

Over the HH-model

| Different currents | Function |
|---|-----------------------------|
| slow K ⁺ | firing frequency adaptation |
| perzistent (non-inactivating) | Na ⁺ burst |
| Ca ²⁺ | burst |
| Ca ²⁺ -(and V-) dependent K ⁺ burst, adaptation | |
| H hyperpolarization activated | pacemaker |

Voltage dependent currents

$$I_x = g_x(t)(E_x - V(t)), \quad g_x(t) = \bar{g}_x \prod_i p_x^{(i)}(t), \quad \frac{dp_x^{(i)}}{dt} = \frac{p_{x\infty}^{(i)}(V(t)) - p(t)}{\tau_{p_x^{(i)}}(V(t))}$$

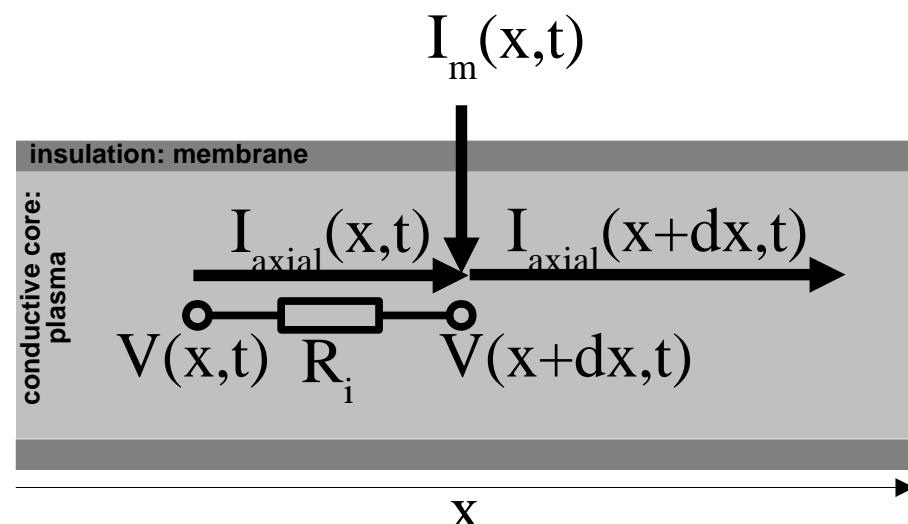
Ca²⁺ concentration

$$\frac{d[Ca]_i}{dt} = \beta I_{Ca}(t) - \frac{[Ca]_i(t)}{\tau_{Ca}}$$

Ca²⁺-dependent gate

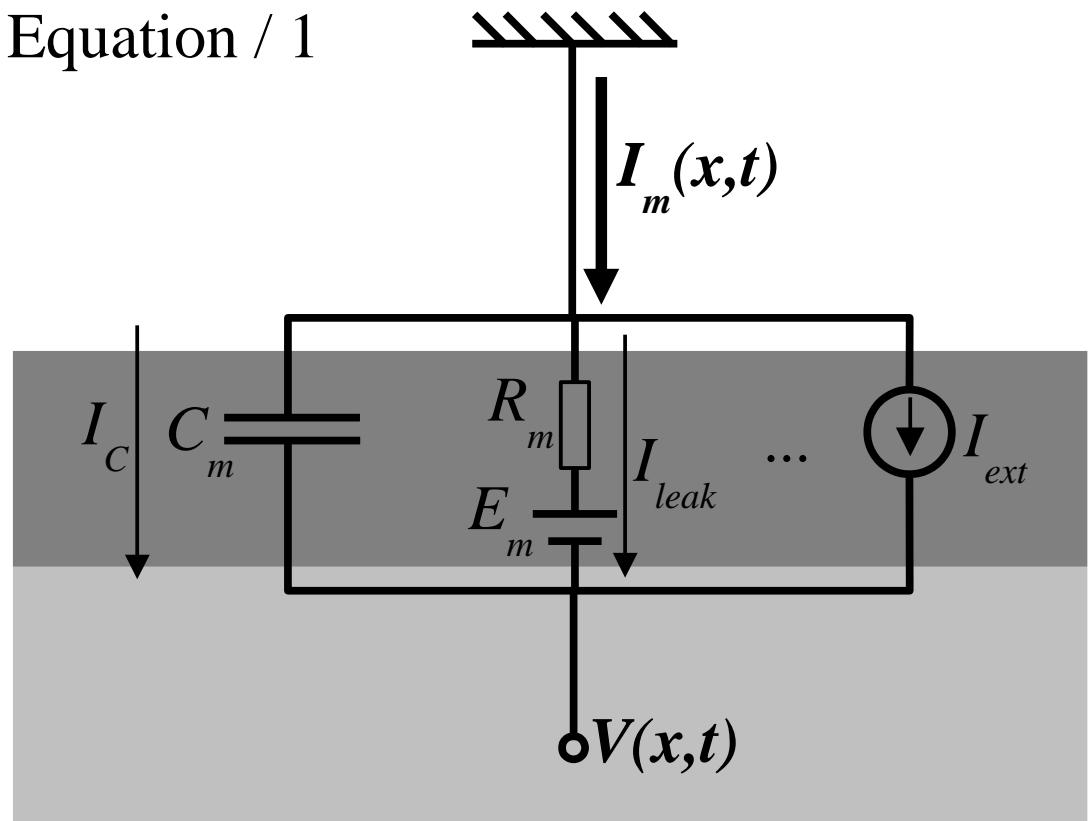
$$\frac{dp_x^{(i)}}{dt} = \frac{p_{x\infty}^{(i)}([Ca]_i(t)) - p(t)}{\tau_{p_x^{(i)}}([Ca]_i(t))}$$

The Cable Equation / 1



$$I_{axial}(x,t) = -\frac{1}{R_i} \frac{\partial V}{\partial x}$$

$$\frac{\partial I_{axial}}{\partial x} = I_m(x,t)$$



$$I_m(x,t) = I_C(x,t) + I_{leak}(x,t) + \dots = -C_m \frac{\partial V}{\partial t} - \frac{V(x,t)}{R_m}$$

simplification: no voltage-dependent currents!

x [cm], t [msec]

V [mV], I_{axial} [μ A], I_m [μ A/cm]

R_i [k Ω /cm], R_m [k Ω cm], C_m [μ F/cm]

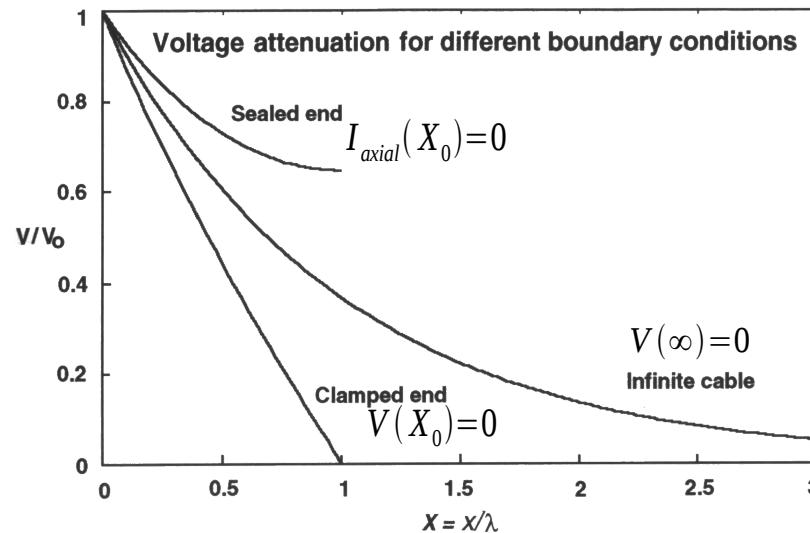
$$\lambda = \sqrt{R_m/R_i} \text{ [cm]}$$

