On the Causality Requirement for Diffusive-Hyperbolic Systems in Non-Equilibrium Thermodynamics

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The classical Newtonian mechanics does not imply any limit for the speeds of propagation of thermomechanical disturbances. However, the physicists firmly believe that the differential equations of nature should exclude instantaneous propagation and, therefore, they should be cast in the hyperbolic form. Yet, some equations of classical continuum mechanics and thermodynamics, those of Navier-Stokes and Fourier, are parabolic. Such an incompatibility has prompted a new thermodynamic theory, developed by several authors in the last two decades, the extended irreversible thermodynamics, [1, 2, 3]. In such a theory the instantaneous propagation is removed by changing in a subtle way many tenets of classical irreversible thermodynamics. Moreover, some of its basic postulates are inspired by the kinetic theory of gases. Such an approach seems to be questionable to those scientists who, along with Truesdell, believe and claim that in a macroscopic framework "not even the form of the constitutive equations is derived from the molecular theory, rather the molecular theory so called is forced into agreement with preconceived phenomenological ideas" ([4], Lecture 7: The Onsager Relations). On the other hand, the classical "paradoxical" theories are able to describe a wide class of physical phenomena. Moreover, as pointed out by Fichera [5] and Day [6, 7] sometimes their paradoxical nature is only apparent. In other cases, such as the second sound propagation at low temperature, these theories are not applicable because the conditions which assure their validity are not verified [8, 9]. From the mathematical point of view the parabolic theories are easier to handle and also allow the application of certain useful mathematical techniques, such as the parabolic regularization method, for searching weak solutions to the hyperbolic systems [10, 11]. The considerations above suggest that in non-equilibrium thermodynamics it would be desirable to obtain systems of governing equations which contain some material parameters whose value determines the nature (parabolic or hyperbolic) of the system itself. To accomplish that task a new formulation of the causality requirement for the constitutive equations, which is compatible with both parabolic and hyperbolic models, seems to be necessary. This problem constitutes the subject of this presentation. Starting from Fichera's remarks [5] we prove that, if the solution of the classical heat equation is interpreted in the light of the basic experimental assumptions, then it leads to a finite speed of propagation of thermal disturbances. The latter result will help us to provide a weak formulation of the Causality Constitutive Principle often postulated in modern non-equilibrium thermodynamics [12]. Then we apply the aforesaid point of view to derive a diffusive-hyperbolic model of heat conduction based on the so called semi-empirical temperature scale, introduced by Cimmelli and Kosiński [13] in the framework of gradient theories with internal variable [14]. An extensive discussion on the diffusive-hyperbolic systems concludes the talk.

References

[1] Müller, I., Ruggeri, T., Rational Extended Thermodynamics, Springer-Verlag, Berlin, 1998.

[2] Jou, D., Casas-Vasquez, J., Lebon, J., Extended Irreversible Thermodynamics, Springer-Verlag, Berlin, 1996.

[3] Jou, D., Casas-Vasquez, J., Extended irreversible thermodynamics and its relation with other continuum approaches, J. Non-Newtonian Fluid Mech., 96 (2001), 77-104.

[4] Truesdell, C., Rational Thermodynamics, 2^{nd} ed., Springer Verlag, Berlin, 1984.

[5] Fichera, G., Is the Fourier theory of heat propagation paradoxical?, Rendiconti del Circolo Matematico di Palermo, Serie II, Tomo XLI (1992), 5-16.

[6] Day, W. A., A note on the propagation of temperature disturbances, Quart. Appl. Math., 55 (1997), 565-572.

[7] Day, W. A., Two lines of defense of Fourier's theory of heat propagation, in: Homage to Gaetano Fichera, Ed. A. Cialdea, pp. 147-159, Aracne, Rome, 2000.

[8] Jackson, H. E., Walker, C. T., Thermal conductivity, second sound, and phonon-phonon interactions in NaF, Phys. Rev. Lett., 3 (1971), 1428-1439.

[9] Narayanamurti, V., Dynes, R. C., Observation of second sound in Bismuth, Phys. Rev. Lett., 28 (1972), 1461-1465.

[10] Dafermos, C. M., Hyperbolic Conservation Laws in Continuum Physics, Springer Verlag, Berlin, 2000.

[11] Smoller, J., Shock Waves and Reaction-Diffusion Equations, Springer Verlag, Berlin, 1983.

[12] Muschik, W., Papenfuss, C., Ehrentraut H., A sketch of continuum thermodynamics, J. Non-Newtonian Fluid Mech., 96 (2001), 255-290.

[13] Cimmelli, V. A., Kosiński, W., Nonequilibrium semi-empirical temperature in materials with thermal relaxation, Arch. Mech., 43 (1991), 753-767.

[14] Kosiński, W., Wojno, W., Gradient generalization to internal state variable approach, Arch. Mech., 47 (1995), 523-536.

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