NONLINEAR $K_\perp$-FACTORIZATION: A NEW PARADIGM FOR AN IN-NUCLEUS HARD QCD *

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We review the origin, and salient features, of the breaking of the conventional linear $k_\perp$-factorization for an in-nucleus hard pQCD processes. A realization of the nonlinear $k_\perp$-factorization which emerges instead is shown to depend on color properties of the underlying pQCD subprocesses. We discuss the emerging universality classes and extend nonlinear $k_\perp$-factorization to AGK unitarity rules for the excitation of the target nucleus.

1. Introduction

An extension of factorization theorems to nuclear targets is of burning urgency - the notion of nuclear gluon densities can be made meaningful only if they furnish a unified description of the whole variety of nuclear hard processes. A large thickness of a target nucleus introduces a new

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scale – the so-called saturation scale Q_A^2 –, which breaks the familiar linear $k_\perp$-factorization theorems for hard scattering in a nuclear environment. Here we review the recent work by the ITEP-Jülich-Landau collaboration in which a new concept of the nonlinear $k_\perp$-factorization has been introduced and elaborated [1,2,3,4,5,6]. To this end, we recall the 1975 observation [7], based on the insight from Gribov [8] and Kancheli [9], that the Lorentz contraction of ultrarelativistic nucleus entails a spatial overlap/fusion/screening of partons from nucleons at the same impact parameter if

$$x \lesssim x_A = 1/R_A m_N, \tag{1}$$

where R_A is the radius of a nucleus of mass number A.

Within the fusion reinterpretation of multiple gluon exchanges, a nuclear glue will be a nonlinear functional of the free nucleon glue: the same sea and glue will be shared by many nucleons. Specifically, we derived the collective glue of j-overlapping nucleons. It is a basis for the definition of the collective nuclear unintegrated glue $\phi(b, \boldsymbol{\kappa})$, per unit area in the impact parameter plane, which describes the coherent diffractive dijet production off nuclei [10,1], for instance, in πA collisions or DIS. We found that the so-defined collective nuclear glue provides the linear $k_\perp$-factorization for the nuclear structure function and the leading single-quark spectrum, but this is rather an exception due to the Abelian nature of these observables. All other single-jet and dijet cross sections prove to be highly nonlinear functionals – quadratures – of the collective nuclear glue [2,3,4,5,6]. Any application of linear $k_\perp$–factorization to nuclei is entirely unwarranted. Here we review how the concrete form of the nonlinearity depends on the relevant pQCD subprocesses and define the universality classes of nonlinear $k_\perp$-factorization for production of hard dijets. We also report the nonlinear $k_\perp$-factorization solution [11] to the Abramovsky-Gribov-Kancheli (AGK) unitarity problem [12].

At the heart of our approach is the equivalence between the parton fusion description of the shadowing introduced in 1975 [7] and the unitarization of the color dipole–nucleus interaction [13]. The major technical problem in the unitarization program is the non-Abelian evolution of color dipoles in a nuclear environment, and we present a closed-form solution based on the multiple-scattering theory for color dipoles [1,14]. A very rich pattern of nonlinear $k_\perp$-factorization relations emerges already for the lowest order pQCD subprocesses considered here.

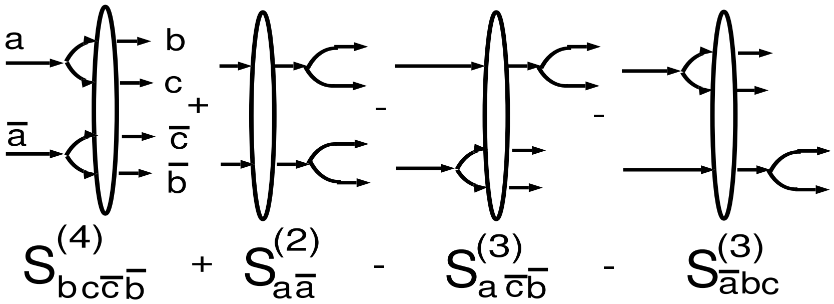

Figure 1. The S-matrix structure of the two-body density matrix for excitation $a \to bc$.

2. The master formula for nuclear dijets

In the laboratory frame, dijet production in the parton-nucleus collisions can be viewed as an excitation $a \to bc$, where $a = \gamma^*, q, g,\ b, c = q, \bar{q}g$. The parton fusion condition (1) amounts to the coherency of excitation over the whole diameter of the nucleus. Excitation of the perturbative $|bc\rangle$ Fock state of the physical projectile $|a\rangle$ by one-gluon exchange, $ag \to bc$, leaves the target nucleon debris in the color excited state. For nuclear targets one has to deal with multiple gluon exchanges which are enhanced by a large target thickness.

