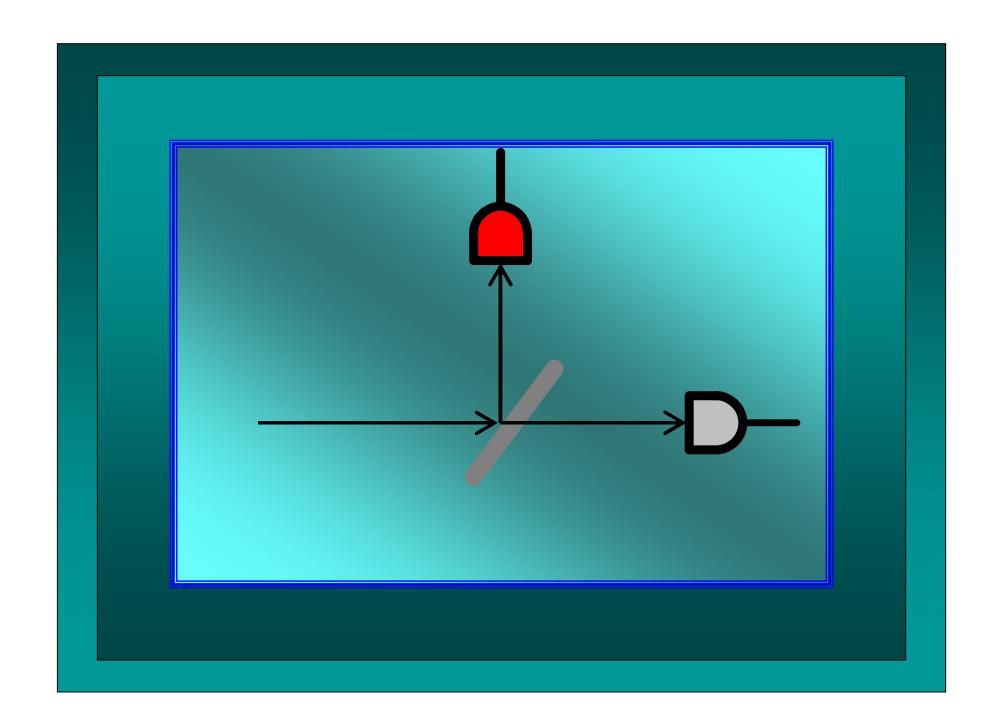
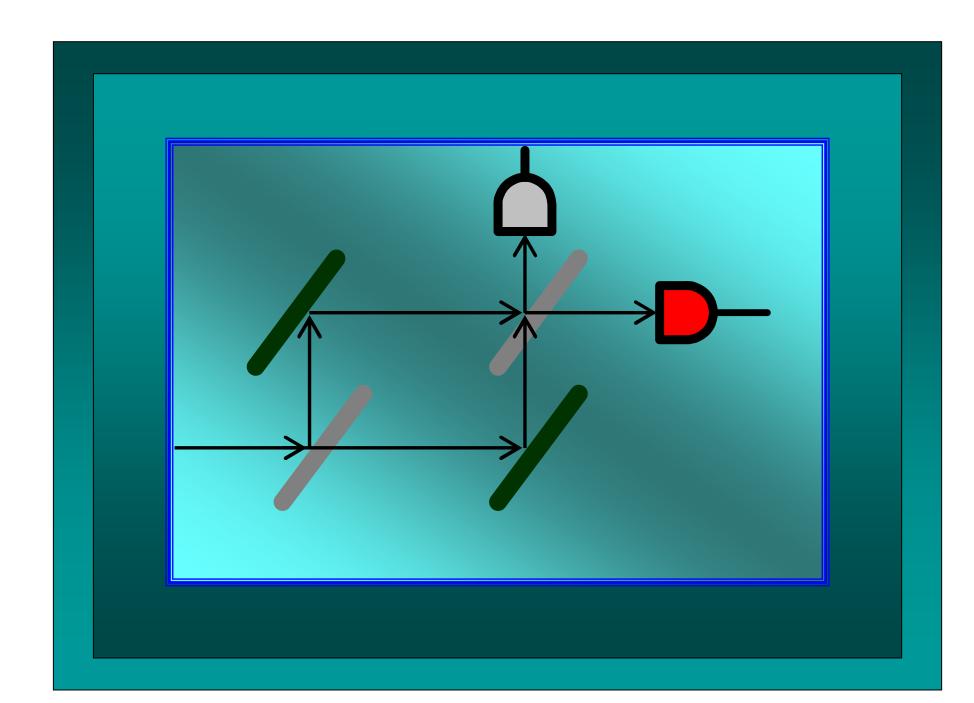
## KVANTUM KOHERENCIA ÉS KVANTUM KORRELÁCIÓK

#### Varró Sándor

ELTE—ELFT Elméleti Fizikai Iskola Tihany, 2010. augusztus 30-szeptember 3.





