Dissipation and differential elliptic flow

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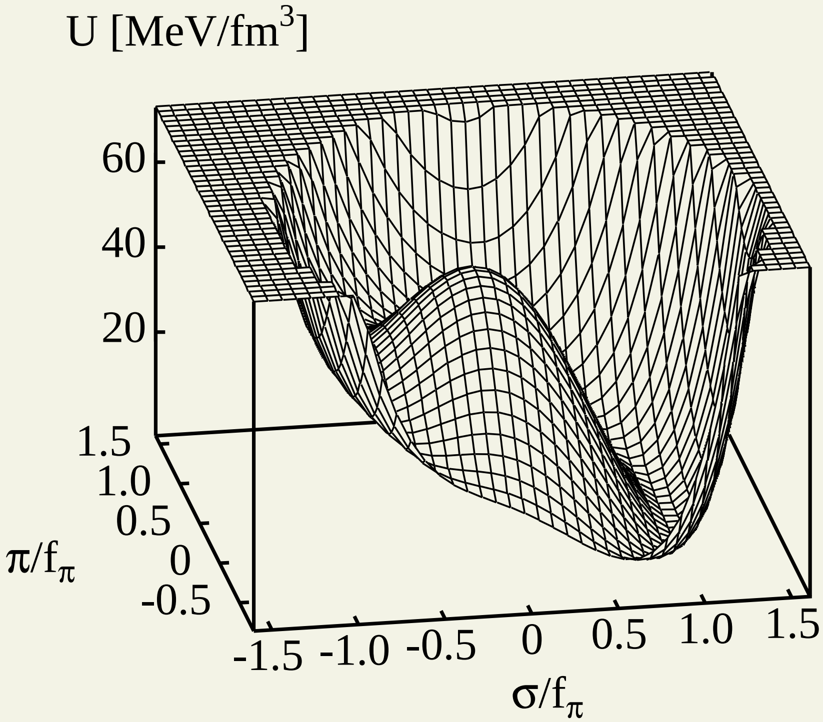
- Thermalization question
  - can pQCD rates do it at RHIC?
  - an ancient beast... or friend perhaps?
  - what if we have the highest possible rates?

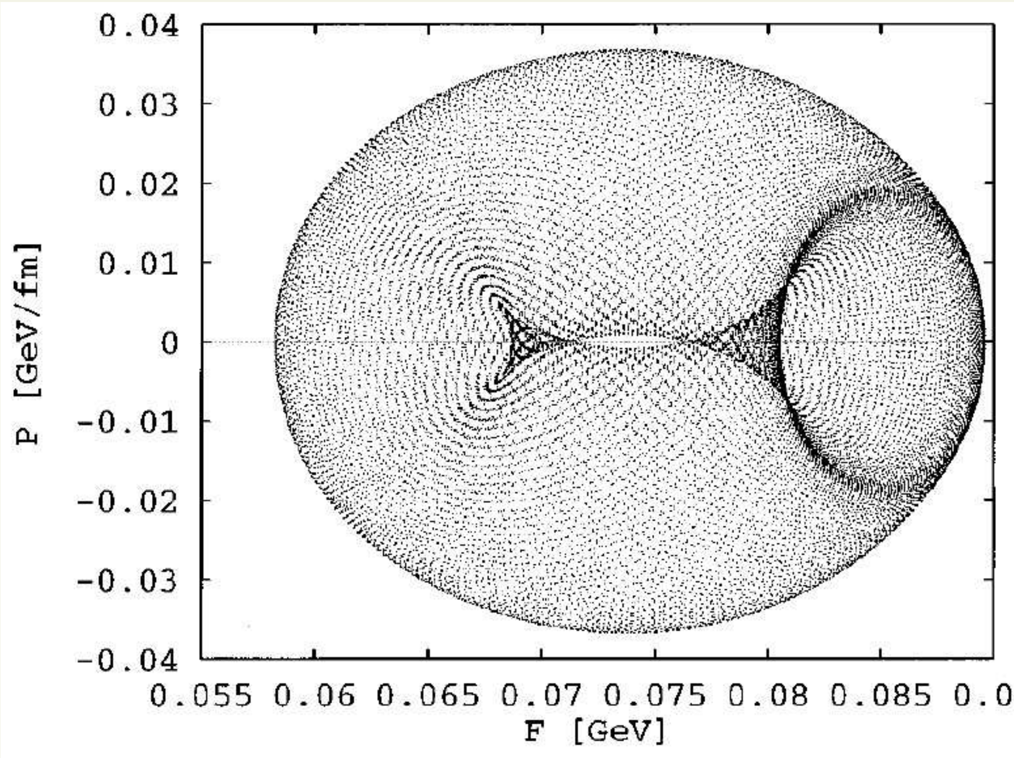
first a bit of history... Great Hunt for DCCs Biro, DM, Feng, Csernai, PRD55 ('97)

$$\mathcal{L} = \bar{\Psi}\left[i\gamma\cdot\partial - g\left(\sigma + i\gamma_5\vec{\tau}\vec{\pi}\right)\right]\Psi + \frac{1}{2}(\partial\vec{\Phi})^2 - U(\vec{\Phi})$$

"Mexican hat"



evolution from large initial ang. mom.



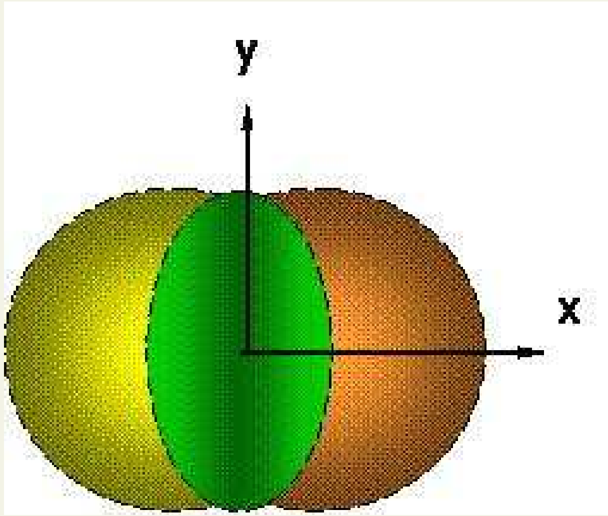
$[F \equiv |\vec{\Phi}|,\ P \equiv \partial_\tau F]$

we had great fun (if only Nature had been kinder)

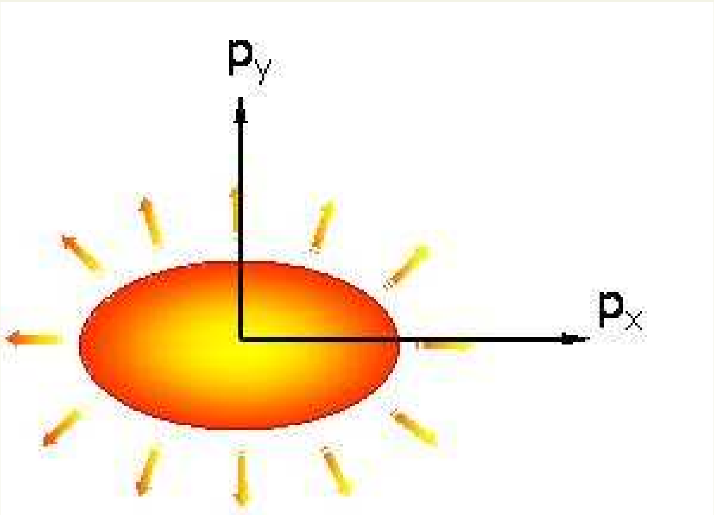
to what degree QCD matter thermalizes in a RHIC collision?

local equilibrium POSTULATE quite successful
but need to understand equilibration dynamics Gyulassy, Pang, Zhang, DM...

- **one measure - "elliptic flow"** (v_2)