According to the general rules of quantum mechanics, the two-particle spectrum is the Fourier transform of the two-body density matrix. The derivation of the master formula for the dijet spectrum, based on the technique developed in [1,14] is found in [3]:

$$\frac{d\sigma(a^* \to bc)}{dz_b d^2\boldsymbol{p}_b d^2\boldsymbol{p}_c} = \frac{1}{(2\pi)^4} \int d^2\boldsymbol{b}_b d^2\boldsymbol{b}_c d^2\boldsymbol{b}'_b d^2\boldsymbol{b}'_c \tag{2}$$
$$\times \exp[-i\boldsymbol{p}_b(\boldsymbol{b}_b - \boldsymbol{b}'_b) - i\boldsymbol{p}_c(\boldsymbol{b}_c - \boldsymbol{b}'_c)]\Psi(z_b, \boldsymbol{b}_b - \boldsymbol{b}_c)\Psi^*(z_b, \boldsymbol{b}'_b - \boldsymbol{b}'_c)$$
$$\times \left\{ S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b, \boldsymbol{b}'_c, \boldsymbol{b}_b, \boldsymbol{b}_c) + S^{(2)}_{\bar{a}a}(\boldsymbol{b}', \boldsymbol{b}) - S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b}, \boldsymbol{b}'_b, \boldsymbol{b}'_c) - S^{(3)}_{\bar{a}bc}(\boldsymbol{b}', \boldsymbol{b}_b, \boldsymbol{b}_c) \right\}.$$

If $\boldsymbol{b}_a = \boldsymbol{b}$ is the projectile's impact parameter, then $\boldsymbol{b}_b = \boldsymbol{b} + z_c\boldsymbol{r}$, $\boldsymbol{b}_c = \boldsymbol{b} - z_b\boldsymbol{r}$, where $z_{b,c}$ stand for the fraction of the lightcone momentum of the projectile a carried by partons b and c, $\Psi(z, \boldsymbol{r})$ stands for the lightcone wave function of the $|bc\rangle$ Fock state of the projectile, its connection to the parton-splitting functions is found in [3]. All $S^{(n)}$ describe a scattering of color-singlet systems of n partons, as indicated in Fig. 1. This is the crucial point - in the course of our derivation of the dijet spectra and single-jet spectra we only deal with infrared-safe observables. $S^{(2)}$ and $S^{(3)}$ are readily calculated in terms of the 2-parton and 3-parton dipole cross sections [13,15,14]. For the dilute-gas nucleus one uses the Glauber-Gribov multiple-scattering theory formula [16,17]

$$\mathsf{S}^{(n)}(\boldsymbol{b}_c{}', \boldsymbol{b}_b{}', \boldsymbol{b}_c, \boldsymbol{b}_b) = \mathsf{S}[\boldsymbol{b}, \Sigma_n] = \exp\{-\frac{1}{2}\Sigma^{(n)}(\boldsymbol{b}_c{}', \boldsymbol{b}_b{}', \boldsymbol{b}_c, \boldsymbol{b}_b)T(\boldsymbol{b})\}, \quad (3)$$

where $T(\boldsymbol{b}) = \int dr_z n_A(r_z, \boldsymbol{b})$ is the optical thickness of a nucleus at the impact parameter $\boldsymbol{b}$ and $n_A(r_z, \boldsymbol{b})$ is the nuclear matter density. The major nontrivial task is a calculation of the coupled-channel operator Σ_4 [1,5,6]. The relevant basis of color states depends on the pQCD subprocess,

$$\begin{aligned}
\gamma^* &\to q\bar{q} : 1_1 + 8_{N_c^2}, \\
g &\to q\bar{q} : 1_1 + 8_{N_c^2}, , \\
q &\to qg : 3_{N_c} + \{6 + 15\}_{N_c^3}, \\
g &\to gg : 1_1 + \{8_A + 8_S\}_{N_c^2} + \{10 + \overline{10} + 27 + R_7\}_{N_c^4},
\end{aligned}$$

where the subscripts indicate the N_c-dependence of the size of the relevant color multiplets. For the description of the two-gluon multiplet R_7 which exists for $N_c > 3$ only, see [6]. These size-dependent subspaces of multiplets prove very useful in the diagonalization of the non-Abelian evolution of color dipoles. In the case of nuclear targets one must distinguish truly inelastic processes, which leave the target nucleus in the color excited state, and coherent diffraction $aA \to (bc)A$ with retention of the target nucleus in the ground state.

3. The $k_\perp$-factorization for DIS off free nucleons

The unintegrated gluon density in the target nucleon,

$$\mathsf{F}(x, \kappa^2) = \frac{\partial G(x, \kappa^2)}{\partial \log \kappa^2}, \quad (4)$$

furnishes a universal description of the proton structure function $F_{2p}(x, Q^2)$ and of the final states. For instance, the linear $k_\perp$-factorization for forward quark-antiquark dijets reads (for applications, see [18] and references therein)

$$\frac{2(2\pi)^2 d\sigma_N(\gamma^* \to Q\overline{Q})}{dz d^2\boldsymbol{p} d^2\boldsymbol{\Delta}} = f(x, \boldsymbol{\Delta}) \left|\Psi(z, \boldsymbol{p}) - \Psi(z, \boldsymbol{p} - \boldsymbol{\Delta})\right|^2 , \quad (5)$$

where $\boldsymbol{\Delta} = \boldsymbol{p}_q + \boldsymbol{p}_{\bar{q}}$ is the jet-jet decorrelation (acoplanarity) momentum, $\boldsymbol{p} \equiv \boldsymbol{p}_{\overline{q}}$, $z \equiv z_{\overline{q}}$ refer to the $\overline{q}$-jet, and

$$f(x, \boldsymbol{\kappa}) = \frac{4\pi\alpha_S}{N_c} \cdot \frac{1}{\kappa^4} \cdot \mathsf{F}(x, \boldsymbol{\kappa}^2) . \quad (6)$$

From the unitarity point of view, Eq. (5) corresponds to the unitarity cuts of diagrams Figs. 2a,b for the forward Compton scattering amplitude. Notice, that the dijet cross section is a direct probe of the free-nucleon unintegrated glue $f(x, \boldsymbol{\Delta})$; the point that the jet-jet decorrelation momentum $\boldsymbol{\Delta}$ comes from the transverse momentum of the exchanged gluon is obvious form Figs. 2a,b for the forward Compton scattering amplitude. Notice, that the dijet cross section is a direct probe of the free-nucleon unintegrated glue $f(x, \boldsymbol{\Delta})$; the point that the jet-jet decorrelation momentum $\boldsymbol{\Delta}$ comes from the transverse momentum of the exchanged gluon is obvious.

