Source Shapes in Heavy-Ion Collisions

P. Danielewicz

Natl Superconducting Cyclotron Lab, Michigan State U

Zimányi 75 Memorial Workshop, 2-4 July, 2007, Hungary

Jozsó's Impact on Me

- Graduate student - 3 wks in January '81: code f/my Ph.D. Green's function calculations developed on KFKI mainframe (Robotron?)
- KFKI Group - fountain of talent: I. Lovas, J. Kuti, B. Lukacs, J. Polonyi, L. Csernai, A. Hasenfratz, P. Hasenfratz, I. Borbely . . .
- Beginnings of quark-gluon plasma physics
- Beginnings of lattice QCD
- Jozsó: elegant concise physics, unrelenting
- Many workshops: Balatonfüred, Siofok, Budapest
- Budapest '04: cartesian harmonic decomposition f/source shapes

Jozsó: $\pi\pi$ correlations: Pratt, Csorgo&Zimanyi PRC42(90)2646

Jozsó's Impact on Me

- Graduate student - 3 wks in January '81: code f/my Ph.D. Green's function calculations developed on KFKI mainframe (Robotron?)
- KFKI Group - fountain of talent: I. Lovas, J. Kuti, B. Lukacs, J. Polonyi, L. Csernai, A. Hasenfratz, P. Hasenfratz, I. Borbely ...
- Beginnings of quark-gluon plasma physics
- Beginnings of lattice QCD
- Jozsó: elegant concise physics, unrelenting
- Many workshops: Balatonfüred, Siofok, Budapest
- Budapest '04: cartesian harmonic decomposition f/source shapes

Jozsó: $\pi\pi$ correlations: Pratt, Csorgo&Zimanyi PRC42(90)2646

Imaging

Source f/correlation: case of imaging. General task:

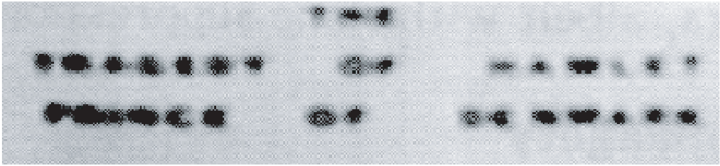
$$C(q) = \int dr\, K(q,r)\, S(r)$$

From data w/ errors, $C(q)$, determine the source $S(r)$.

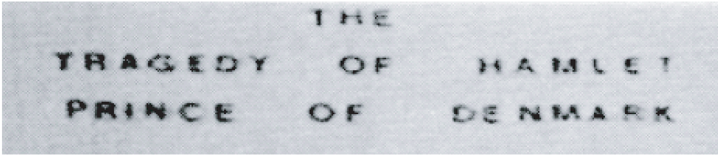
Requires inversion of the kernel K.

Optical recognition: K - blurring function, max entropy method

C:

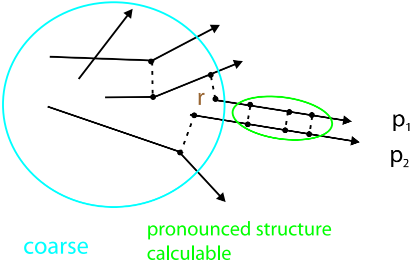


S:



Electron scattering: $d\sigma/dq \rightarrow \rho(r)$

Factorization of Final-State Amplitude in Reactions



2-ptcle inclusive cross section at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2} = \int d\mathbf{r}\, S'_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

data — source — 2-ptcle wf

S': distribution of emission points in 2-ptcle CM

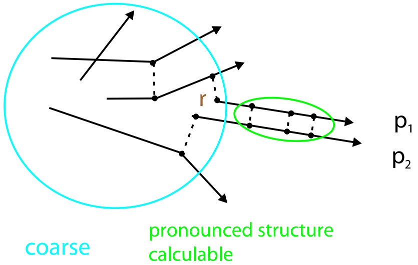
Normalizing with 1-ptcle cross sections yields correlation f:

$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1}\,\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_2}} = \int d\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int d\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r}) = 1$.

Note: C may only give access to the density of relative emission points in 2-ptcle CM, integrated there over time

Factorization of Final-State Amplitude in Reactions



2-ptcle inclusive cross section at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2} = \int \mathrm{d}\mathbf{r}\, S'_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

data source 2-ptcle wf

S': distribution of emission points in 2-ptcle CM

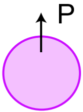
Normalizing with 1-ptcle cross sections yields correlation f:

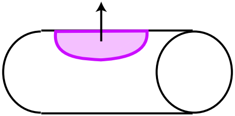
$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1}\,\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_2}} = \int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r}) = 1$.

