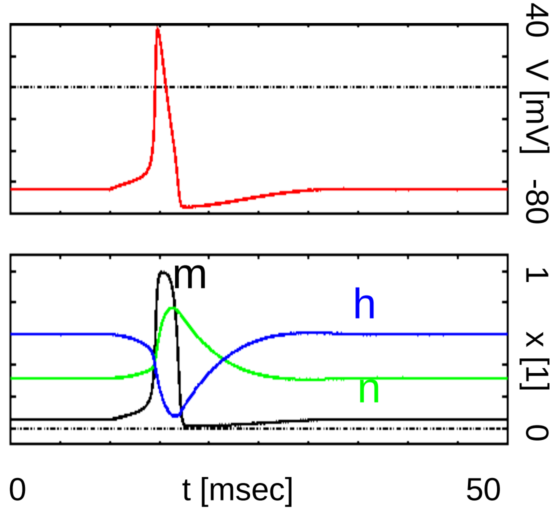
Simplified neuron models

Reminder: the HH-model

$$C_m \frac{dV}{dt} = \bar{g}_{Na}\left(E_{Na} - V(t)\right) m^3(t) h(t) + \bar{g}_K \left(E_K - V(t)\right) n^4(t) +$$

$$+ g_{leak}\left(E_{leak} - V(t)\right) + I_{external}(t)$$

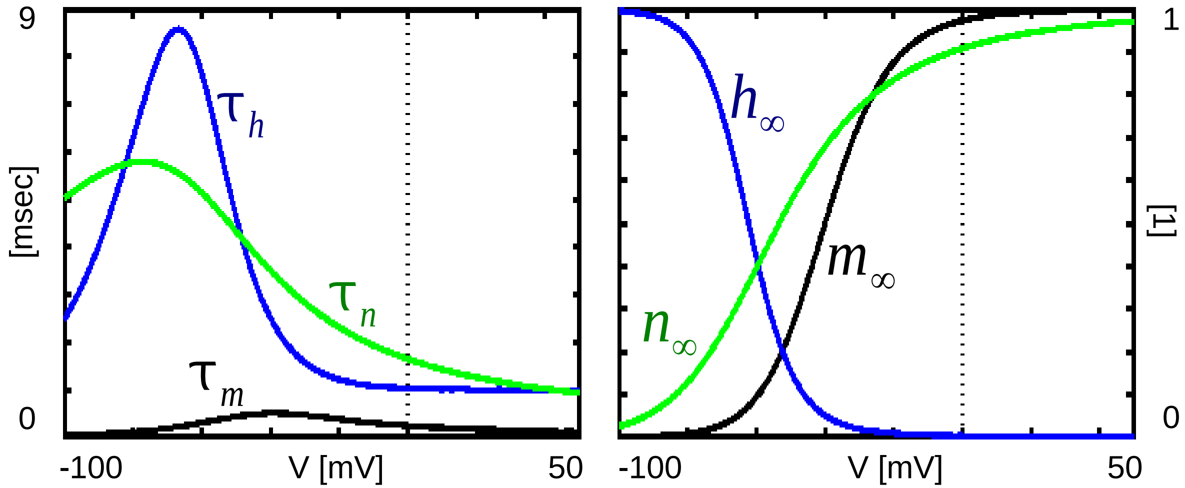
One spike



$$\frac{dm}{dt} = \frac{m_\infty\left(V(t)\right) - m(t)}{\tau_m\left(V(t)\right)}$$

$$\frac{dh}{dt} = \frac{h_\infty\left(V(t)\right) - h(t)}{\tau_h\left(V(t)\right)}$$

$$\frac{dn}{dt} = \frac{n_\infty\left(V(t)\right) - n(t)}{\tau_n\left(V(t)\right)}$$