The focus of the further discussion is on how linear $k_\perp$-factorization (5) is modified by multiple gluon exchanges predominant in hard production off nuclear targets.

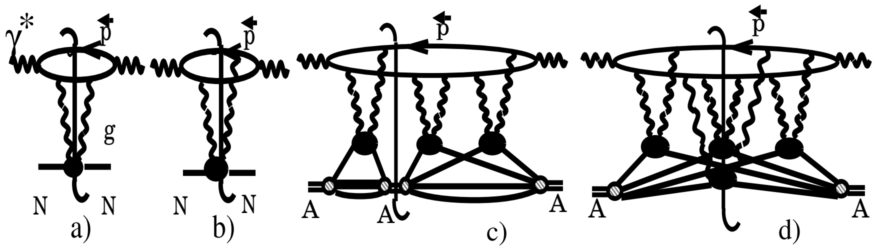

Figure 2. The typical unitarity cuts and dijet final states in DIS : (a),(b) - free-nucleon target, (c) - coherent diffractive DIS off a nucleus, (d) - truly inelastic DIS with multiple color excitation of the nucleus.

4. What is the collective unintegrated nuclear glue?

While seeking the factorization properties of an in-nucleus hard QCD, one must define the collective glue in terms of a certain observable such that the so-defined glue enters in a universal manner a description of all other nuclear processes. Taking DIS as an example, the same nuclear glue must describe the total DIS cross section, its coherent diffractive unitarity cuts, Fig. 2c, and multiple color excitation unitarity cuts, Fig. 2d, of the nuclear forward Compton scattering amplitude.

DIS off a nucleus at $x \lesssim x_A$ can be described in terms of the color dipole-nucleus cross-section [13]

$$\sigma_A(\boldsymbol{r}) = 2\int d^2\boldsymbol{b}\Big\{1 - \exp[-\frac{1}{2}\sigma(x,\boldsymbol{r})T(\boldsymbol{b})]\Big\}\,, \qquad (7)$$

where

$$\sigma(x,\boldsymbol{r}) = \int d^2\boldsymbol{\kappa} f(x,\boldsymbol{\kappa})[1-\exp(i\boldsymbol{\kappa}\boldsymbol{r})] \tag{8}$$

is the $q\bar{q}$ dipole cross section [13]. The x-dependence of $\sigma(x,\boldsymbol{r})$ is governed by the color-dipole evolution equation [15,19]; for the sake of brevity, here-below we suppress the x-dependence. The nuclear dipole cross section (7) sums in the compact form the Glauber-Gribov multiple-scattering diagrams and is a basis for the quantitative description of nuclear shadowing in DIS [20].

It is remarkable, that although a deposition of dozen MeV energy will break any heavy nucleus, at $x \lesssim x_A$ such a coherent diffraction makes $\approx 50\%$ of the total cross section of DIS off heavy nucleus [21]. To the lowest order in pQCD, the coherent diffractive final state consists of the back-to-back dijet with vanishing transverse momentum transfer $\boldsymbol{\Delta}$ to the target nucleus and the large transverse momentum $\boldsymbol{p}$ of dijets in πA collisions comes entirely from gluons exchanged with target nucleons [10,22]. Consequently, one can take the partial wave of the diffraction amplitude, i.e., the nuclear profile function, for the definition of the collective nuclear glue per unit area in the impact parameter space, $\phi(\boldsymbol{b},\boldsymbol{\kappa})$ [10,1]:

$$\Gamma_A(\boldsymbol{b},\boldsymbol{r}) = [1-\exp(-\frac{1}{2}\sigma(\boldsymbol{r})T(\boldsymbol{b}))] = \int d^2\boldsymbol{\kappa}\phi(\boldsymbol{b},\boldsymbol{\kappa})\{1-\exp[i\boldsymbol{\kappa}\boldsymbol{r}]\}\,. \tag{9}$$

In can be expanded as

$$\phi(\boldsymbol{b},\boldsymbol{\kappa}) = \frac{1}{\sigma_0}\sum_{j=1}^{\infty} w_j(\boldsymbol{b}) f^{(j)}(\boldsymbol{\kappa}), \tag{10}$$

where the collective glue $f^{(j)}(\boldsymbol{\kappa})$ of j overlapping nucleons of the Lorentz-contracted nucleus satisfies the convolution representation

$$f^{(j)}(\boldsymbol{\kappa}) = \frac{1}{\sigma_0}(f^{(j-1)}\otimes f)(\boldsymbol{\kappa}), \tag{11}$$

and a probability to find j overlapping nucleons equals

$$w_j(\boldsymbol{b}) = \frac{\nu_A^j(\boldsymbol{b})}{j!}\exp\left[-\nu_A(\boldsymbol{b})\right], \quad \nu_A(\boldsymbol{b}) = \frac{1}{2}\sigma_0 T(\boldsymbol{b}), \tag{12}$$

$\sigma_0 = \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa})$ is the dipole cross section $\sigma(\boldsymbol{r})$ for large dipoles $\boldsymbol{r}$.

