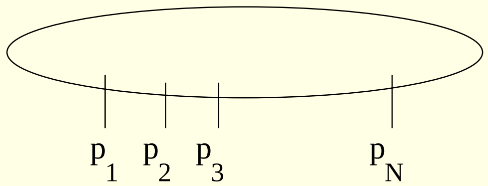
BY quantization: $\theta_0 = \hat{\theta}_n + \delta\theta$

$$mL\sinh\theta_0 + \delta\Phi = 2i\pi n \quad ; \qquad \delta\Phi = \int\frac{d\theta'}{2\pi}\frac{d}{d\hat{\theta}}S(\frac{i\pi}{2}+\theta_0-\theta')e^{-mL\cosh\theta'}$$

Energy

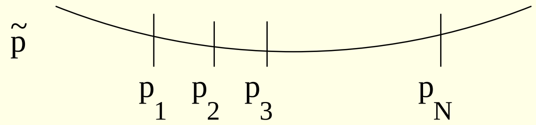
$$E_{\{n\}}(L) = m\cosh\hat{\theta}_n + \delta\theta\, m\sinh\hat{\theta}_n - m\int\frac{d\theta}{2\pi}\cosh\theta\, S(\frac{i\pi}{2}+\theta-\hat{\theta}_n)e^{-mL\cosh\theta}$$

Finite size correction to a multiparticle state



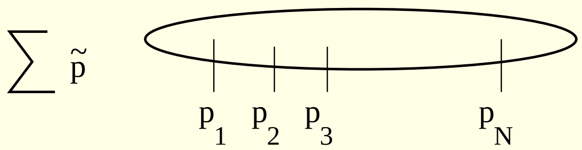
$$2n_k\pi = p_kL - i\log\left[\prod_{k\neq j} S^{aa}_{aa}(p_k,p_j)\right] + \delta\Phi_k$$

where the correction to the asymptotic BA is



$$\delta\Phi_k = -\int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}(-1)^F Tr_{a_1}\left[S^{a_2a}_{a_1a}(\tilde{p},p_1)\dots\frac{dS^{a_{k+1}a}_{a_ka}(\tilde{p},p_k)}{d\tilde{p}}\dots S^{a_1a}_{a_Na}(p,p_1)\right]e^{-\tilde{\epsilon}(\tilde{p})L}$$

and the correction to the energy



$$\begin{aligned} E^j(L) \;&=\; \sum_k \epsilon(p_k) - \sum_{j,k}\frac{d\epsilon(p_k)}{dp_k}\left(\frac{\delta BY_k}{\delta p_j}\right)^{-1}\delta\Phi_j \\ &\quad - \int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}(-1)^F Tr\left[S^{a_2a}_{a_1a}(\tilde{p},p_1)S^{a_3a}_{a_2a}(\tilde{p},p_2)\dots S^{a_1a}_{a_Na}(p,p_1)\right]e^{-\epsilon_{\tilde{a}_1}(\tilde{p})L} \end{aligned}$$

Scatterings for AdS/CFT: $Psu(2,2|4) \xrightarrow{Tr(ZZZZ)} su(2|2)^2 \supset su(2) \otimes su(2)$

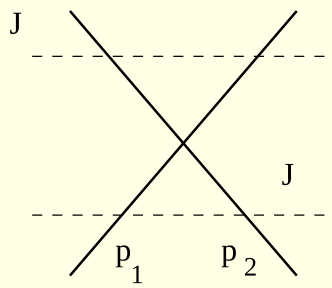
Fundamental representation $\{w_1, w_2, \theta_3, \theta_4\} = \{w_a, \theta_\alpha\} \leftrightarrow \mathcal{V}^1 = \frac{1}{2} \otimes 0 + 0 \otimes \frac{1}{2}$
Q-magnon $\{w_{a_1} \dots w_{a_Q}, w_{a_1} \dots w_{a_{Q-1}} \theta_\alpha, w_{a_1} \dots w_{a_{Q-2}} \theta_\alpha \theta_\beta\} \leftrightarrow \mathcal{V}^Q = \frac{Q}{2} \otimes 0 + \frac{Q-1}{2} \otimes \frac{1}{2} + \frac{Q-2}{2} \otimes 0$

$$L_a^b = w_a \frac{\partial}{\partial w_b} - \frac{1}{2} \delta_a^b w_c \frac{\partial}{\partial w_c} \quad ; \quad R_\alpha^\beta = \theta_\alpha \frac{\partial}{\partial \theta_\beta} - \frac{1}{2} \delta_\alpha^\beta \theta_\gamma \frac{\partial}{\partial \theta_\gamma}$$

$$Q_\alpha^a = a\, \theta_\alpha \frac{\partial}{\partial w_a} + b \epsilon^{ab} \epsilon_{\alpha\beta} w_b \frac{\partial}{\partial \theta_\beta} \quad ; \quad Q_a^{+\alpha} = d\, w_a \frac{\partial}{\partial \theta_\alpha} + c \epsilon^{\alpha\beta} \epsilon_{ab} \theta_\beta \frac{\partial}{\partial w_b}$$