$$\tau = R_m C_m \text{ [msec]}$$

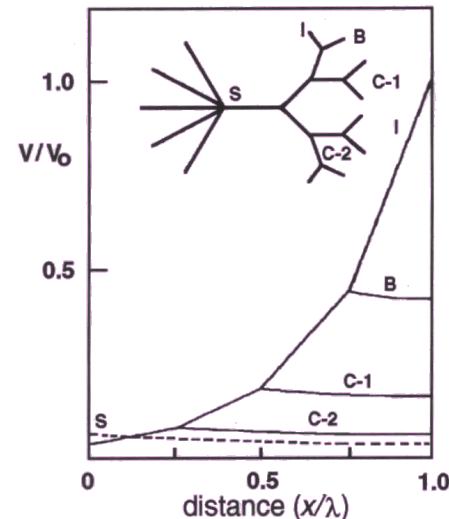
$$\frac{1}{R_i} \frac{\partial^2 V}{\partial x^2} - C_m \frac{\partial V}{\partial t} - \frac{V(x,t)}{R_m} = 0$$

$$\lambda^2 \frac{\partial^2 V}{\partial x^2} - \tau \frac{\partial V}{\partial t} - V(x,t) = 0$$

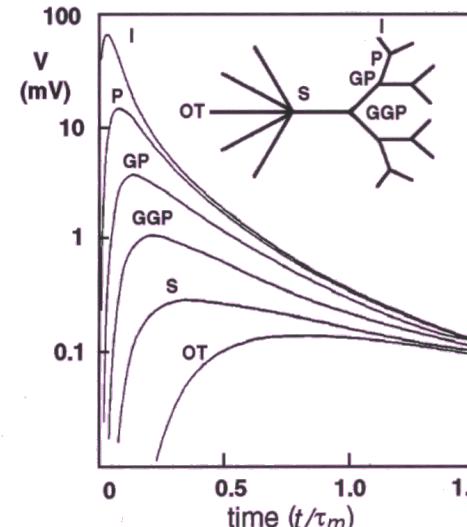
The Cable Equation / 2



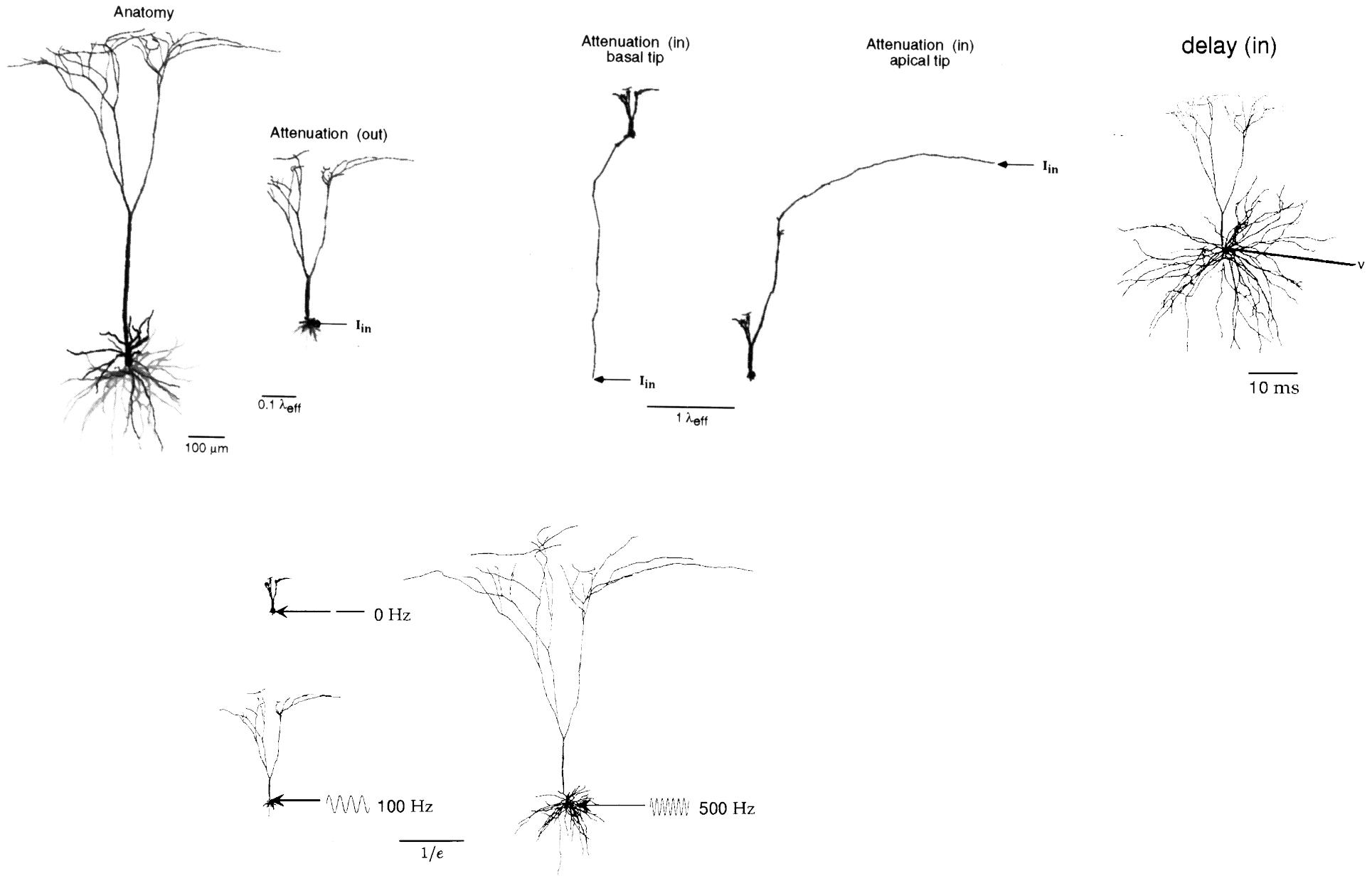
Constant current injection:
steady-state **spatial**
voltage spread



