Ferromagnetism in quark matter and origin of the magnetic field in compact stars

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Two magnetic aspects of quark matter, ferromagnetism and spin density wave, are discussed in the temperature-density plane. Some implications of ferromagnetism are suggested on relativistic heavy-ion collisions and compact stars.

1. INTRODUCTION

So far, many studies have been devoted to understand high density and/or high temperature QCD. In this talk we are concentrated on magnetic aspects of quark matter. We shall discuss the possibility of ferromagnetism and spin density wave in quark matter. These magnetic properties of quark matter should be closely related to phenomena observed in relativistic heavy-ion collisions or compact stars. Actually our study has been stimulated by the recent discovery of magnetars [1], which have huge magnetic field of 10^{15}G and come into a new class of compact stars. It has been first estimated by $P-\dot{P}$ curve for pulsars and more recently suggested by observation of the cyclotron absorption lines.

The origin of the magnetic field in compact stars has been a long-standing problem since the first discovery of pulsars in early seventies. Instead of asking for the origin, many people have taken a simple working hypothesis, conservation of magnetic flux during the stellar evolution. However, if we apply this hypothesis to explain the magnetic field of magnetars, we immediately have a contradiction that their radius should be much less than the Schwarzschild radius. Thus, the discovery of magnetars seems to give a chance to reconsider the origin of the magnetic field in compact stars. Since there is widely developed hadronic matter inside compact stars, we consider the spin-polarization in hadronic matter as a microscopic origin of the magnetic field in compact stars.

2. RELATIVISTIC FERROMAGNETISM

We first studied the possibility of ferromagnetism in quark matter in the perturbative way, in analogy with the Bloch mechanism for itinerant electron system [2]. Consider the spin-polarized electron gas interacting with the Coulomb force. Then total kinetic energy is increased, while the interaction energy is decreased due to the Pauli principle;

an electron pair with the same spin cannot approach each other to effectively avoid the Coulomb repulsion.

In the relativistic theories, spin is not a good quantum number. Still, we can consider a ferromagnetic state with recourse to the Pauli-Luvansky vector and the spin four vector a^μ , which is specified not only by the spin direction vector $\boldsymbol{\zeta}$ in the rest frame of each quark but also by its momentum $\mathbf{p}$, different from non-relativistic theories. Thereby there are many choices for the spin configuration in the phase space of quark matter, which are all reduced to a constant $\boldsymbol{\zeta}$ configuration in the non-relativistic limit [3]. Using the standard spin configuration given by the Lorentz transformation of $\boldsymbol{\zeta} = \pm\hat{\mathbf{z}}$ for each quark, we can see a weakly first-order phase transition to ferromagnetic state at low densities n_q (Fig. 1). We also see that there is metamagnetic state even in the paramagnetic phase.

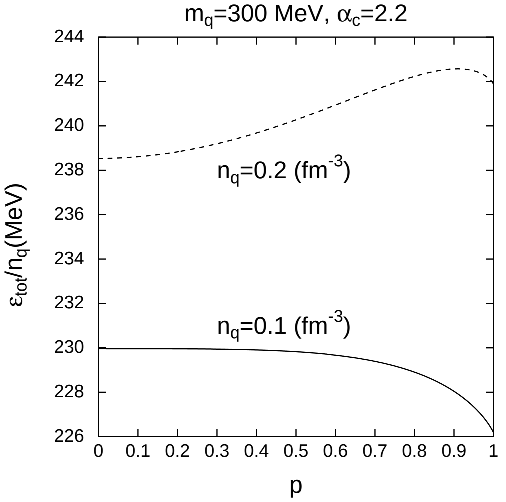

Figure 1. First-order phase transition with MIT bag-model parameters. $p \equiv (n_+ - n_-)/n_q$

To get more insight into the ferromagnetic transition we must treat it in a non-perturbative way; the Hartree-Fock approximation may be the next step [4,5]. When we consider the Fock self energy V for quarks, the propagator can be simply written as

$$G^{-1}(p) = \not{p} - m_q + V(p), \tag{1}$$

where the self-energy consists of the mean-fields in various channels. Among them the axial-vector mean-field $\mathbf{V}_A$ is responsible to magnetization. Taking only $\mathbf{V}_A$ for simplicity along the z-axis, $V_A = -\gamma^5\gamma^3 U_A$, we find the single-particle energy,

$$\epsilon_n = \pm\left[\mathbf{p}^2 + m_q^2 + U_A^2 + \zeta U_A\sqrt{m_q^2 + p_z^2}\right]^{1/2}, \tag{2}$$

for each spin state $\zeta = \pm 1$. Here we can clearly see that it is split depending on the spin state [1] and rotation symmetry is broken due to the coupling of momentum and $\mathbf{V}_A$. Thus we have two Fermi seas with different shapes: the Fermi sea of majority particle is deformed in the "prolate" shape, while that of minority particle in the "oblate" shape [4]. Since U_A should be given by $G(p)$,

$$U_A(k) = -\frac{g^2}{2}\frac{N_c^2-1}{4N_c^2}\frac{1}{N_f}\int\frac{d^4p}{(2\pi)^4}D(k-p)\mathrm{tr}\left[\gamma^5\gamma^3 G(p)\right], \tag{3}$$

it may still be a complicated task to solve Eq. (3) in a self-consistent manner. Here we use an effective model with a *zero-range* interaction instead of the original OGE,

$$\mathcal{H}_{\mathrm{int}} = G^2\bar{q}\gamma_\mu\gamma^5 q\bar{q}\gamma^\mu\gamma^5 q. \tag{4}$$

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[1] It is called as the exchange splitting in the non-relativistic case.