Note: C may only give access to the density of relative emission points in 2-ptcle CM, integrated there over time

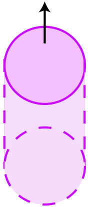
Expectations Regarding the Source S_P

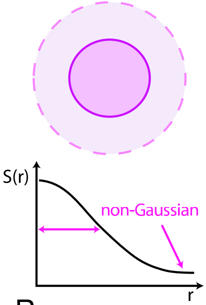




Instantaneous volume surface freeze-out.

Prolonged emission from source moving in pair cm.





Resonance emission from a source stationary in pair frame.

Any determination of source characteristics from data, unaided by reaction theory, is an imaging.

Integral Relation

Of interest the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int \mathrm{d}\mathbf{r} \left(|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int \mathrm{d}\mathbf{r}\, K(\mathbf{q}, \mathbf{r})\, S(\mathbf{r})$$

Learning about S possible when scat wf $|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair. For some ptcle pairs, it may easier to learn about S than for others. Ease of learning about shape anisotropy may depend on a pair.

For pure interference, π^0's or γ's, $\Phi_{\mathbf{q}}^{(-)}(\mathbf{r}) = \frac{1}{\sqrt{2}} \left(e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}} \right)$, kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$:

$$K(\mathbf{q}, \mathbf{r}) = |\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 = \cos(2\mathbf{q}\mathbf{r})$$

$$\Rightarrow \quad \mathcal{R}(\mathbf{q}) = \int \mathrm{d}\mathbf{r} \cos(2\mathbf{q}\mathbf{r}) S(\mathbf{r})$$

3D cosine-transform source-correlation relation.
Brown&PD, PLB398(97)252

Integral Relation

Of interest the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int \mathrm{d}\mathbf{r} \left(|\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int \mathrm{d}\mathbf{r}\, K(\mathbf{q},\mathbf{r})\, S(\mathbf{r})$$

Learning about S possible when scat wf $|\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair. For some ptcle pairs, it may easier to learn about S than for others. Ease of learning about shape anisotropy may depend on a pair.

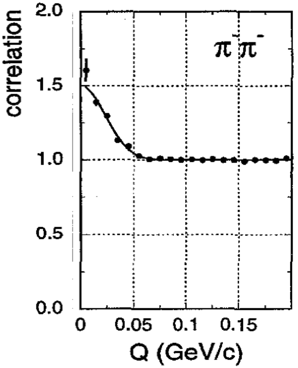
For pure interference, π^0's or γ's, $\Phi^{(-)}_{\mathbf{q}}(\mathbf{r}) = \frac{1}{\sqrt{2}} \left(e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}} \right)$, kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$:

$$K(\mathbf{q},\mathbf{r}) = |\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2 - 1 = \cos\left(2\mathbf{q}\mathbf{r}\right)$$

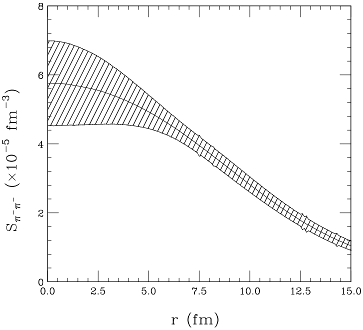
$$\Rightarrow \quad \mathcal{R}(\mathbf{q}) = \int \mathrm{d}\mathbf{r} \cos\left(2\mathbf{q}\mathbf{r}\right) S(\mathbf{r})$$

3D cosine-transform source-correlation relation.
Brown&PD, PLB398(97)252

Fourier-Transform of Angle-Averaged Correlation



Coulomb-corrected 1D π^--π^- correlation-function (Miskowiec *et al.*)



restored source: relative dstr of π^--π^- emission pts in central Au+Au at 10.8 GeV/c (E877)

Brown, PD PLB398(97)252

$S(r \to 0)$: entropy, freeze-out density (Brown, PD, Panitkin . . .)