Subsequently we will use also $\Phi(\boldsymbol{b},\boldsymbol{\kappa}) = \phi(\boldsymbol{b},\boldsymbol{\kappa}) + w_0(\boldsymbol{b})\delta(\boldsymbol{\kappa})$, which is the Fourier transform of the S-matrix for an elementary dipole,

$$\mathsf{S}[\boldsymbol{b},\sigma(\boldsymbol{r})] = \int d^2\boldsymbol{\kappa}\Phi(\boldsymbol{b},\boldsymbol{\kappa})\exp[i\boldsymbol{\kappa}\boldsymbol{r}]\,. \tag{13}$$

The antishadowing properties [1,10] of the so-defined collective glue $\phi(\boldsymbol{b},\boldsymbol{\kappa})$ are responsible for the familiar Cronin effect [3]. It furnishes the linear $k_\perp$-factorization for the nuclear structure function $F_{2A}(x,Q^2)$ and the forward single jets in DIS off nuclei, precisely the same as for the free-nucleon target in terms of $f(\boldsymbol{\kappa})$. This linear $k_\perp$-factorization is rather an exception because of their special Abelian features. Dijets in DIS, and dijet and single-jet spectra in hA collisions, however, prove to be highly nonlinear functionals of $\Phi(\boldsymbol{b},\boldsymbol{\kappa})$ [1,2,3,4,5,6]. We dubbed this property the nonlinear $k_\perp$-factorization.

An important point is that color dipole cross section depends on the $SU(N_c)$ representation the partons belong to. For this reason the collective nuclear glue can not be described by one scalar function, it is rather a density matrix in the space of color representations, for examples see [3,6].

5. The origin of nonlinear $k_\perp$-factorization

The fundamental point is that the calculation of the single-jet and dijet cross sections can be reduced to an interaction with the nucleus of color-singlet multiparton systems [14], as exemplified by our master formula (3). The interaction cross-section for such systems of n partons is a (matrix) superposition of elementary dipole cross-sections. Upon the diagonalization of the non-Abelian evolution problem, the matrix elements of nuclear S-matrices $\mathsf{S}^{(n)}$, $n \geq 3$, in (3) will be a (matrix) products of nuclear S-matrices for elementary dipoles. In the calculation of the Fourier transform of the two-body density matrix, nuclear S-matrices for elementary dipoles will be replaced by their Fourier transforms (13). Consequently, the Fourier transforms of $\mathsf{S}^{(n)}$ would give rise to multiple convolutions of $\Phi(\boldsymbol{b},\boldsymbol{\kappa})$, and the nuclear dijet spectra will be highly nonlinear quadratures of collective nuclear glue. The specific expansion for eigenvalues of $\mathsf{S}^{(n)}$ in term of elementary dipole cross sections, and the specific form of the emerging nonlinear $k_\perp$-factorization thereof, depends on the color representations of the initial-state parton and final-state dijet, still there emerges a well-defined pattern of the universality classes.

6. Why collective glue for slices of a nucleus is a must?

In the realm of pQCD with partons in the fundamental and adjoint multiplets, one finds an exact product representations for nuclear S-matrices for $n = 2, 3$ in terms of the S-matrices for elementary dipoles. The case of $n = 4$ is much more tricky. It is remarkable how the forbidding three-channel for qg dijets, and seven-channel for gg dijets, non-Abelian intranuclear evolution can be diagonalized exactly for large N_c due the block-diagonal structure of the four-parton cross-section operator $\hat{\Sigma}^{(4)}$. We explicitly represent it in the form

$$\hat{\Sigma}^{(4)} = \hat{\Sigma}^{(0)} + \hat{\omega}\,, \tag{14}$$

where $\hat{\Sigma}^{(0)}$ is the diagonal operator, while the off-diagonal $\hat{\omega}$, proportional to the transition cross section $\Omega(\mathbf{s},\boldsymbol{r},\boldsymbol{r}')$ introduced in [1,5], describes the $1/N_c$ suppressed transitions between the subspaces formed by the states having equal dimension at large N_c as expounded in Sec. 2. The expansion (14) is a a basis for the systematic large-N_c perturbation theory. One can address it, for instance, making use of the Sylvester expansion [1], which has a generic form (we show the result for the case of gg-dijets, the detailed description of eigenstates $|e_i\rangle$ and eigenvalues Σ_i is found in [6])

$$\begin{aligned}
&\mathsf{S}[\boldsymbol{b},\hat{\sigma}^{(4)}(\mathbf{s},\boldsymbol{r},\boldsymbol{r}')] = |e_2\rangle\langle e_2|\exp[-\frac{1}{2}\Sigma_2 T(\boldsymbol{b})] \\
&+|e_4\rangle\langle e_2|\frac{\Omega(\mathbf{s},\boldsymbol{r},\boldsymbol{r}')}{\sqrt{2}N_c(\Sigma_2-\Sigma_4)}\left\{\exp[-\frac{1}{2}\Sigma_4 T(\boldsymbol{b})]-\exp[-\frac{1}{2}\Sigma_2 T(\boldsymbol{b})]\right\}.
\end{aligned} \tag{15}$$

Here $\Sigma_{2,4}$ – the relevant eigenvalues of $\hat{\Sigma}^{(0)}$ – are certain superpositions of the elementary dipole cross sections, While the Fourier transform of the diagonal term in (16) will be a multiple convolution of the collective nuclear glue, for the off-diagonal transitions, which excite gg-dijets in higher color multiplets, $10, \overline{10}, 27, R_7$, this is the dead end, though. The denominator $(\Sigma_2 - \Sigma_4)$ in the Sylvester expansion (16) blocks further analytic calculations. For instance, in their closely related study, Kovchegov and Jalilian-Marian [23] stop at precisely this point; in order to proceed further one has to rely upon the brute force numerical Fourier transform, which masks important distinction between different production mechanisms and salient features of the production dynamics.