THE

# PRINCIPLES OF QUANTUM MECHANICS

BY

P. A. M. DIRAC

... "Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the intensity of a beam is connected with the probable number of photons in it, we should have half the total number of photons going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components.

Each photon then interferes only with itself.

Interference between two different photons never occurs." [ § 3, p. 9. ] [ Third Edition 1947 ]

## OPTIKA:FÉNY, ELEKTRON, NEUTRON...

"All kinds of particles are associated with waves in this way and conversely all wave motion is associated with particles. Thus all particles can be made to exhibit interference effects and all wave motions has its energy in the form of quanta."

[ P. A. M. DIRAC: THE PRINCIPLES OF QUANTUM MECHANICS.

(Clarendon Press, Oxford, 1930, 1935, 1947) § 9. p. 9-10.]

"Even assuming that the [electron] emission was to some extent directed, so that the angular spread was less than  $2\pi$ , it is clear that the occupation numbers were very small even in these rather extreme experiments. Hence the almost complete identity of light optics and electron optics, in spite of the extreme difference between Einstein-Bose and Fermi-Dirac statistics."

[ D. GABOR: LIGHT AND INFORMATION\*.

Progress in Optics Vol. 1 (Ed. E. Wolf), pp. 109-153. (North-Holland, Amsterdam, 1961) ]

"The <u>similarities of classical optics and neutron optics</u> are easy to understand and anticipate, since the time-independent Schrödinger equation is formally equivalent to the Helmholtz scalar wave equation which accounts for the behaviour of light waves (aside from polarization effects)."

[ H. RAUCH and S. A. WERNER: NEUTRON INTERFEROMETRY.

Lessons in Experimental Quantum Mechanics. (Clarendon Press, Oxford, 2000) ]

- Az intenzitás-intenzitás korrelációkról általában, a Hanbury Brown és Twiss ( HBT ) féle kísérlet.
- A "hullám-részecske kettősség" történetéből, "öninterferencia". Korrelációk és koherencia.
- A klasszikus és "kvantum valószínűségről", egyenlőtlenségek.
- "HBT Reneszánsz" napjainkban. Fény, Röntgen, gamma, elektron, neutron, atomizotópok.
- Esettanulmány I.: Foton antikorreláció (Aspect).
- Esettanulmány II.: Termikus neutronnyaláb.
- Foton elektron összefonódás, Compton-szórás extrém intenzív lézerfényben, evaneszcens terek.

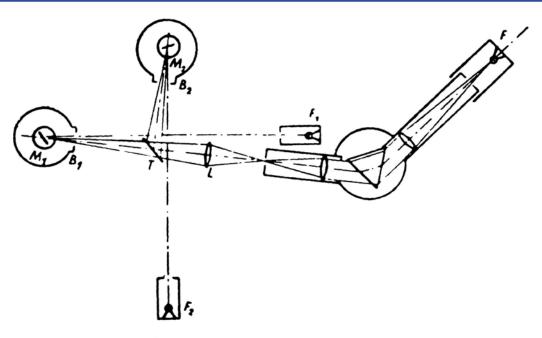


FIG. 1. Experimental arrangement of Ádám, Jánossy, and Varga. Light from source F is focused through a monochromator onto photomultipliers  $M_1$  and  $M_2$  via beam splitter T. (Figure after Ádám, Jánossy, and Varga.)

Ádám, Jánossy és Varga (1955), Brannen and Ferguson (1956)

Hanbury Brown and Twiss (1956), Rebka and Pound (1957)

Arecchi (1963), Arecchi, Gatti and Sona (1965)

Farkas, Jánossy, Náray és Varga (1965)

## HBT II. $\Delta I_1 - \Delta I_2$ KORRELÁCIÓ (1956)<sub>a</sub>

No. 4497 January 7, 1956

NATURE

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The nitrogen mustards act most strikingly upon cells in which growth and division are rapid. The selectivity of the derivatives studied here is based on at least two cellular variables—growth-rate and enzyme content. It seems probable that a further increase in selectivity could be introduced by using inactivating agents introducing dependence upon yet a third variable. For example, acylation of add dep introducing dependence upon yet at third variable. For example, acylation of add ids.