$S(0) \searrow \Leftrightarrow$ entropy $\nearrow$

Multipole Decomposition

$$\boxed{\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\mathbf{r}\, K(\mathbf{q},\mathbf{r})\, S(\mathbf{r})}$$

source-correlation relation

but a spin-averaged kernel depends on the relative angle only

$$K(\mathbf{q},\mathbf{r}) = K(q,r,\cos\theta_{\mathbf{qr}}) = \sum_\ell (2\ell+1)\, K_\ell(q,r)\, P^\ell(\cos\theta_{\mathbf{qr}})$$

With

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})\,, \quad S(\mathbf{r}) = \sqrt{4\pi} \sum S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D relations for different deformation coefficients:

$$\boxed{\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, S^{\ell m}(r)}$$

Different deformation coefficients for the source and correlation functions are directly related to each other.

For pure interference $K_\ell(q,r) = (-1)^{\ell/2} j_\ell(2qr)$, for even ℓ only, where j_ℓ is spherical Bessel function.

Multipole Decomposition

$$\boxed{\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\mathbf{r}\, K(\mathbf{q},\mathbf{r})\, S(\mathbf{r})}$$

source-correlation relation

but a spin-averaged kernel depends on the relative angle only

$$K(\mathbf{q},\mathbf{r}) = K(q,r,\cos\theta_{\mathbf{qr}}) = \sum_{\ell} (2\ell+1)\, K_\ell(q,r)\, P^\ell(\cos\theta_{\mathbf{qr}})$$

With

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})\,, \qquad S(\mathbf{r}) = \sqrt{4\pi} \sum S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D relations for different deformation coefficients:

$$\boxed{\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, S^{\ell m}(r)}$$

Different deformation coefficients for the source and correlation functions are directly related to each other.

For pure interference $K_\ell(q,r) = (-1)^{\ell/2} j_\ell(2qr)$, for even ℓ only, where j_ℓ is spherical Bessel function.

Imaging of Angle-Averaged Sources

The $\ell = 0$ 1D relation connects angle-averaged source $S(r)$ and correlation $\mathcal{R}(q)$ functions, in terms of angle-averaged kernel K_0:

$$\mathcal{R}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_0(q,r)\, S(r)$$

Source discretization strategy

1. Discretize integral

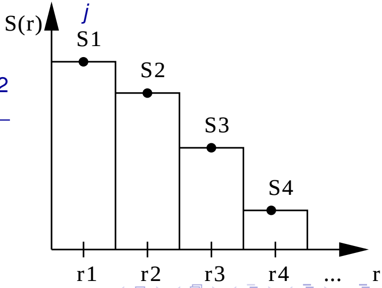
$$\mathcal{R}_i = \sum_j 4\pi\, \Delta r\, r_j^2\, K_0(q_i, r_j)\, S(r_j) \equiv \sum_j K_{ij}\, S_j$$

2. Vary $S(r_j)$ to minimize χ^2:

$$\chi^2 = \sum_i \frac{(\sum_j K_{ij}\, S_j - \mathcal{R}_i^{exp})^2}{\sigma_i^2}$$

3. Mtx result from minimization:

$$S = (K^\top K)^{-1}\, K^\top\, \mathcal{R}^{exp}$$

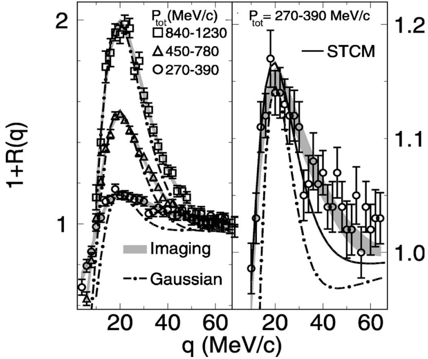


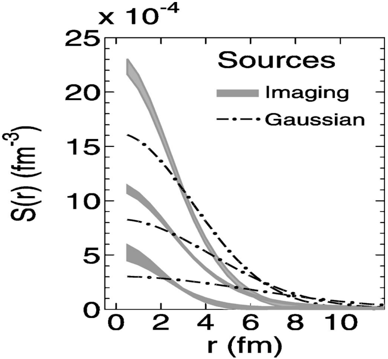
Brown&PD, PRC57(98)2474

pp Imaging

Imaging altered the interpretation of C_{pp}:

Verde PRC65(02)054609 ^{14}N+^{197}Au at 75 MeV/u



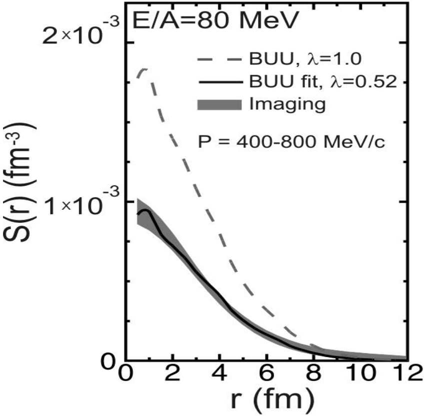


Gauss par: quickly changing radii. Imaging: quickly changing preequilibrium fraction, non-Gaussian source shapes!