The way to the explicit analytic quadratures is paved by our integral representation [1], which to the first order in the off-diagonal perturbation $\hat{\omega}$ takes the form (similar representation is found for higher orders of large-N_c perturbation theory [1])

$$\mathsf{S}[\boldsymbol{b},\hat{\Sigma}^{(0)}+\hat{\omega}] - \mathsf{S}[\boldsymbol{b},\hat{\Sigma}^{(0)}] = -\frac{1}{2}T(\boldsymbol{b})\int_0^1 d\beta\,\mathsf{S}[\boldsymbol{b},(1-\beta)\hat{\Sigma}^{(0)}]\,\hat{\omega}\,\mathsf{S}[\boldsymbol{b},\beta\hat{\Sigma}^{(0)}]. \tag{16}$$

This integral representation is much more than a technical trick to achieve the convolution representation for the four-body S-matrix. Specifically, $\hat{\omega}$

describes the hard excitation of dijets in higher color multiplets at the depth β from the front face of the nucleus, while $\mathsf{S}[\boldsymbol{b},\beta\hat{\Sigma}^{(0)}]$ and $\mathsf{S}[\boldsymbol{b},(1-\beta)\hat{\Sigma}^{(0)}]$ describe the Initial (ISI) and Final State Interaction (FSI) in slices $[0,\beta]$ and $[\beta,1]$ of the nucleus, respectively. Now, the color multiplet of the dijet is different from that of the incident parton, and the resulting distinction between ISI and FSI is an integral part of the dijet production dynamics, which can not be described entirely in terms of the classical gluon field of a whole nucleus [24]. The emergence of $\mathsf{S}[\boldsymbol{b},\beta\hat{\Sigma}^{(0)}]$ and $\mathsf{S}[\boldsymbol{b},(1-\beta)\hat{\Sigma}^{(0)}]$ calls upon the nuclear glue defined for the slice $[0,\beta]$ of the nucleus ($0\leq\beta\leq 1$)

$$\exp\left[-\frac{1}{2}\beta\sigma(\boldsymbol{r})T(b)\right] = \int d^2\boldsymbol{\kappa}\Phi(\beta;\boldsymbol{b},\boldsymbol{\kappa})\exp(i\boldsymbol{\kappa}\boldsymbol{r}), \tag{17}$$

and the corresponding overlap probabilities $w_j(\beta;\boldsymbol{b})$. We will need also the wave function of bc Fock states coherently distorted in this slice,

$$\Psi(\beta;z,\boldsymbol{p}) \equiv \int d^2\boldsymbol{\kappa}\Phi(\beta;\boldsymbol{b},\boldsymbol{\kappa})\Psi(z,\boldsymbol{p}+\boldsymbol{\kappa}). \tag{18}$$

Now we are in the position to present our explicit results on nonlinear $k_\perp$-factorization. We start with the short digression into the single-jet problem.

7. The fate of $k_\perp$-factorization for single-jet spectra in pA collisions

The integration over the transverse momentum of the unobserved parton in the master formula (3) is straightforward. The unobserved parton and its antiparton will enter at the same impact parameter and multiparton color singlet states simplify to the two-parton ones. Still, the non-Abelian features of QCD manifest themselves in the breaking of linear $k_\perp$-factorization. Here we show the the large-N_c result for the radiation of gluons from quarks, $q^*\to qg$ [3], which is directly relevant to jet production in the proton hemisphere of pA collisions at RHIC [25,26]. The linear $k_\perp$-factorization holds for the free-nucleon target,

$$\frac{(2\pi)^2 d\sigma_A(q^*\to gq)}{dz_g d^2\boldsymbol{p}_g} = \frac{1}{2}\int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) \tag{19}$$
$$\times\Big\{\big|\Psi(z_g,\boldsymbol{p}_g)-\Psi(z_g,\boldsymbol{p}_g+\boldsymbol{\kappa})\big|^2+\big|\Psi(z_g,\boldsymbol{p}_g+\boldsymbol{\kappa})-\Psi(z_g,\boldsymbol{p}_g+z_g\boldsymbol{\kappa})\big|^2\Big\},$$

while the same spectrum for the nuclear target is of the two-component form

$$\frac{(2\pi)^2 d\sigma_A(q^* \to gq)}{dz_g d^2\boldsymbol{p}_g d^2\boldsymbol{b}} = S[\boldsymbol{b},\sigma_0]\int d^2\boldsymbol{\kappa}\phi(\boldsymbol{b},\boldsymbol{\kappa}) \tag{20}$$

$$\times\Big\{|\Psi(z_g,\boldsymbol{p}_g)-\Psi(z_g,\boldsymbol{p}_g+\boldsymbol{\kappa})|^2+|\Psi(z_g,\boldsymbol{p}_g+\boldsymbol{\kappa})-\Psi(z_g,\boldsymbol{p}_g+z_g\boldsymbol{\kappa})|^2\Big\}+$$

$$\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2\phi(\boldsymbol{b},\boldsymbol{\kappa}_1)\phi(\boldsymbol{b},\boldsymbol{\kappa}_2)|\Psi(z_g,\boldsymbol{p}_g+z_g\boldsymbol{\kappa}_1)-\Psi(z_g,\boldsymbol{p}_g+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)|^2\,.$$

