THE STATUS OF NONPERTURBATIVE QCD

I. A pure dynamical theory of gluon confinement

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What QCD is about?

$$L_{QCD} = L_{YM} + L_{qg}$$

$$L_{YM} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + L_{g.f.} + L_{gh.}$$

$$F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$L_{qg} = i \bar{q}^j_\alpha D_{\alpha\beta} q^j_\beta + \bar{q}^j_\alpha m^j_0 q^j_\beta$$

$$D_{\alpha\beta} q^j_\beta = (\delta_{\alpha\beta}\partial_\mu - ig(1/2)\lambda^a_{\alpha\beta} A^a_\mu)\gamma_\mu q^j_\beta \qquad (covariant\ derivative)$$

$$\alpha, \beta = 1, 2, 3. \qquad j = 1, 2, 3, ...N_f$$

$$a = N_c^2 - 1 = 1, 2, 3, ...8, \quad N_c = 3$$

and λ^a are $SU(N_c)$ matrices.

Repeated indices are always summed over.

QCD Phase Transitions

I. The Confinement Phase Transition.

a). The absence of gluons in the IR

b). The absence of the pole-type singularities in the quark propagator, i.e.,

$$S(p) \neq \frac{const.}{\hat{p} - m}$$

c). A discrete spectrum only

II. The Chiral Phase Transition or usually PCAC

Dynamical (spontaneous) chiral symmetry breaking

$SU_L(N_f) \times SU_R(N_f)$ is spontaneously broken to $SU(N_f)$ in the ground state. The Goldstone theorem then implies $N_f^2 - 1$ plet of massless pseudoscalar particles (bosons)

III. No Higgs phase

$SU(N_c)$ is exact and gluons remain massless.

We still do not know the interaction between quarks and gluons!

The Lagrangian is not enough, it is necessary to know the structure of the QCD ground state.

Two nonperturbative approaches to QCD

I. Lattice QCD

II. Dynamical equations of motion approach

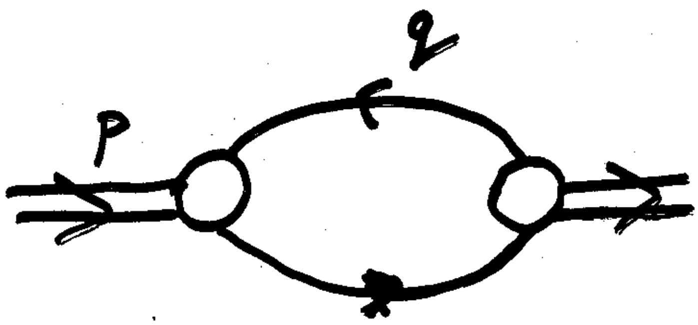
" Progress towards 21 century is impossible without solution of the dynamical equations of motion for particles and fields"