$S(r \to 0)$: preequilibrium fraction, entropy, freeze-out ρ...

Imaged pp Source Compared to Transport

Verde PRC67(03)034606: Ar+Sc central collisions at 80 MeV/u, fast 400 $< P_{tot} <$ 800 MeV/c pairs



Nucleon-based transport reproduces correctly the *shape* of the preequilibrium source.

The transport *cannot* describe correctly the preequilibrium pair fraction.

Kernel Properties

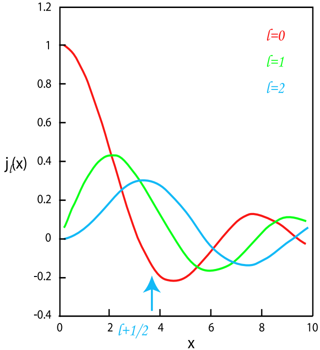
- Pure interference

$$K_\ell(q,r) = (-1)^{\ell/2} j_\ell(2qr)$$

- Classical Coulomb

$$K_\ell(q,r) = K_\ell(r/r_c)$$

where $r_c \propto 1/q^2$ distance of closest approach; i.e. $r/r_c \propto r\,q^2$
e.g. $K_0 = \Theta(r - r_c)\sqrt{1 - r_c/r} - 1$



- Strong interactions in asymptotic region

$$K(q,r,\cos\theta_{qr}) \approx \frac{1}{r^2}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\pi - \theta_{qr}) - \frac{\sigma}{2\pi r^2}\,\delta(1 + \cos\theta_{qr})$$

PD&Pratt PRC75(07)034907

Conclusions on Kernels

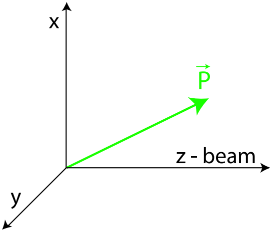
1. Low/large $q \longrightarrow$ large/low r true for identity interference & Coulomb only.
2. Identity interference, Coulomb and strong interactions all can provide access to shape deformation.
3. $qr \gtrsim \ell$ generally needed for accessing rank-ℓ deformation.
4. Strong interaction particularly effective in resonances.
5. *s*-wave resonances can provide access to deformation.

Basis in Spherical Angle

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})$$

$$S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q, r)\, S^{\ell m}(r)$$



Problem: Why giving up real quantities, R & S, for imaginary, $R^{\ell m}$ & $S^{\ell m}$?

Another basis??

Take the direction vector: $\hat{n}_\alpha = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Rank-ℓ tensor product:

$$(\hat{n}^\ell)_{\alpha_1 \ldots \alpha_\ell} \equiv \hat{n}_{\alpha_1}\, \hat{n}_{\alpha_1} \ldots \hat{n}_{\alpha_\ell} = \sum_{\ell' \leq \ell, m} c_{\ell' m}\, \mathrm{Y}^{\ell' m}$$

$\mathcal{P}^{(\ell,\ell)}$ projection operator that, within the space of rank-ℓ cartesian tensors, removes $\mathrm{Y}^{\ell' m}$ components with $\ell' < \ell$:

$$(\mathcal{P}\hat{n}^\ell)_{\alpha_1 \ldots \alpha_\ell} = \sum_m c_{\ell m}\, \mathrm{Y}^{\ell m}$$

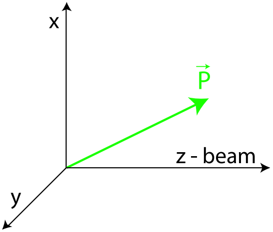
The components $\mathcal{P}\hat{n}^\ell$ are real and can be used to replace $\mathrm{Y}^{\ell m}$

Basis in Spherical Angle

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})$$

$$S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, S^{\ell m}(r)$$



Problem: Why giving up real quantities, R & S, for imaginary, $R^{\ell m}$ & $S^{\ell m}$?

