Jozsó's Legacy:
Chemical and kinetic freeze-out in heavy-ion collisions*

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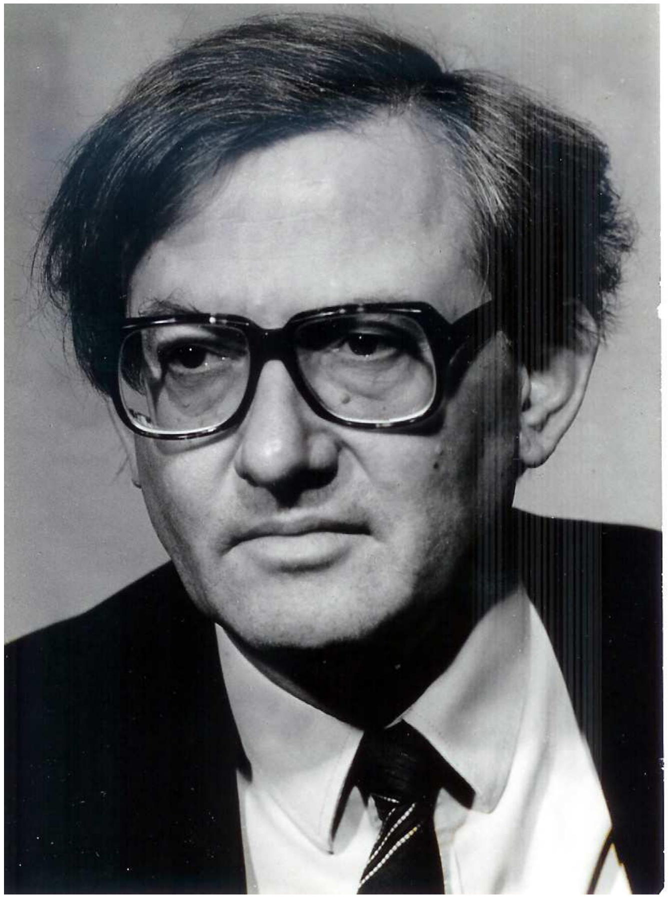
1977: Jozsó's first nuclear theory paper:



J. P. Bondorf, S. I. A. Garpman. J. Zimányi,
"A simple analytical hydrodynamic model for expanding fireballs",
Nuclear Physics A **296** (1978) 320-332

Discovers simple scaling solution of non-relativistic Euler equations for spherically symmetric fireballs with power-law radial density and velocity profiles

Section 3:
"Geometric concept of the break-up":

Really a **dynamic** freeze-out concept: Introduces ideas of competition between "separation velocity" (flow velocity gradients) and thermal velocity and between expansion rate and scattering rate as driving freeze-out.

Key point: thermal velocity and scattering rate depend on particle species
$\Longrightarrow$ **"differential freeze-out"**

1985/86: UH meets JZ in Stony Brook

UH works with Kang Seok Lee and Mark Rhoades-Brown on QGP hadronization (HLR)

JZ works with Nandor Balacz on the same problem

B. Lukácz, J. Zimányi, N.L. Balazs, PLB 183 (1987) 27
C. Greiner, P. Koch, H. Stöcker, PRL 58 (1987) 1825
HLR, Mod. Phys. Lett. A 2 (1987) 153

discover independently strangeness separation in a first-order QGP-HG phase transition at large net baryon density

(Jozsó first mentions analogy with distilling alcohol and coins phrase "strangeness distillation")