A. Salam

(i). The distribution nature of the Green's functions

(ii). The self-consistency of any truncation scheme

(iii). A manifest gauge invariance



$$D_{\pi^+}(p) \sim \int d^4q Tr[\gamma_5 S_u(p+q)\gamma_5 S_{\bar{d}}(q)],$$

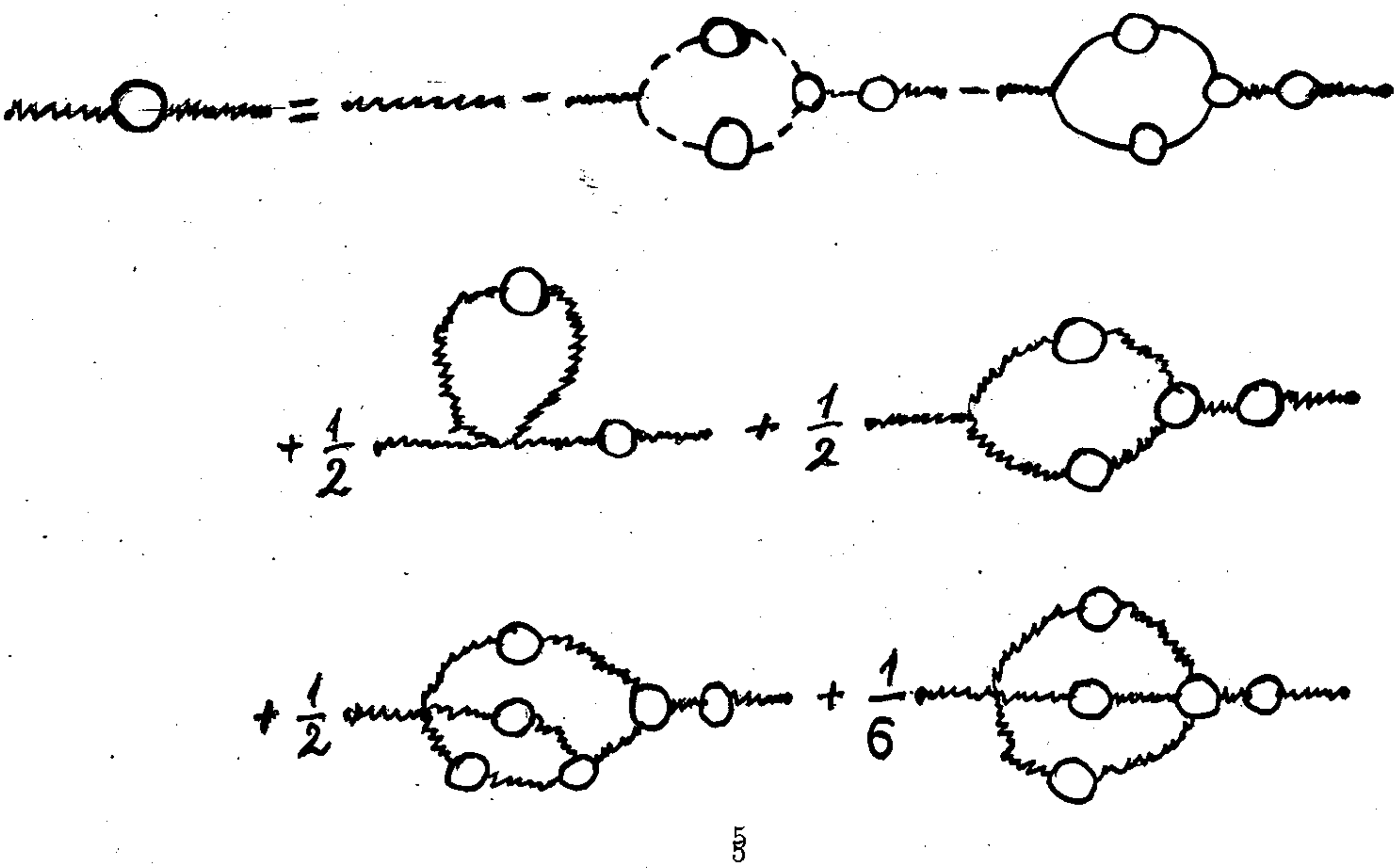
II. Gluon propagator and its SD equation

$$D_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q)d(q^2,\xi) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2},$$

$$T_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = g_{\mu\nu} - L_{\mu\nu}(q)$$

$$D^0_{\mu\nu}(q) = i\left\{T_{\mu\nu}(q) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}.$$

$$D(q) = D^0(q) - D^0(q)T_{gh}(q)D(q) - D^0(q)T_q(q)D(q) + D^0(q)T_g[D](q)D(q),$$