Another basis??

Take the direction vector: $\hat{n}_\alpha = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Rank-ℓ tensor product:

$$(\hat{n}^\ell)_{\alpha_1 \ldots \alpha_\ell} \equiv \hat{n}_{\alpha_1}\, \hat{n}_{\alpha_1} \ldots \hat{n}_{\alpha_\ell} = \sum_{\ell' \leq \ell, m} c_{\ell' m}\, \mathrm{Y}^{\ell' m}$$

$\mathcal{P}^{(\ell,\ell)}$ projection operator that, within the space of rank-ℓ cartesian tensors, removes $\mathrm{Y}^{\ell' m}$ components with $\ell' < \ell$:

$$(\mathcal{P}\hat{n}^\ell)_{\alpha_1 \ldots \alpha_\ell} = \sum_m c_{\ell m}\, \mathrm{Y}^{\ell m}$$

The components $\mathcal{P}\hat{n}^\ell$ are real and can be used to replace $\mathrm{Y}^{\ell m}$.

Low-ℓ Cartesian Harmonics

$$\begin{aligned}
\mathcal{P}\hat{n}^0 &= 1 \\
(\mathcal{P}\hat{n}^1)_\alpha &= \hat{n}_\alpha \\
(\mathcal{P}\hat{n}^2)_{\alpha_1\,\alpha_2} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2} - \frac{1}{3}\delta_{\alpha_1\,\alpha_2} \\
&\vdots
\end{aligned}$$

Applequist Theor. Chem. Acc 107(02)103

PD&Pratt PLB618(05)60

$\mathcal{P}$ can be called a detracing operator as

$$\sum_\alpha (\mathcal{P}\hat{n}^\ell)_{\alpha\,\alpha\,\alpha_3\ldots\alpha_\ell} = 0$$

Completeness relation ($\mathcal{P} = \mathcal{P}^\top = \mathcal{P}^2$):

$$\begin{aligned}
\delta(\Omega' - \Omega) &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!}\sum_{\alpha_1\ldots\alpha_\ell}(\mathcal{P}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell}\,(\mathcal{P}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \\
&= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!}\sum_{\alpha_1\ldots\alpha_\ell}(\mathcal{P}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell}\,\hat{n}_{\alpha_1}\ldots\hat{n}_{\alpha_\ell}
\end{aligned}$$

Low-ℓ Cartesian Harmonics

$$\begin{aligned} \mathcal{P}\hat{n}^0 &= 1 \\ (\mathcal{P}\hat{n}^1)_\alpha &= \hat{n}_\alpha \\ (\mathcal{P}\hat{n}^2)_{\alpha_1\,\alpha_2} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2} - \frac{1}{3}\delta_{\alpha_1\,\alpha_2} \\ &\vdots \end{aligned}$$

Applequist Theor. Chem. Acc 107(02)103

PD&Pratt PLB618(05)60

$\mathcal{P}$ can be called a detracing operator as

$$\sum_\alpha (\mathcal{P}\hat{n}^\ell)_{\alpha\,\alpha\,\alpha_3\ldots\alpha_\ell} = 0$$

Completeness relation ($\mathcal{P} = \mathcal{P}^\top = \mathcal{P}^2$):

$$\begin{aligned} \delta(\Omega' - \Omega) &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!}\sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{P}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell}\,(\mathcal{P}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \\ &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!}\sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{P}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell}\,\hat{n}_{\alpha_1}\ldots\hat{n}_{\alpha_\ell} \end{aligned}$$

Low-ℓ Cartesian Harmonics

$$\begin{aligned} \mathcal{P}\hat{n}^0 &= 1 \\ (\mathcal{P}\hat{n}^1)_\alpha &= \hat{n}_\alpha \\ (\mathcal{P}\hat{n}^2)_{\alpha_1\,\alpha_2} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2} - \frac{1}{3}\delta_{\alpha_1\,\alpha_2} \\ &\vdots \end{aligned}$$

Applequist Theor. Chem. Acc 107(02)103

PD&Pratt PLB618(05)60

$\mathcal{P}$ can be called a detracing operator as

$$\sum_\alpha (\mathcal{P}\hat{n}^\ell)_{\alpha\,\alpha\,\alpha_3\ldots\alpha_\ell} = 0$$