\*\*Koherens: kis spektrális\*\*

\*\*Example acylation of add ids.\*\*

\*\*Example acylation of acylation of add ids.\*\*

\*\*Example acylation of acylation of

We are indebted to Boots Pure Drug Co., Ltd., for the gift of compounds (I), (II) and (III), to Dr. W. C. J. Ross, who synthesized compound (IV) for us, to Mr. J. A. Marsh for help and advice with the tumour tests, and to Prof. A. Haddow and Dr. L. N. Owen for advice on many points. One of us (P. H.) has held a studentship of the Medical Research Council while carrying out this work. [Oct. 10]

- Danielli, J. F., "Pharmacology and Cell Physiology" (Elsevier, Amsterdam, 1950); Nature, 170, 863 (1952); "Leukemia Research" (Ciba Foundation, London, 1954).
- <sup>2</sup> Haddow, A., Kon, G. A. R., and Ross, W. C. J., Nature, 162, 824 (1948).
- <sup>3</sup> Ross, W. C. J., "Advances in Cancer Research", 1, 397 (1953).

## CORRELATION BETWEEN PHOTONS IN TWO COHERENT BEAMS OF LIGHT

By R. HANBURY BROWN

University of Manchester, Jodrell Bank Experimental Station

AND

R. Q. TWISS

Services Electronics Research Laboratory, Baldock

In an earlier paper<sup>1</sup>, we have described a new type of interferometer which has been used to measure the angular diameter of radio stars<sup>2</sup>. In this instrument the signals from two aerials  $A_1$  and  $A_2$  (Fig. 1a) are detected independently and the correlation between the low-frequency outputs of the detectors

reasons a laboratory experiment was carried out as described below.

The apparatus is shown in outline in Fig. 2. A light source was formed by a small rectangular aperture, 0.13 mm.  $\times 0.15$  mm. in cross-section. on which the image of a high-pressure mercury are

## HBT II. $\Delta I_1 - \Delta I_2$ KORRELÁCIÓ (1956)

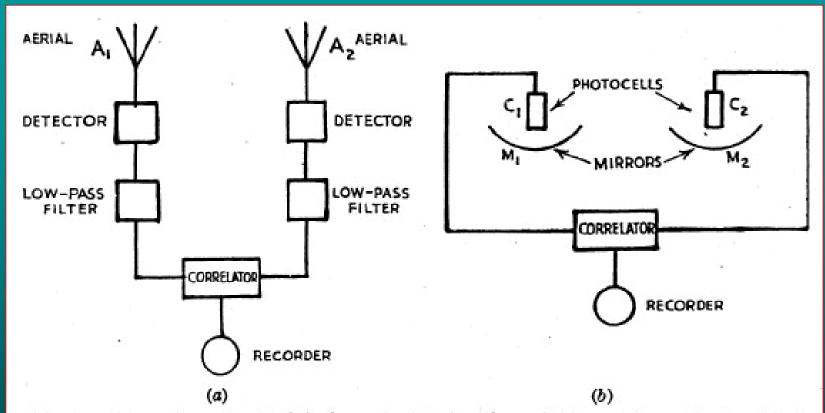


Fig. 1. A new type of radio interferometer (a), together with its analogue (b) at optical wave-lengths

Megjegyzés: Az antennák felülete mozaik elemekből állt. Igy a fotocellákba irányuló térszögekbe kevesebb transzverzális módus (M. szabadsági fok) jutott, ez növelte az 1/M

kontrasztot a koincidencia-görbén.

#### HBT II. Brannen & Ferguson (1956)a

No. 4531 September 1, 1956 NATURE

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seem to be worth studying, so that biologists interested in this kind of work may find it worth while to watch the biotype in its various stages of development.