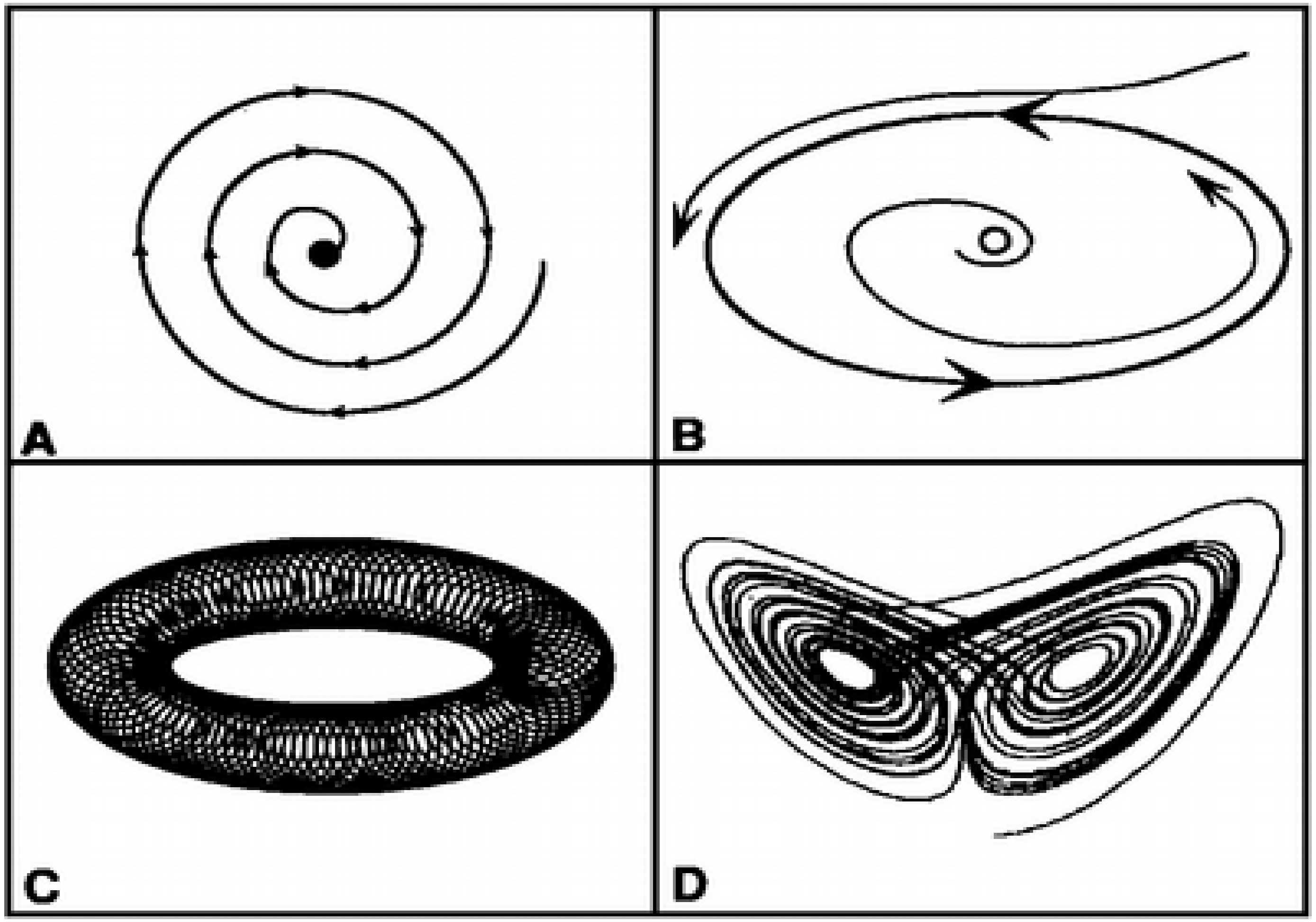
Types of attractors

A: Fixed point

B: Limit cycle

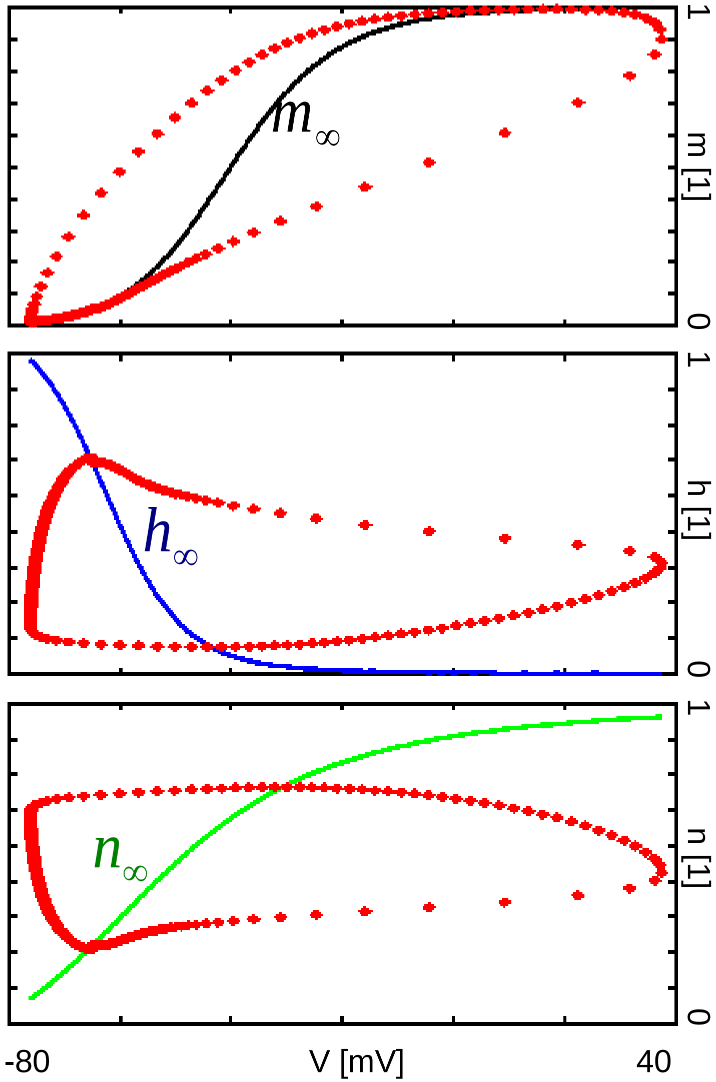
C: Quasi periodic: requires irrational ratio for at least two osscillations.

D: Chaotic: requires at least 3 dimensions.



The dimension reduction of the HH-model

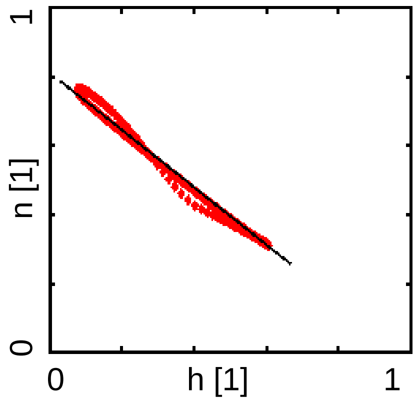
One spike in the state space (three projections)



FAST ACTIVATING VARIABLES *(V)*

$$C_m \frac{dV}{dt} = \bar{g}_{Na}\left(E_{Na} - V(t)\right) m_\infty^3(t)\left(1 - W(t)\right) + \bar{g}_K\left(E_K - V(t)\right)\left(\frac{W(t)}{s}\right)^4 + g_{sziv}\left(E_{leak} - V(t)\right) + I_{external}(t)$$

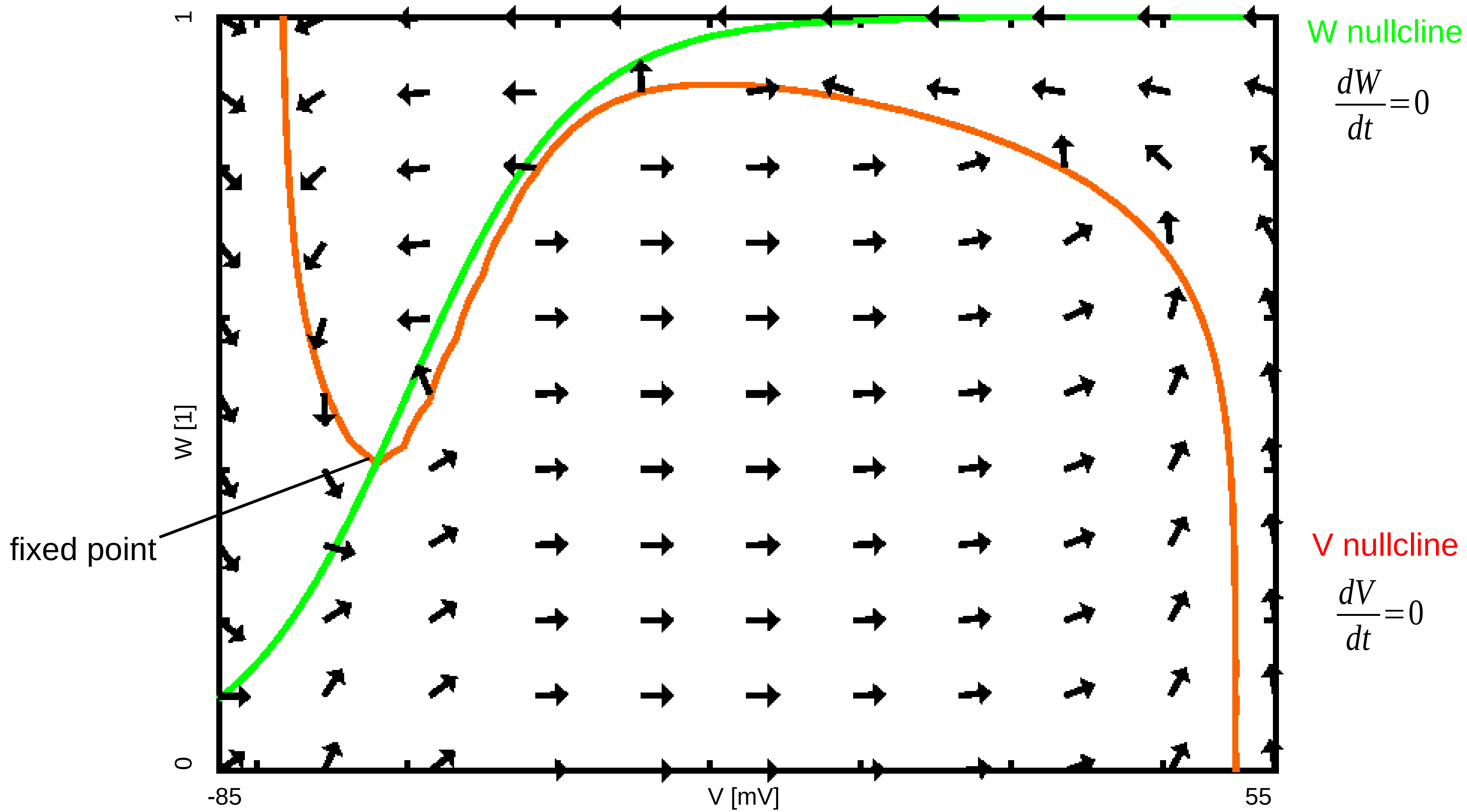
$$\frac{dW}{dt} = \frac{W_\infty\left(V(t)\right) - W(t)}{\tau_W\left(V(t)\right)}$$