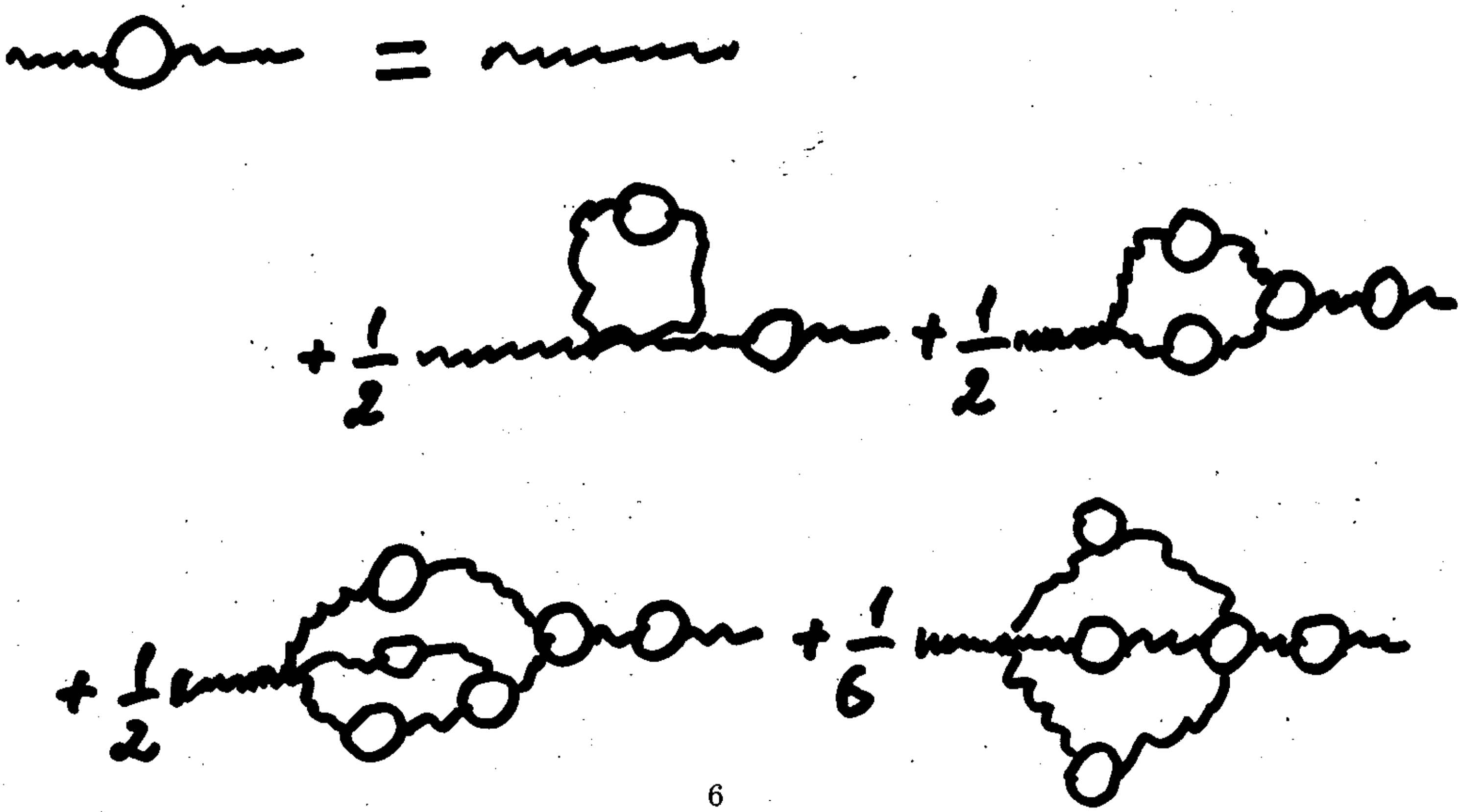


$$D(q) = \tilde{D}^0(q) + \tilde{D}^0(q) T_g[D](q) D(q) = \tilde{D}^0(q) + D^{NL}(q),$$

$$\tilde{D}^0(q) = \frac{D^0(q)}{1 + [T_{gh}(q) + T_q(q)] D^0(q)},$$

$$T_{gh}(q) = g^2 \int \frac{id^4k}{(2\pi)^4} k_\nu G(k) G(k-q) G_\mu(k-q, q),$$

$$T_q(q) = -g^2 \int \frac{id^4p}{(2\pi)^4} Tr[\gamma_\nu S(p-q) \Gamma_\mu(p-q, q) S(p)].$$



In general, these quantities can be decomposed as follows:

$$T_{gh}(q) \equiv T^{gh}_{\mu\nu}(q) = \delta_{\mu\nu} q^2 T^{(1)}_{gh}(q^2) + q_\mu q_\nu T^{(2)}_{gh}(q^2)$$

$$T_q(q) \equiv T^q_{\mu\nu}(q) = \delta_{\mu\nu} q^2 T^{(1)}_q(q^2) + q_\mu q_\nu T^{(2)}_q(q^2),$$

where all invariant functions $T^{(n)}_{gh}(q^2)$ and $T^{(n)}_q(q^2)$ at $n = 1, 2$ are dimensionless ones with a regular behavior at zero. Obviously, $\delta_{\mu\nu} \to T_{\mu\nu}$ and $q_\mu q_\nu \to L_{\mu\nu} q^2$.