Transient current injection:
temporal development
of voltage spread

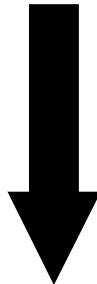


The Cable Equation / 3



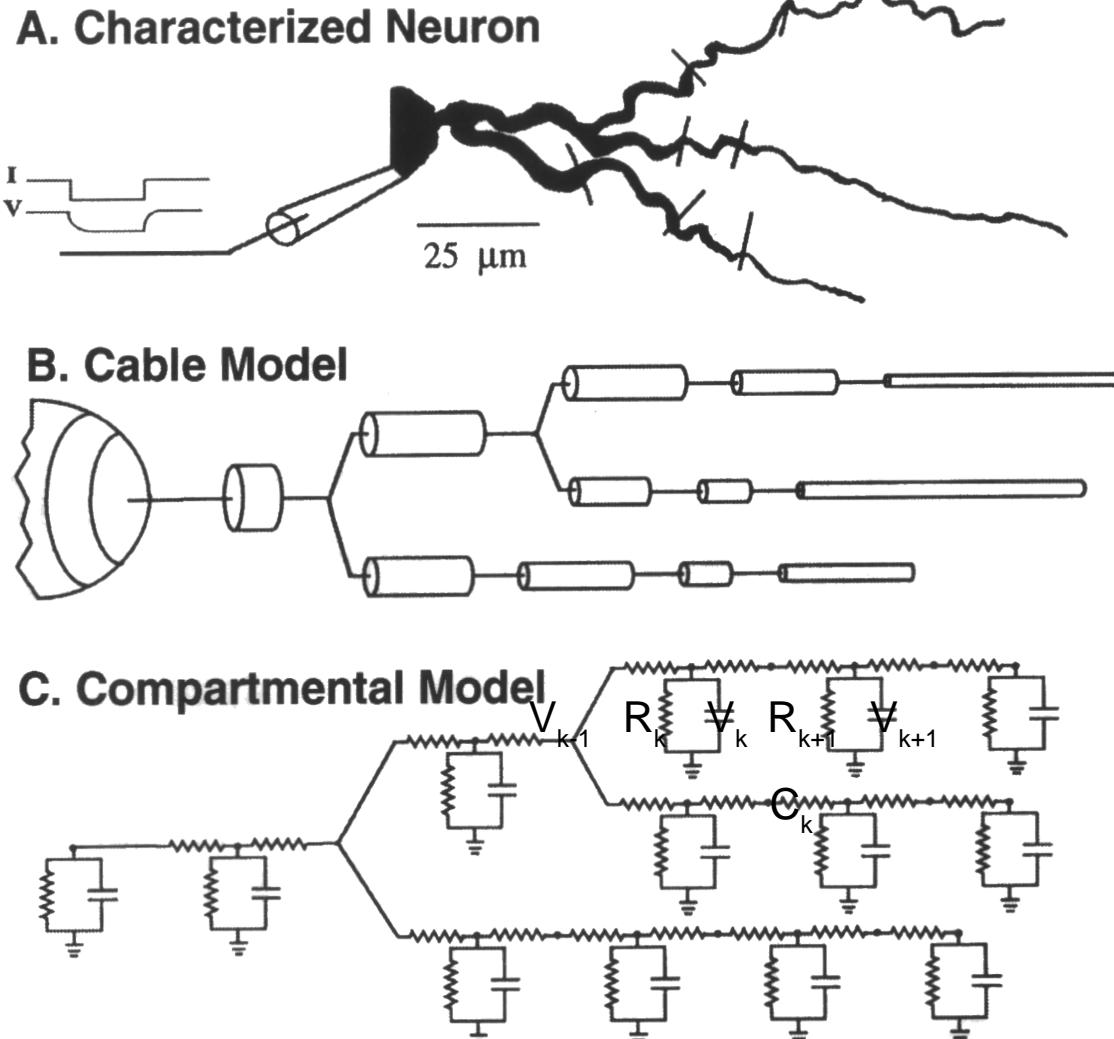
Multicompartmental modeling

$$\frac{1}{R_i} \frac{\partial^2 V}{\partial x^2} - C_m \frac{\partial V}{\partial t} - \frac{V(x, t)}{R_m} = 0$$



$$C_k \frac{dV_k}{dt} = \underbrace{I_k(t)} + \frac{V_{k-1}(t) - V_k(t)}{R_k} + \frac{V_{k+1}(t) - V_k(t)}{R_{k+1}}$$

all sorts of ionic currents (HH, etc)



Detailed cell models: Why?

I. Reproducing different phenomena (how does it works?)

Traub & Miles (1991, 1994) hippocampal pyramidal cell model

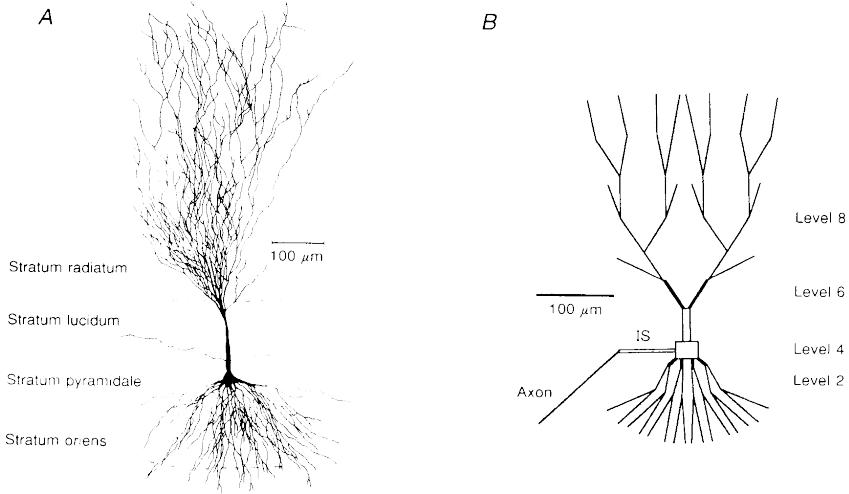
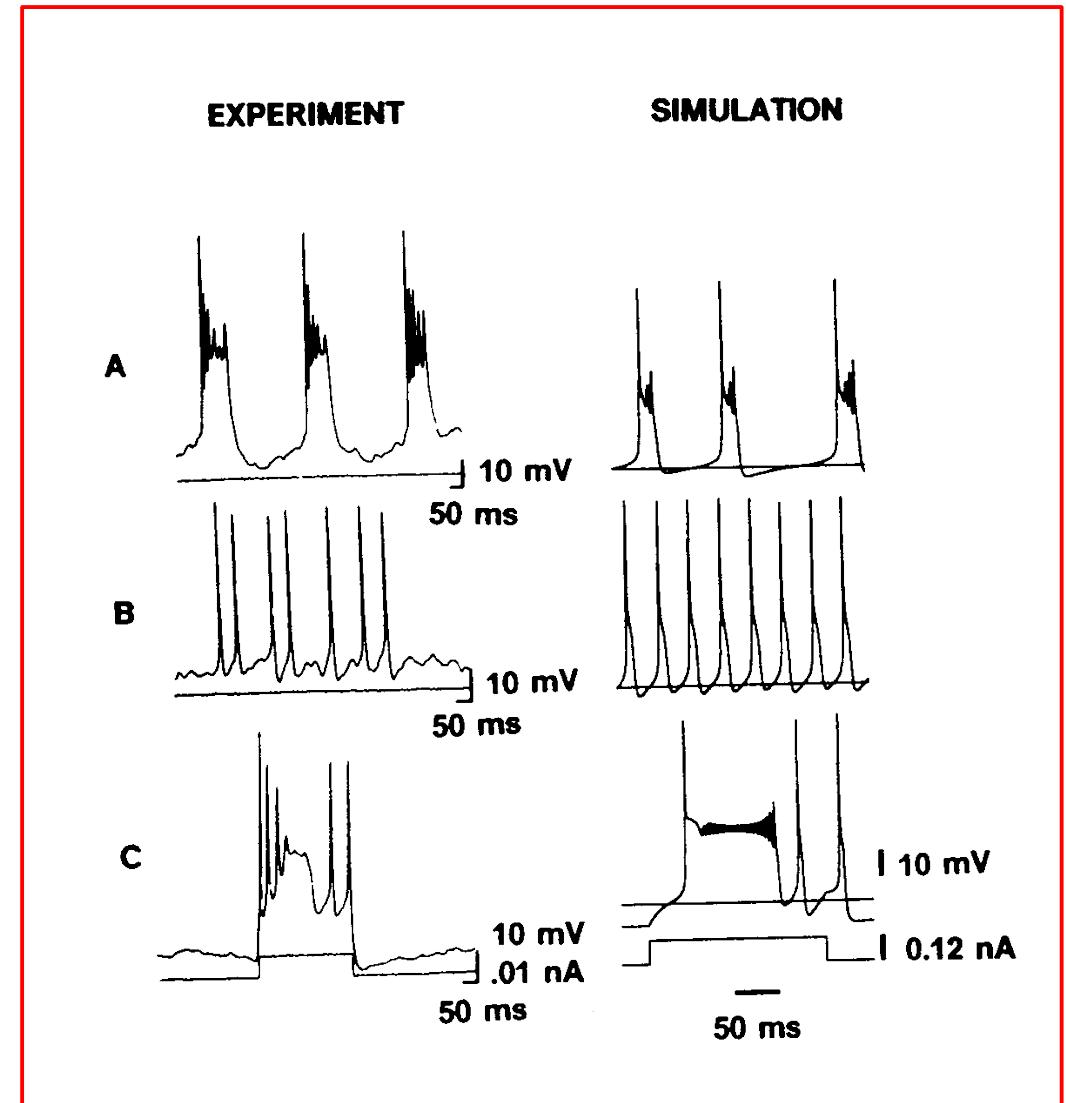