SLOW, INACTIVATING VARIABLES *(W)*

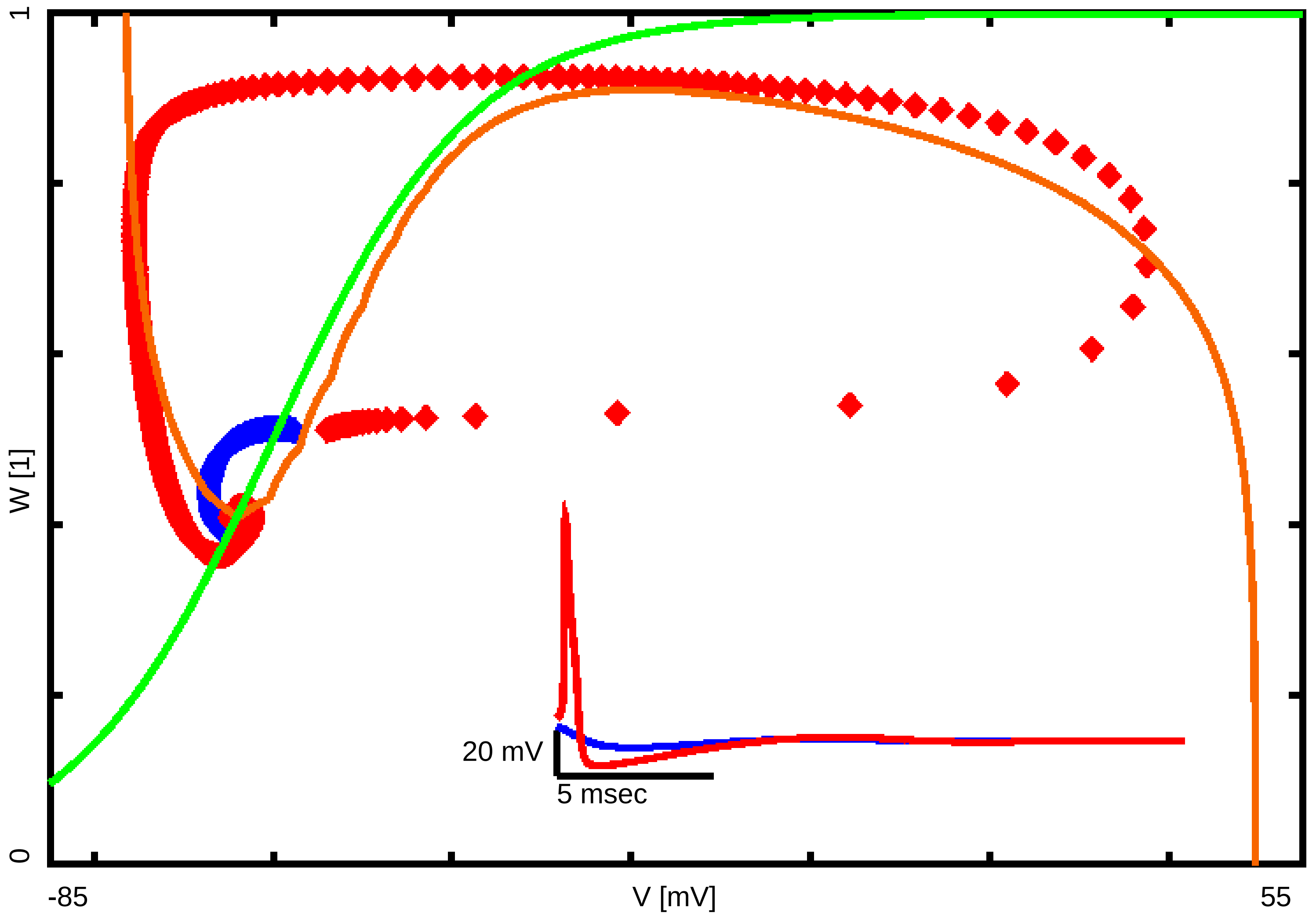
Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 1

