New results on the pion-laser

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Introduction

Motivation:

Bose-Einstein correlations of emitted particles

• Stellar interferometry: HBT effect

• Heavy ion physics: GGLP effect

Connection between the initial geometry and the final momentum correlations – a theoretical challenge

Simple formula:

$$\tilde{S}(q,K)=\int d^4x\, S(x,K)e^{-iqx} \qquad C(q)=1+\frac{\left|\tilde{S}(q)\right|^2}{\left|\tilde{S}(0)\right|^2}$$

Drawbacks: plane-wave approximation, i.e. no final state interactions (FSI), and two-particle approximation (no multi-particle correlations taken into account)

Introduction

Possible improvements:

- FSI: using the final state wave-function: imaging method

Present topic:

- Multi-particle correlations: symmetrize the wave-function in ALL variables
  - In a single heavy ion collision: hundreds of identical pions
  - Calculation time usually goes with $n!$: NP-hard problem

Pion-laser model:

- In a special case, reduces the NP-hard problem to a polinomial problem (symmetrization to recurrence relations): S. Pratt, 1993

- Solution to the recurrence relations (utilizing wave-packets): J. Zimányi and T. Csörgő, 1997

The pion-laser model

System consisting of arbitary number of bosons characterized by wave-packets

- Density matrix:

$$\hat{\rho} = \sum_{n=0}^{\infty} p_n \hat{\rho}_n \qquad \hat{\rho}_n = \int d\alpha \, \rho_n\left(\alpha_1 \ldots \alpha_n\right) \left|\alpha_1 \ldots \alpha_n\right\rangle \left\langle \alpha_1 \ldots \alpha_n\right|$$

- Multi-particle wave-packet states:

$$\alpha \equiv \left(\xi, \chi, \sigma\right) \qquad \left|\alpha_1 \ldots \alpha_n\right\rangle = \left(\sum_{\sigma} \prod_{i=1}^{n} \left\langle \alpha_i \middle| \alpha_{\sigma_i} \right\rangle\right)^{-1/2} \hat{\alpha}_1^{\dagger} \ldots \hat{\alpha}_n^{\dagger} \left|0\right\rangle$$

- Single-particle wave-packet states:

$$\hat{\alpha}_i^{\dagger} = \int dk \left\langle k \middle| \alpha_i \right\rangle \hat{a}^{\dagger}(k) \qquad \left\langle k \middle| \alpha_i \right\rangle = \left(\sigma^2 \pi\right)^{-3/4} e^{-\frac{(k-\chi)^2}{2\sigma^2} - i\xi(k-\chi)}$$

The pion-laser model

- Assumptions for the density function (stimulated emission):

$$\rho_n\left(\alpha_1 \ldots \alpha_n\right)=\left(\sum_\sigma \prod_{i=1}^n\left\langle\alpha_i \middle| \alpha_{\sigma_i}\right\rangle\right) \prod_{j=1}^n \rho_1\left(\alpha_j\right)$$

- Exclusive spectra:

$$N_m^{(n)}\left(k_1, \ldots, k_m\right)=\operatorname{Tr}\left\{\hat{\rho}_n \hat{a}^{\dagger}(k_1) \ldots \hat{a}^{\dagger}(k_m) \hat{a}(k_1) \ldots \hat{a}(k_m)\right\}$$

- Inclusive spectra:

$$N_m\left(k_1, \ldots, k_m\right)=\operatorname{Tr}\left\{\hat{\rho} \hat{a}^{\dagger}(k_1) \ldots \hat{a}^{\dagger}(k_m) \hat{a}(k_1) \ldots \hat{a}(k_m)\right\}$$

$$N_m\left(k_1, \ldots, k_m\right)=\sum_{n=m}^{\infty} p_n N_m^{(n)}\left(k_1, \ldots, k_m\right)$$

- Measured correlation functions: obtained from inclusive spectra
- Calculation of spectra: permutation sums (Wick's theorem)

Analytically solvable multi-particle symmetrization

- Permutations: ordered by number of orbits

$$N_1^{(n)} = \frac{1}{\omega_n} \sum_{i=1}^{n} G_i(k_1, k_1)\omega_{n-i}$$

$$N_2^{(n)} = \frac{1}{\omega_n} \sum_{i=2}^{n} \sum_{l=1}^{i-1} \left\{ G_l(1,1)G_{i-l}(2,2) + G_l(1,2)G_{i-l}(2,1) \right\} \omega_{n-i}$$

- Auxiliary functions:

$$\overline{\rho}(p,q) = \int d\alpha \left\langle \alpha \middle| p \right\rangle \rho_1(\alpha) \left\langle q \middle| \alpha \right\rangle$$

$$G_n(p,q) = \int dk_1 ... dk_{n-1} \overline{\rho}(p,k_1)\overline{\rho}(k_1,k_2)...\overline{\rho}(k_{n-1},q)$$