Table 2. Active conductance densities (mS cm^{-2})

| Level | Na^+ | Ca^{2+} | K(DR) | K(AHP) | K(C) | K(A) |
|-------|---------------|------------------|----------------|-----------------|---------------|---------------|
| 1 | — | 1.0 | — | 0.8 | 4.0 | — |
| 2 | — | 1.0 | — | 0.8 | 4.0 | 0.5 |
| 3 | 1.0 | 1.0 | 15 | 0.8 | 8.0 | 0.5 |
| 4 | 100 | 1.0 | 135 | 0.8 | 20 | 0.5 |
| 5 | 3.0 | 1.0 | 20 | 0.8 | 8.0 | 0.5 |
| 6 | 3.0 | 1.0 | 20 | 0.8 | 8.0 | 0.5 |
| 7 | — | 2.0 | — | 0.8 | 4.0 | — |
| 8 | — | 3.0 | — | 0.8 | 12 | — |
| 9 | — | 3.0 | — | 0.8 | 12 | — |
| 10 | — | 1.0 | — | 0.8 | 4.0 | — |
| 11 | — | 1.0 | — | 0.8 | 4.0 | — |
| IS | 500 | — | 250 | — | — | — |
| Axon | 500 | — | 250 | — | — | — |



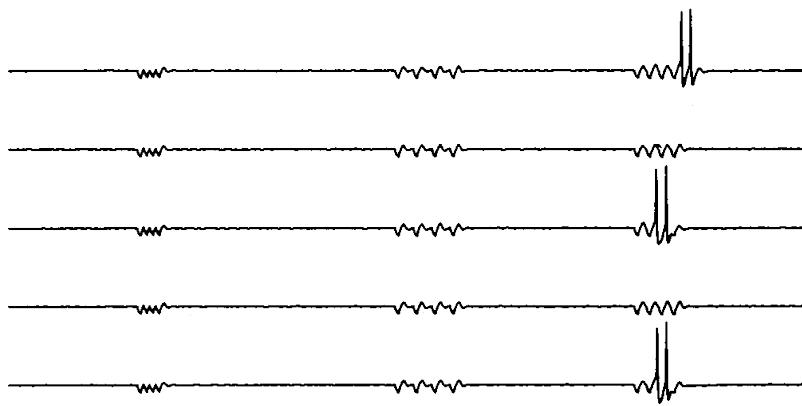
Detailed cell models: Why?

II. Revealing computational functions (What it is good for?) / 1

What is a burst good for?

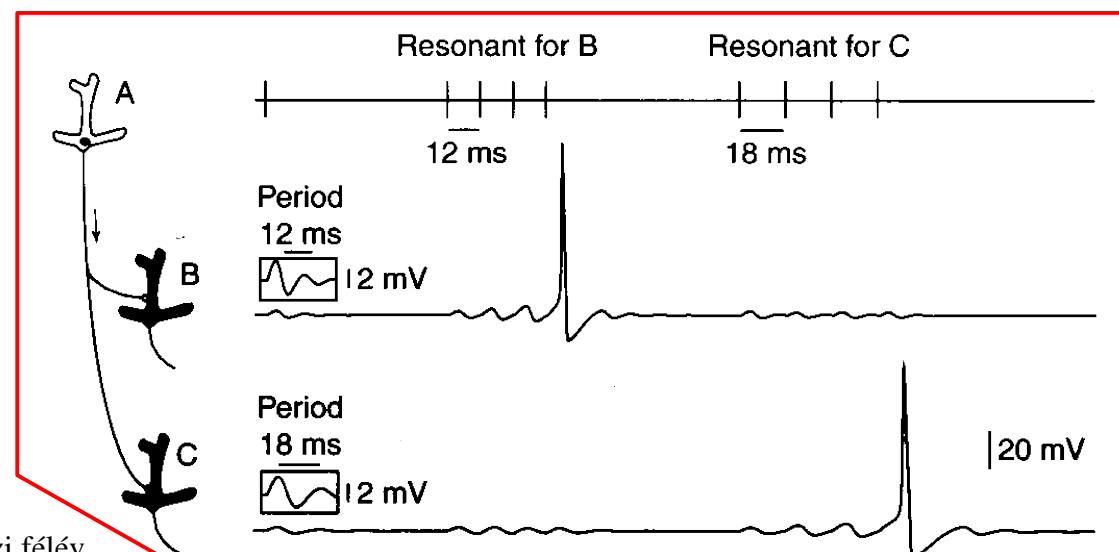
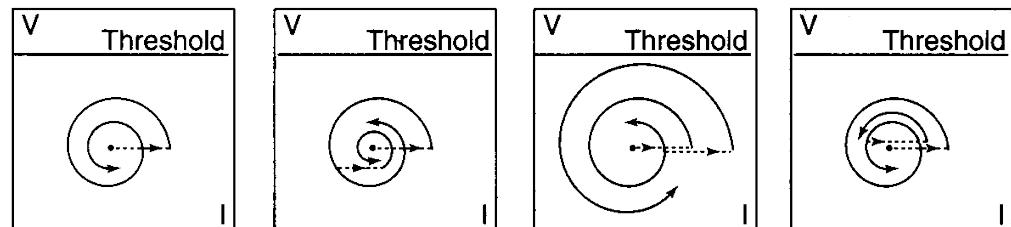
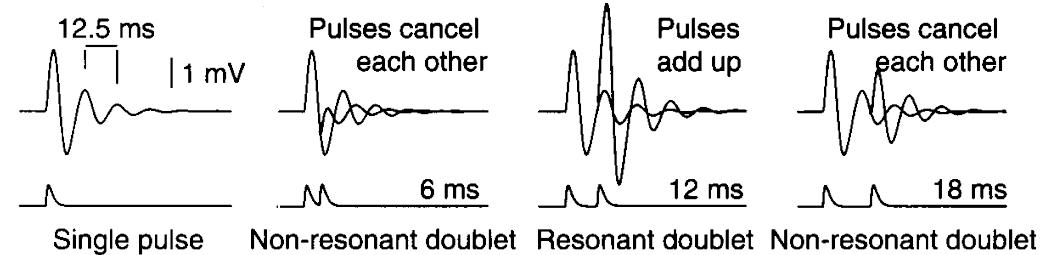
1. Common sense (Lisman): more robust transmission

2. Izhikevich: selective communication with resonance



Inhibitory input

5 ms 15 ms 10 ms
Non-resonant burst Non-resonant burst Resonant burst



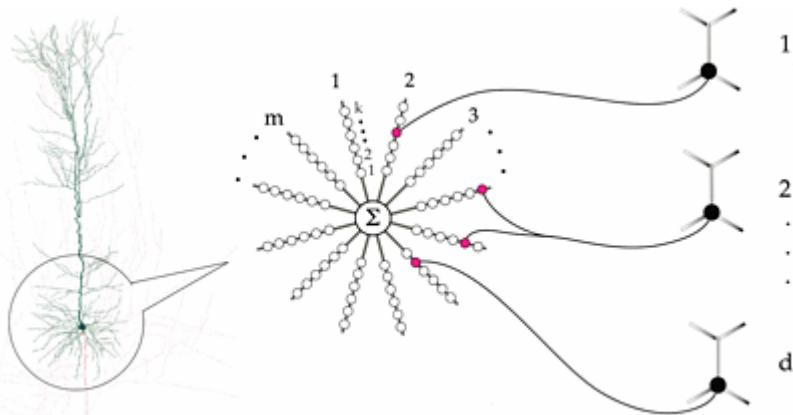
Detailed cell models: why?