The phase plane with the direction field

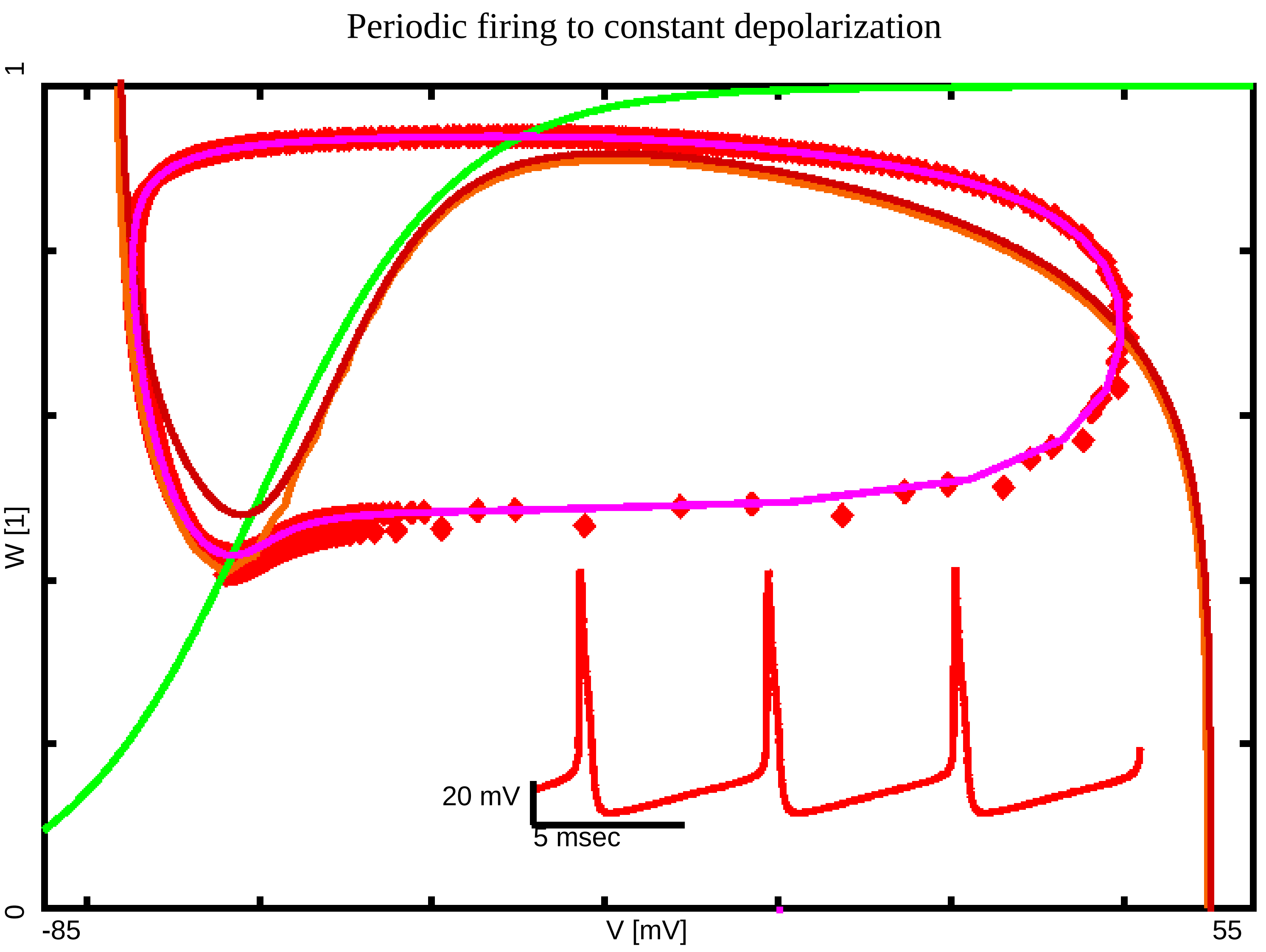


Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 2

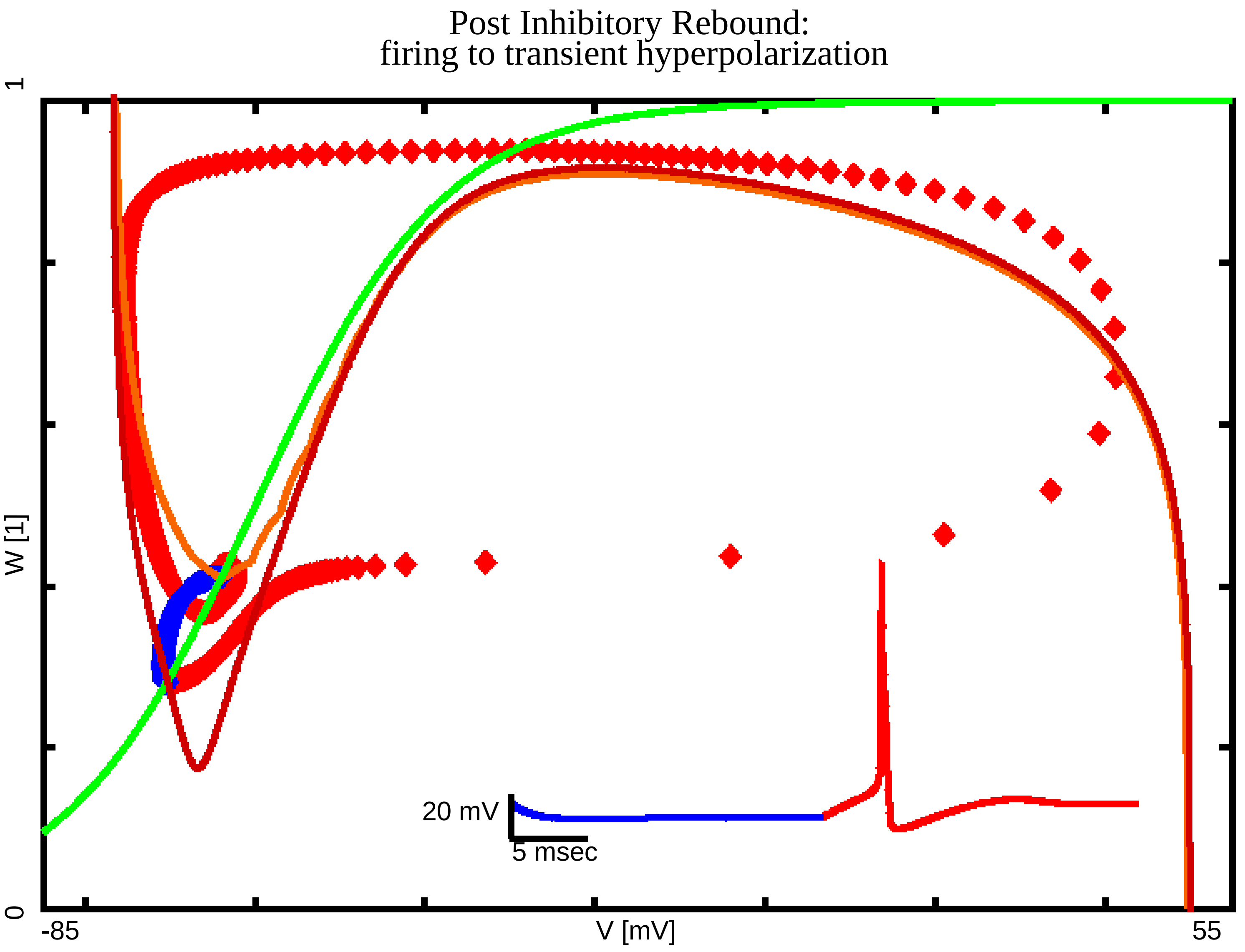
Excitability: firing is a threshold phenomenon (all-or-none)