Itt is: Koherens: kis spektráli

1 Pringshe (1947) Itt is: Koherens: kis spektrális szélesség:  $\Delta v \le v$ !!

- <sup>2</sup> Pringsheim, E. G., J. Gen. Microbiol., 5, 124 (1951).
- <sup>3</sup> Baker, F., Ann. App. Biol., 30, 230 (1943).
- <sup>4</sup> Pringsheim, E. G., Arch. Microbiol., 16, 18 (1951). Cüster, E., Arch. Protistenk., 2, 351 (1908). ringsheim, E. G., J. Protozool., 2, 137 (1945). [arold, R., and Stanier, R. Y., Bact. Rev., 19, 49 (1955).

## THE QUESTION OF CORRELATION BETWEEN PHOTONS IN COHERENT LIGHT RAYS

By PROF. ERIC BRANNEN and H. I. S. FERGUSON

Department of Physics, University of Western Ontario, London, Canada

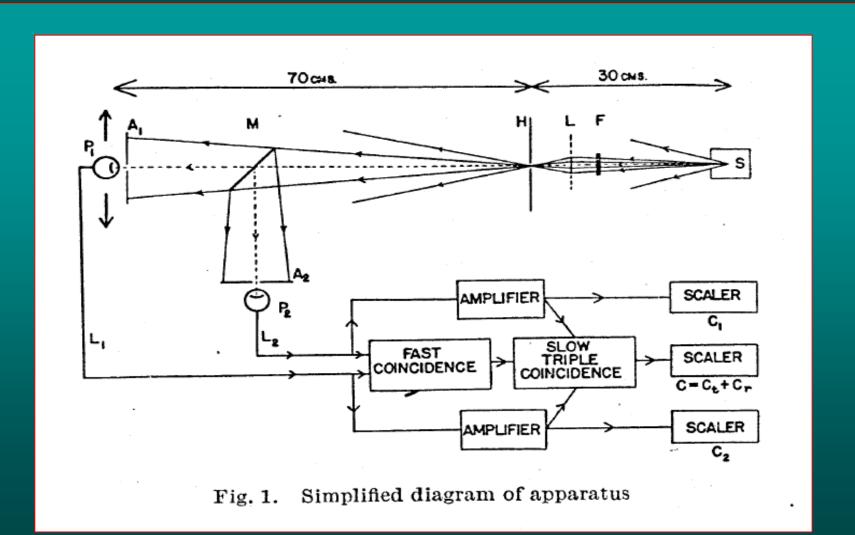
CONSIDERABLE interest has been aroused in this question, especially since the experiments of R. Hanbury Brown and R. Q. Twiss¹ showed a correlation, unexpected by many, between the current outputs from two photoe'ectric detectors viewing coherent light rays. On the other hand, A. Adám, L. Jánossy and P. Varga² performed a similar experiment using "an amplifier with a resolving time of 2 µsec." and observed no such correlation.

In this laboratory we have been performing coincidence experiments<sup>3</sup> on photons emitted in cascade transitions from optical sources using apparatus of resolving time  $2\tau = 10^{-8}$  sec. With

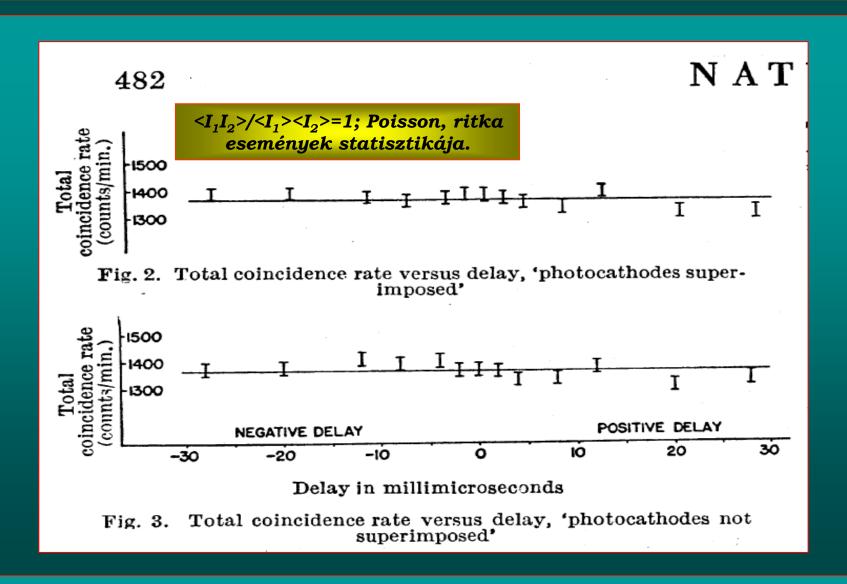
experiment by insertion of a time delay in one channel much greater than the resolving time of the apparatus. This means that such macroscopic intensity fluctuations in the source would not affect the results of our experiment.