II. Revealing computational functions (What it is good for?) / 2

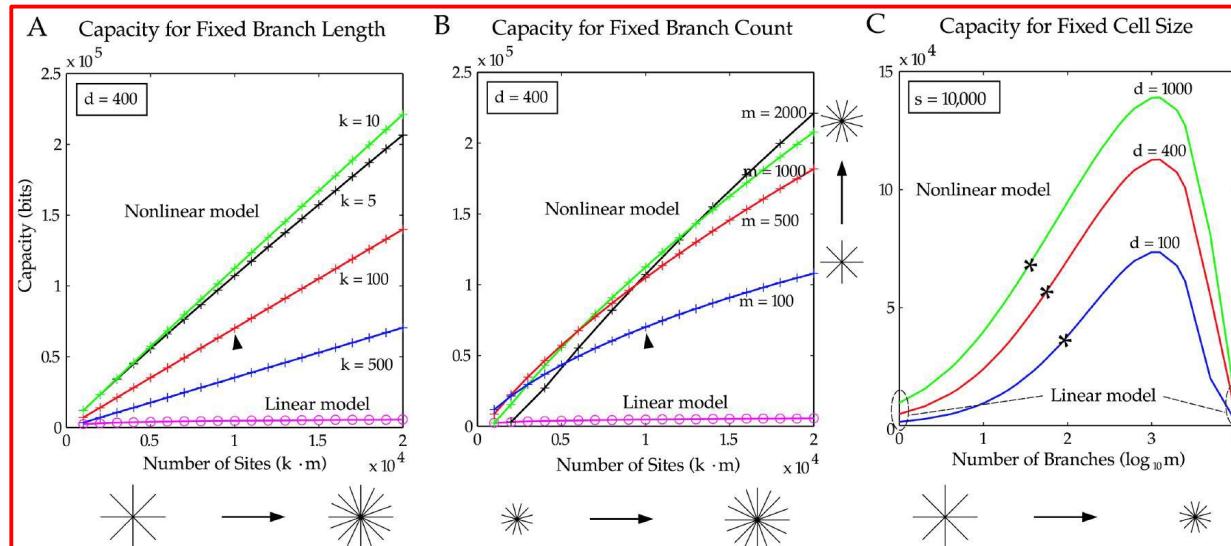
What is the role of the dendrites?

1. Common sense (Cook&Johston): amplification of distal synaptic effects

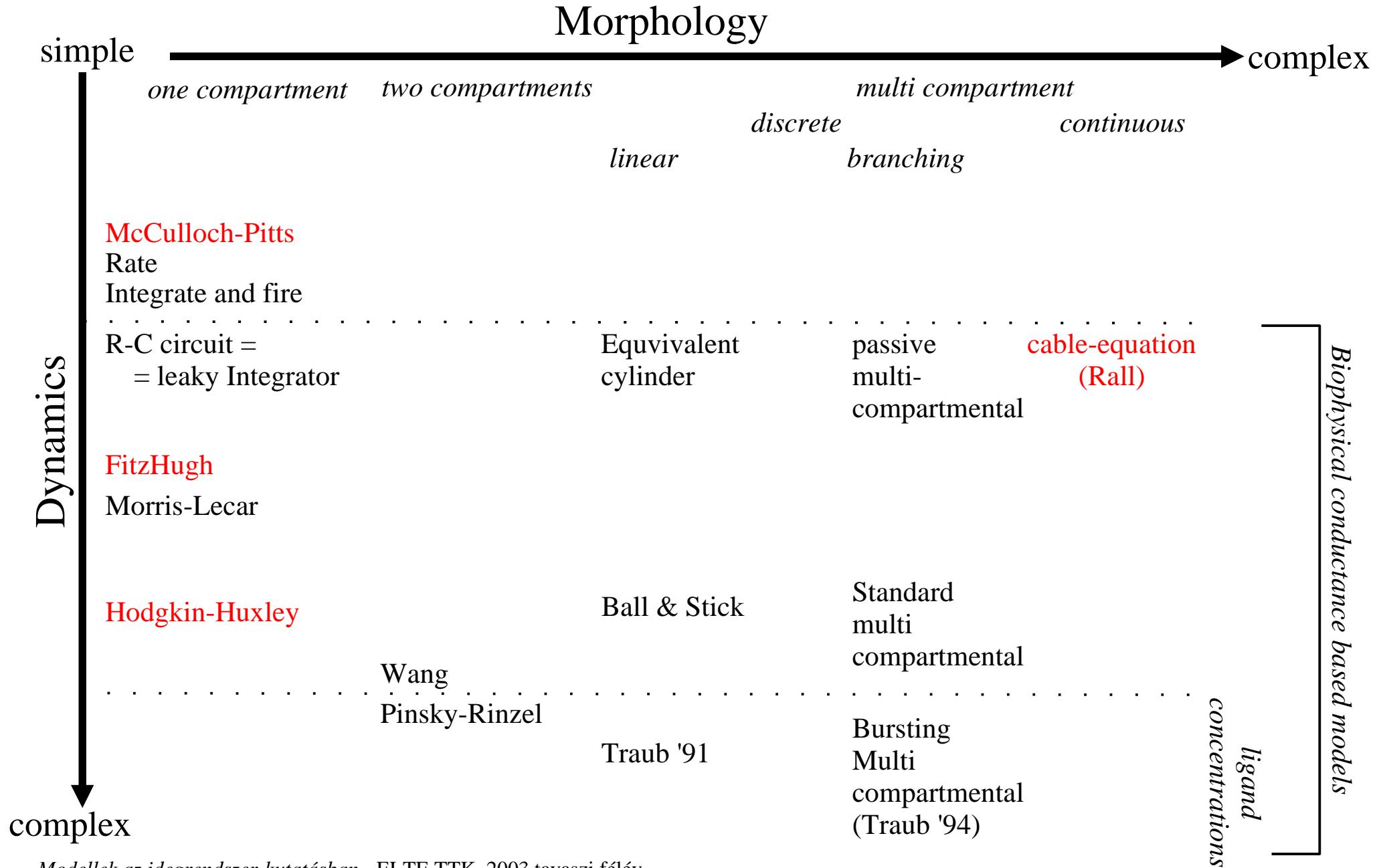
2. Mel: increasing storage capacity



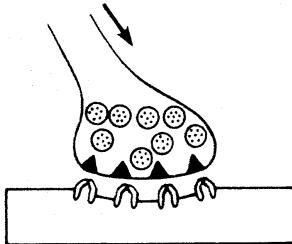
| | Linear Cell | Nonlinear Cell |
|--|----------------------|--|
| Wiring Configurations | $a_L(x)$ | $a_N(x)$ |
| (1) | $4x_1 + 3x_2 + 2x_3$ | $b(2x_1 + x_2) + b(2x_1 + x_2) + b(x_2 + 2x_3)$ |
| (2) | $4x_1 + 3x_2 + 2x_3$ | $b(2x_1 + x_3) + b(x_1 + 2x_2) + b(x_1 + x_2 + x_3)$ |
| Total number of distinct i/o functions | 110 | 220 |