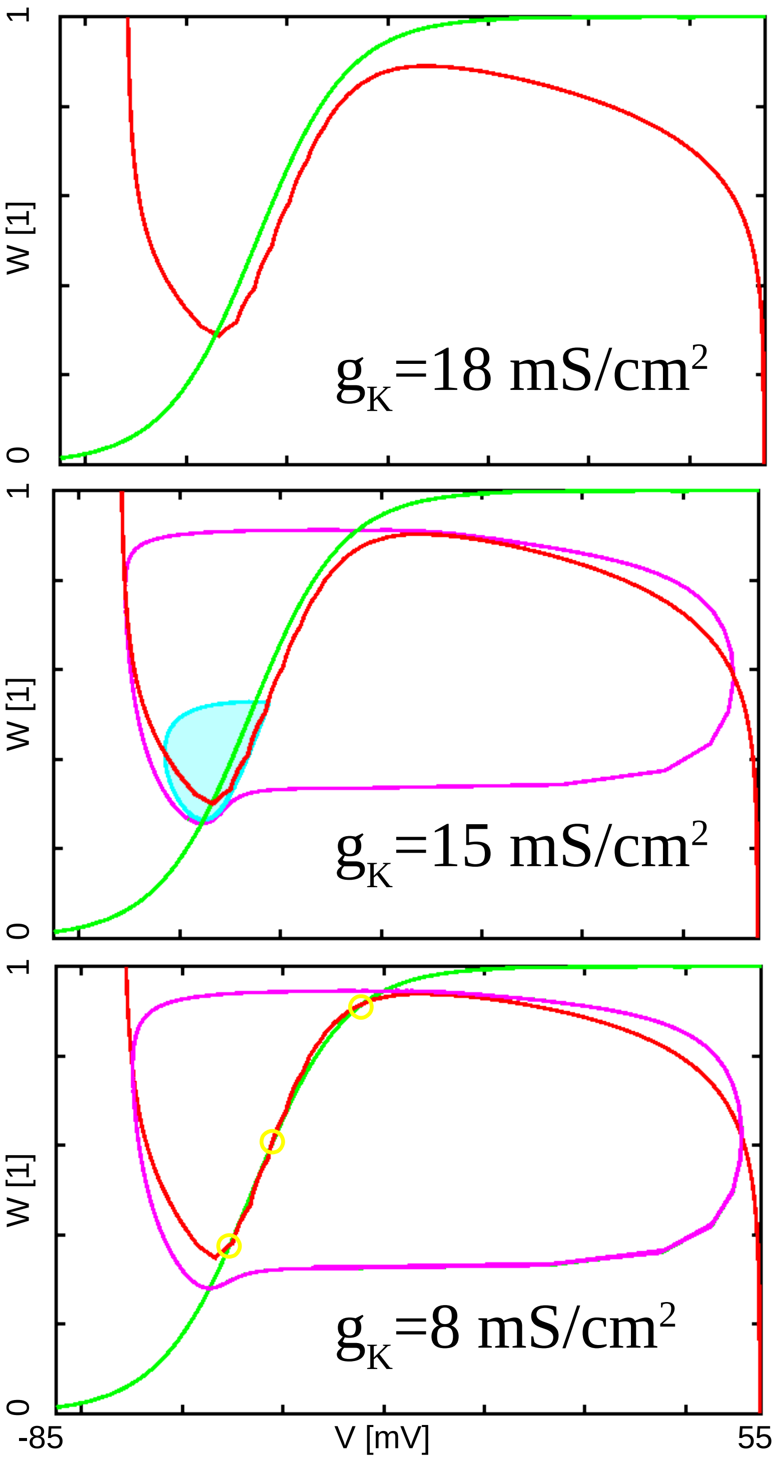
Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 3