$\longrightarrow$

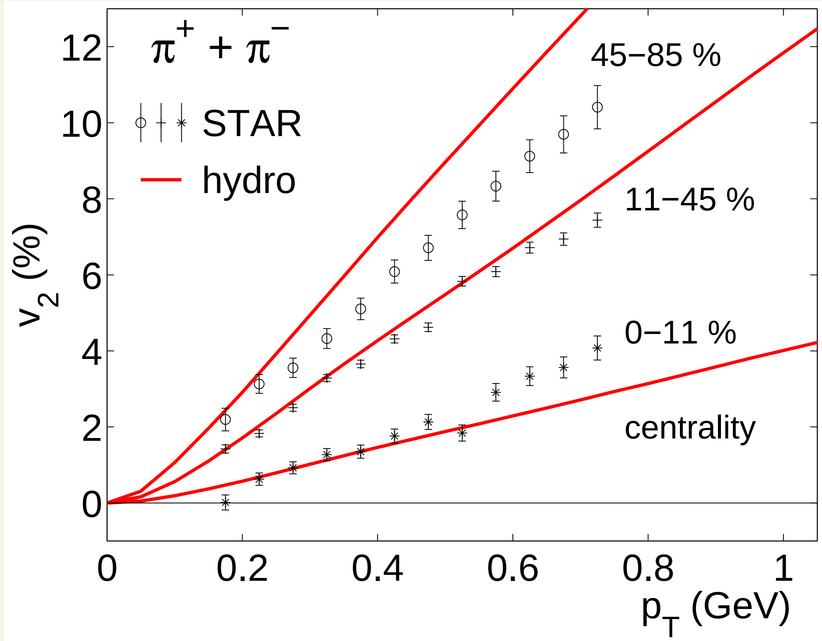


$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

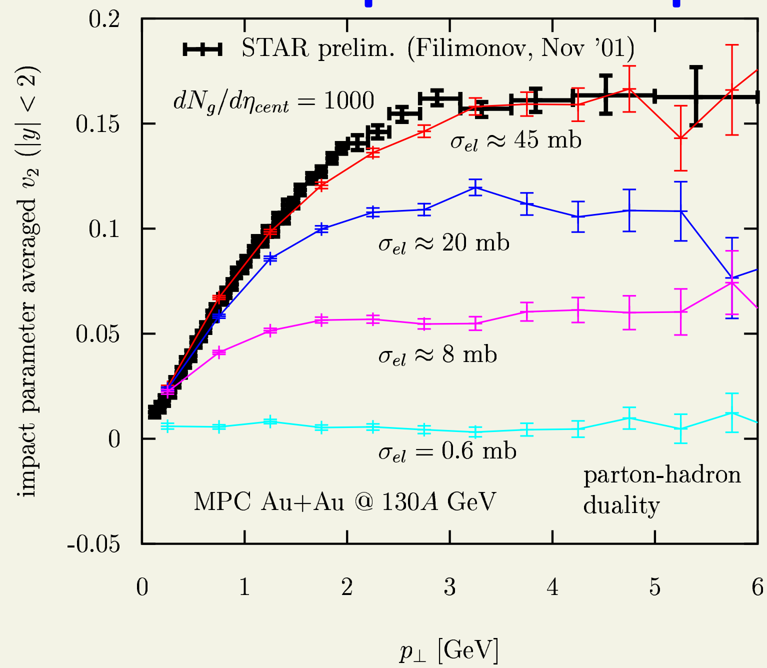
Kolb, Heinz et al

ideal hydrodynamics



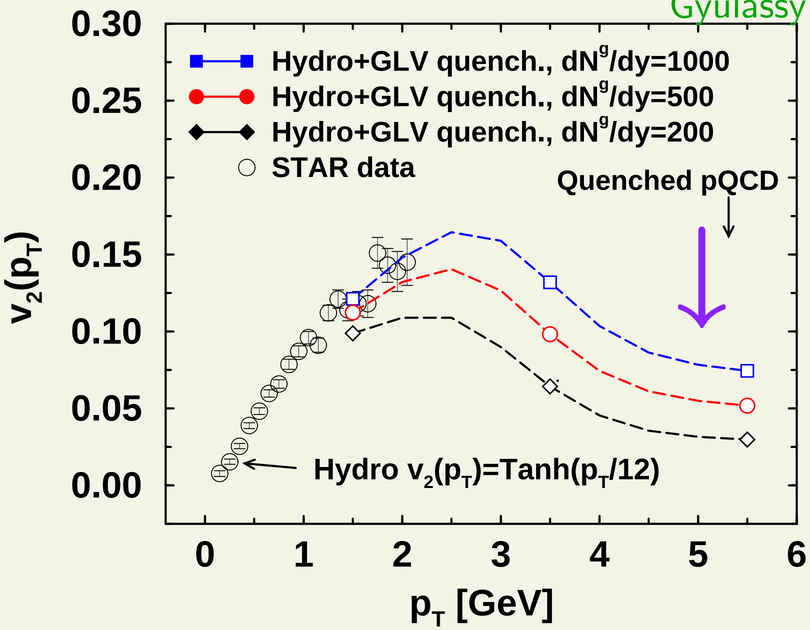
DM & Gyulassy

covariant parton transport



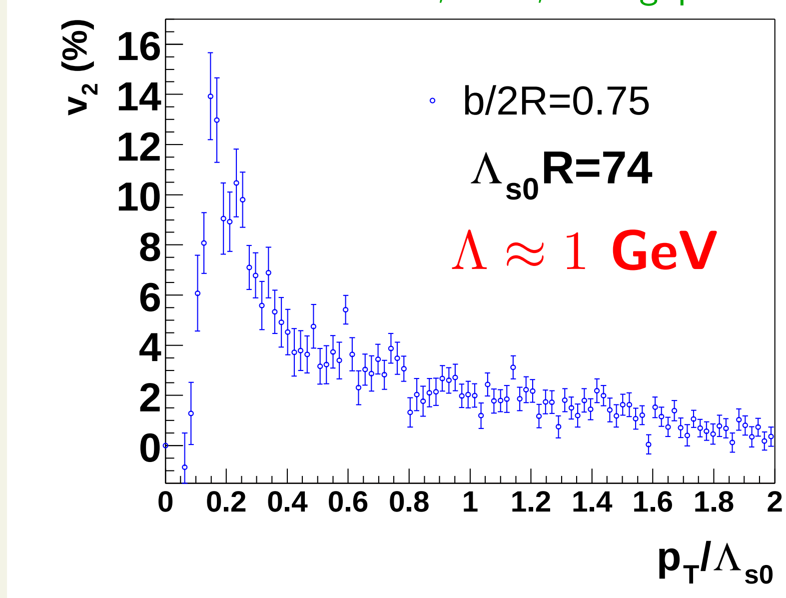
parton energy loss...

Gyulassy & Vitev



classical Yang-Mills ...

Krashnitz, Nara, Venugopalan



Covariant transport

Boltzmann ..., Israel, Stewart, de Groot, ... Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, Xu, Greiner ...

Covariant, causal, nonequil. approach - formulated in terms of local rates.

$$\Gamma_{2\to 2}(x) \equiv \frac{dN_{scattering}}{d^4x} = \sigma v_{rel} \frac{n^2(x)}{2}$$

transport eqn.: $f_i(\vec{x}, \vec{p}, t)$ - **phase space distributions**

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = \overbrace{S_i(\vec{x}, \vec{p}, t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x}, \vec{p}, t)}^{2\to 2\ (\text{ZPC}, \text{GCP}, ...)} + \overbrace{C_i^{inel.}[f](\vec{x}, \vec{p}, t)}^{2\leftrightarrow 3\ (\text{MPC}, \text{Xu}-\text{Greiner})} + ...$$

algorithms: OSCAR code repository @ http://nt3.phys.columbia.edu/OSCAR

HERE: utilize MPC algorithm DM, NPA 697 ('02)

rate is a local and manifestly covariant scalar

for particles with momenta p_1 and p_2:

$$\Gamma(x) = \sigma\, v_{rel}\, n_1(x) n_2(x) = \sigma\, \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}\, n_1(x) n_2(x)$$

(n/E is a scalar)

an equivalent alternative form is $v_{rel} = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}$

[in cascade algorithms, rate is evaluated in the pair c.o.m. frame, where $\vec{v}_1 || \vec{v}_2$ and thus $v_{rel} = |\vec{v}_1 - \vec{v}_2|$]

Example: Molnar's Parton Cascade

Elementary processes: elastic $2 \to 2$ **processes** + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

Equation for $f^i(x, \vec{p})$: $\quad i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$$
\begin{aligned}
p_1^\mu \partial_\mu \tilde{f}^i(x, \vec{p}_1) \quad = \quad & \frac{\pi^4}{2} \sum_{jkl} \int\limits_2 \int\limits_3 \int\limits_4 \left(\tilde{f}_3^k \tilde{f}_4^l - \tilde{f}_1^i \tilde{f}_2^j \right) \left| \bar{\mathcal{M}}_{12\to 34}^{i+j\to k+l} \right|^2 \delta^4(12-34) \quad \swarrow 2 \to 2 \\
& + \frac{\pi^4}{12} \int\limits_2 \int\limits_3 \int\limits_4 \int\limits_5 \left(\frac{\tilde{f}_3^i \tilde{f}_4^i \tilde{f}_5^i}{g_i} - \tilde{f}_1^i \tilde{f}_2^i \right) \left| \bar{\mathcal{M}}_{12\to 345}^{i+i\to i+i+i} \right|^2 \delta^4(12-345) \quad \swarrow 2 \leftrightarrow 3 \\
& + \frac{\pi^4}{8} \int\limits_2 \int\limits_3 \int\limits_4 \int\limits_5 \left(\tilde{f}_4^i \tilde{f}_5^i - \frac{\tilde{f}_1^i \tilde{f}_2^i \tilde{f}_3^i}{g_i} \right) \left| \bar{\mathcal{M}}_{45\to 123}^{i+i\to i+i+i} \right|^2 \delta^4(123-45) \quad \swarrow 3 \leftrightarrow 2 \\
& + \tilde{S}^i(x, \vec{p}_1) \quad \leftarrow \textbf{initial conditions}
\end{aligned}
$$

with shorthands:

$$
\tilde{f}_i^q \equiv (2\pi)^3 f_q(x, \vec{p}_i), \quad \int\limits_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1 + p_2 - p_3 - p_4) \equiv \delta^4(12-34)
$$

Hydrodynamic limit

mean free path:

$$\lambda(x) \equiv \frac{1}{\text{cross section} \times \text{density}(\text{x})}$$

- **Ideal fluid limit** $\lambda \to 0$**: local equilibrium**

$$T^{\mu\nu}_{id} = (e+p)u^\mu u^\nu - p\, g^{\mu\nu}$$

$$\partial_\mu S^\mu = 0 \quad \Rightarrow \text{ entropy conserved}$$

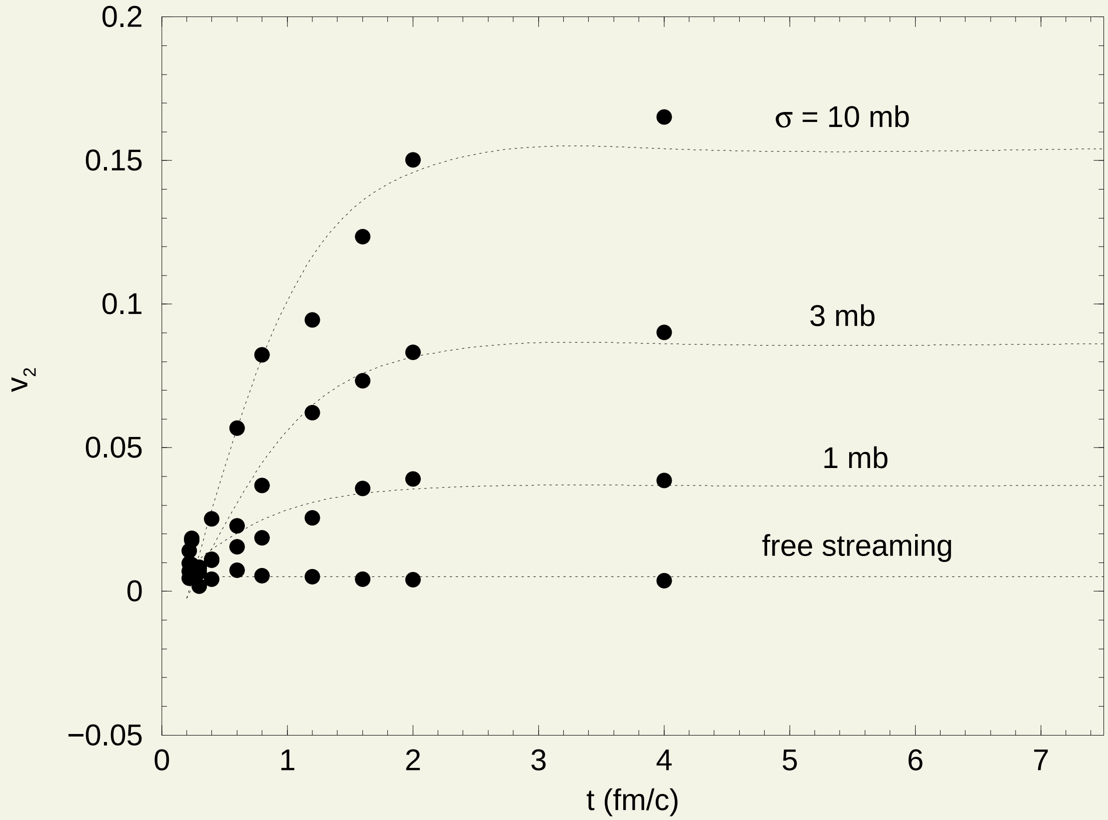
- **Viscous hydro** $\lambda \ll \textit{length \& time scales}$**: near local equilibrium**