Completeness relation ($\mathcal{P} = \mathcal{P}^\top = \mathcal{P}^2$):

$$\begin{aligned} \delta(\Omega' - \Omega) &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{P}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell}\,(\mathcal{P}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \\ &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{P}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell}\,\hat{n}_{\alpha_1}\ldots\hat{n}_{\alpha_\ell} \end{aligned}$$

Consequences

- $$\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\Omega' \, \delta(\Omega' - \Omega) \, \mathcal{R}(\mathbf{q}') = \sum_{\ell} \sum_{\alpha_1 \ldots \alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) \, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

  where $$\mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) = \frac{(2\ell+1)!!}{\ell!} \int \frac{\mathrm{d}\Omega_{\mathbf{q}}}{4\pi} \, \mathcal{R}(\mathbf{q}) \, (\mathcal{P} \hat{q}^{\ell})_{\alpha_1 \ldots \alpha_\ell}$$

- Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

  $$\boxed{\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int \mathrm{d}r \, r^2 \, K_\ell(q, r) \, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)}$$

- For weak anisotropies, only lowest-ℓ matter:

  $$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}^{(1)}_{\alpha}(q) \, \hat{q}_\alpha + \sum_{\alpha_1 \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q) \, \hat{q}_{\alpha_1} \hat{q}_{\alpha_2} + \ldots$$

  monopole, dipole, quadrupole…

Consequences

- $$\mathcal{R}(\mathbf{q}) = \int d\Omega' \, \delta(\Omega' - \Omega) \, \mathcal{R}(\mathbf{q}') = \sum_{\ell} \sum_{\alpha_1 \ldots \alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) \, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

  where $$\mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) = \frac{(2\ell+1)!!}{\ell!} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \, \mathcal{R}(\mathbf{q}) \, (\mathcal{P}\hat{q}^{\ell})_{\alpha_1 \ldots \alpha_\ell}$$

- Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

  $$\boxed{\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int dr \, r^2 \, K_\ell(q, r) \, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)}$$

- For weak anisotropies, only lowest-ℓ matter:

  $$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}^{(1)}_{\alpha}(q) \, \hat{q}_\alpha + \sum_{\alpha_1 \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q) \, \hat{q}_{\alpha_1} \hat{q}_{\alpha_2} + \ldots$$

  monopole, dipole, quadrupole...

Consequences

- $$\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\Omega' \, \delta(\Omega' - \Omega) \, \mathcal{R}(\mathbf{q}') = \sum_{\ell} \sum_{\alpha_1 \ldots \alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) \, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

  where $$\mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) = \frac{(2\ell+1)!!}{\ell!} \int \frac{\mathrm{d}\Omega_{\mathbf{q}}}{4\pi} \mathcal{R}(\mathbf{q}) \, (\mathcal{P}\hat{q}^{\ell})_{\alpha_1 \ldots \alpha_\ell}$$

- Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

  $$\boxed{\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int \mathrm{d}r \, r^2 \, K_\ell(q,r) \, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)}$$

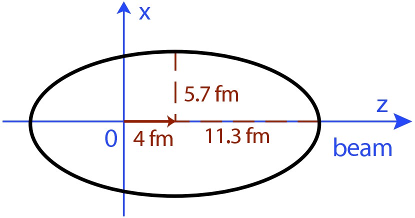
- For weak anisotropies, only lowest-ℓ matter:

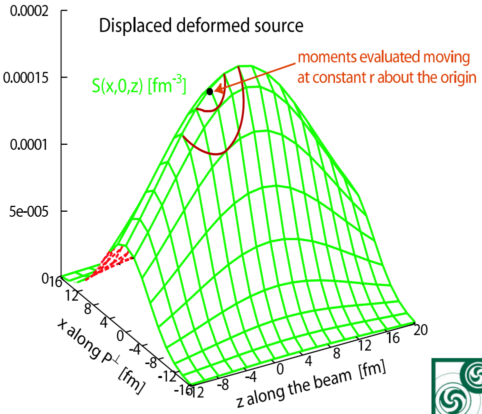
  $$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}^{(1)}_{\alpha}(q) \, \hat{q}_{\alpha} + \sum_{\alpha_1 \, \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q) \, \hat{q}_{\alpha_1} \, \hat{q}_{\alpha_2} + \ldots$$

  monopole, dipole, quadrupole...