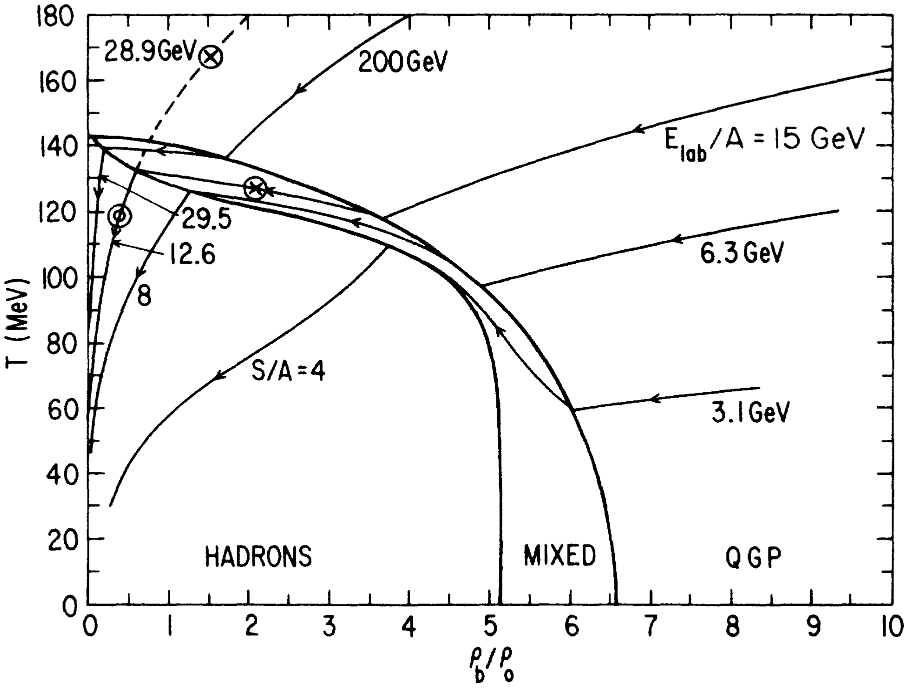
HLR pick up on ideas of S. Nagamiya (PRC 24 (1981) 971) to explain $T_p > T_{K^+} > T_{K^-} > T_\pi$ at the BEVALAC.

Want to use different mean free paths of K^- and K^+ in baryon-rich nuclear matter to see distillation process through different slopes

JZ mentions his 1978 work on kinetic freeze-out in expanding fireball

HLR, "K^+ and K^- slope parameters as a signature for deconfinement atfinite baryon density", PRL 58 (1987) 2292

Discusses interplay of differential freeze-out and radial flow, using BGZ's 1978 hydrodynamic model in $\tau_{\rm exp} = \frac{1}{\partial\cdot u} < \tau_{\rm scatt} = \frac{1}{\rho\langle\sigma v\rangle}$



∼1991: Freeze-out in HICs driven by transverse expansion!

Ekkard Schnedermann: Diploma Thesis 1989, Ph.D. Thesis 1992:

"Global (relativistic) hydrodynamics"

(E. Schnedermann, UH, PRC 47 (1993) 1738; (+J. Sollfrank) PRC 48 (1993) 2462; PRC 50 (1994) 1675)