dissipative dynamics in terms of transport coefficients and relaxation times

$$e.g., \quad \text{shear viscosity } \eta \approx 0.8 \frac{T}{\sigma_{tr}}\ , \qquad \text{relaxation time } \tau_\pi \approx 1.2 \lambda_{tr}$$

Israel, Stewart ('79) ...

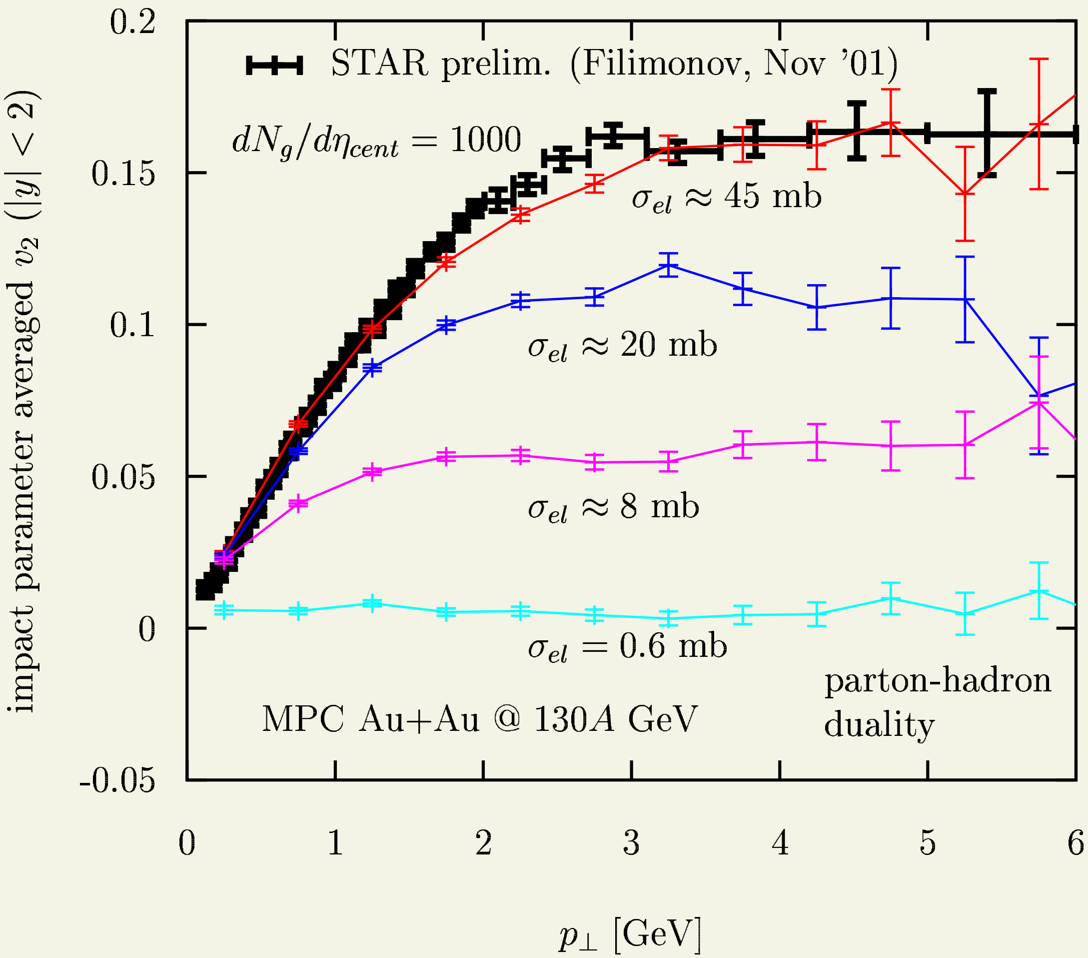
Zhang, Gyulassy & Ko, PLB455 ('99): **ZPC algorithm**



sharp cylinder $R = 5$ fm, $\tau_0 = 0.2$ fm$/c$, $b = 7.5$ fm, $dN^{cent}/dy = 300$

anisotropy increases with cross section, and developes early ($\sim 1 - 2$ fm$/c$)

DM & Gyulassy, NPA 697 ('02): **$v_2(p_T, \chi)$ at RHIC**



parton transport model MPC

diffuse nuclear geometry

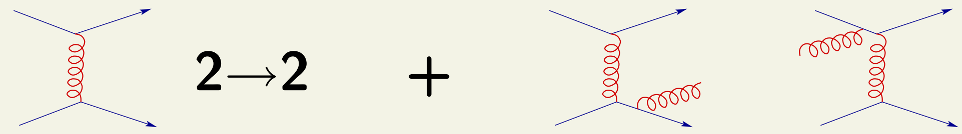
$dN/d\eta$ based on EKRT saturation

Au+Au @ 130 GeV, $b = 8$ fm

- HIJING (minijet+radiation) initconds
- binary transverse profile
- 1 parton $\longrightarrow$ 1 π hadronization

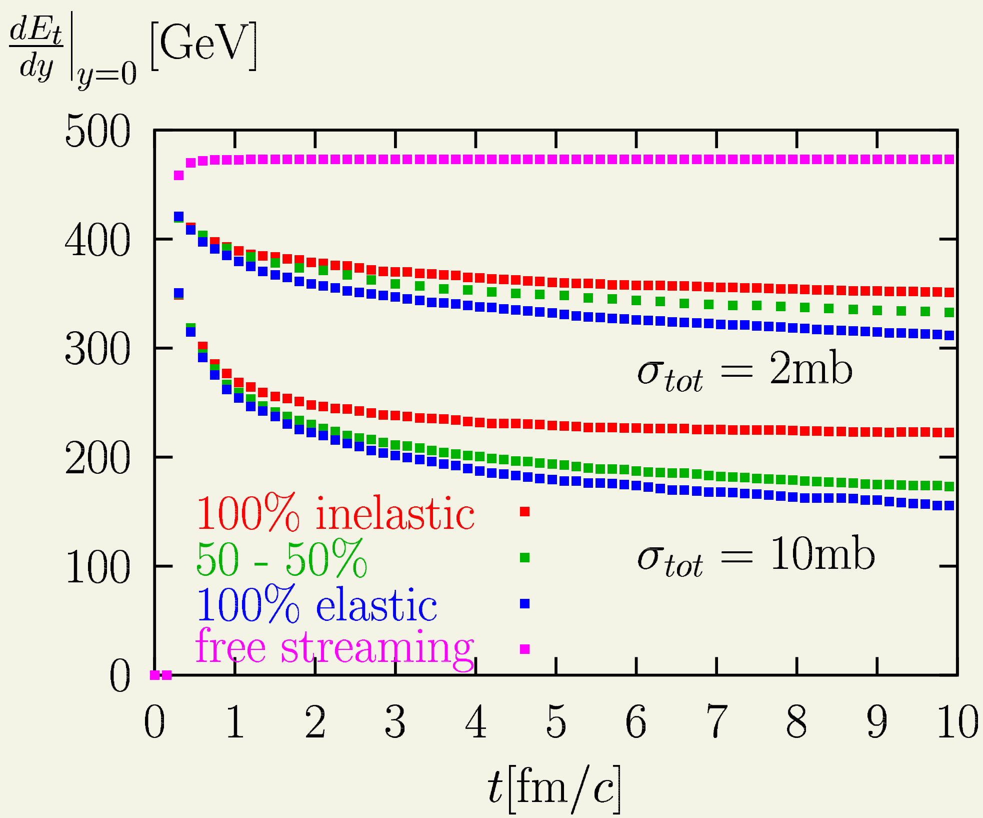
large RHIC v_2: perturbative $2 \to 2$ rates insufficient, need $15\times$ higher

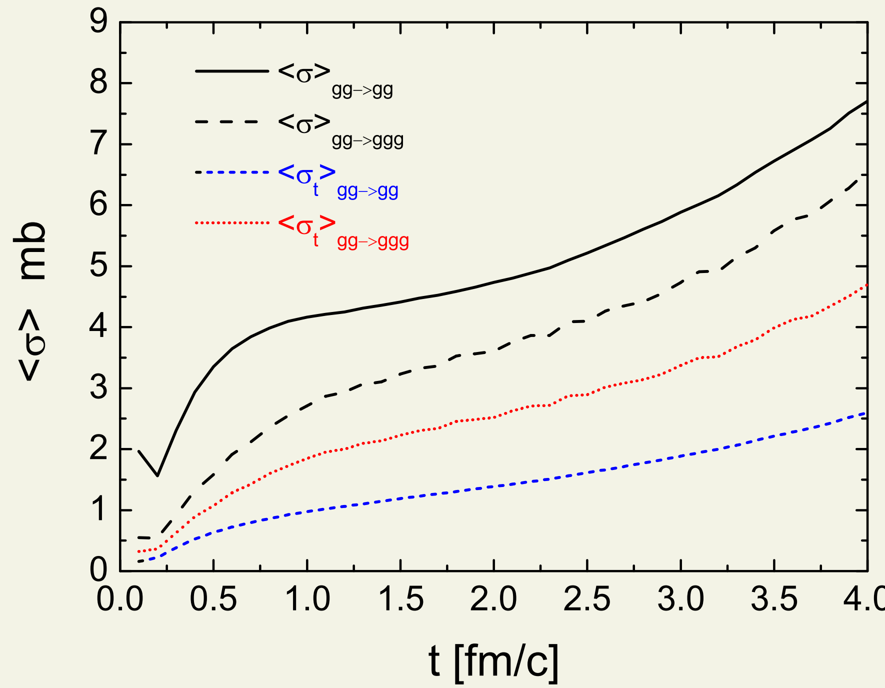
radiative transport:



DM & Gyulassy, NPA 661 ('99): $p\,dV$ **cooling**

Greiner & Xu, PRC71 ('05): **transport xsec**





mainly increase in σ_{tr} matters

about $3\times$ larger with $3 \to 2$

$\Rightarrow$ **big help but likely not enough (need $v_2(p_T)$ results)**

Another important angle in the story of thermalization...