The experimental arrangement is shown in Fig. 1. S is a high-pressure Hilger mercury arc. The image of the arc is focused by means of an F2 Zeiss Biotar lens L on to a small pinhole H of diameter 0.25 mm. The light is filtered by a Baird interference filter ( $\lambda 4358$  A.) and the appropriate Wratten filter, at F, so that essentially monochromatic light falls on the pinhole. It was found that a good lens free of aber-

## HBT II. Brannen & Ferguson (1956)b



## HBT II. Brannen & Ferguson (1956)c

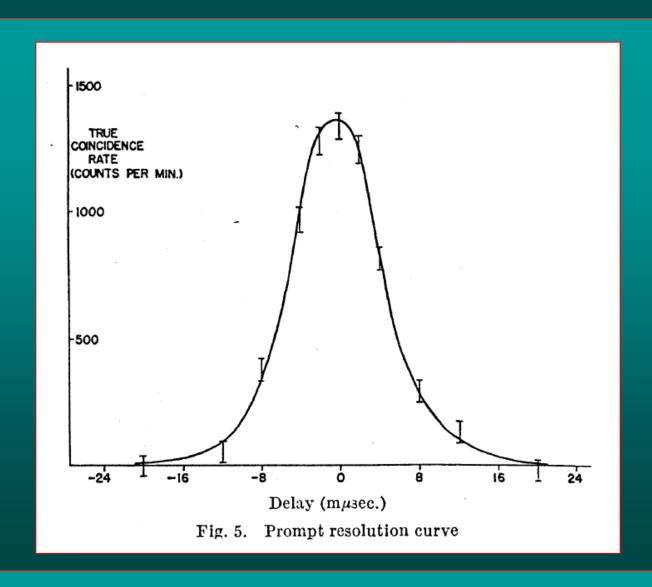


## HBT II. Brannen & Ferguson (1956)d

Note added in proof. It would appear to the authors, and also to Prof. Jánossy (private communication), that if such a correlation did exist, it would call for a major revision of some fundamental concepts in quantum mechanics. This was, of course, the reason why these experiments were performed.

- 1 Hanbury Brown, R., and Twiss, R. Q., Nature, 177, 27 (1956).
- <sup>2</sup> Adám, A., Jánossy, L., and Varga, P., Acta Phys. Hungary, 4, No. 4, 301 (1955).
- <sup>3</sup> Brannen, E., Hunt, F. R., Adlington, R. H., and Nicholls, R. W., Nature, 175, 810 (1955).

## HBT II. Brannen & Ferguson (1956)e



#### No. 4594 November 16, 1957 NATURE

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#### TIME-CORRELATED PHOTONS

By G. A. REBKA, JUN.,\* and PROF. R. V. POUND Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

SEVERAL studies of the time correlation of photons in coherent light beams have recently been reported in this journal<sup>1-8</sup>. We have undertaken to demonstrate such correlation using pulse and coincidence techniques in contrast to the continuous-wave technique used by Brown and Twiss in their first study. Our experiment differs from their more recent one<sup>9</sup> in that counts from each detector are recorded. These are used for reducing each observation to a measurement of effective resolving time.

Following the formulation of Purcell<sup>4</sup>, the number of coincidences  $N_c$  in the counting period T can be written

$$N_c = (N_1 N_2/T) \{ 2\tau_R + np\alpha \tau_0 f(\Delta) \}$$

The quantities  $N_1$  and  $N_2$  are the counts recorded from each detector,  $\tau_R$  is the resolving time of the coincidence circuit, while  $\tau_0$  is the correlation time of the light and is related to the width and shape of the spectral line used, in the manner discussed by Purcell. The factor n accounts for the presence of pulses of other origin than the light beam, which are assumed random. In our experiment a typical value of n was 0.951. Similarly p accounts for polarization of the light beam. It would have a value of  $\frac{1}{2}$  for com-

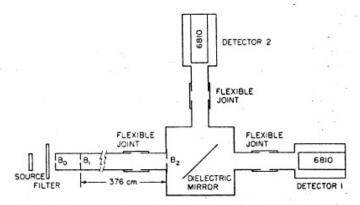


Fig. 1. A representation of the optical system. Baffles  $B_1$  and  $B_2$  define the geometrical coherence

Brown and Twiss<sup>1,9</sup>, assures geometrical coherence at the two detectors without critical adjustment. Consequently we were not able to vary the correlation by changing the geometrical coherence.