$a = \sqrt{\frac{g}{Q}} \eta \quad ; \quad b = \sqrt{\frac{g}{Q}} \frac{i e^{i2\xi}}{\eta} \left(\frac{x^+}{x^-} - 1\right) \quad ; \quad c = -\sqrt{\frac{g}{Q}} \frac{\eta e^{-2i\xi}}{x^+} \quad ; \quad d = \sqrt{\frac{g}{Q}} \frac{x^+}{i\eta} \left(1 - \frac{x^-}{x^+}\right)$

The tensor product is irreducible $\mathcal{V}^1 \otimes \mathcal{V}^Q = \mathcal{W}^{Q+1}$



$$S(p_1, p_2) \left[J(p_1, e^{ip_2}) + J(p_2, 1) \right] =$$

$$\left[J(p_1, 1) + J(p_2, e^{ip_1}) \right] S(p_1, p_2)$$

fixes the S-matrix completely

the S-matrix in terms of $su(2)\otimes su(2)$ invariants: $\Lambda^i_j = v^i D_j$

$(\frac{1}{2}\otimes 0 + 0\otimes\frac{1}{2})\quad (\frac{Q}{2}\otimes 0 + \frac{Q-1}{2}\otimes\frac{1}{2} + \frac{Q-2}{2}) = \frac{Q+1}{2}\otimes 0 + 3\frac{Q-1}{2}\otimes 0 + 2\frac{Q}{2}\otimes\frac{1}{2} + 2\frac{Q-2}{2}\otimes\frac{1}{2} + \frac{Q-1}{2}\otimes 1 + \frac{Q-3}{2}\otimes 0$

$S = a^1_1\Lambda^1_1 + \sum^4_{i,j=2} a^j_i\Lambda^i_j + + \sum^6_{i,j=5} a^j_i\Lambda^i_j + \sum^8_{i,j=7} a^j_i\Lambda^i_j + a^9_9\Lambda^9_9 + a^{10}_{10}\Lambda^{10}_{10}$

$$v_1 D^1 = \frac{1}{Q+1}(w^1_a w^2_{a_1}\dots w^2_{a_Q} + \dots + w^2_a w^2_{a_1}\dots w^1_{a_Q})\frac{1}{Q!}\frac{\partial^{Q+1}}{\partial w^1_a \partial w^2_{a_1}\dots\partial w^2_{a_Q}}$$

$a^1_1 = 1$ normalization $a^5_5 = \frac{x^+_1 - x^+_2}{x^+_1 - x^-_2}\frac{\tilde\eta_1}{\eta_1}\;\; a^6_5 = a^5_6 = \sqrt{Q}\frac{(x^+_1 - x^-_1)}{(x^+_1 - x^-_2)}\frac{\tilde\eta_2}{\eta_1}\quad ;$

$a^6_6 = Q\frac{x^-_1 - x^-_2}{x^+_1 - x^-_2}\frac{\tilde\eta_2}{\eta_2}\; a^9_9 = \frac{x^-_1 - x^+_2}{x^+_1 - x^-_2}\frac{\tilde\eta_1}{\eta_1}\frac{\tilde\eta_2}{\eta_2}\; a^7_7 = \frac{2}{Q}\frac{x^-_2(x^-_1 - x^+_2)(1 - x^-_1 x^+_2)}{x^+_2(x^+_1 - x^-_2)(1 - x^-_1 x^-_2)}\frac{\tilde\eta_2}{\eta_2}$

$a^8_8 = \frac{x^-_1(x^-_1 - x^+_2)(1 - x^+_1 x^-_2)}{2x^+_1(x^+_1 - x^-_2)(1 - x^-_1 x^-_2)}\frac{\tilde\eta_1}{\eta_1}\frac{\tilde\eta^2_2}{\eta^2_2}\qquad a^7_8 = a^8_7 = \frac{i}{\sqrt{Q}}\frac{x^-_1 x^-_2(x^-_1 - x^+_2)}{x^+_1 x^+_2(x^+_1 - x^-_2)(1 - x^-_1 x^-_2)}\frac{\tilde\eta_1\tilde\eta^2_2}{\eta_2}$