Evaluating Eq. (3) over not only the Fermi seas but also the Dirac sea, we find that the phase transition is of second order at the critical chemical potential,

$$\mu_c = \mu_c^F \Lambda / m_q e^{-\gamma_E/2}, \quad \mu_c^F = \exp\left[\frac{4\pi^2}{3N_f N_c G^2 m_q^2}\right], \tag{5}$$

for $m_q \ll \Lambda$, where μ_c^F is given by the genuine Fermi-sea effect and other factors by the vacuum polarization effect after the proper-time regularization with a cutoff parameter Λ[5]. We can clearly see the effect of quark mass as well as a non-perturbative nature with respect to the coupling strength. Thus quark matter is in a ferromagnetic phase above μ_c. It would be interesting to compare this result with the previous one given by the perturbative calculation, where we have seen that a ferromagnetic phase develops at rather low densities. This difference should come by the difference of the interaction range: the infinite range in the latter case and the zero range in the present case. The order of the phase transition is also changed from the first order to the second order. These qualitative differences may be very important and need further investigations [6].

3. COLOR MAGNETIC SUPERCONDUCTIVITY

When the ferromagnetic phase is realized in quark matter, its relation with the color superconductivity should be elucidated. We have discussed the possibility of the coexistence of both phase in quark matter [4]. Starting with the OGE interaction again, we leave the particle-particle pair mean-field Δ as well as the particle-hole one V; the former is responsible to color superconductivity, while the latter ferromagnetism. Considering the $\bar{3}$ pair field both in the color and flavor space for Δ, we assume quark pairs with the same spin state $\zeta = \pm 1$ in each Fermi surface. Then we found that the gap functions Δ_ζ have angular dependence on the Fermi surfaces like in the A phase of the liquid ^{3}He: they take nodes on the both poles and maximum around the equator. Consequently we have shown that the coexistence of ferromagnetism and color superconductivity is possible, while the latter somewhat suppresses magnetism [4].

4. CHIRAL SYMMETRY RESTORATION AND SPIN DENSITY WAVE

In the recent papers we have discussed another magnetic aspect in the intermediate densities [7], where chiral symmetry restoration has been expected. We consider a density-wave state with

$$\langle \bar{q}q \rangle = \Delta \cos \mathbf{q}\cdot\mathbf{r}, \quad \langle \bar{q}i\gamma^5\tau_3 q \rangle = \Delta \sin \mathbf{q}\cdot\mathbf{r}, \tag{6}$$

for two-flavor quark matter. Then we can see that the non-trivial spatial dependence of the chiral angle $\theta = \mathbf{q}\cdot\mathbf{r}$ induces the kinetic term of the density wave $\propto q^2$ by the vacuum polarization as well as the axial-vector coupling term with quarks. The latter term gives an energy gain due to the *nesting* of the Fermi surface If it is superior to the energy increase due to the former effect, such a density-wave state is favored. We call it as *dual chiral-density wave* (DCDW) state. Physically the amplitude gives a dynamical mass to quarks and is closely related to the chiral-symmetry restoration, while the phase gives a magnetic ordering to quark matter; the mean spin value vanishes in this phase, but total magnetic moment is spatially oscillating. So, this phase is a kind of the *spin density wave*. We have explored the possible region of the DCDW state in the temperature-density plane, by explicitly using the NJL model [7]. It should be also interesting to consider that the ansatz (6) describes another path for chiral-symmetry restoration, different from the usual one ($\mathbf{q} = 0$). In this view chiral-symmetry restoration is delayed by the presence of DCDW.

5. SUMMARY AND CONCLUDING REMARKS

We have discussed two magnetic aspects expected in quark matter: ferromagnetism and spin density wave. For the latter we may notice a resemblance of DCDW to pion condensation in hadron matter. Actually, the symmetry structure is common, and thereby magnetic property is the same: the pion condensed phase has been known to have anti-ferromagnetic order [8]. So, one may expect a hadron-quark continuity between two phases.

Ferromagnetism has some implications on phenomena in relativistic heavy-ion collisions, cosmology or compact stars. If the strange matter is absolutely stable, strangelets should be magnetized like small magnets, or magnetized quark lumps may be left as relics of QCD phase transition in early universe to produce a primordial magnetic field in the galaxies. More direct implication may be given for magnetic fields in compact stars. Assuming that magnetars are mostly occupied by quark matter, we can easily estimate the magnitude of the surface magnetic field B to be $O(10^{15-17}\mathrm{G})$[2]. As another implication, ferromagnetism may give a scenario about the hierarchy of the magnetic field observed in compact stars; there are located three colonies of pulsars with different B in the $P - \dot{P}$ plane. One may need some dynamical effects to explain it like destruction and formation of magnetic domain or metamagnetic state, depending on change of the external parameters such as rotation, mass accretion or temperature.

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REFERENCES

1. P.M. Woods and C.J. Thompson, astro-ph/0406133.
2. T. Tatsumi, Phys. Lett. **B489** (2000) 280.
3. T. Maruyama and T. Tatsumi, Nucl. Phys. **A693** (2001) 710.
4. E. Nakano, T. Maruyama and T. Tatsumi, Phys. Rev. **D68** (2003) 105001; T. Tatsumi, T. Maruyama and E. Nakano, Prog. Theor. Phys. Suppl. **153** (2004) 190; hep-ph/0312351.
5. T. Tatsumi, E. Nakano and K. Nawa, hep-ph/0506002.
6. T. Maruyama and T. Tatsumi, in progress.
7. T. Tatsumi and E. Nakano, hep-ph/0408294; E. Nakano and T. Tatsumi, Phys. Rev. **D71** (2005) 114006.
8. T. Takatsuka et al., Prog. Theor. Phys. **59**(1978) 1933.