The photomultiplier tubes were cooled throughout the experiment with solid carbon dioxide, and electrostatic shields at cathode potential were used to minimize dark current. The tubes were operated with an anode-cathode potential of 2,450 volts, and

#### ARECCHI, GATTI AND SONA [ 1965 ]a

interval  $\tau$ . The number of pulses occurring in  $\Delta \tau$  is counted and recorded in the memory of a pulse-height analyzer. The source used was either a Gaussian source obtained by randomization of a laser light [3] by means of a rotating ground glass disc and observed within a coherence area, or the laser light itself coming from a single axial mode of a He-Ne laser (with a cavity length of 20 cm and TEM<sub>OO</sub> emission). The experimental results are shown in fig. 1 and agree

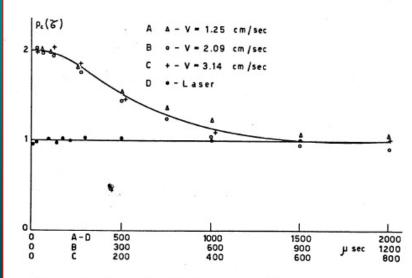


Fig. 1. Conditional probability  $p_{\rm C}(\tau)$  of a second count occurring at a time  $\tau$  after a first has occurred at time  $\tau = 0$ 

tem with different coherence times.

Next we discuss experiment b). The time intervals distribution between two successive pulses can be derived from the above conditional probability as follows:

$$p_{01}(\tau) = \alpha I \exp \left[-\int_{0}^{\tau} p_{c}(x) dx\right]$$
 (2)

where  $p_{01}$  is the probability density of the second event occurring at  $\tau$  while the first one has occurred at  $\tau = 0$ . A start-stop system was used to perform the measurement and the experimental results are reported in fig. 2a and b on a semilogarithmic plot.

While the laser light gives a single exponential, the thermal light has a short-time dependence which tends to an exponential with an initial time constant twice as large as that of the long-time asymptotic exponential, in accordance with (1) and (2).

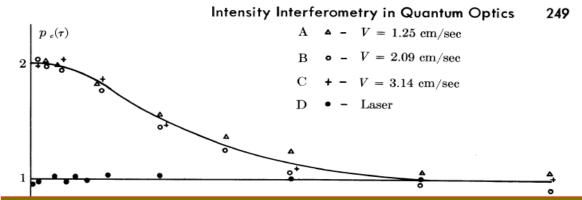
For the Gaussian field comparison with the spectral amplitude distribution  $I(\omega)$  of I(t) measured with a wave analyzer gives the correct correspondence between  $I(\omega)$  and  $|\gamma_{11}(\tau)|^2$  that is

$$|\gamma_{11}(\tau)|^2 = \int_0^\infty I^2(\omega) \cos \omega \tau \, d\omega$$
,

within the experimental errors.

The same experiment has been previously performed with a two meter laser going on two

#### ARECCHI, GATTI AND SONA [ 1965 | b



Glauber: Quantum Theory of Optical Coherence [ 1963 ]

#### Klauder & Sudarshan: Fundamentals of Quantum Optics [ 1968 ]

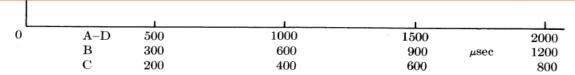


Fig. 10-1 The relative intensity correlation versus time for thermal and coherent sources. Experimental results apply to a laser and to an artificially synthesized chaotic source created by passing the laser radiation through ground glass which is rotated at speeds of 1.25 cm/sec, 2.09 cm/sec, and 3.14 cm/sec, respectively. The relative intensity correlation decays from 2 to 1 in a time characteristic of the spectral bandwidth of the chaotic radiation. The equilibrium value of unity is reached in a shorter time interval for a higher rate of rotation of the ground glass indicating thereby a correspondingly wider band of frequencies. In the case of laser light, essentially no intensity correlation was observed compatible with the absence of intensity fluctuations. [After F. T. Arecchi, E. Gatti, and A. Sona, Phys. Rev. Letters 20, 27 (1966); reprinted with permission.]