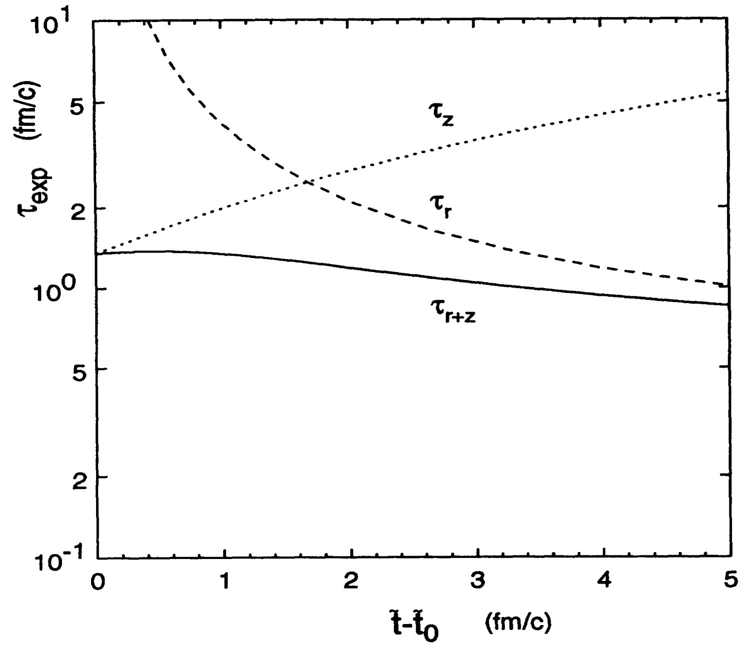


FIG. 6. The expansion time scale (here for the B scenario

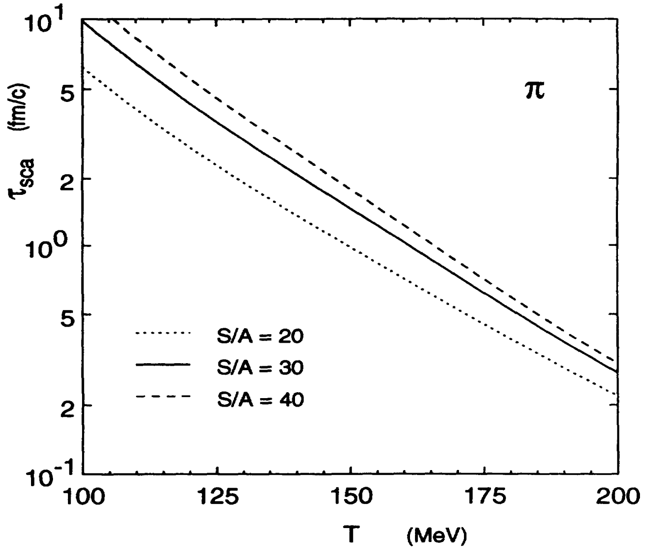


FIG. 7. The scattering time scale of pions in a hadron gas varies with S/A because of the different baryon content. Its exponential decrease at higher temperatures, combined with the near constancy of $\tau_{\rm exp}$, results in freeze-out temperatures

Hubble radius $\sim \tau_{\rm exp}$ of expanding fireball is much smaller than geometric radius
$\Longrightarrow$ freeze-out driven by dynamics (similar to Big Bang) and dominated by transverse expansion.

Steep T-dependence of $\tau_{\rm scatt} \Longrightarrow$ Freeze-out happens at $T_{\rm dec} \approx \text{const.}$

Early 1990's: Differential chemical and thermal freeze-out

Kinetic freeze-out is governed by the competition between local scattering and expansion rates (Bondorf, Garpman, Zimányi 1978):

$$\tau^{(i)}_{\rm scatt} \equiv \frac{1}{\sum_j \rho_j \langle \sigma_{ij} v_{ij} \rangle} = \zeta\, \tau_{\rm exp} \equiv \zeta\, \frac{1}{\partial\cdot u(x)}$$

Here $\zeta = O(1)$ is an unknown parameter, and $\partial\cdot u = \gamma_\perp \left(\frac{1}{\tau} + \nabla_\perp\cdot\boldsymbol{v}_\perp\right) + \left(\partial_\tau + \boldsymbol{v}_\perp\cdot\nabla_\perp\right)\gamma_\perp$ is the local expansion rate, computed from the hydrodynamic flow profile.

Chemical freeze-out $\leftrightarrow \sigma^{\rm inel}_{ij}$

Thermal freeze-out $\leftrightarrow \sigma^{\rm tot}_{ij}$

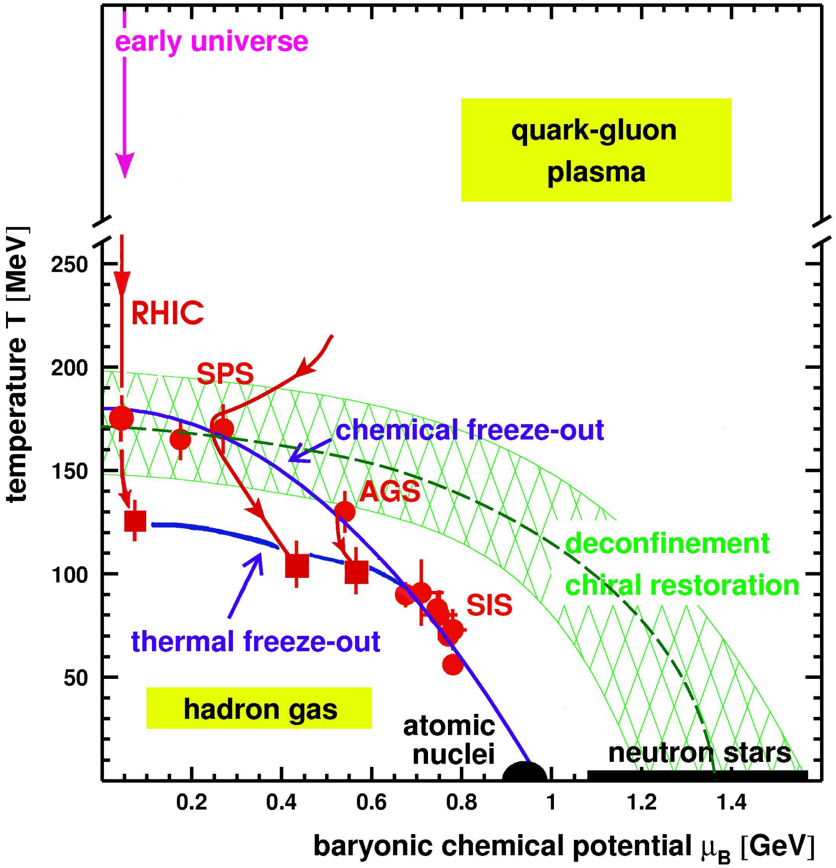
$\sigma^{\rm inel}_{ij} < \sigma^{\rm tot}_{ij} \Longrightarrow$ chemical processes decouple earlier

$\Longrightarrow$ abundances freeze out earlier than momentum spectra

$\Longrightarrow$ $T_{\rm chem} > T_{\rm therm}$! (UH, NPA 566 (1994) 563; NATO ASI Series B346 (1995) 413)

The scattering rate is particle specific. Different hadrons should freeze out at different temperatures. Differential thermal freeze-out is difficult to implement in hydrodynamics (loss terms?). Fortunately, usually one species dominates (N at low $\sqrt{s}$, π at high $\sqrt{s}$) whose freeze-out triggers all others.