Ein sibenköpfige schlang.

ra an ficta esset, quærere debebat. Mihi cum Erythræo planè commentum artis uidetur. Auriculę, lingua, nasus, facies, toto genere à serpentium natura discrepant. quòd si figmenti author, rerum naturæ (quæ in ipsis etiam monstris plerunq; non undiquaq; degenerat) nō imperitus fuisset, multò artificiosiùs potuisset imponere spectatoribus.

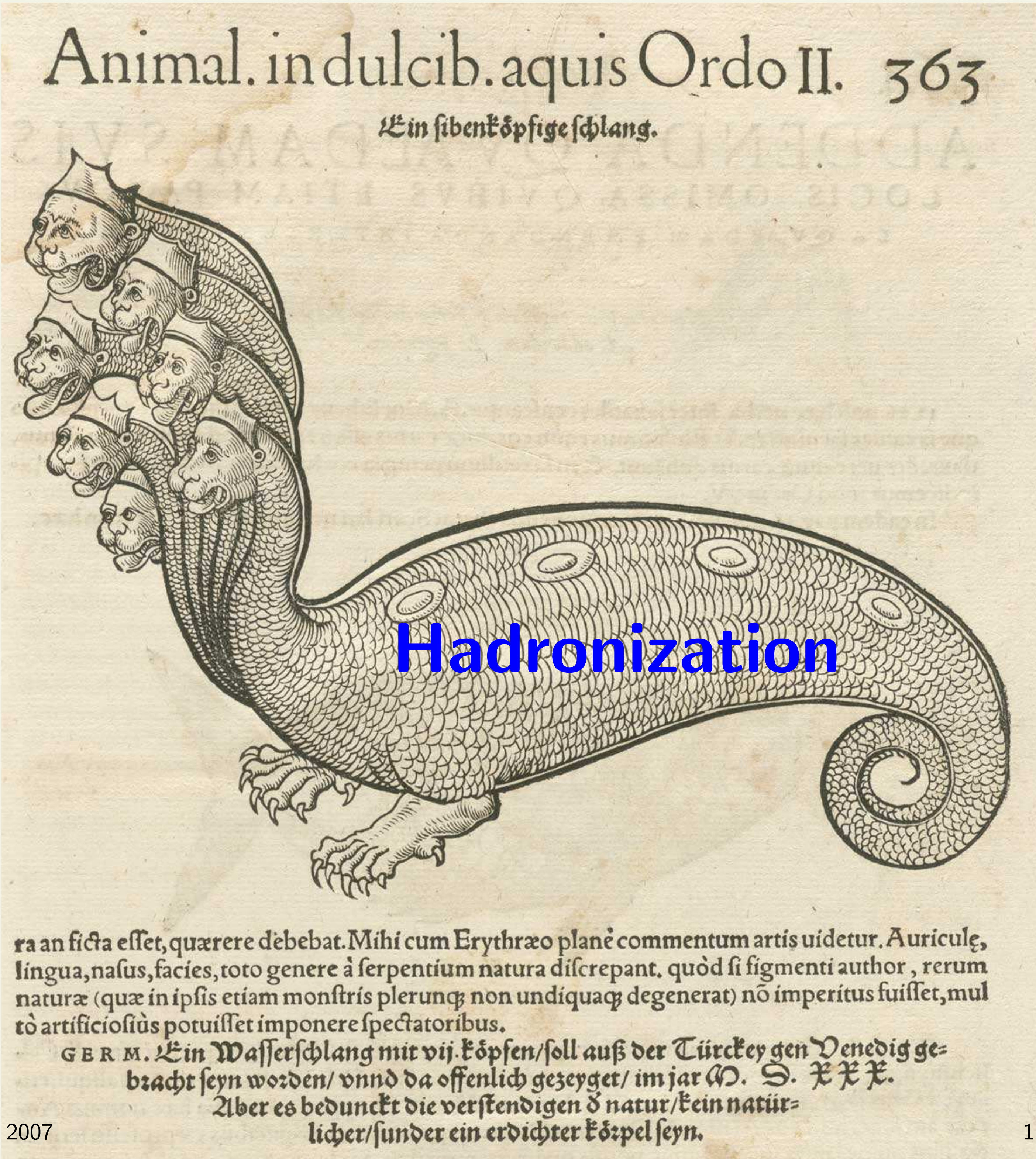
GERM. Ein Wasserschlang mit vij. köpfen/ soll auß der Türckey gen Venedig ge=
bracht seyn worden/ vnnd da offenlich gezeyget/ im jar M. D. XXX.
Aber es beduncket die verstendigen d natur/ kein natür=
licher/ sunder ein erdichter körpel seyn.

Lund model →

indep. frag →

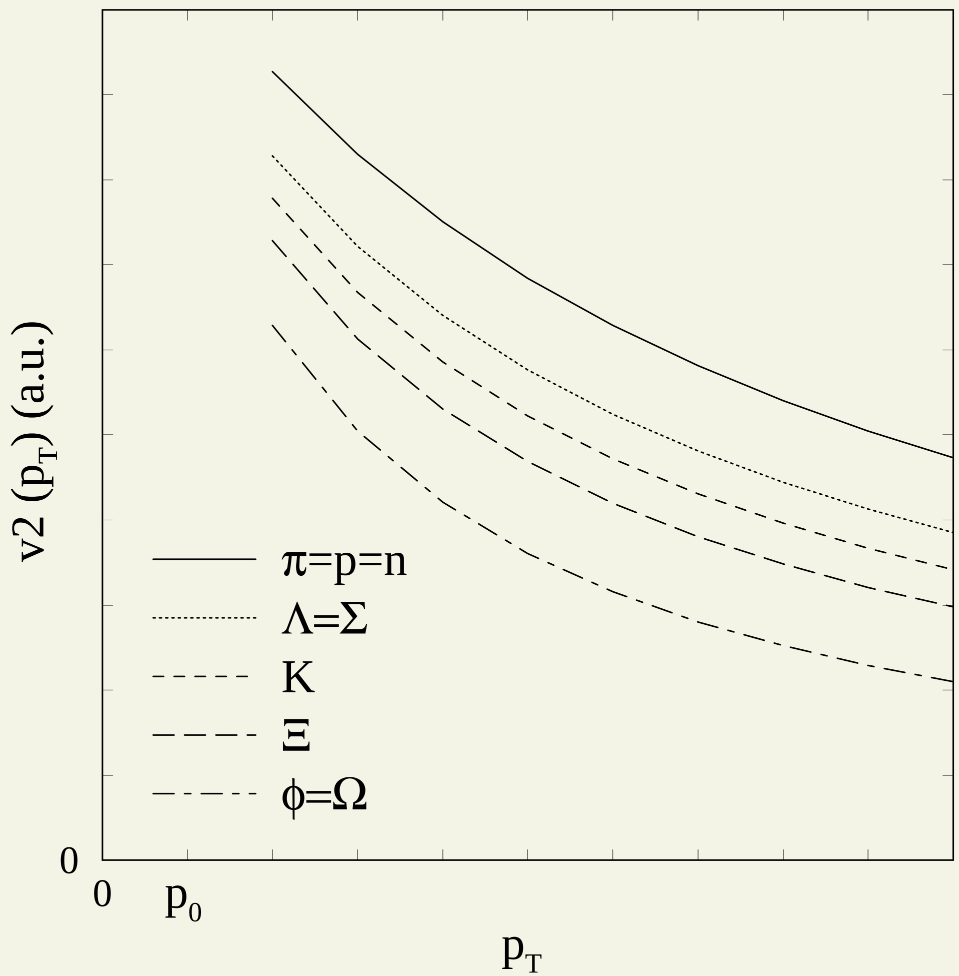
parton-hadron →
duality

coalescence →




MICOR
ALCOR

Ko & Lin, nucl-th/020714 [PLR89 ('02)]: **suggested flavor ordering of elliptic flow**



$q\bar{q} \rightarrow meson$, $qqq \rightarrow baryon$
assuming fast quarks pick up partner(s) at REST(?!)