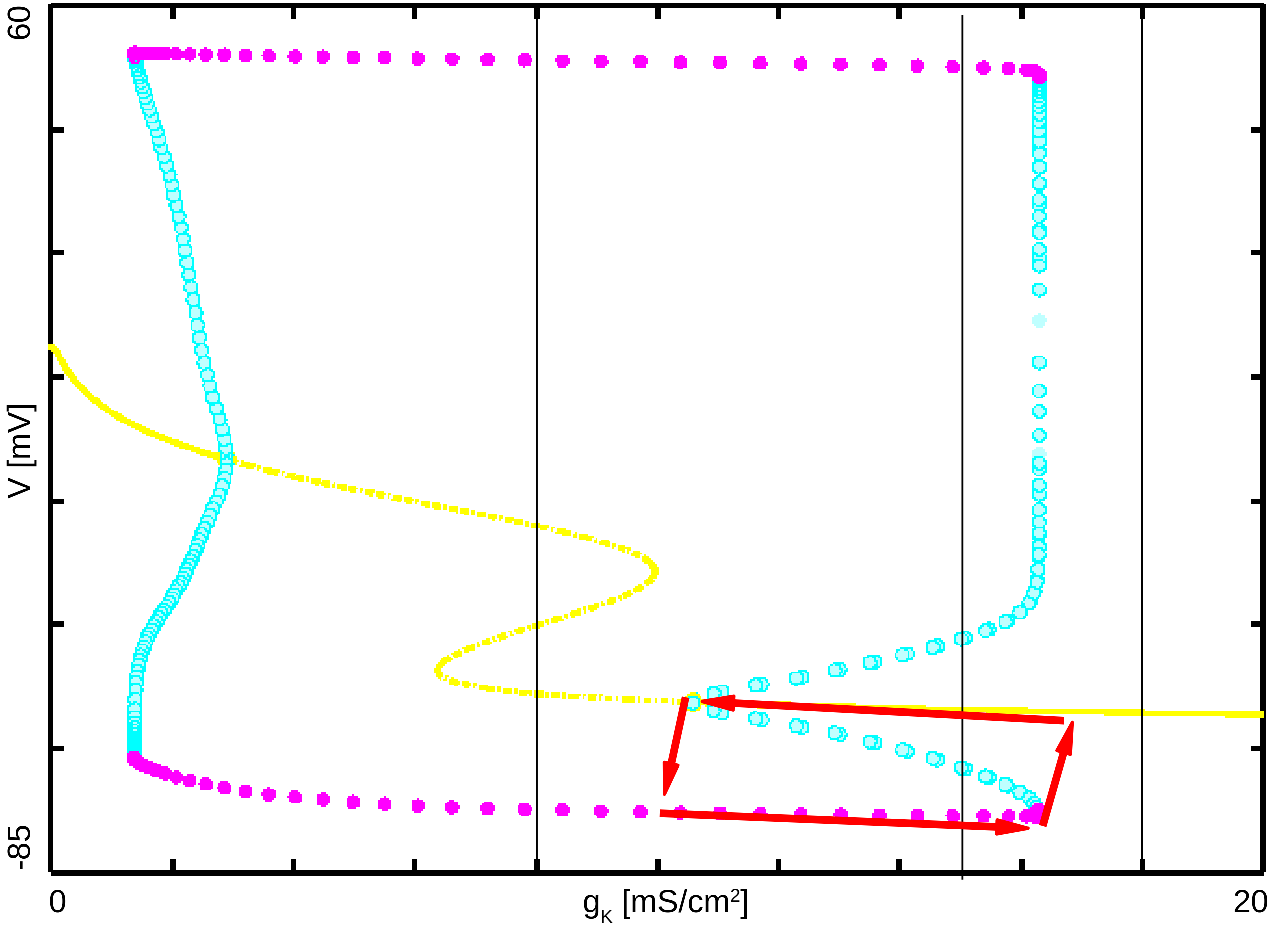


Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 4



Phase Plane Analysis: Bursting / 1

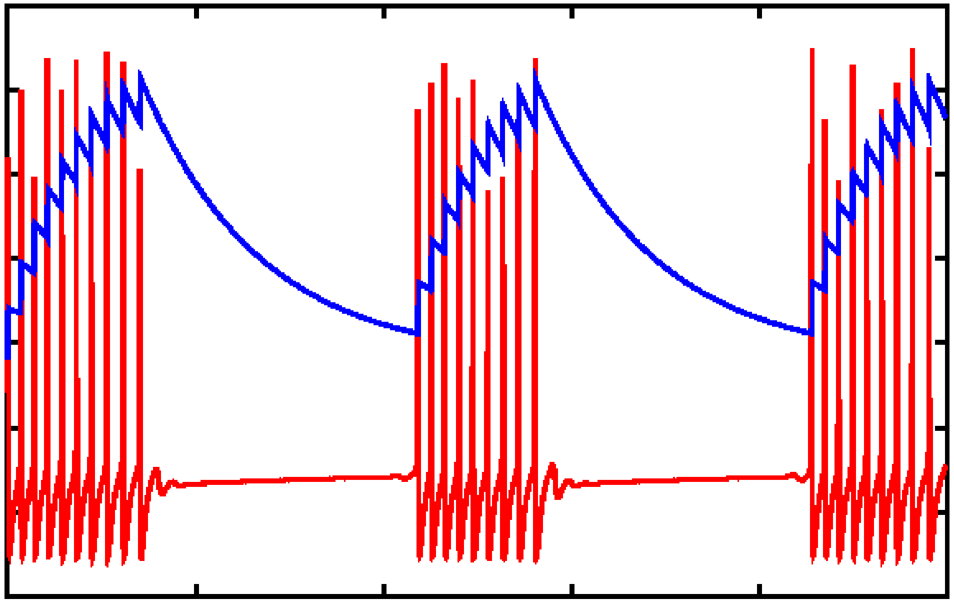




$$\frac{d(g_K)}{dt} = Z(V)(V - V_{rest}) - \frac{g_K - g_K^{rest}}{\tau_{g_K}}$$

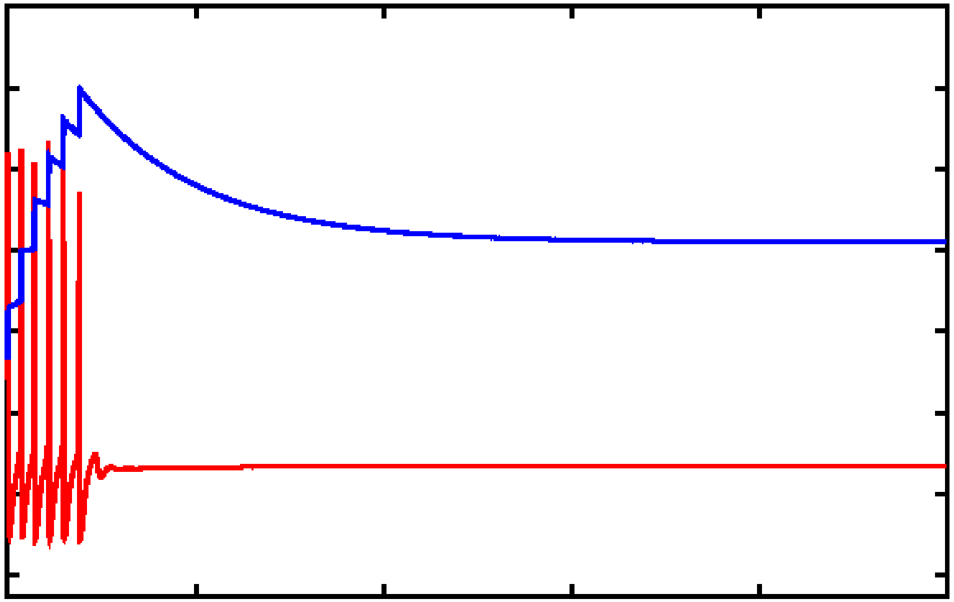
Phase Plane Analysis: Bursting / 2

Endogeneous Burster



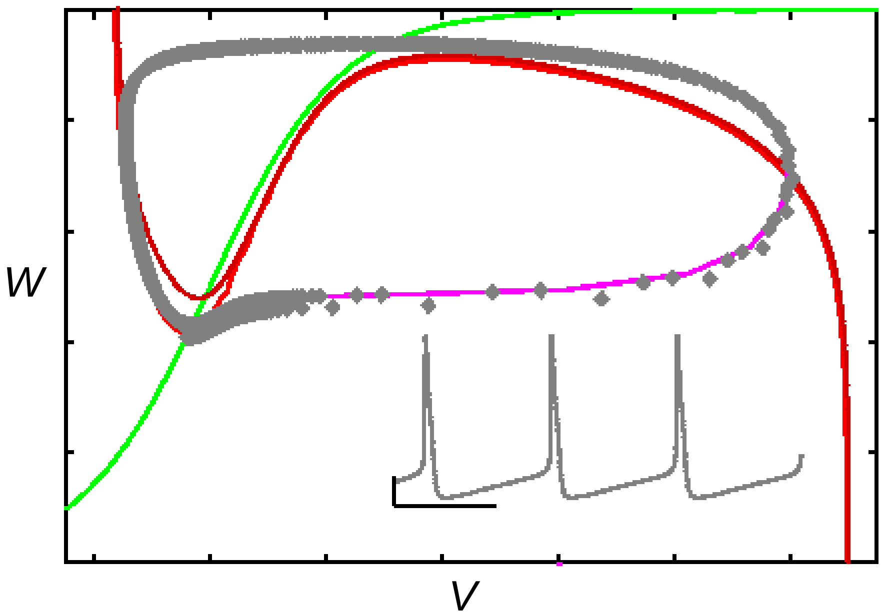
$g_K^{rest} = 8\,mS/cm^2$