Sample Gaussian Source

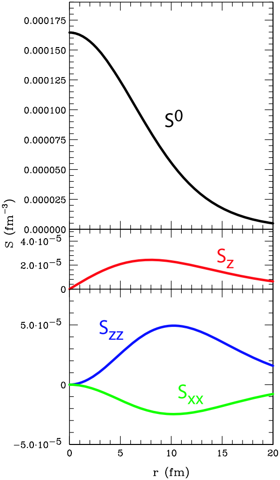
Anisotropic Gaussian, elongated and displaced along the beam axis



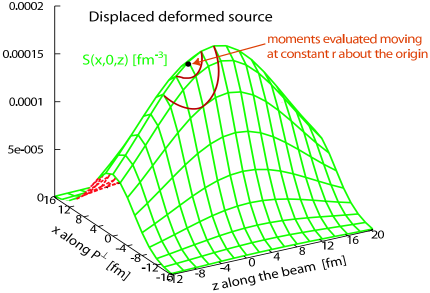


Low-ℓ Characteristics

3D info in terms of 1D plots



$S^{(\ell)} \propto r^\ell$



Along z-axis:

$$S(r) = S^0(r) + S_z(r) + S_{zz}(r) + \dots$$

In xz plane, at 45° to z-axis:

$$S(r) = S^0(r) + \frac{1}{\sqrt{2}} S_z(r) + \frac{1}{2}\left(S_{xx}(r) + S_{zz}(r)\right)$$

$$S_{xx} + S_{yy} + S_{zz} = 0$$

Source & Correlation Symmetries

Large-statistics NA49 $\pi^- - \pi^-$ data from the midrapidity region of 20, 40, 80, 160 AGeV Pb+Pb collisions at SPS:

- Identical ptcles: $\vec{r} \to -\vec{r} \Leftrightarrow$ even-ℓ only
- Midrapidity: $z \to -z$ (beam dir) $\Leftrightarrow$ even-z moments only
- Reaction-plane averaging: $y \to -y$ (sideways) $\Leftrightarrow$ even-y moments only
- In the end, also: $x \to -x$ (outward) $\Leftrightarrow$ even-x moments only

- $\ell = 0$
- $\ell = 2$: x^2, y^2, z^2 (only 2 independent)
- $\ell = 4$: x^4, y^4, z^4, $x^2\,y^2$, $x^2\,z^2$, $y^2\,z^2$ (only 3 independent)

Source & Correlation Symmetries

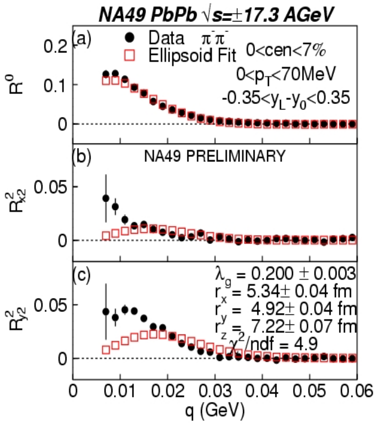
Large-statistics NA49 $\pi^- - \pi^-$ data from the midrapidity region of 20, 40, 80, 160 AGeV Pb+Pb collisions at SPS:

- Identical ptcles: $\vec{r} \rightarrow -\vec{r} \Leftrightarrow$ even-ℓ only
- Midrapidity: $z \rightarrow -z$ (beam dir) $\Leftrightarrow$ even-z moments only
- Reaction-plane averaging: $y \rightarrow -y$ (sideways) $\Leftrightarrow$ even-y moments only
- In the end, also: $x \rightarrow -x$ (outward) $\Leftrightarrow$ even-x moments only

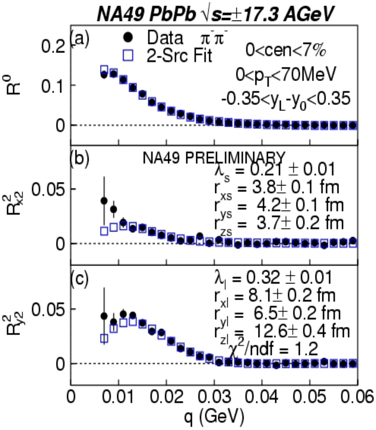
- $\ell = 0$
- $\ell = 2$: x^2, y^2, z^2 (only 2 independent)
- $\ell = 4$: x^4, y^4, z^4, $x^2\,y^2$, $x^2\,z^2$, $y^2\,z^2$ (only 3 independent)