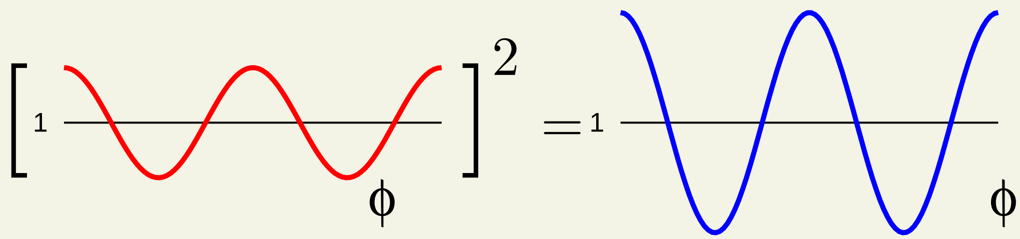
Elliptic flow scaling

coalescence of comoving quarks: $q\bar{q} \Longrightarrow M \qquad 3q \Longrightarrow B$

DM & Voloshin, PRL91 ('03)

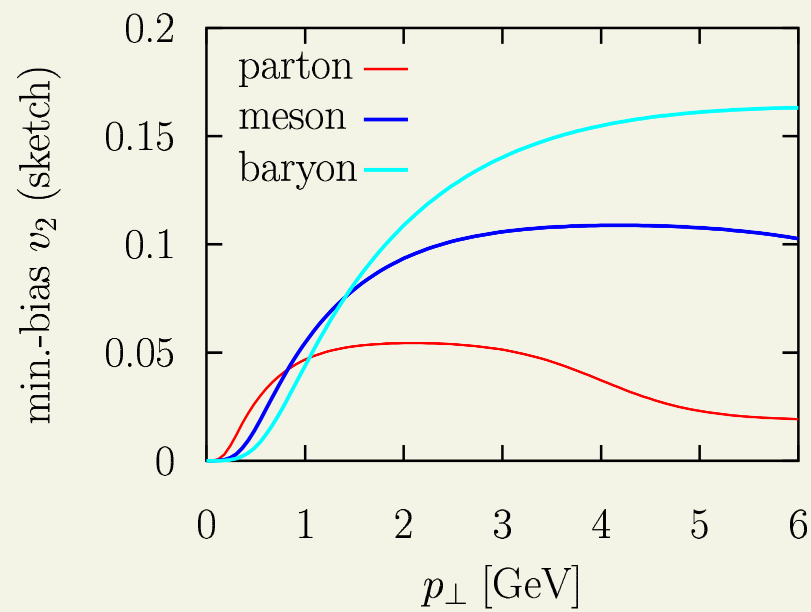
$$\frac{dN_M(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/2)}{d\phi}\right]^2$$

$$\frac{dN_B(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/3)}{d\phi}\right]^3$$



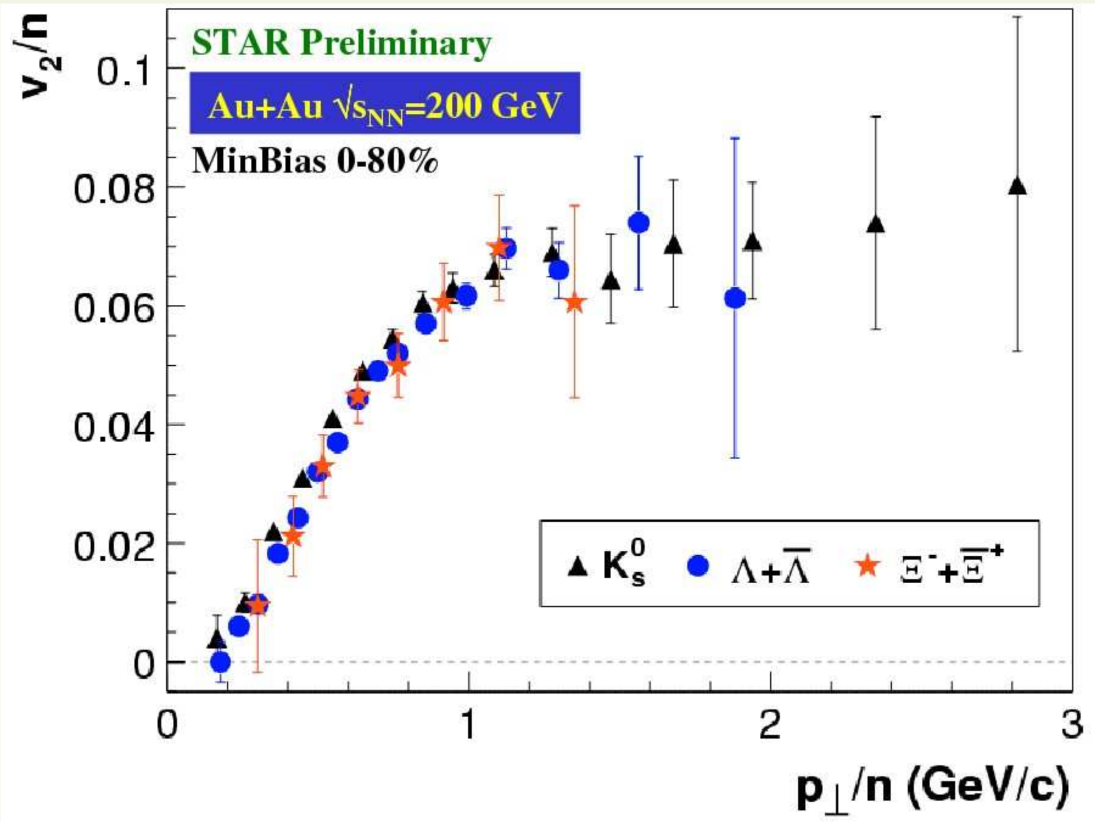
squared/cubed probability → amplified v_2

$$v_2^{hadron}(p_\perp) \approx n \times v_2^{quark}(p_\perp/n)$$

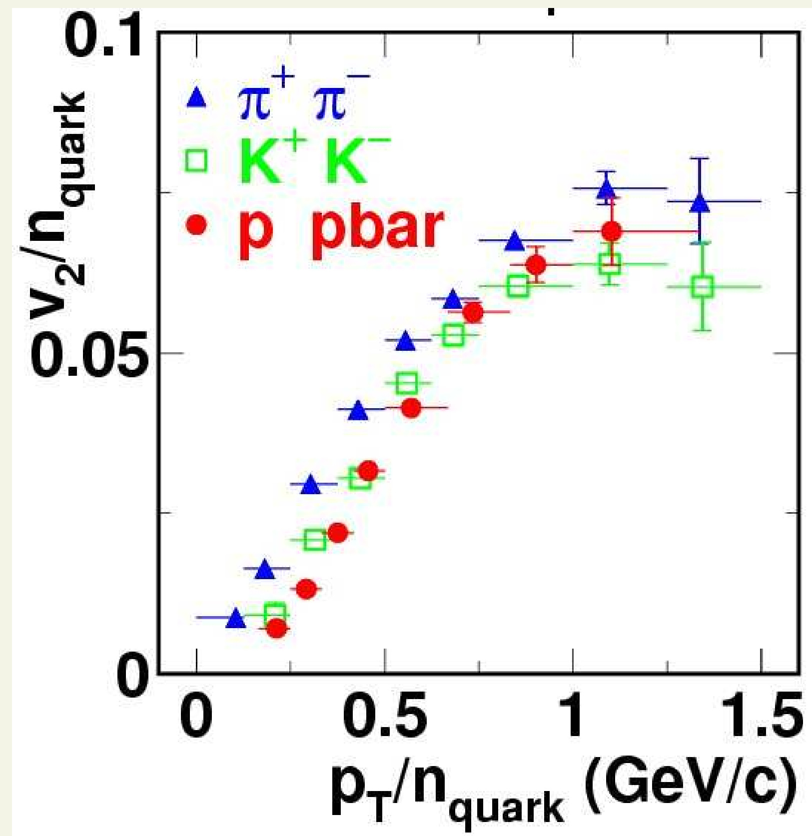


simple but naive DM '04: **ignores space-time, other hadronization channels**

Castillo [STAR] '03



Esumi [PHENIX] '03



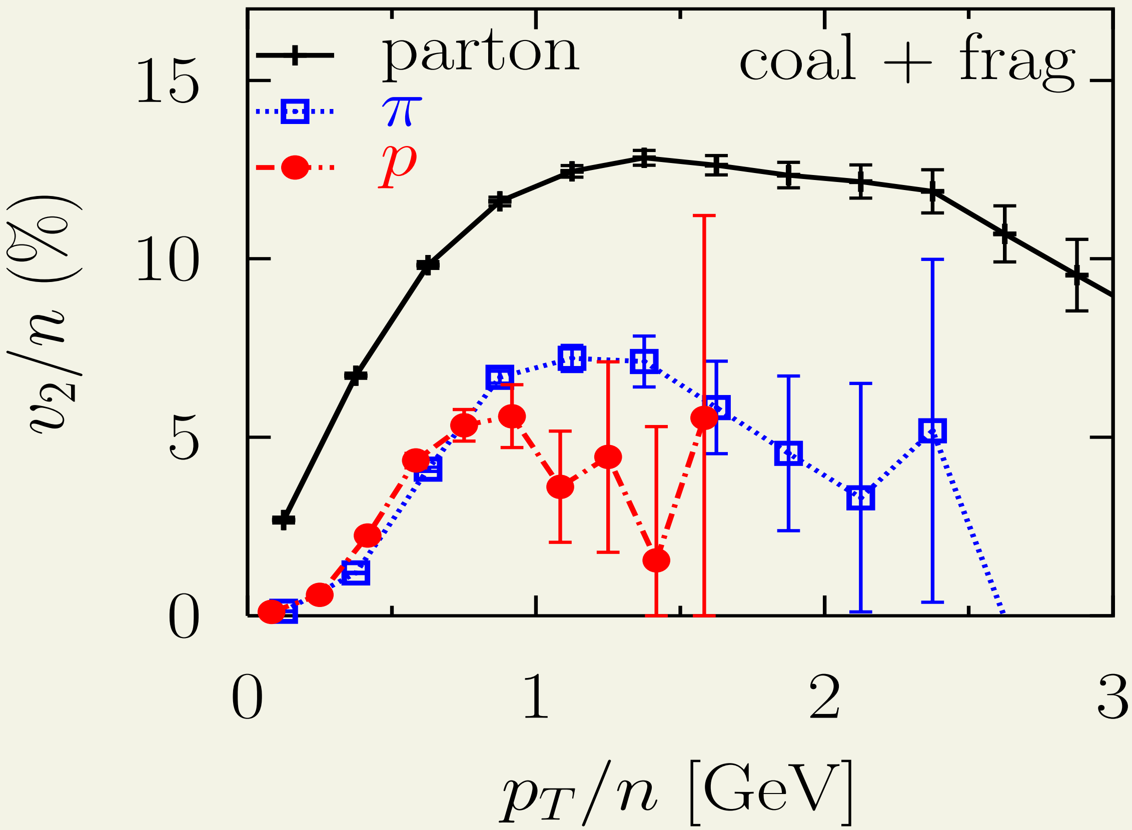
we all love it - simple & works **(not exact)**

coalescence idea very plausible → "must be right"