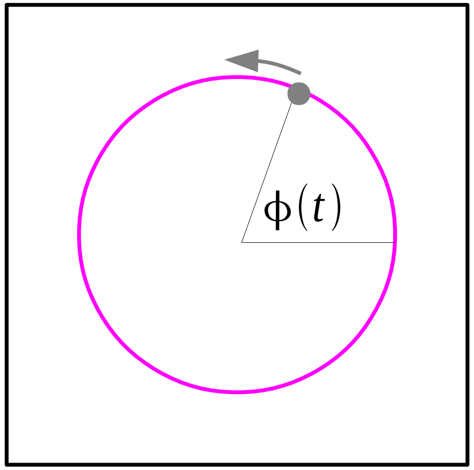
Conditional Burster



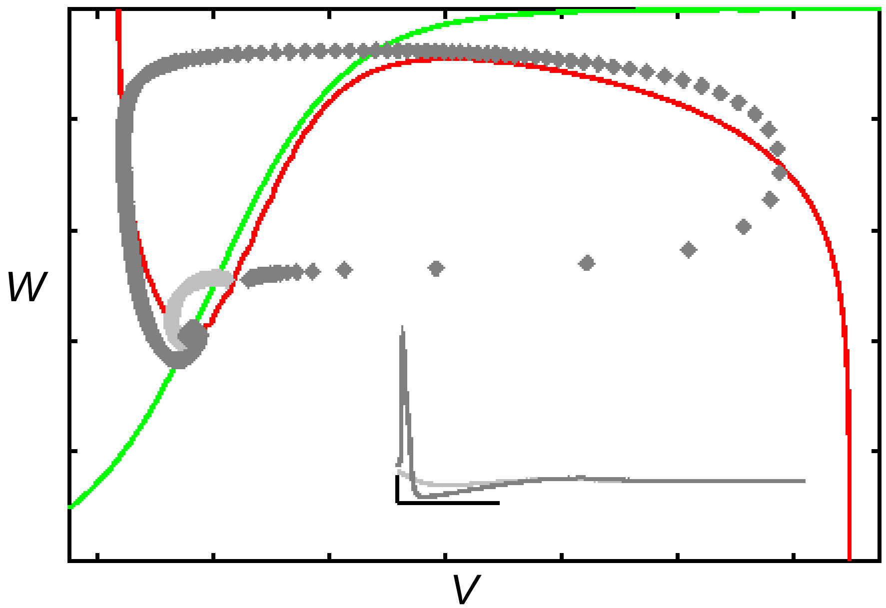
$g_K^{rest} = 12\,mS/cm^2$

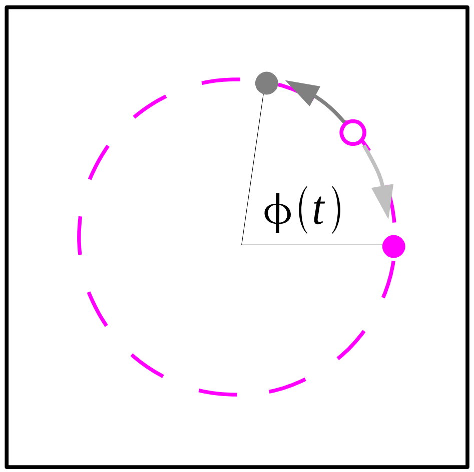
The phase-model





$$\frac{d\phi}{dt}=\omega\big(I(t)\big)+h\big(\phi(t),\phi(t)'\big)$$

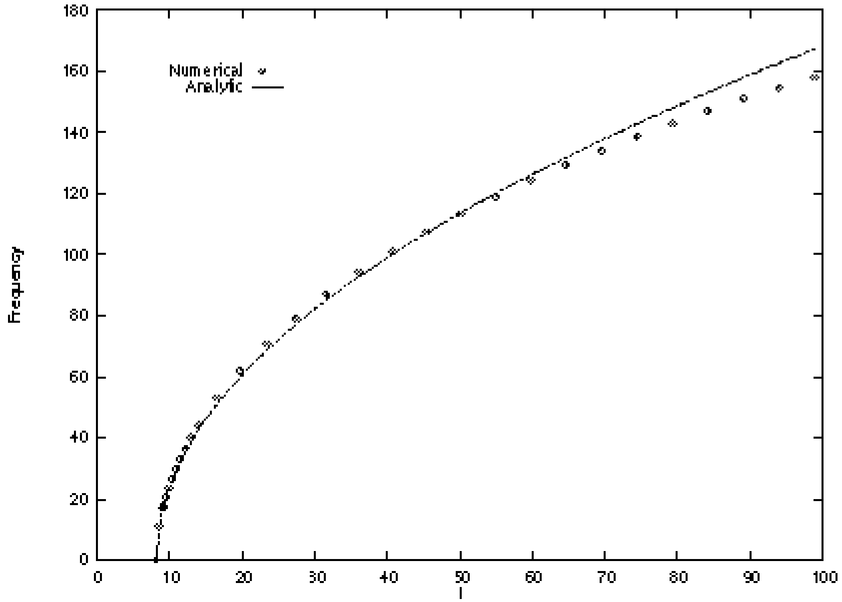




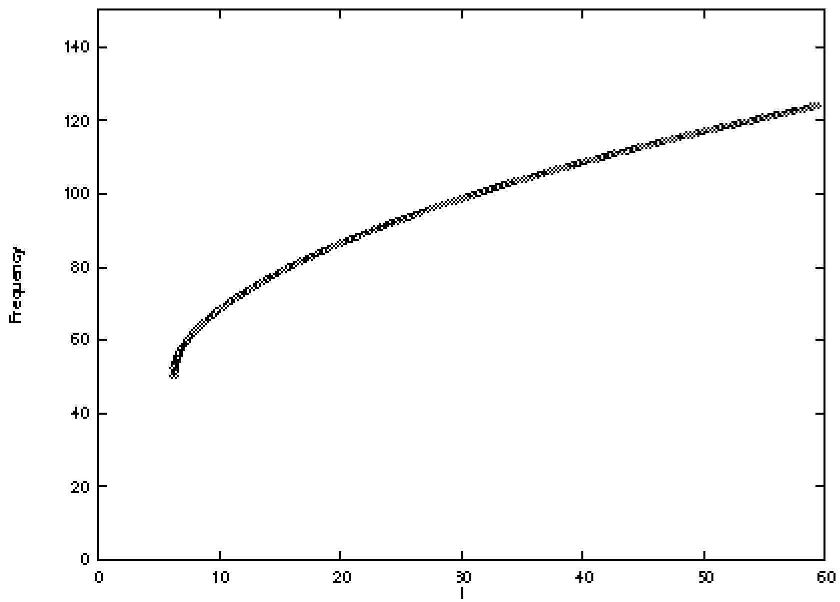
$$\frac{d\phi}{dt}=\omega\big(I(t),\phi(t)\big)+h\big(\phi(t),\phi(t)'\big)$$