2000: ~ 15 years of ultra-relativistic heavy-ion experiments
Chemical and thermal freeze-out vs. the QCD phase transition



Observations:

Different chemical and kinetic freeze-out temperatures (Cleymans & Redlich, Dobler & Heinz, Tomašik et al.)

For $\sqrt{s} \gtrsim 10\,A\,\mathrm{GeV}$, same "universal" chemical freeze-out temperature observed in e^+e^-, pp, $p\bar{p}$, and $A+A$ collisions (only difference: level of strangeness saturation) (Becattini et al.)

For $\sqrt{s} \gtrsim 10\,A\,\mathrm{GeV}$, $T_{\rm chem}$ agrees with T_c from lattice QCD

Hadronic cascades (RQMD, URQMD, . . .) show that hadronic rescattering alters momentum distributions and resonance populations, but not stable hadron yields (Bass et al., Bravina et al.)

$\Longrightarrow$ $T_{\rm chem}$ reflects statistical hadron production from a prehadronic medium, measures ε_c (Becattini & Heinz 1997, Stock 1999)

The controversy

Camp I:

(Becattini & UH, ZPC 76 (1997) 269;
UH, NPA 661 (1999) 140;
Stock PLB 456 (1999) 277)

- Hadron production = statistical process associated with a phase transition
- No inelastic reactions among hadrons – momentum distributions evolve through quasi-elastic resonance scattering
- $T_{\rm chem}$ = Lagrange multiplier reflecting critical energy density of the hadronization phase transition via Maximum Entropy Principle
- $T_{\rm chem}$ and $T_{\rm kin}$ are conceptually different – $T_{\rm kin}$ characterizes a kinetic freeze-out process but $T_{\rm chem}$ does not

Camp II:

(C. Greiner & Leupold, JPG 27 (2001) L95;
Braun-Munzinger, Stachel, Wetterich, PLB 596
(2004) 61; C. Greiner et al., JPG 31 (2005) S725)

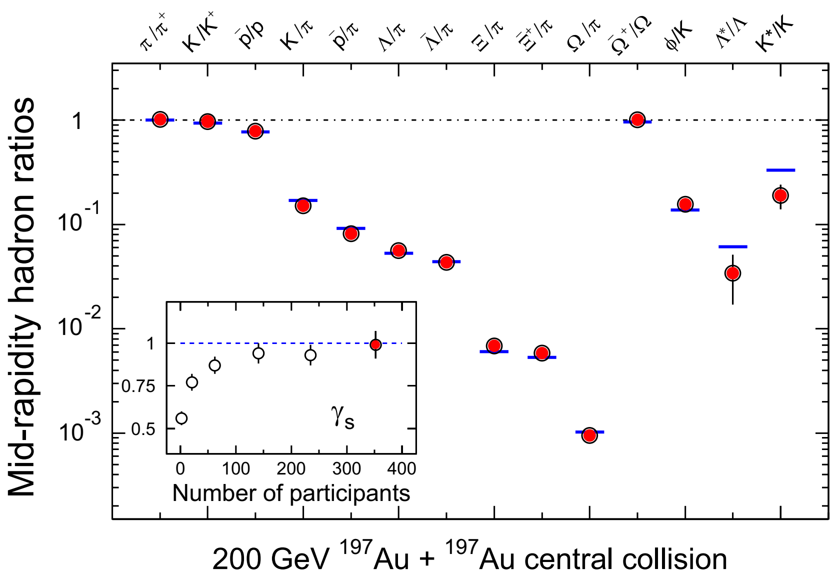
- Hadron freeze-out = kinetic process within the hadronic phase
- Hadrons driven into chemical equilibrium by inelastic multi-hadron processes and frozen out by global expansion
- $T_{\rm chem}$ = real temperature at which forward and backward chemical reactions last balance each other
- $T_{\rm chem}$ and $T_{\rm kin}$ are conceptually identical, but numerically differ due to difference between inelastic and total scattering cross sections

This issue can be resolved!