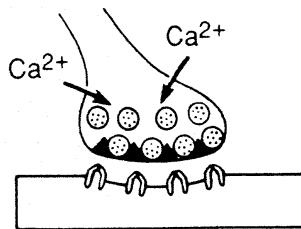
2D taxonomy of single cell modells



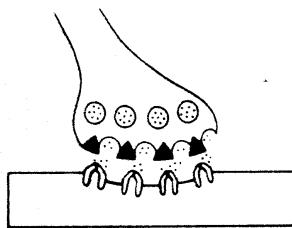
Synaptic models



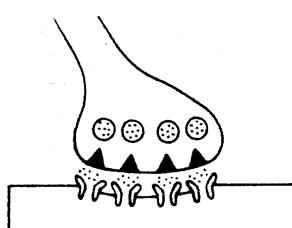
1. presynaptic action potential



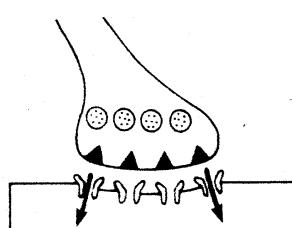
2. Ca²⁺ influx



3. transmitter release from the vesicles



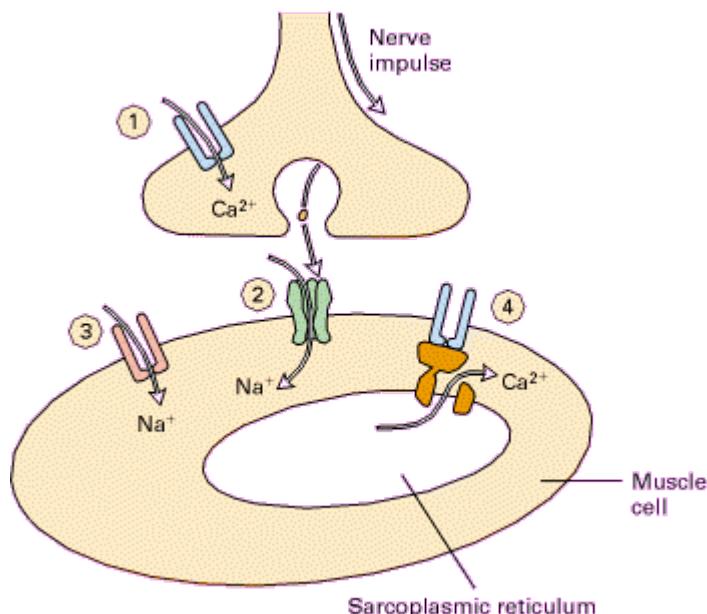
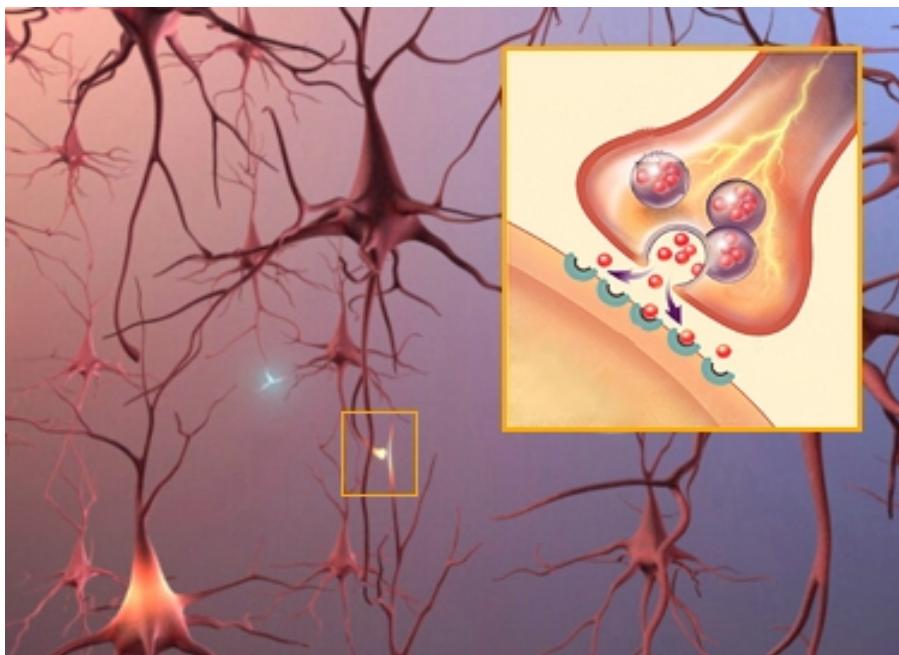
4. transmitter-receptor binding



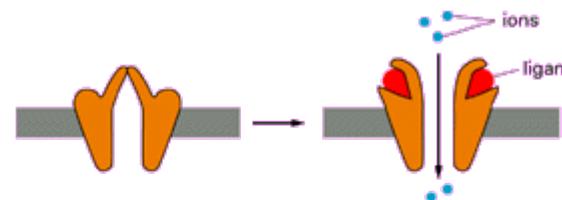
5. postsynaptic conductance ("PSG"),
current (PSC) and potential changes (PSP)

The aim synaptic models:
To calculate the postsynaptic potential changes, based on the presynaptic activity.

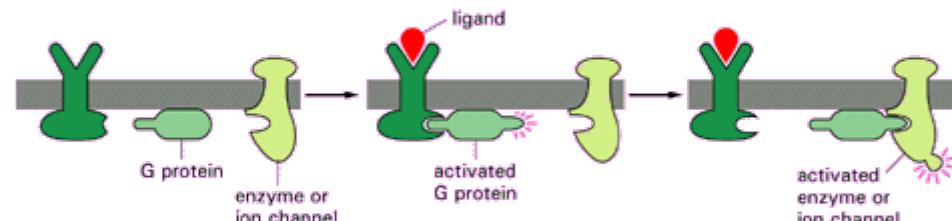
Between two neuron: The synapse



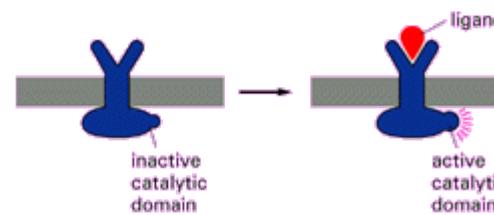
(A) ION-CHANNEL-LINKED RECEPTOR



(B) G-PROTEIN-LINKED RECEPTOR



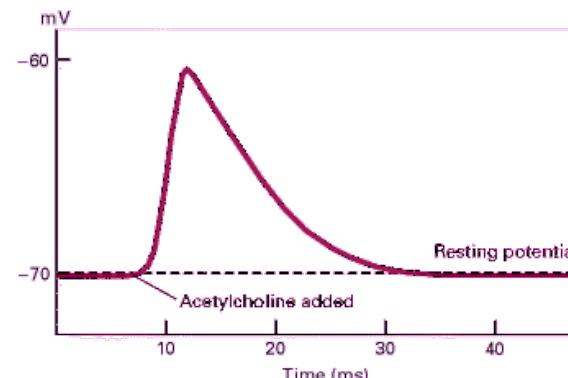
(C) ENZYME-LINKED RECEPTOR



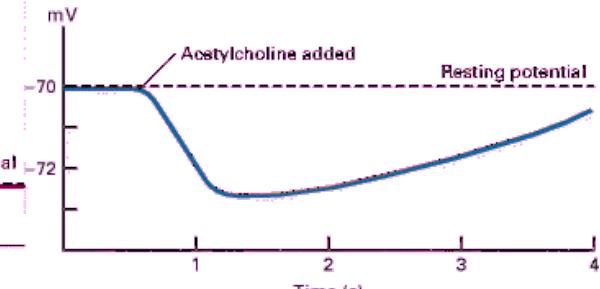
Ionotropic (A) and metabotropic (B,C) receptors

Excitatory and inhibitory postsynaptic potentials

(a) Excitatory synapse



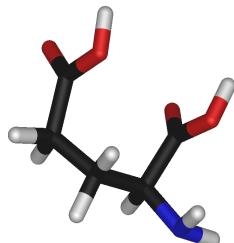
(b) Inhibitory synapse



Excitatory and inhibitory neurotransmitters

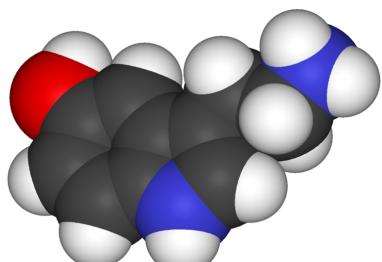
Glutamat

(information transmission)