$$\tilde{D}^0(q) = \frac{D^0(q)}{1 + [T_{gh}(q) + T_q(q)] D^0(q)},$$

$$D^0_{\mu\nu}(q) = \delta_{\mu\nu}(i/q^2)$$

$$\tilde{D}^0(q) = D^0(q) A(q^2),$$

$$A(q^2) = \frac{1}{1 + T(q^2)}.$$

$$D(q) = \tilde{D}^0(q) + \tilde{D}^0(q) T_g[D](q) D(q) = \tilde{D}^0(q) + D^{NL}(q),$$

$$T_g[D](q) = \frac{1}{2} T_t(q) + \frac{1}{2} T_1(q) + \frac{1}{2} T_2(q) + \frac{1}{6} T_2'(q),$$

$$T_t(q) = g^2 \int \frac{i d^4 q_1}{(2\pi)^4} T_4^0 D(q_1),$$

$$T_1(q) = g^2 \int \frac{i d^4 q_1}{(2\pi)^4} T_3^0(q, -q_1, q_1 - q) T_3(-q, q_1, q - q_1) D(q_1) D(q - q_1),$$

$$T_2(q) = g^4 \int \frac{i d^4 q_1}{(2\pi)^4} \int \frac{i d^n q_2}{(2\pi)^4} T_4^0 T_3(-q_2, q_3, q_2 - q_3) \times$$
$$T_3(-q, q_1, q_3 - q_2) D(q_1) D(-q_2) D(q_3) D(q_3 - q_2),$$

$$T_2'(q) = g^4 \int \frac{i d^4 q_1}{(2\pi)^4} \int \frac{i d^4 q_2}{(2\pi)^4} T_4^0 T_4(-q, q_1, -q_2, q_3) D(q_1) D(-q_2) D(q_3).$$

In the last two equations

$$q - q_1 + q_2 - q_3 = 0$$

is assumed as usual.

The formal iteration solution of the gluon SD equation looks

$$D(q) = D^{(0)}(q) + \sum_{k=1}^{\infty} D^{(k)}(q)$$

$$= D^{(0)}(q)+\sum_{k=1}^{\infty}\Big\{D^{(0)}(q)T_g[\sum_{m=0}^{k-1} D^{(m)}](q)(\sum_{m=0}^{k-1} D^{(m)}(q))-\sum_{m=1}^{k-1} D^{(m)}(q)\Big\},$$

where, for example, explicitly the first four terms are:

$$D^{(0)}(q) = \tilde{D}^0(q)$$

$$D^{(1)}(q) = \tilde{D}^0(q)T_g[\tilde{D}^0](q)\tilde{D}^0(q),$$

$$D^{(2)}(q) = \tilde{D}^0(q)T_g[\tilde{D}^0 + D^{(1)}](q)(\tilde{D}^0(q) + D^{(1)}(q)) - D^{(1)}(q),$$

$$D^{(3)}(q) = \tilde{D}^0(q)T_g[\tilde{D}^0+D^{(1)}+D^{(2)}](q)(\tilde{D}^0(q)+D^{(1)}(q)+D^{(2)}(q))$$
$$-D^{(1)}(q) - D^{(2)}(q),$$

and so on. The order of iteration does not coincide with the order of PT in the coupling constant squared. Any iteration (even zero) contains ghost and quark degrees of freedom in all orders of PT. There is no hope to sum up an infinite series presented in this equation. The most important general feature of the iteration solution is that each subsequent iteration contains all the preceding ones.

b. Severe IR structure of the gluon propagator

Let us show explicitly now that the QCD vacuum is really beset with the severe IR singularities if standard (or even modified) PT is applied. "But it is to just this violent IR behavior that we must look for the key to the low energy and large distance hadron phenomena. In particular, the absence of quarks and other colored objects can only be understood in terms of the IR divergences in the self-energy of a color bearing objects" [Susskind, Kogut].

A. NP IR singularity is more severe than $1/q^2$ at $q^2 \to 0$.

B. PT IR singularity is as much singular as $1/q^2$ at $q^2 \to 0$.

The full gluon propagator up to first iteration is

$$D_{\mu\nu}(q) = \tilde{D}^0_{\mu\nu}(q) + D^{(1)}_{\mu\nu}(q) + ...$$

$$= \tilde{D}^0_{\mu\nu}(q) + \tilde{D}^0_{\mu\nu_1}(q) T^g_{\nu_1\mu_1}[\tilde{D}^0](q) \tilde{D}^0_{\mu_1\nu}(q) + ...$$

and for simplicity $T_3 = T_3^0, \ T_4 = T_4^0$.

$$T'^2_{\nu_1\mu_1}(q) = g^4 \int \frac{id^4q_1}{(2\pi)^4} \int \frac{id^4q_2}{(2\pi)^4} T^0_{\nu_1\rho\lambda\sigma} T^0_{\mu_1\rho_1\lambda_1\sigma_1} \times$$

$$\tilde{D}^0_{\rho\rho_1}(q_1) \tilde{D}^0_{\lambda\lambda_1}(-q_2) \tilde{D}^0_{\sigma\sigma_1}(q - q_1 + q_2).$$