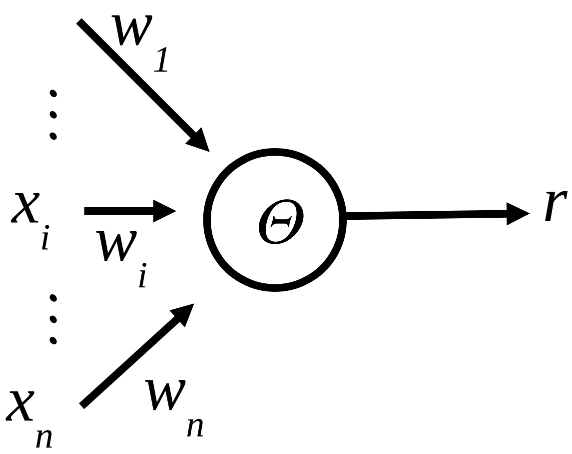
The rate-model

Type I

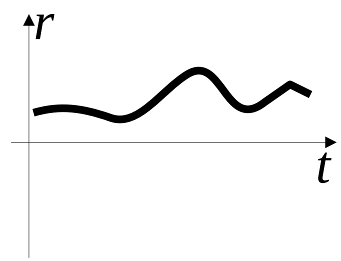


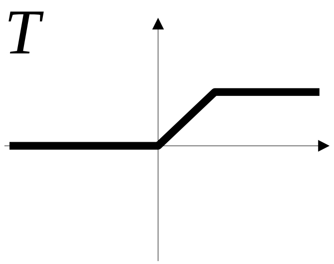
Type 2 (HH)



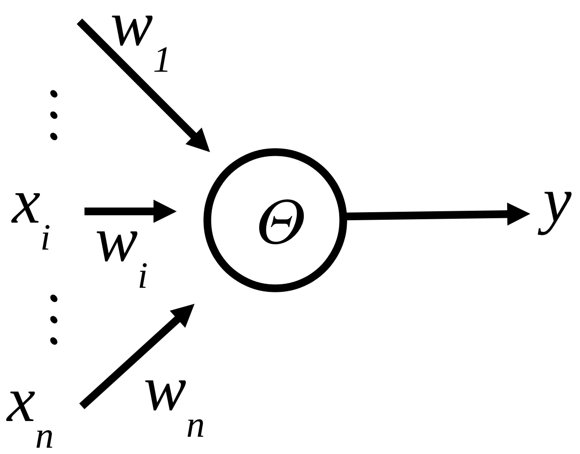


$$r(t) = T\left(\sum_{i=1}^{n} w_i \cdot x_i(t) - \Theta\right)$$

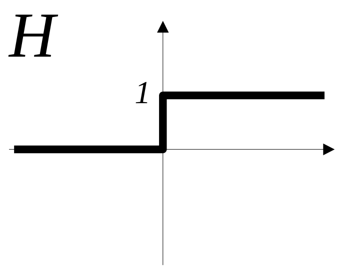




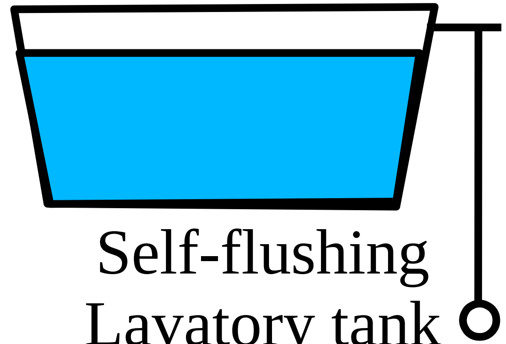
The McCulloch-Pitts model



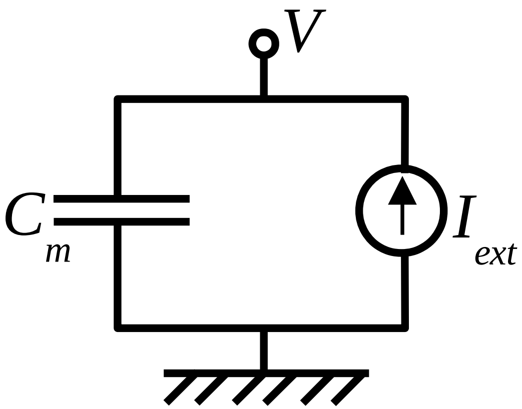
$$y(t)=H\left(\sum_{i=1}^{n} w_i\cdot x_i(t)-\Theta\right)$$



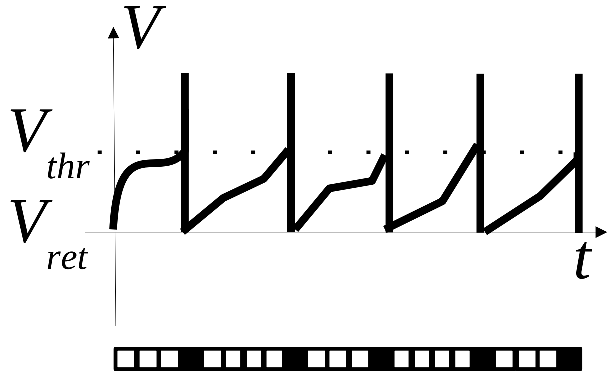
Integrate & Fire neuron



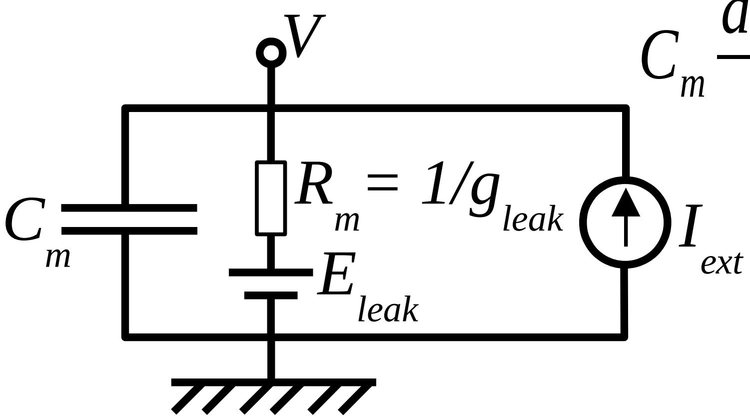
ha $V(t) > V_{thr} \longrightarrow$ ***spike*** $\longrightarrow V(t) := V_{ret}$



$$C_m \frac{dV(t)}{dt} = I_{ext}(t)$$



Leaky Integrator



$$C_m \frac{dV(t)}{dt} = \frac{E_{leak} - V(t)}{R_m} + I_{ext}(t)$$