Chemical Freeze-out at $T_{\rm had} \simeq 170\,$MeV

Au+Au @ 200 A GeV

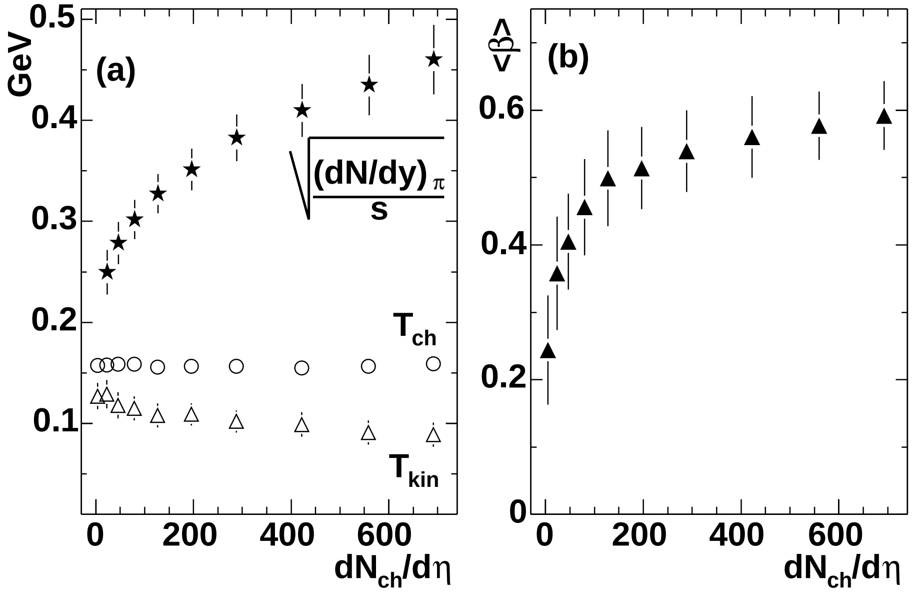
STAR Coll., NPA 757 (2005) 102



Abundance ratios of stable hadrons decouple in maximum entropy state of "apparent chemical equilibrium" with $T_{\rm chem} = 163{\pm}4\,$MeV $\simeq T_{\rm had}$, $\mu_B = 24{\pm}4\,$MeV, and $\gamma_s({\rm central}) = 0.99{\pm}0.07$.

$T_{\rm chem}$ insensitive to expansion rate:

STAR Coll., PRL 92 (2004) 112301



- Radial flow $\langle\beta\rangle$ increases with centrality
- $T_{\rm kin}$ decreases with centrality
- γ_s increases with centrality, approaching 1 in central collisions
- $T_{\rm chem}$ independent of centrality!

Can we understand the different centrality dependences?

Kinetic freeze-out in an expanding system

Kinetic freeze-out is governed by the competition between local scattering and expansion rates (Bondorf, Garpman, Zimányi 1978):

$$\frac{1}{\tau_{\mathrm{scatt}}} = \xi \, \frac{1}{\tau_{\mathrm{exp}}} = \xi \, \partial \cdot u(x)$$