Because of singular dependence on q^2 in the integrand function in the region of very small q_1^2 and q_2^2 loop momenta, it is useful to decompose its general structure as follows:

$$T'^2_{\nu_1\mu_1}(q) = \delta_{\nu_1\mu_1}[\frac{\Delta_2^4}{q^2} T_2^{(1)}(q^2) + \Delta_2^2 T_2^{(2)}(q^2) + q^2 T_2^{(3)}(q^2)]$$

$$+ q_{\nu_1} q_{\mu_1} [\frac{\Delta_2^2}{q^2} T_2^{(4)}(q^2) + T_2^{(5)}(q^2)].$$

Let us emphasize the inevitable appearance of the mass gap Δ_2^2. Evidently, this is due to the above-mentioned singular dependence on q^2 in the integrand function. It characterizes the nontrivial dynamics in the IR.

$$T^g_{\nu_1\mu_1}[\tilde{D}^0](q) \equiv T^g_{\nu_1\mu_1}(q) = \delta_{\nu_1\mu_1}[\frac{\Delta^4}{q^2} T_g^{(1)}(q^2) + \Delta^2 T_g^{(2)}(q^2) + q^2 T_g^{(3)}(q^2)]$$

$$+ q_{\nu_1} q_{\mu_1} [\frac{\Delta^2}{q^2} T_g^{(4)}(q^2) + T_g^{(5)}(q^2)],$$

The corresponding mass gap is denoting as Δ^2.

Using this decomposition and omitting some tedious algebra, the full gluon propagator up to first iteration becomes

$$D_{\mu\nu}(q) = \delta_{\mu\nu}[\frac{\Delta^2}{q^4}a_1 + \frac{\Delta^4}{q^6}a_2 + ...] + D^{PT}_{\mu\nu}(q),$$

where a_1, a_2 are some finite numbers and $D^{PT}_{\mu\nu}(q)$ denotes all terms, which are as much singular as $1/q^2$ in the IR. It may also depend on the mass gap Δ^2 (the so-called shifted terms, see below), so that when it formally goes to zero, then the PT part survives, nevertheless.

Thus one can conclude in that in the region of small momentum the gluon propagates like this, and not like the almost free one

$$\tilde{D}^0_{\mu\nu}(q^2) = D^0_{\mu\nu}(q^2)A(q^2)$$

though we just started from it.

(i). The zero momentum modes enhancement (**ZMME**) effect – at any covariant gauge.

(ii). It requires the existence of a mass gap

(iii). Neither quark skeleton loop nor ghost skeleton loop can cancel this behavior.

Confinement criterion for gluons

On general ground one has

$$D^{INP}(q^2,\Delta^2) = \sum_{k=0}^{\infty}(q^2)^{-2-k}(\Delta^2)^{k+1}\sum_{m=0}^{\infty} a_{k,m}(\xi)g^{2m},$$

$$(q^2)^{-2-k} = \frac{1}{\epsilon}a(k)[\delta^4(q)]^{(k)} + f.t., \quad \epsilon \to 0^+,$$

It is possible to show that $g^2 = \bar{g}^2$ and $\xi = \bar{\xi}$), then the IR renormalization of a mass gap as $\Delta^2 = \epsilon\bar{\Delta}^2$ solves the problem. The simplest NP IR singularity will survive in the $\epsilon \to 0^+$ limit, namely

$$D^{INP}(q^2,\Delta^2) = \Delta^2(q^2)^{-2}\sum_{m=0}^{\infty} a_{0,m}(\xi)g^{2m},$$

I. If there is an explicit integration over the gluon momentum

$$D^{INP}(q,\bar{\Delta}^2) = \bar{\Delta}^2\pi^2\delta^4(q).$$

II. If there is no explicit integration over the gluon momentum

$$D^{INP}(q,\bar{\Delta}^2) \sim \epsilon, \quad \epsilon \to 0^+.$$

This means that any amplitude for any number of soft-gluon emissions (no integration over their momenta) will vanish in the IR limit in our picture. In other words, there are no gluons in the IR, i.e., at large distances (small momenta) there is no possibility to observe gluons experimentally as free particles. So color gluons can never be isolated. This behavior can be treated as the gluon confinement criterion. Evidently, this behavior is a manifestly gauge-invariant.