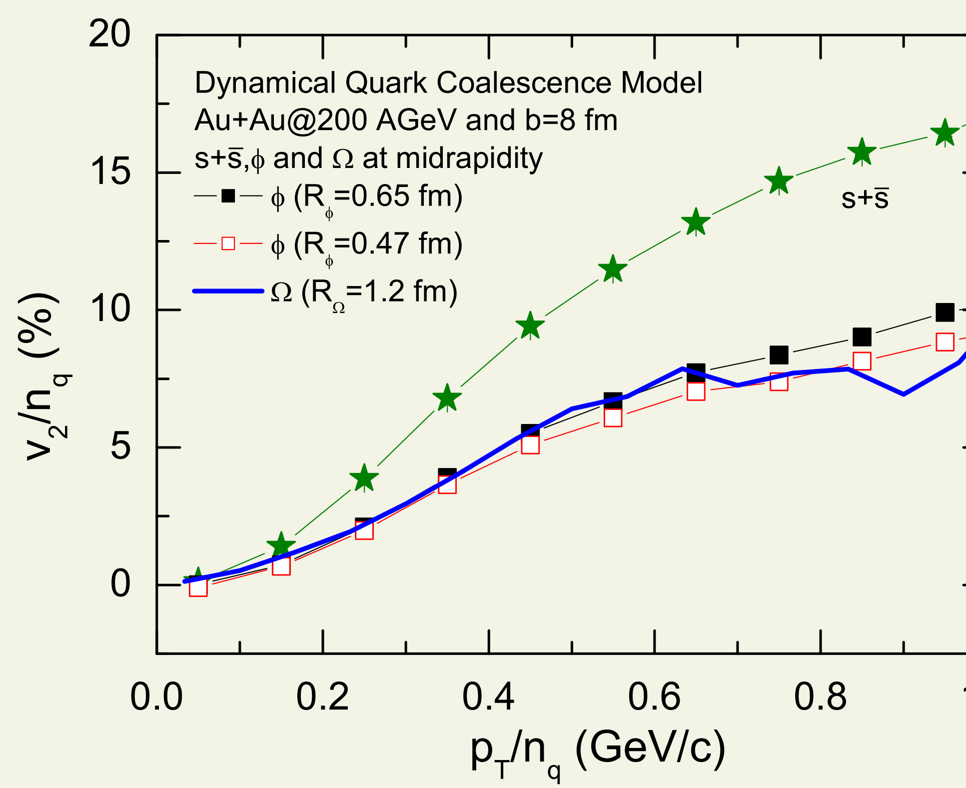
but life is complicated...

dynamical coalescence: scaled $v_2(p_T)$ is roughly half of underlying parton v_2

DM, NPA774 ('06): MPC, coal + frag



Ko et al, PRC73 ('06) AMPT, coal only



Most recent direction:

instead of perturbative dynamics...

study evolution for highest possible scattering rates (quantum limit)

Classical transport rates get arbitrarily large as $\lambda_{MFP} \to 0$

BUT, quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

+ kinetic theory: $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

$$\eta \approx 4/5 \cdot T/\sigma_{tr}$$

$$s \approx 4n$$

gives minimal viscosity: $\eta/s = \frac{\lambda_{tr} T}{5} \geq 1/15$

$\mathcal{N} = 4$ **SYM + gauge-gravity duality:** $\eta/s \geq 1/4\pi$

Policastro, Son, Starinets, PRL87 ('02)
Kovtun, Son, Starinets, PRL94 ('05)

might be a universal lower bound - but general proof lacking

$\Rightarrow$ **no ideal fluids - "most perfect" are those with minimal viscosity**

["most" is crucial - perfect $\equiv$ ideal already since '50s]

two main frameworks for near-equilibrium evolution:

causal viscous hydrodynamics Israel, Stewart; ... Muronga, Rischke; Romatschke et al; Heinz et al...

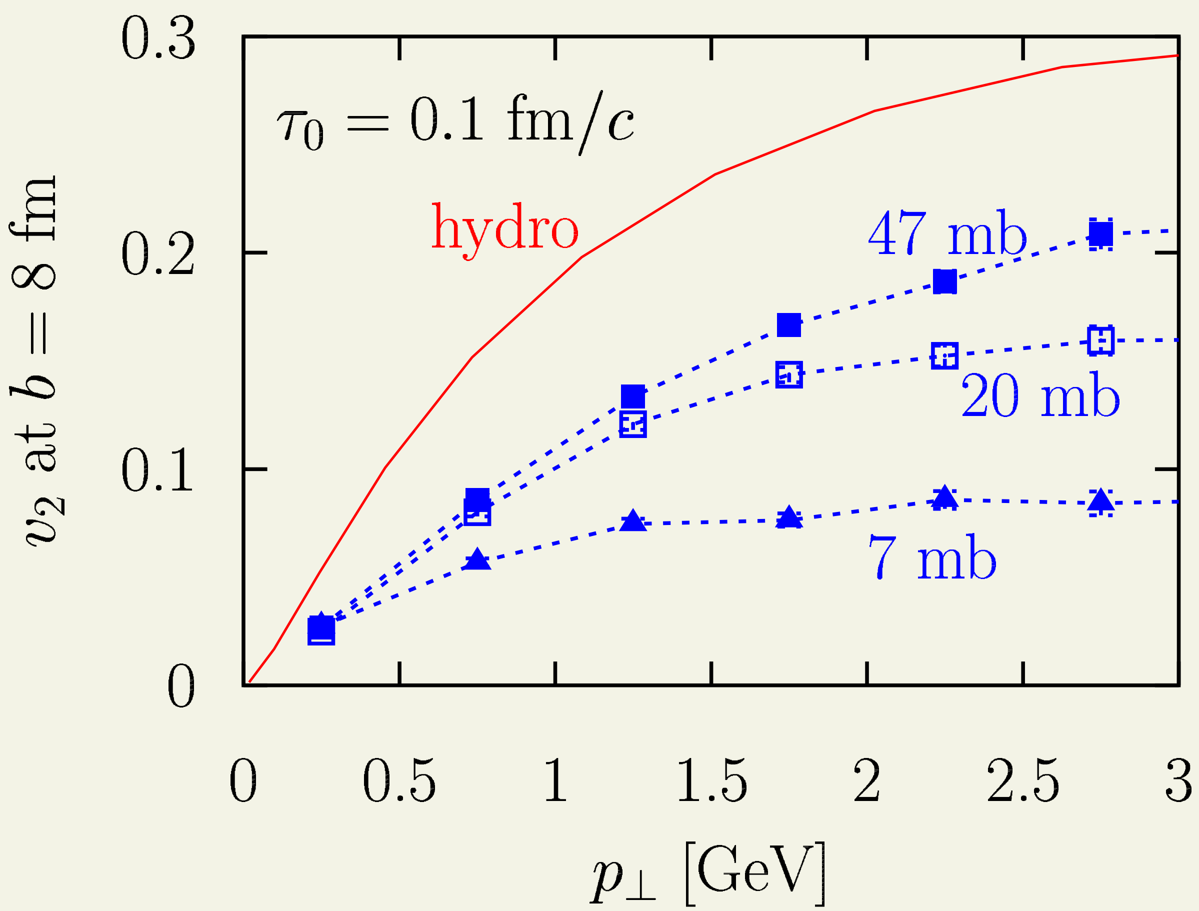
main challenge - acausality and instability