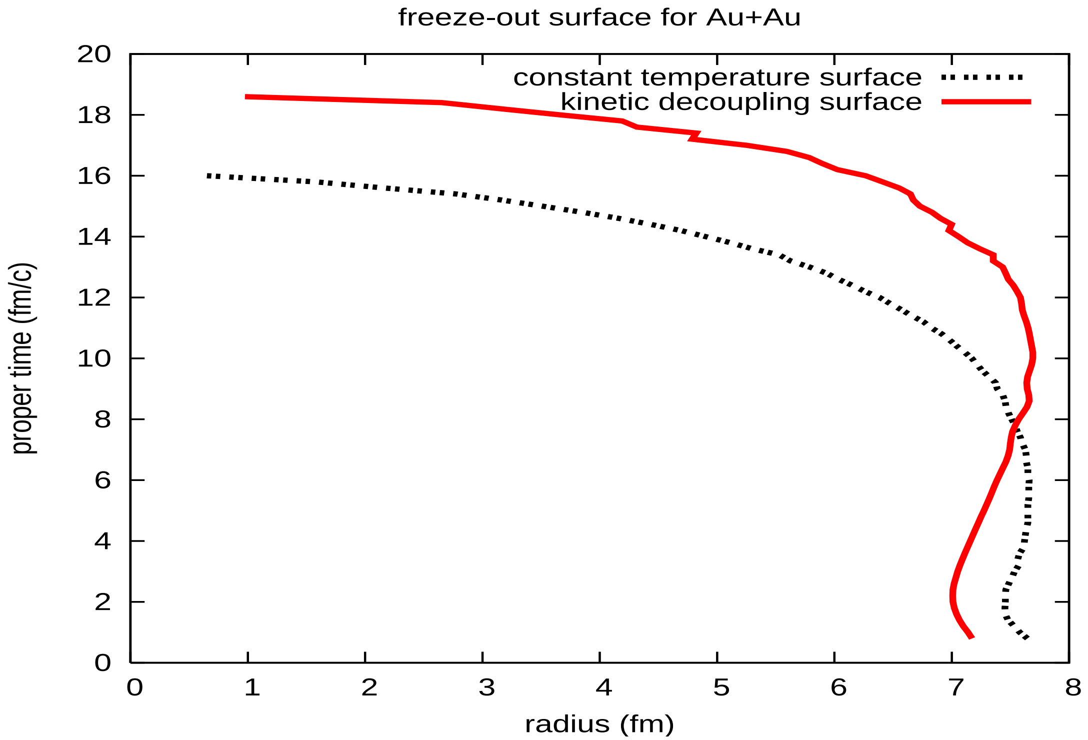
Here $\xi = \zeta^{-1} = O(1)$ is an unknown parameter, and $\partial{\cdot}u = \gamma_\perp \left(\frac{1}{\tau}+\nabla_\perp{\cdot}\boldsymbol{v}_\perp\right) + \left(\partial_\tau+\boldsymbol{v}_\perp{\cdot}\nabla_\perp\right)\gamma_\perp$ is the local expansion rate, computed from the hydrodynamic flow profile.

At RHIC the fireball is pion dominated. Let's assume that all hadrons decouple when pions freeze out, and take the pion scattering rate from Hung & Shuryak (PRC 57 (1998) 1891), parametrized as

$$\frac{1}{\tau^{\pi}_{\mathrm{scatt}}} = (59.5\,\mathrm{fm}^{-1}) \left(\frac{T}{1\,\mathrm{GeV}}\right)^{3.45}$$

Kinetic freeze-out surface

$b = 0$ Au+Au @ 200 A GeV, $\xi^{-1} = 2.9$



For $\xi^{-1} = 2.9$, the kinetic freeze-out surface at $b = 0$ has an average temperature $\langle T_{\rm kin}\rangle = 115$ MeV.

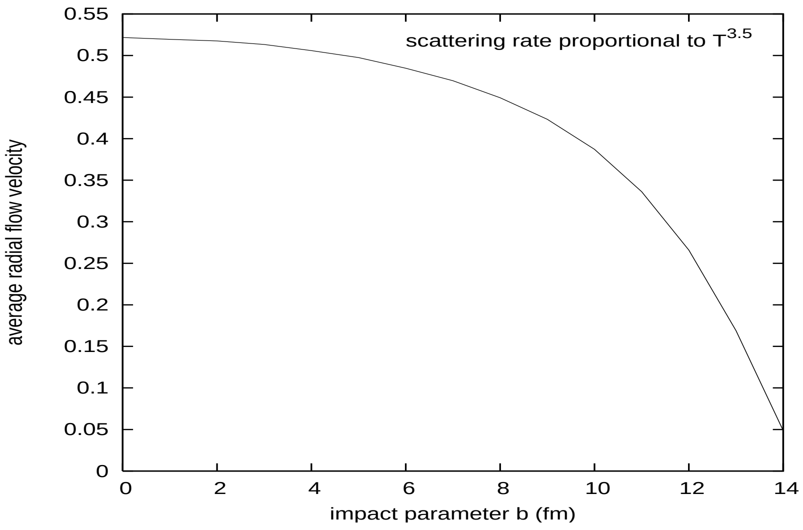
The temperature is higher near the edge where the expansion is larger.