The first component is an exact counterpart of the free-nucleon spectrum: it is linear $k_\perp$-factorizable, but is suppressed by the nuclear absorption factor $S[\boldsymbol{b},\sigma_0(x)]$. For central interactions of the main experimental interest, the gluon spectrum is entirely dominated by the second component which is a non-linear – quadratic – functional of the collective nuclear glue. A full compendium of nonlinear $k_\perp$-factorization results for single jets from all possible pQCD subprocesses is found in [3].

There is a conspicuous difference between the z_g-dependence of the free-nucleon and nuclear spectra. This amounts to the $\boldsymbol{p}_g$-dependence of the Landau-Pomeranchuk-Migdal effect; the same applies to the spectrum of leading quarks and nuclear quenching of forward jets in pA collisions [3].

8. Nonlinear $k_\perp$-factorization for dijets: the case of DIS

Here we report closed-form analytic results for quark-antiquark dijets in DIS to the leading order in $1/N_c$, higher orders can be derived following Ref. [1]. In DIS off nuclei, dipoles propagate as color-singlets until at depth β from the front face of a nucleus they excite into the octet state; the non-Abelian evolution in the slice $[\beta,1]$ consists of color rotations within the octet state. Combining the truly inelastic and diffractive components of DIS,

$$\begin{aligned}\frac{(2\pi)^2 d\sigma_A(\gamma^*\to Q\overline{Q})}{d^2\boldsymbol{b}dzd^2\boldsymbol{p}d^2\boldsymbol{\Delta}} &= \frac{1}{2}T(\boldsymbol{b})\int_0^1 d\beta\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}\\ &\times f(\boldsymbol{\kappa})\Phi(1-\beta,\boldsymbol{b},\boldsymbol{\Delta}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\Phi(1-\beta,\boldsymbol{b},\boldsymbol{\kappa}_1)\\ &\times\Big|\Psi(\beta;z,\boldsymbol{p}-\boldsymbol{\kappa}_1)-\Psi(\beta;z,\boldsymbol{p}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\Big|^2\\ &+\delta^{(2)}(\boldsymbol{\Delta})\Big|\Psi(1;z,\boldsymbol{p})-\Psi(z,\boldsymbol{p})\Big|^2\,.\end{aligned} \tag{21}$$

The diffractive component of Eq. (21), $\propto \delta^{(2)}(\boldsymbol{\Delta})$, gives exactly back-to-back dijets (for the $\boldsymbol{\Delta}$-dependence for finite-size nuclei see Ref. [10]). It is a quadratic functional of the collective nuclear glue. The first component in

(21) – truly inelastic DIS with excitation of color-octet dijets – is of fifth order in gluon field densities: a linear one of the free-nucleon glue $f(\boldsymbol{\kappa})$ which describes the hard singlet-to-octet transition and a quartic functional of the collective nuclear glue for the two slices of a nucleus: the coherent ISI's of color-singlet dipoles in the slice $[0, \beta]$ and incoherent FSI's of color-octet dipoles in the slice $[\beta, 1]$.

9. Nonlinear $k_\perp$-factorization for dijets: universality classes

We start with the universality class of coherent diffraction. The origin of coherent diffractive DIS is a difference between the in-vacuum and intranuclear-distorted wave function of the $q\bar{q}$ state of the photon:

$$\frac{(2\pi)^2 d\sigma_A(\gamma^* \to q\bar{q})}{d^2b dz d^2\boldsymbol{p} d^2\boldsymbol{\Delta}} = \delta^{(2)}(\boldsymbol{\Delta}) \Big|\Psi(1; z, \boldsymbol{p}) - \Psi(z, \boldsymbol{p})\Big|^2 . \qquad (22)$$

In the case of $q \to qg$ the incident partons are colored, and intranuclear attenuation of the incident quark wave entails the suppression of coherent diffraction by the factor $w_0^2(b)$, similar, but stronger intranuclear attenuation is found for $g \to gg$ [6]. This nuclear suppression of coherent diffraction can be identified with Bjorken's diffractive gap survival probability [27], its process dependence makes clear that the concept of hard factorization for the nuclear pomeron makes no sense. In all above cases coherent diffraction is of leading order in $1/N_c$, while coherent diffraction $g \to q\bar{q}$ is suppressed, which adds further case against the hard factorization for nuclear pomerons.