NA49 Pb+Pb $\pi\pi$ Correlations

Paul Chung *et al.*



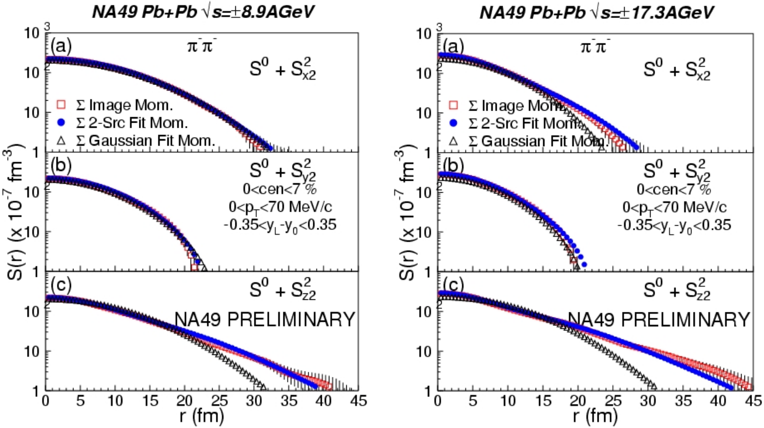
Fit w/anisotropic Gaussian



Fit w/2-Gaussian source

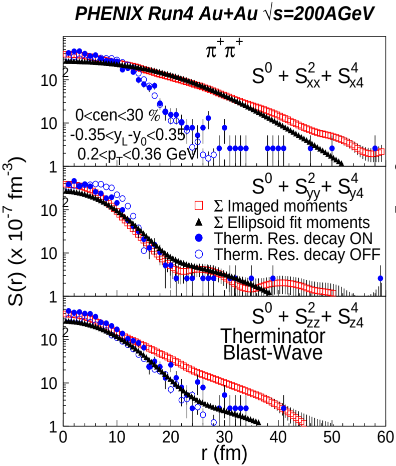
NA49 Pb+Pb $\pi\pi$ Sources

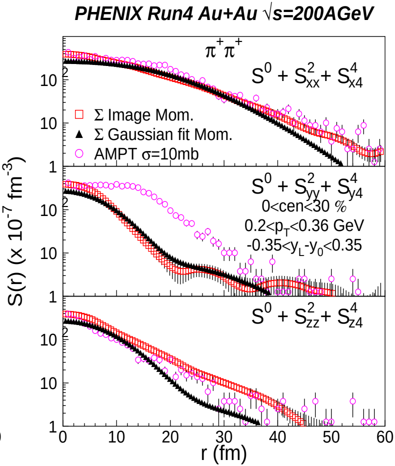
Paul Chung *et al.*



Exponential tails in the beam direction. An exponential tail develops in the outward direction between $\sqrt{s} = 9\,\text{GeV/u}$ and $\sqrt{s} = 17\,\text{GeV/u}$ and persists at the higher RHIC energies.

PHENIX Au+Au $\pi\pi$ Sources





Comparison to Therminator Comparison to AMPT

Final Remarks

- Correlations at low relative velocities yield access to source spatial characteristics in the pair CM! Temporal information in the pair CM is not directly accessible.
- Different final-state effects can provide information on source asymmetry, including identity interference, Coulomb and strong interactions.
- Even *s*-wave resonances can provide information on source deformation, due to wave interference.
- Cartesian harmonics provide easy means f/representing & manipulating info in functions dependent on spherical angle, such as correlation and source functions.
- High-stat $\pi\pi$ data yield $\pi\pi$ sources that are compact in side dir & exhibit nongaussian tails in other directions.

Jozsó: Persistence may get you there!

Final Remarks

- Correlations at low relative velocities yield access to source spatial characteristics in the pair CM! Temporal information in the pair CM is not directly accessible.
- Different final-state effects can provide information on source asymmetry, including identity interference, Coulomb and strong interactions.
- Even *s*-wave resonances can provide information on source deformation, due to wave interference.
- Cartesian harmonics provide easy means f/representing & manipulating info in functions dependent on spherical angle, such as correlation and source functions.
- High-stat $\pi\pi$ data yield $\pi\pi$ sources that are compact in side dir & exhibit nongaussian tails in other directions.

Jozsó: Persistence may get you there!