covariant transport DM

much more difficult numerically but fully stable and causal

No, still not ideal fluid

DM & Huovinen, PRL94 ('05): **final $v_2(p_T)$**

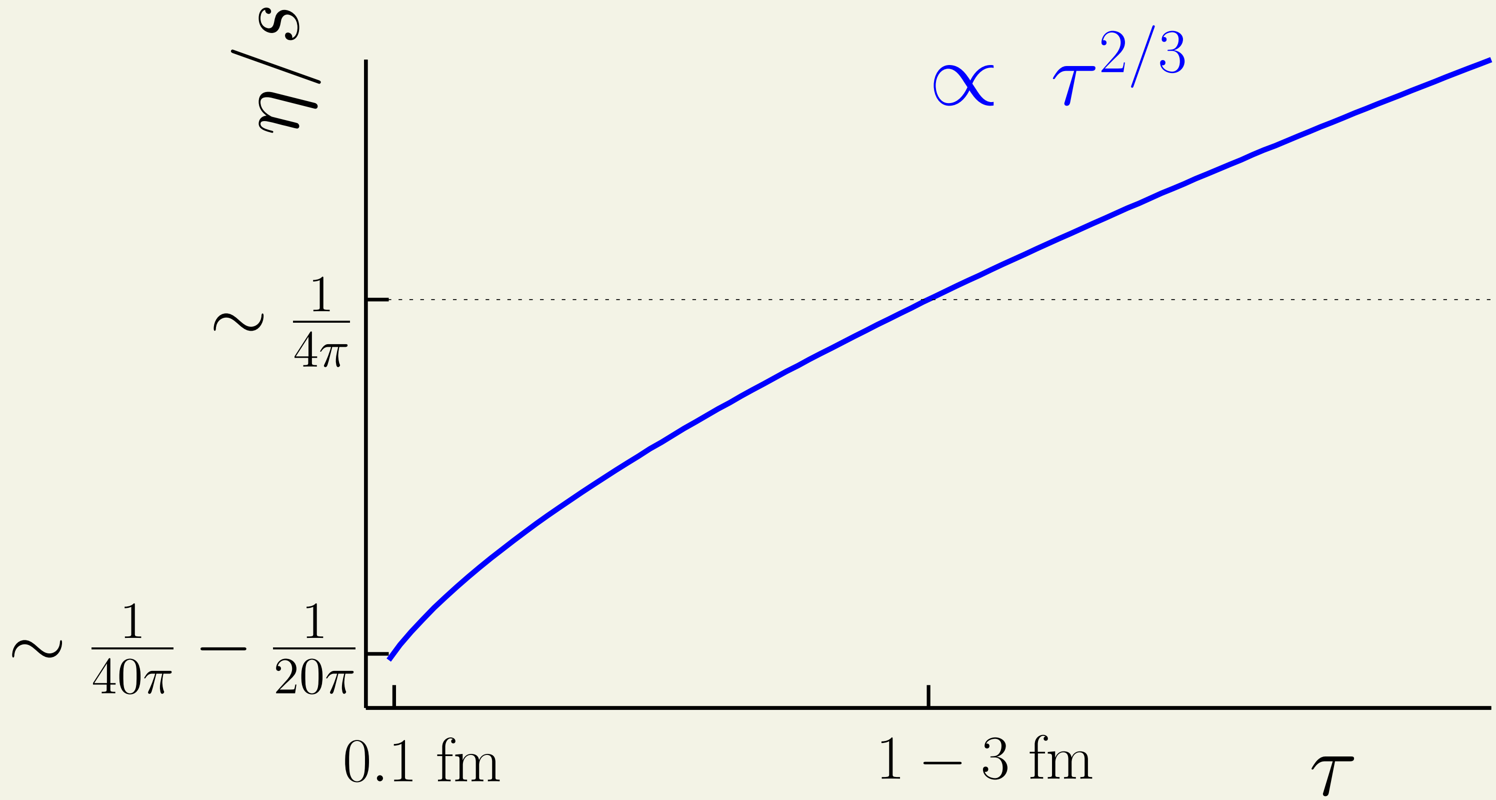


large gradients

$\Rightarrow$ **even a tiny viscosity matters**

[identical RHIC Au+Au initconds, $b = 8$ fm, binary profile, $T_0 = 0.7$ GeV, e=3p EOS]

$\sigma \approx 47$ **mb dynamics corresponds to** $\eta/s \sim \lambda_{tr} T \sim 1/(\sigma T^2)$



initially "better than perfect", after $\tau \sim 1-3$ fm "less than perfect"

$\Rightarrow \eta/s = const$ **needs growing** $\sigma(\tau) \propto 1/T^2 \propto \tau^{2/3}$

η/s for transport

"minimal" viscosity - corresponds to $\lambda_{tr} \approx 1/(3T_{eff}) \approx 0.1$ **fm** at $\tau_0 = 0.1$ fm

estimate from average density: $\lambda_{tr} = \frac{1}{\langle n \rangle \sigma_{tr}}$

for $b = 8$ fm @ RHIC, transport with 47 mb gives

$$\lambda_{tr}(\tau_0) = \frac{\tau_0 A_T}{\sigma_{tr} dN/d\eta} \sim 1 - 2 \times 10^{-2} \text{ fm}$$

estimate from transport opacity χ**:** assuming 1D Bjorken expansion

$$\chi = \int dz\, \rho(z)\sigma_{tr} \sim \int d\tau \rho_0 \frac{\tau_0}{\tau}\sigma_{tr} = \frac{\tau_0}{\lambda_{tr}(\tau_0)} \ln \frac{L}{\tau_0}$$

for $b = 8$ fm @ RHIC, transport with 47 mb gives $\chi \approx 20$

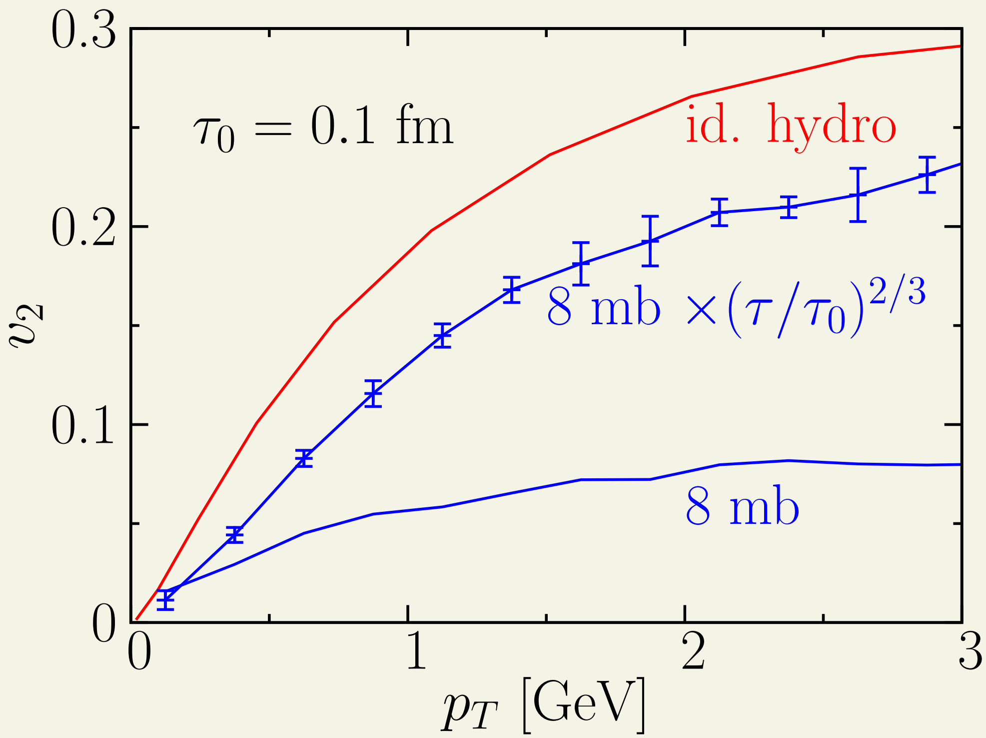
$$\rightarrow \lambda_{tr}(\tau_0) \sim 1.5 - 2 \times 10^{-2} \text{ fm (!)}$$

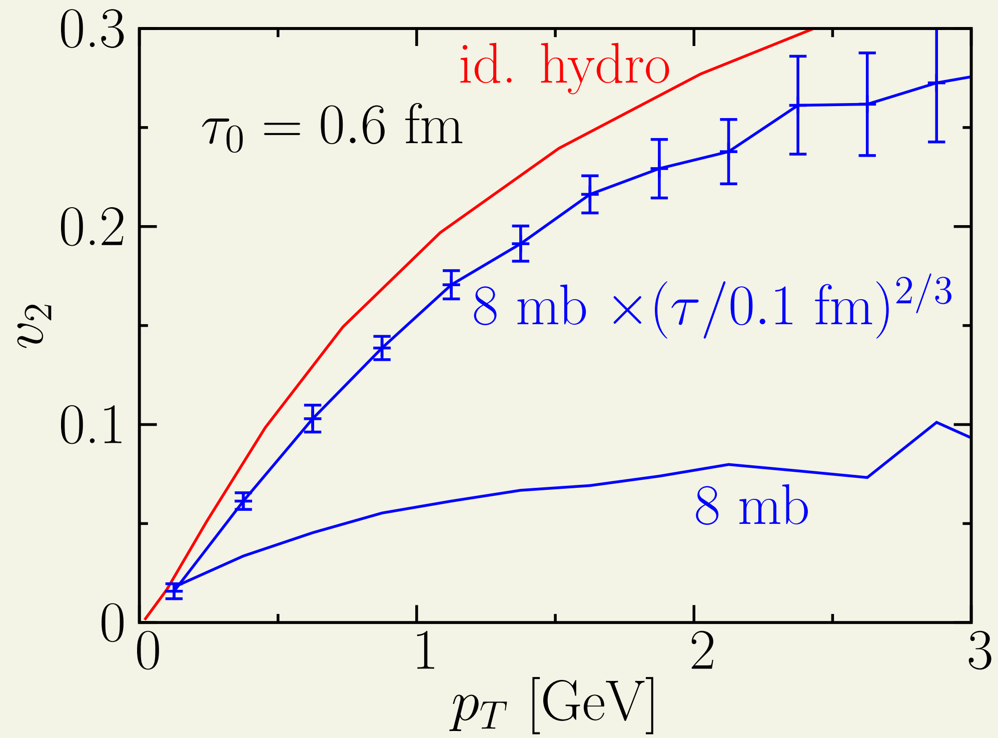
$\Rightarrow \sigma_{gg} \approx 50$ **mb is already better than best-case scenario**

hydro/transport RHIC comparison, now with "minimal viscosity"

$\Rightarrow \sigma_{gg}(\tau = 0.1 \text{ fm}) \sim 4 - 9$ **mb** [4 mb for center of collision zone]

DM '06: $b = 8$ **fm**





$\Rightarrow$ **still $20-30$% drop in v_2 due to dissipation, even at low p_T**

Now apply this at LHC ...

and predict $v_2(p_T)$ for "minimum viscosity" system, i.e., maximal scattering rates

from a transport perspective, there are 3 relevant scales:

$\sigma_{tr} \cdot dN/d\eta$, T_{eff}, **and** τ_0/R

[DM & Gyulassy, NPA697 ('01)]

RHIC vs LHC

I. nuclear geometry identical (gold $\simeq$ lead)

II. larger $dN_{ch}/d\eta \sim 1200 - 2500$, highly uncertain but irrelevant(!)

$\lambda_{tr} \propto \sigma_{tr} \cdot dN/d\eta$ **fixed by minimal viscosity requirement**