The universality class of dijets in higher color miltiplets excited from partons in lower multiplet is the most interesting one. As an example we cite the large-N_c result [5]

$$\frac{d\sigma\big(q^* \to qg(6+15)\big)}{d^2b dz d^2\boldsymbol{\Delta} d^2\boldsymbol{p}} = \frac{1}{2(2\pi)^2} T(b) \int_0^1 d\beta$$
$$\times \int d^2\boldsymbol{\kappa} d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 d^2\boldsymbol{\kappa}_3 \delta(\boldsymbol{\kappa} + \boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2 + \boldsymbol{\kappa}_3 - \boldsymbol{\Delta})$$
$$\times \underbrace{\Phi(\beta; \boldsymbol{b}, \boldsymbol{\kappa}_3)}_{Quark\ \ ISI} \underbrace{f(\boldsymbol{\kappa}) \left|\Psi(\beta; z, \boldsymbol{p} - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_3) - \Psi(\beta; z, \boldsymbol{p} - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa}_3 - \boldsymbol{\kappa})\right|^2}_{Hard\ \ Excitation}$$
$$\times \underbrace{\Phi(1-\beta; \boldsymbol{b}, \boldsymbol{\kappa}_1)}_{Quark\ \ FSI} \underbrace{\Phi(\frac{C_A}{C_F}(1-\beta); \boldsymbol{b}, \boldsymbol{\kappa}_2)}_{Gluon\ \ FSI}, \qquad (23)$$

where we indicated the rôle of different factors in the integrand. Notice the sixth order nonlinearity of the dijet spectrum (23) in nuclear and free-nucleon glue. The most nontrivial point here is how incoherent distortions of the incident quark wave come along with the coherent distortions of the qg wave function in the same slice $[0,\beta]$ of the nucleus.

The ratio of Casimirs C_A/C_F in $\Phi(\frac{C_A}{C_F}(1-\beta),\boldsymbol{b},\boldsymbol{\kappa}_2)$ for FSI of gluons is a reminder that the collective nuclear glue derives from a density matrix in color space [1,3]. In DIS, $\gamma^*\to q\bar{q}(8)$, ISI of the photon vanishes and $\Phi(\beta;\boldsymbol{b},\boldsymbol{\kappa}_3)=\delta^{(2)}(\boldsymbol{\kappa}_3)$, for the incident gluons, $g\to gg(10+\overline{10}+27+R_7)$, the quark FSI factor must be swapped for the gluon FSI factor, and there will be a slight modification of incoherent ISI distortions - at large N_c the gluon behaves like the uncorrelated quark-antiquark pair [3,6]. The striking distinction between ISI and FSI requires the collective nuclear glue for slices of the nucleus – nonlinear $k_\perp$-factorization can not be described by the classical gluon field of the whole nucleus. The nonlinear $k_\perp$-factorization entails a nuclear enhancement of the decorrelation of dijets [1,5,6].

Universality class of dijets in the same lower color multiplet as the beam parton has its own unique features. For instance, in the case of color-triplet qg dijets [5]

$$\frac{d\sigma\big(q^*A\to qg(3)\big)}{d^2bdzd^2\boldsymbol{\Delta}d^2\boldsymbol{p}}=\frac{1}{(2\pi)^2}\phi(b,\boldsymbol{\Delta})\left|\Psi(1;z,\boldsymbol{p}-\boldsymbol{\Delta})-\Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})\right|^2. \tag{24}$$

The free-nucleon cross section has exactly the same form in terms of $f(\boldsymbol{\Delta})$ and the in-vacuum wave functions. Recalling that $\Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})$, which has a collinear singularity, is the probability amplitude to find the qg state in the physical quark, we reinterpret this result as a fragmentation of the quark, scattered quasi-elastically with the differential cross section $\propto\phi(b,\boldsymbol{\Delta})$, see also below. The change from the free-nucleon to a nuclear target,

$$|\Psi(z,\boldsymbol{p}-\boldsymbol{\Delta})-\Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})|^2\Longrightarrow|\Psi(1;z,\boldsymbol{p}-\boldsymbol{\Delta})-\Psi(z,\boldsymbol{p}-z\boldsymbol{\Delta})|^2,$$

must be interpreted as a nuclear modification of the hard fragmentation function of the quark.

Apart from the fact that ISI of incident gluons looks like an ISI of the uncorrelated quark-antiquark pair, the nonlinear $k_\perp$-factorization for $g\to q\bar{q}(8)$, $g\to gg(8_A+8_S)$, $g\to gg(8_S)$ is very similar to that of $q\to qg(3)$ [6]. In $g\to gg(8_A+8_S)$ too there emerges the nuclear modified hard fragmentation function of the gluon.

10. Nonlinear $k_\perp$-factorization for dijets: unitarity cuts and AGK rules for color excitation of the target nucleus

Our technique can readily be extended [11] to partial cross sections $d\sigma_j$ for final states with j color-excited nucleons of the target nucleus – in the language of the so-called AGK unitarity rules that corresponds to j cut pomerons [12]. This multiplicity j controls the hadronic multiproduction in the nucleus hemisphere, i.e., the collision centrality, as well as the nonperturbative energy-loss contribution to the quenching of forward jets. The simplest case is the AGK rule for the universality class of coherent diffraction:

$$d\sigma_j = \delta_{j0} d\sigma_D. \tag{25}$$

Now we notice that the differential cross section of quark-nucleon quasi-elastic scattering $qN \to q'N^*$ equals

$$\frac{d\sigma_{qN}}{d^2\boldsymbol{\kappa}} = \frac{1}{2} f(\boldsymbol{\kappa}). \tag{26}$$

Here the target debris N^* is in the color-excited state. Then, the convolution property (11) of the collective glue $f^{(j)}(\boldsymbol{\kappa})$ relates it to the differential cross section of j-fold incoherent quasi-elastic scattering,

$$f^{(j)}(\boldsymbol{\kappa}) \propto \frac{d\sigma^{(j)}}{d^2\boldsymbol{\kappa}}. \tag{27}$$

Evidently, the latter is simply a j-fold convolution of single scattering cross sections. This observation paves a way to a simple interpretation of $f^{(j)}(\boldsymbol{\kappa})$ in terms of j cut pomerons.