Serotonin

(mood, wake/sleep)



Acetylcholin

(neuromuscular junction)

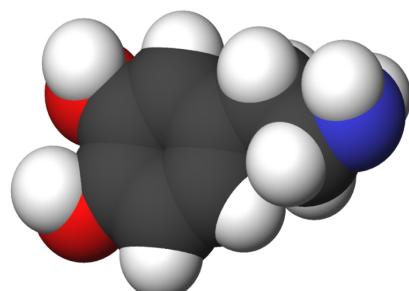


Noradneraline

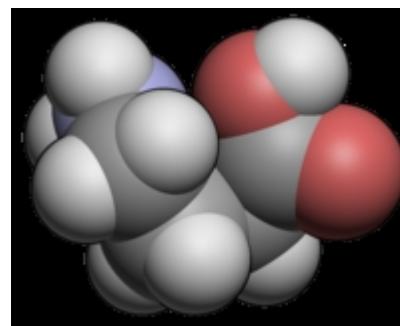
(arousal)

Dopamine

(reward system,
Parkinson disease,
schizophrenia)

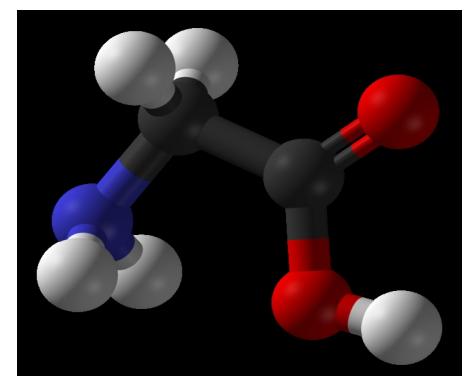


GABA-gamma aminobutyric acid
(in the central neural system)



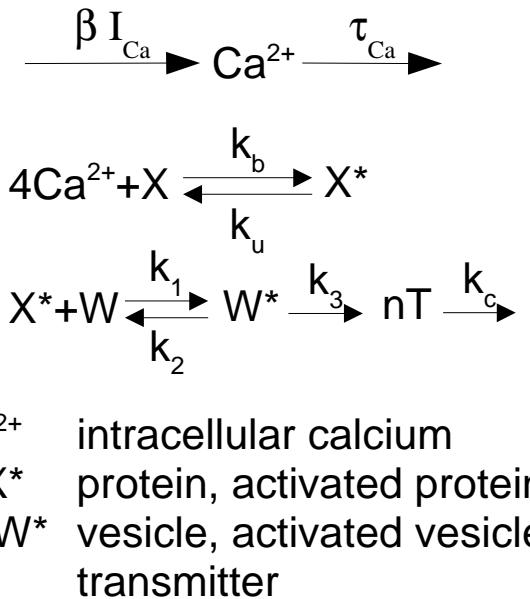
Glycine

(in the periphery)



Detailed kinetic synaptic models: the presynaptic side (1-3.)

kinetic schema (example)

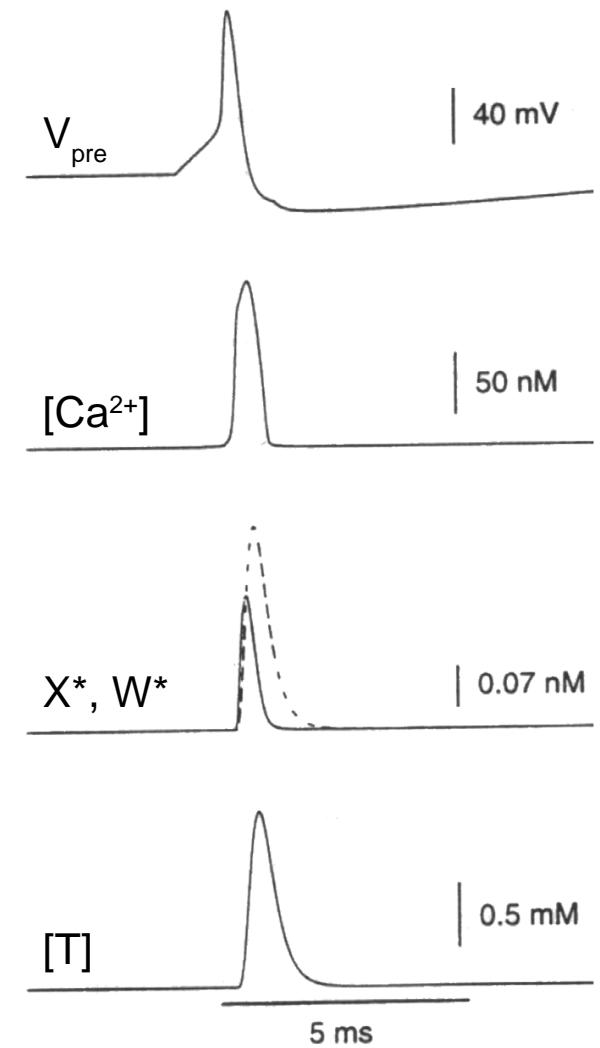


Reminder:

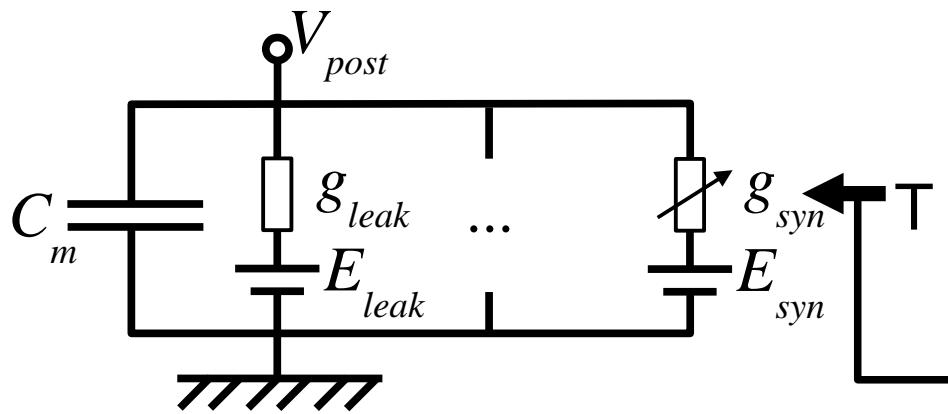
$$I_{Ca}(t) = \bar{g}_{Ca} s(t) (E_{Ca} - V_{pre}(t))$$

$$\frac{ds}{dt} = \frac{s_\infty(V_{pre}(t)) - s(t)}{\tau_s(V_{pre}(t))}$$

$$\begin{aligned} \frac{d[Ca^{2+}]}{dt} &= \beta I_{Ca}(t) - \frac{[Ca^{2+}](t)}{\tau_{Ca}} - \\ &\quad - 4k_b [Ca^{2+}](t) X(t) + 4k_u X^*(t) \\ \frac{dX}{dt} &= -k_b [Ca^{2+}](t) X(t) + k_u X^*(t) \\ \frac{dX^*}{dt} &= k_b [Ca^{2+}](t) X(t) - k_u X^*(t) - \\ &\quad - k_1 X^*(t) W(t) + k_2 W^*(t) \\ \frac{dW}{dt} &= -k_1 X^*(t) W(t) + k_2 W^*(t) \\ \frac{dW^*}{dt} &= k_1 X^*(t) W(t) - k_2 W^*(t) - k_3 W^*(t) \\ \frac{dT}{dt} &= k_3 n W^*(t) - k_c [T](t) \end{aligned}$$