III. higher typical momenta

for massless dynamics, momenta scale with initial T_{eff} ($\langle p_T \rangle$, or for saturation model Q_{sat})

provided there are no other scales in the problem

$\Rightarrow$ **universal $v_2(\frac{p_T}{Q_s})$, i.e.,**

$$v_2^{LHC}(p_T) \approx v_2^{RHIC}\left(p_T \frac{Q_s^{RHIC}}{Q_s^{LHC}}\right)$$

estimate Q_s^{RHIC}/Q_s^{LHC} from saturation condition

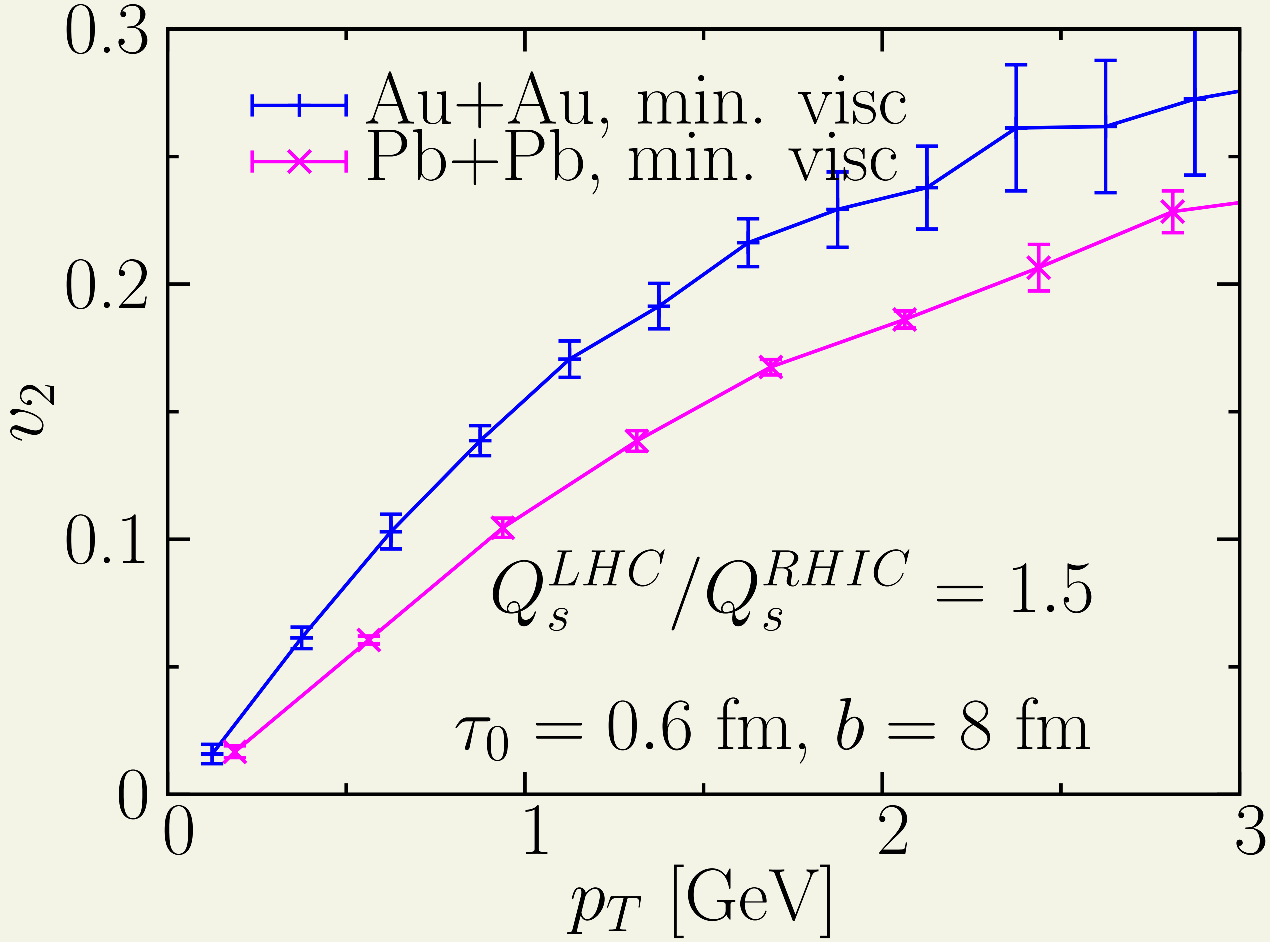
$$Q_s^2 = \frac{2\pi^2}{C_F}\alpha_S(Q_s^2)\ xG(x = \frac{Q_s}{\sqrt{s}}, Q_s^2)\ T_A$$

$\Rightarrow Q_s^{LHC}/Q_s^{RHIC} \approx 1.5$ (central collisions)

refine for $b \neq 0$ with $\langle p_T^2 \rangle$ from k_T-factorized GLR as in Adil et al, PRD73 ('06)

$$\frac{dN_g}{d^2x_T dp_T d\eta} = \frac{4\pi}{C_F}\frac{\alpha_s(p_T^2)}{p_T}\int d^2k_T\, \phi_A(x_1, \vec{p}_1, \vec{x}_T)\, \phi_B(x_2, \vec{p}_2, \vec{x}_T)$$

$\Rightarrow Q_s^{LHC}/Q_s^{RHIC} \sim \sqrt{\frac{\langle p_T^2 \rangle^{LHC}}{\langle p_T^2 \rangle^{RHIC}}} \approx\ 1.3 - 1.5$ **for $b = 8$ fm**



at a given pT, v_2 at LHC will be smaller than at RHIC

in contrast, SPS → RHIC it stayed about same

IV. higher T_{eff} also means higher σ, since $\lambda_{tr} \approx \frac{1}{3T_{eff}}$ quantum bound

i.e., need $v_2(p_T)$ for $1.3 - 1.5\times$ larger σ

$\Rightarrow$ **small $5 - 10$% INCREASE in $v_2(p_T)$ relative to naive scaling**

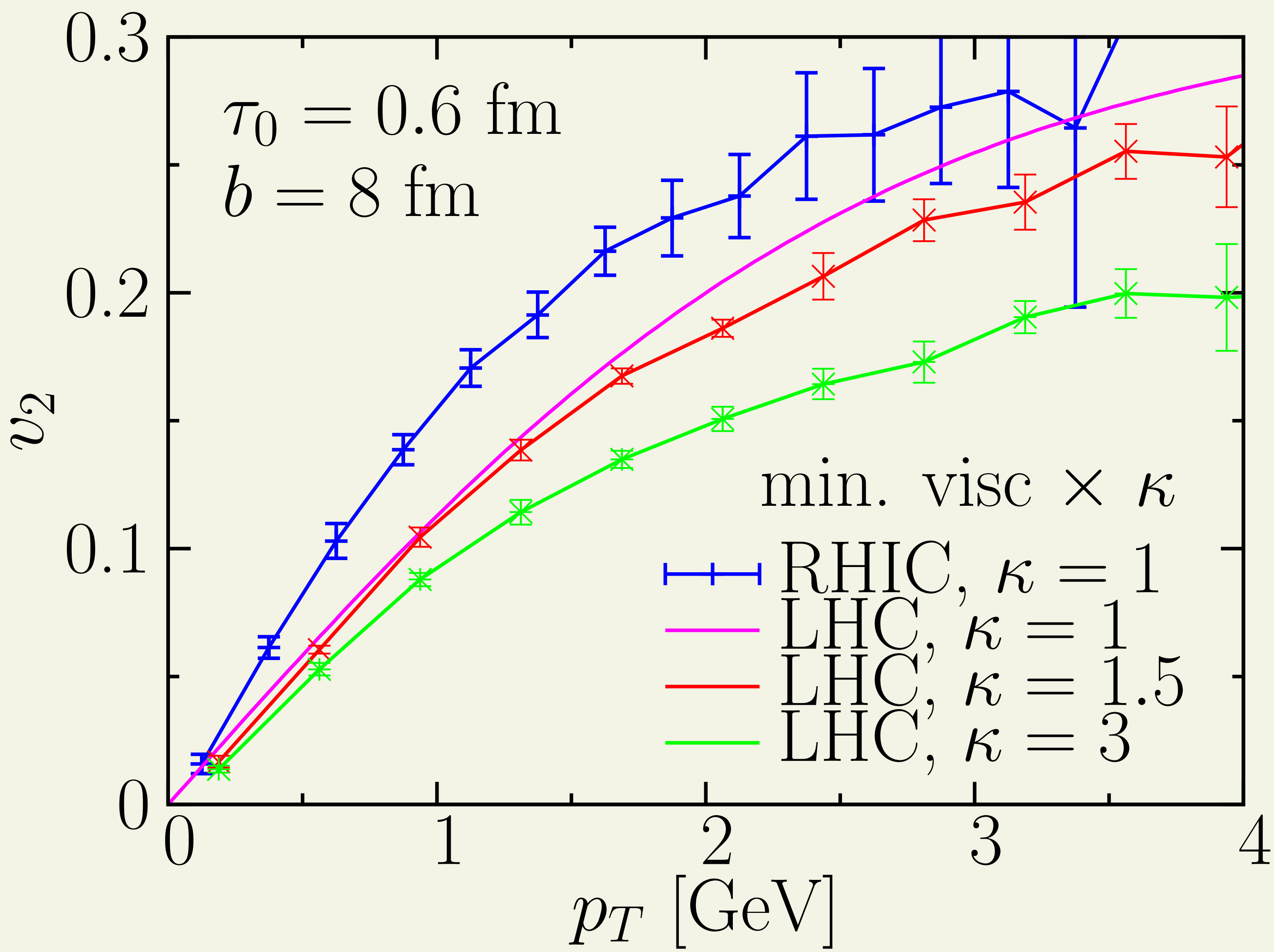
V. higher Q_{set} also (likely) means faster thermalization $\tau_0 \sim 1/Q_s$

involves the last scale τ_0/R - controls interplay between longitudinal and transverse dynamics

DM ('07): **factor 6 decrease in τ_0 gives only about 20% decrease in v_2**

$\Rightarrow$ **rather insensitive, only a few-% effect**

DM ('07): $\eta/s \approx \kappa/(4\pi)$



Conclusions

perturbative rates and large v_2 at RHIC: $2 \to 2$ is insufficient but $3 \leftrightarrow 2$ may work (still open)

there is a $20 - 30$% dissipative reduction of elliptic flow at RHIC even if scattering rates saturate their quantum bounds ("minimal viscosity" $\eta/s = 1/(4\pi)$)

<u>if</u> LHC and RHIC plasma are both "minimally viscous", expect

$$v_2^{LHC,5500}(p_T) \approx v_2^{RHIC,200}(p_T \cdot k)$$

with $k \approx 1.3 - 1.5$ (GLR estimate for $b = 8$ fm).

hadronization is a significant theory uncertainty
- need more great champions to tame it

Open issues

initial geometry (eccentricity ε)

- **gluon saturation models can give $\sim 1.3\times$ larger ε than for binary profile** (depends on model details)

this mainly affects interpretation because $v_2 \sim \varepsilon$ (allows for larger η/s)

missing $3 \leftrightarrow 2$ processes

not a big issue here because our viscosity is FIXED by the entropy. Extra scattering channels decrease η below the quantum bound, unless all cross sections are reduced at the same time.