- For Gaussian sources: the $G_n(p,q)$ functions are Gaussians

- Recurrence relations: determine $G_n(p,q)$ from $G_{n-1}(p,q)$

Solutions: the exclusive spectra

- Solution trick: mapping into linear recurrences

$$G_n(p,q) = h_n \exp\left(-\alpha_n\left(p^2+q^2\right)+\gamma_n pq\right)$$

- Coefficients have an **analytic** expression
- Auxiliary quantities: ω_n : integrated „orbit-loops''

$$\omega_n = \frac{1}{n}\sum_{l=1}^{n-1} l C_l \omega_{n-l} \qquad C_n = \int dk\, G_n(k,k)$$

- Solution to this recurrence:

$$\sum_{m=0}^{\infty} \omega_m z^m = \exp\left(\sum_{l=1}^{n} C_l z^l\right)$$

Solutions: the exclusive spectra

- Exclusive spectra thus have an analytic form, without numerical difficulties
- Towards inclusive quantities: multiplicity distribution p_n
- Expression of ω_n : the same as a probability distribution by means of its combinants
- So, first idea for the multiplicity distribution:

$$p_n = \frac{\omega_n \bar{n}_0^n}{\sum_{i=0}^{\infty} \omega_i \bar{n}_0^i}$$

- Limiting cases are exactly solvable
- Inclusive spectra have simple form

Multiplicity distributions: calculating inclusive spectra

- Limiting cases for the multiplicity distribution
  - Rare gas: almost Poissonian distribution with $\bar{n}_0$ mean:

$$p_n \sim \frac{\bar{n}_0^{\,n}}{n!} e^{-\bar{n}_0}$$

  - Dense gas: geometrical distribution …

$$p_n \sim \frac{\langle n \rangle^n}{\left(\langle n \rangle + 1\right)^{n+1}} \qquad \langle n \rangle = \frac{\bar{n}_0}{1-\bar{n}_0}$$

… or Bose-Einstein-condensation (with finite energy constraint)

$$p_n \sim \delta_{n,n_f}$$

Multiplicity distributions: calculating inclusive spectra

- Inclusive spectra:

$$N_1\left(k_1\right)=G\left(k_1,k_1\right)$$

$$N_2\left(k_1,k_2\right)=G\left(k_1,k_1\right)G\left(k_2,k_2\right)+G\left(k_1,k_2\right)G\left(k_2,k_1\right)$$

$$N_m\left(k_1,\ldots,k_m\right)=\sum_{\sigma(m)}\prod_{i=1}^{n}G\left(k_i,k_{\sigma(m)_i}\right)$$

$$G\left(p,q\right)\equiv\sum_{i=1}^{\infty}G_i\left(p,q\right)\overline{n}_0^i$$

- Simple forms: special to this multiplicity distribution
- Poissonian limiting case: a flaw of the model

Alternative choices for the multiplicity distribution

- Generalizing multiplicity distribution: by means of integral transformation:
  - Poissonian transformation:

$$p_n = \int_0^\infty dy\, H(y) \frac{y^n}{n!} e^{-y}$$

  - Generalized transformation:

$$p_n = \int_0^\infty dy\, H(y) \frac{\omega_n y^n}{\sum \omega_m y^m}$$

- Example: $H(y) = 1/\langle n \rangle \exp\left(-y/\langle n \rangle\right)$: multiplicity distribution is a geometrical (i.e. special negative binomial) in the rare-gas limiting case with $\langle n \rangle$ mean
  - In this case opposite limiting case is normalizable as well: no need for additional finit energy constraint

Alternative choices for the multiplicity distribution

• Inclusive spectra: more complicated expressions & unphysical behavior of correlation functions (intercept values)

$$N_2 = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{G_n(1,1)G_m(2,2)+G_n(1,2)G_m(2,1)\right\} f_{n+m}$$

$$f_n = \int_0^{\infty} dy\, y^n H(y)$$

• If f_n obeys the property $f_{n+m} = f_n f_m$, the model is solvable in terms of the inclusive spectra as well

• Intercept behavior is physical in rare-gas limiting case, if and only if $f_{n+m} = f_n f_m$, it follows, that $H(y) \sim \delta\left(y - \overline{n}_0\right)$

• Conclusion: No simple analytically solvable model (MD) exists except of the already known one

Summary and outlook

- Pion-laser model: an analytically solvable multi-particle system
- Symmetrization can be done with $\sim n^0$ computing time (recurrence relations and their solutions)
- Exclusive spectra: full solution exists
- Pro- and contras of the present model:
  - Inclusive spectra: analytic expressions only for one special multiplicity distribution
  - Uniqueness proven
  - More general multiplicity distribution is needed
- Possible generalizations (future tasks):
  - Other type of source functions (not so important)
  - Other type of factorization for the n-particle density matrix

Thank you for your attention!