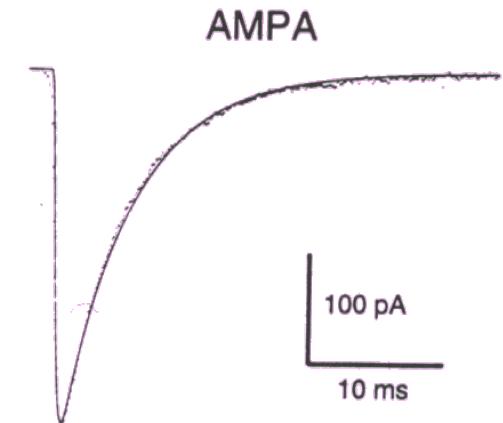
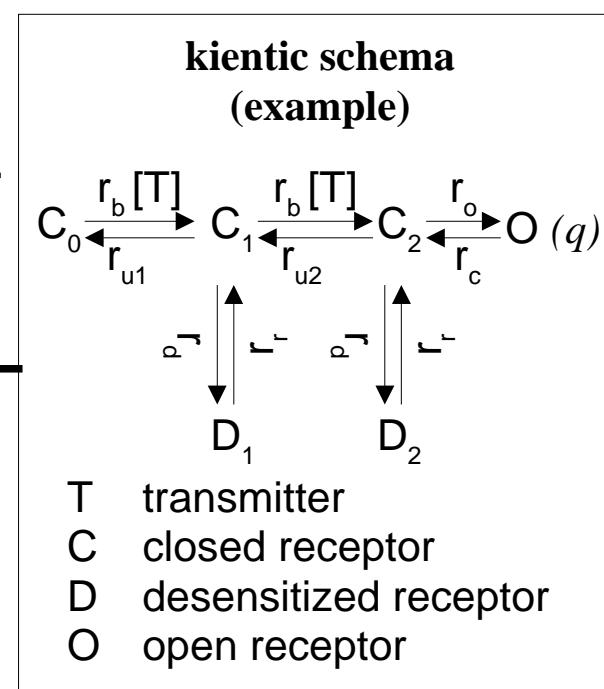


Detailed kinetic synaptic models: the postsynaptic side (4-5.)



Reminder:

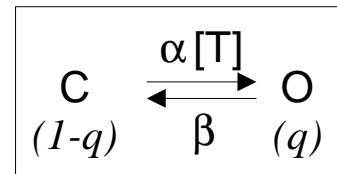
$$I_{syn}(t) = \bar{g}_{syn} q(t) (E_{syn} - V_{post}(t))$$



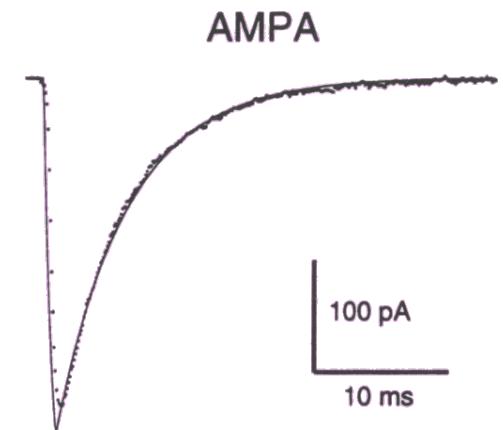
presynaptic side

$$[T](V_{pre}(t)) = \frac{T_{max}}{1 + e^{-\frac{V_{pre}(t) - V_\Theta}{K_\Theta}}}$$

postsynaptic side



$$\frac{dq}{dt} = \alpha [T](t)(1 - q(t)) - \beta q(t)$$

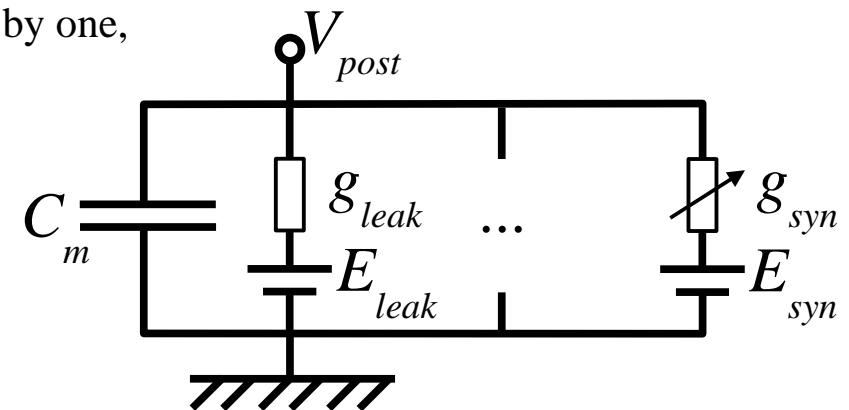


Phenomenological synaptic models

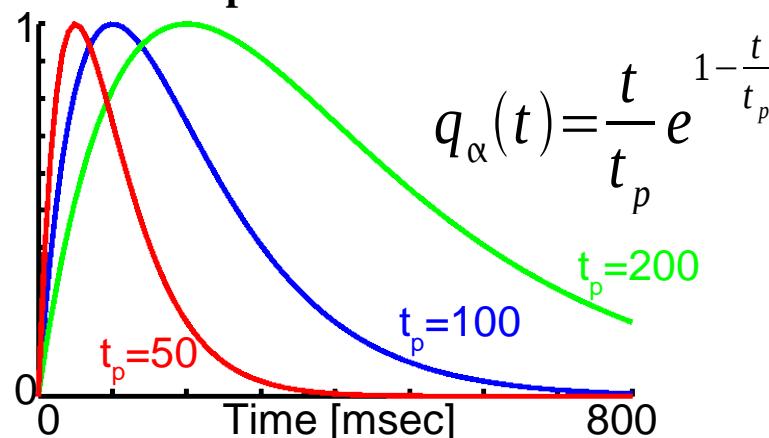
The postsynaptic conductance change, caused by one, single presynaptic action potential

$$I_{syn}(t) = \bar{g}_{syn} (E_{syn} - V_{post}(t)) \underbrace{\int_0^t \Delta(V_{pre}(t_0) - V_\Theta) \cdot q(t-t_0) dt_0}_{\text{Linear summation of the individual conductance changes}}$$

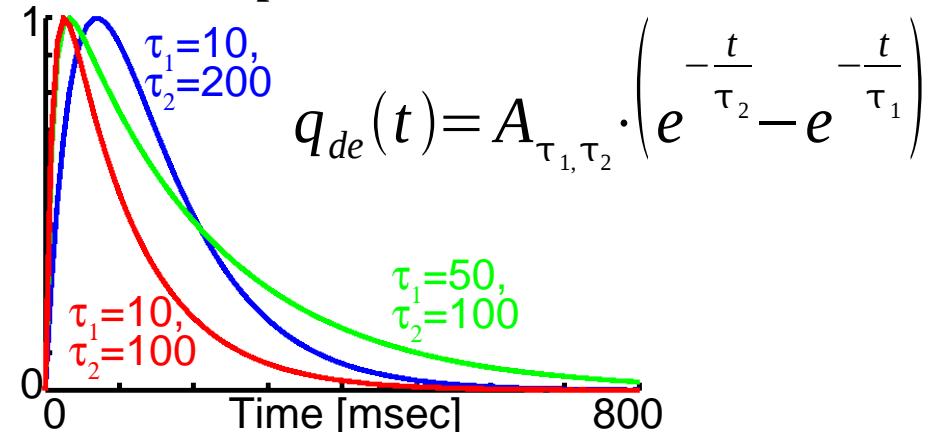
(convolution by Dirac-delta function)



alpha-function



double exponential function



Synaptic models: summary

| | | detailed | simplified | phenomenological |
|----------------------|-----------------------|----------|------------|------------------|
| Number of variables | presynaptic | ≥ 5 | 0 | 0 |
| | postsynaptic | ~ 5 | 1 | |
| reproduced phenomena | sensitisation | yes | no | no |
| | desensitisation | yes | no | no |
| | saturation (PSG, PSC) | yes | no | no |
| | saturation (PSP) | yes | yes | no |

Excitation or inhibition?