and so on but fix the scalar factor from bootstrap

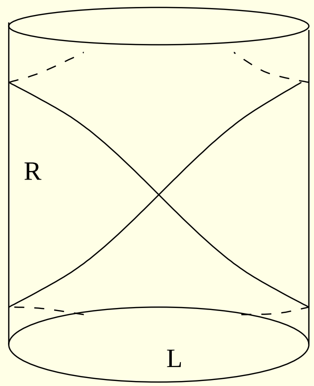
The Luscher correction to Konishi: two $Q=1$ particle $(3,\dot{3})$ with $p=\frac{2\pi}{3}+\dots$

$$E^j(L)=2\epsilon(p)-\frac{2}{L}\epsilon'(p)\delta\Phi_j-\int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}(-1)^F\sum_{Q,Q'}\left[S^{Q'3\dot{3}}_{Q3\dot{3}}(\tilde{p},p)S^{Q3\dot{3}}_{Q'3\dot{3}}(\tilde{p},-p)\right]e^{-\tilde{\epsilon}_Q(\tilde{p})L}$$

why mirror particles? AF $\boxed{\epsilon_E^2+16g^2\sin^2\frac{p_E}{2}+1=0}$ $\quad \epsilon_E^2+p_E^2+m^2=0$

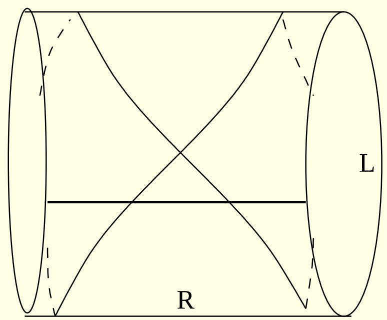
$(\epsilon,p)=(i\epsilon_E,p_E)$

$\epsilon(p)=\sqrt{1+16g^2\sin^2\frac{p}{2}}$



Mirror $(\tilde{\epsilon},\tilde{p})=(ip_E,\epsilon_E)$

$\tilde{\epsilon}(\tilde{p})=2\,\mathrm{arcsin\,h}\frac{1+\tilde{p}^2}{4g}$

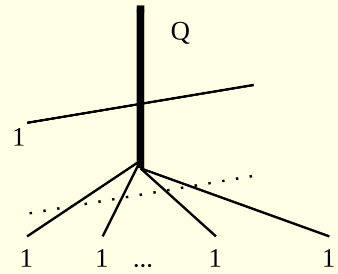


$$Tr\left(e^{-H(L)R}\right)=e^{-E_0(L)R}+\dots=Tr\left(e^{-H(R)L}D\right)=\sum_{|n\rangle\in\mathcal{H}}\frac{\langle n|D|n\rangle e^{-E_n(R)L}}{\langle n|n\rangle}$$

Defect operator: $\langle\tilde{p}|D|\tilde{p}\rangle=S^{Q'3\dot{3}}_{Q3\dot{3}}(\tilde{p},p)S^{Q3\dot{3}}_{Q'3\dot{3}}(\tilde{p},p)$

Why the Luscher works at all: AJK $e^{-\tilde{\epsilon}(\tilde{p})}\longrightarrow^{g\to 0}\longrightarrow\frac{4g^2}{1+\tilde{p}^2}$ Qmagnon $\frac{4g^2}{Q^2+\tilde{p}^2}$

The final result: Fix the scalar part from fusion: N.Dorey $p_1 = q_1 + q_2 + \cdots + q_Q$



$$S^0_Q(p_1, p_2) = S^0_{BDS}(q_1, p_2) S^0_{BDS}(q_2, p_2) \dots S^0_{BDS}(q_Q, p_2)$$

Integrand $\sum_{Q,Q'} \left[S^{Q'3\dot{3}}_{Q3\dot{3}}(\tilde{p}, p) S^{Q3\dot{3}}_{Q'3\dot{3}}(\tilde{p}, -p) \right] e^{-\tilde{\epsilon}(\tilde{p})L}$

$$\frac{147456 Q^2 \left(3q^2+3Q^2-4\right)^2}{\left(q^2+Q^2\right)^4} \cdot \frac{g^8}{(9q^4+6(3Q(Q+2)+2)q^2+(3Q(Q+2)+4)^2)(Q \leftrightarrow Q-2)}$$

After integration

$$\frac{-7776Q(-2+3Q^2)(5-177Q^2+261Q^4+7047Q^6-13608Q^8-21141Q^{10}+35721Q^{12}-21870Q^{14}+6561Q^{16})}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)} + \frac{864}{Q^3} - \frac{1440}{Q^5}$$

After summing up from 1 to ∞

$$E = 324 + 864\zeta(3) - 1440\zeta(5)$$