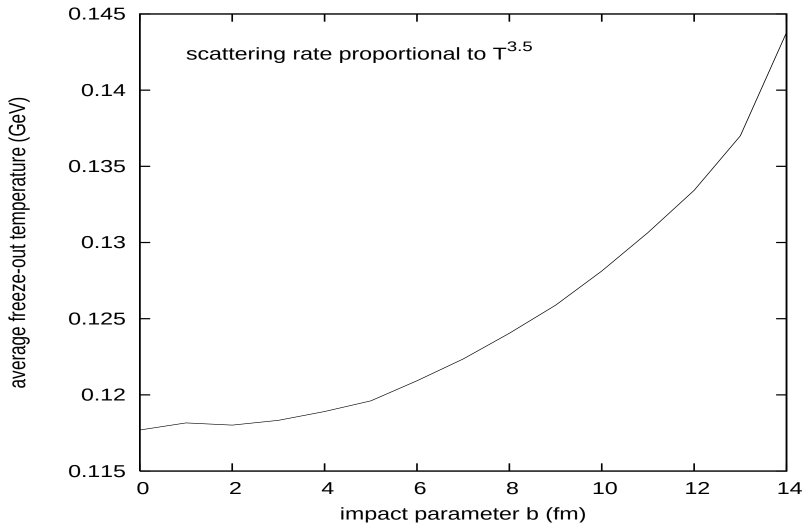
In the interior, the expansion rate and temperature are smaller, and it takes longer to reach freeze-out than for a constant-T surface with $T = \langle T_{\rm kin}\rangle = 115$ MeV.

For the same ξ, we can now predict how freeze-out changes with impact parameter:

Impact parameter dependence of kinetic freeze-out

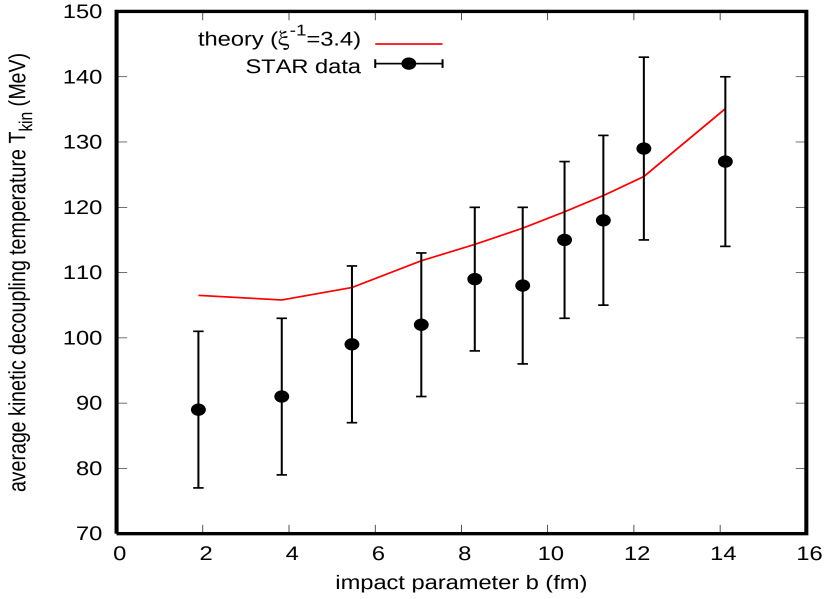
Au+Au @ 200 A GeV, $\xi^{-1} = 2.9$





Qualitative agreement with STAR data, but $\langle\beta\rangle$ is a bit low while $\langle T_{\rm kin}\rangle$ is a bit high.

Fine-tune $\xi^{-1} \to 3.38$:



Chemical decoupling as a kinetic freeze-out process??

Some inelastic reactions that control chemical equilibration are:

$$\pi + N \leftrightarrow K + Y$$

$$\pi + Y \leftrightarrow \bar{K} + N$$

$$\Omega + \bar{K} \leftrightarrow \Xi + \pi$$

$$\Omega + \pi \leftrightarrow \Xi + K$$

$$\Omega + \bar{N} \leftrightarrow 2\pi + 3K$$

$$N + \bar{N} \leftrightarrow 5\pi$$

$$K + \bar{K} \leftrightarrow \phi + \pi$$

$$\pi + \pi \leftrightarrow K + \bar{K}$$

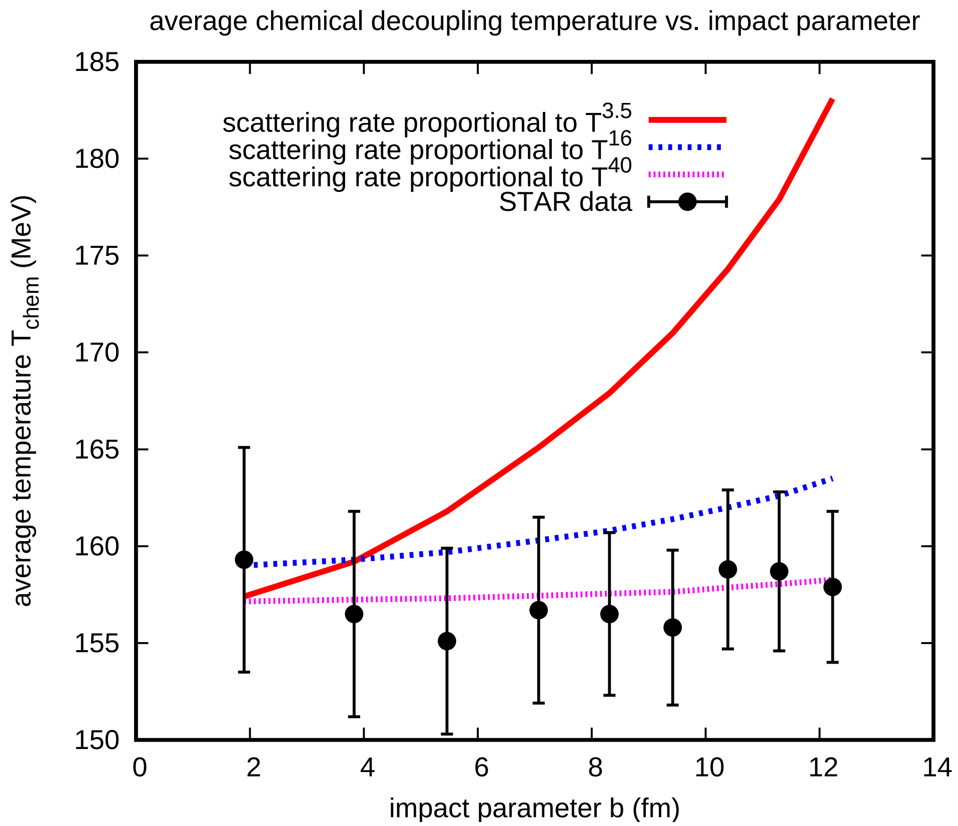
Rates for processes with n_{in} ingoing hadrons are $\sim \Pi_{i=1}^{n_{\mathrm{in}}} n_i(T)$ where $n_i(T)$ grows with T at least as T^3.

Particle yields for hadrons requiring collisions of many abundantly available particles for their production/destruction $(\bar{p}, \Omega, \dots)$ thus tend to freeze out at higher T than particle yields for hadrons whose abundances can be changed by two-body reactions $(\pi, K, \phi, \dots)$.

In an expanding, cooling system, simultaneous freeze-out of all hadron yields at a common temperature requires a conspiracy of rates with widely differring T-dependences.

The punchline:

It is impossible to maintain such a conspiracy over the entire impact parameter range:



To reproduce the STAR chemical decoupling data, all inelastic scattering rates must grow with T as T^n with $n \gtrsim 20$!

This is impossible to understand within a hadronic rescattering picture.

Conclusions:

Jozsó's Legacy: Kinetic decoupling processes in a hydrodynamically expanding fireball necessarily lead to impact parameter dependent decoupling temperatures.

While the observed impact parameter dependence of the average temperature and flow at thermal freeze-out can be quantitatively understood as a kinetic decoupling process in a hydrodynamically expanding source, the measured impact parameter independence of $T_{\rm chem}$ can not.

The only consistent interpretation of the STAR hadron yield data is that the hadrons are "born into chemical equilibrium" (Stock) in an environment that is too dilute and expands too fast to allow for any further inelastic hadronic reactions.

$T_{\rm kin}$ and $T_{\rm chem}$ thus stand on conceptually different footings. $T_{\rm chem}$ is a Lagrange multiplier related by the Maximum Entropy Principle to the critical energy density ε_c for hadronization.

Its universality in e^+e^-, pp, and AA collisions of all centralities shows that at ε_c a phase transition occurs. Hadrons are formed during this transition in a statistical process subject to the Principle of Maximum Entropy.

The absence of inelastic hadronic rescattering processes allows the direct measurement of T_c through $T_{\rm chem}$ and thus the experimental observation of the phase transition.

Conclusions (contd.)

The increase of the strangeness saturation factor γ_s from pp to central Au+Au collisions at RHIC shows that the lifetime of the QGP (and thus the time for chemically equilibrating strange with light quarks) is still limited. Only for midcentral to central Au+Au collisions γ_s has sufficient time to saturate.

Corrollary/Prediction:

At lower energies (low SPS and AGS) $T_{\rm chem} < T_c$. In this case the phase transition can not be the origin of the observation of chemical equilibrium yields; hadronic chemical reactions must be responsible for lowering the freeze-out temperature below T_c.

Since the present work has shown that the kinetic decoupling of hadronic chemical reaction rates is influenced by the fireball expansion rate, which again depends on collision centrality, I expect to see impact parameter dependence of $T_{\rm chem}$ whenever it is measured to be well below T_c.