We illustrate the emerging AGK rule for the universality class of dijets in the same lower color representation as the beam parton on an example of the reaction $q \to qg(3)$ with j color-excited nucleons of the target nucleus:

$$\frac{d\sigma_j\big(q^*A \to qg(3)\big)}{d^2b dz d^2\boldsymbol{\Delta} d^2\boldsymbol{p}} = \\ = \frac{1}{(2\pi)^2\sigma_0} w_j(b) f^{(j)}(\boldsymbol{\Delta}) \left|\Psi(1; z, \boldsymbol{p}-\boldsymbol{\Delta}) - \Psi(z, \boldsymbol{p} - z\boldsymbol{\Delta})\right|^2. \tag{28}$$

It makes the interpretation of this final state as a result of fragmentation of the quasi-elastically scattered quark an obvious one - the nuclear-distorted fragmentation function, its collinear pole included, does not depend on the multiplicity of cut pomerons j. Notice the uncut pomerons which enter the intranuclear distortion of $\Psi(1; z, \boldsymbol{p}-\boldsymbol{\Delta})$.

As an example of AGK rules from the universality class of dijets in higher color multiplet excited from partons in lower multiplet we show the large-N_c result for $q \to qg(6+15)$ with j color-excited nucleons of the target nucleus:

$$
\begin{aligned}
&\frac{d\sigma_j\big(q^* \to qg(6+15)\big)}{d^2b dz d^2\boldsymbol{\Delta} d^2\boldsymbol{p}} = \frac{1}{(2\pi)^2} T(b) \int_0^1 d\beta \\
&\times \int d^2\boldsymbol{\kappa} d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 d^2\boldsymbol{\kappa}_3 \delta(\boldsymbol{\kappa} + \boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2 + \boldsymbol{\kappa}_3 - \boldsymbol{\Delta}) \\
&\times \sum_{n,k,m} \delta(j-n-k-m-1) \underbrace{w_m(\beta;\boldsymbol{b}) \frac{f^{(m)}(\boldsymbol{\kappa}_3)}{\sigma_0}}_{Quark\ \ ISI} \\
&\times \underbrace{f(\boldsymbol{\kappa}) \left|\Psi(\beta; z, \boldsymbol{p} - \boldsymbol{\kappa}_3 - \boldsymbol{\kappa}_2) - \Psi(\beta; z, \boldsymbol{p} - \boldsymbol{\kappa}_3 - \boldsymbol{\kappa}_2 - \boldsymbol{\kappa})\right|^2}_{Hard\ \ excitation} \\
&\times \underbrace{w_k\Big(\frac{C_A}{C_F}(1-\beta); \boldsymbol{b}\Big) \frac{f^{(k)}(\boldsymbol{\kappa}_2)}{\sigma_0}}_{Gluon\ \ FSI} \times \underbrace{w_n\Big((1-\beta); \boldsymbol{b}\Big) \frac{f^{(n)}(\boldsymbol{\kappa}_1)}{\sigma_0}}_{Quark\ \ FSI} \qquad (29)
\end{aligned}
$$

It corresponds to the following counting of cut pomerons: one cut pomeron for hard excitation at the depth β; m cut pomerons for m-fold quasi-elastic intranuclear scattering of the incident quark in the slice $[0, \beta]$; k cut pomerons for k-fold quasi-elastic intranuclear scattering of the final-state gluon in the slice $[\beta, 1]$ and n cut pomerons for n-fold quasi-elastic intranuclear scattering of the final-state quark in the slice $[\beta, 1]$ of the nucleus. There are more uncut pomerons which describe the coherent distortion of the qg wave function in the slice $[0, \beta]$. The functional form of the hard excitation factor, which can also be re-interpreted as the (non-local) gluon radiation vertex, is preserved. However, upon the convolution with partial collective glue for initial-state and final-state slices of the nucleus, its numerical value will depend on the multiplicity of cut pomerons in the intimal-state and final-state channels.

New features of our AGK rules for an in-nucleus hard QCD are noteworthy: (i) we established simple relationship between cut pomerons, quasi-elastic scattering of partons and collective glue for overlapping nucleons of the Lorentz-contracted nucleus, (ii) the concrete realization of the AGK rules depends on the universality class the specific pQCD subprocess belongs to, (iii) the coherent nuclear distortions described by uncut pomerons persist in all universality classes, (iv) the gluon radiation vertex depends on the multiplicity of cut pomerons.

Conclusions

Hard processes in a nuclear environment must be described by nonlinear $k_\perp$-factorization in terms of the collective nuclear glue defined through the coherent diffractive dijet production. Any application of the familiar linear $k_\perp$-factorization would be entirely erroneous. We reported explicit quadratures for single-jet to dijet spectra from all pQCD subprocesses. Indispensable virtue of nonlinear $k_\perp$-factorization is a nontrivial interplay of incoherent rescatterings and coherent distortions of the dijet wave function. Still another important virtue is that it requires collective glue for slices of a nucleus - nonlinear $k_\perp$-factorization can not be described by a classical gluon field of the whole nucleus. Our connection between the collective glue for overlapping nucleons and the cross section of multiple quasi-elastic scattering of partons gives a simple interpretation of the AGK unitarity rules for excitation of the target nucleus. These unitarity rules can be applied to evaluation of the energy loss and quenching of forward jets.

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