## "KVANTUM KOHERENCIA ELMÉLET, KVANTUM KORRELÁCIÓK"

in textbook form. Indeed, some of our analysis is original and has not been previously published elsewhere.

There are, of course, several sources that are basic to the subject and from which the reader may gain additional insight into similar or related topics. Notable among these are the following:

- 1. M. Born and E. Wolf, *Principles of Optics*, Pergamon, Oxford, 3rd ed. (1965).
- 2. R. J. Glauber, Quantum Optics and Electronics (C. DeWitt, A. Blandin, and C. Cohen-Tannoudji, eds.), Gordon and Breach, New York (1964).
- 3. L. Mandel and E. Wolf, "Coherence properties of optical fields," Rev. Modern Phys. 37, 231 (1965).

Indeed, the last article has a veritable wealth of references to the recent literature which we do not attempt to duplicate here. Occasionally in our chapter bibliographies we include texts or articles not explicitly cited in which further relevant discussion may be found.

#### GOLDBERGER, WATSON & LEWIS [ 1963 -... ]

## Kvantált sugárzási tér

$$\hat{\vec{E}} = i \sum_{k} \sqrt{2\pi \hbar \omega_{k}} \left[ \vec{u}_{k}(\vec{r}) \hat{a}_{k} e^{-i\omega_{k}t} - \vec{u}_{k}^{*}(\vec{r}) \hat{a}_{k}^{\dagger} e^{+i\omega_{k}t} \right] \equiv \hat{\vec{E}}^{(+)} + \hat{\vec{E}}^{(-)}$$

$$(\nabla^2 + k^2)\vec{u}_k = 0$$

$$[\hat{a}_k, \hat{a}_l^+] = \delta_{kl}$$

$$(1/8\pi) \int d^3r (\hat{E}^2 + \hat{B}^2) = \sum_k \hbar \omega_k (\hat{a}_k^+ \hat{a}_k + 1/2)$$

$$\hat{a}_k^+ \hat{a}_k | n_k \rangle = n_k | n_k \rangle, \quad n_k = 0,1,2,..., \forall k$$

$$|\psi_k\rangle = \sum_{n_k=0}^{\infty} c_{n_k} |n_k\rangle$$

$$|\psi_{k}\rangle = \sum_{n_{k}=0}^{\infty} c_{n_{k}} |n_{k}\rangle |\psi_{k'k''}\rangle = \sum_{n_{k'}=0}^{\infty} \sum_{n_{k''}=0}^{\infty} c_{n_{k'},n_{k''}} |n_{k'}\rangle |n_{k''}\rangle |\hat{\mathcal{D}} = \sum_{\psi} |\psi\rangle \Pr_{\psi}\langle \psi|$$

$$\hat{\rho} = \sum_{\psi} |\psi\rangle \Pr_{\psi} \langle \psi|$$

#### **Fotondetektálás**

$$T_{fi} = \frac{ie}{\hbar} \int_{0}^{t} dt' \langle \varphi_{g} | \vec{x}(t) | \varphi_{a} \rangle \cdot \langle \psi_{f} | \vec{E}(\vec{r}, t) | \psi_{i} \rangle$$

$$s \sim |\mu|^2, \ \mu = \vec{d} \cdot \vec{\varepsilon},$$
 $\vec{d} \cdot \vec{\varepsilon} \equiv e \langle \varphi_g | \vec{x} | \varphi_a \rangle$ 

$$w_{i} = \sum_{f} w_{fi} = s \langle \psi_{i} | E^{(-)}(\vec{r}, t) E^{(+)}(\vec{r}, t) | \psi_{i} \rangle =$$

$$s \langle E^{(-)}(\vec{r}, t) E^{(+)}(\vec{r}, t) \rangle$$

## Glauber kvantum koherenciafüggvényei

#### LINEÁRIS FOTOELEKTRON-KIVÁLTÁS VALÓSZINŰSÉGE

$$w_1 = s \langle E^{(-)}(\vec{r}, t) E^{(+)}(\vec{r}, t) \rangle \sim \langle a^+ a \rangle \sim I(\vec{r}, t)$$

KÉT FOTOELEKTRON KIVÁLTÁSÁNAK VALÓSZINŰSÉGE

$$w_{2} = s^{2} \langle E^{(-)}(\vec{r}_{1}, t_{1}) E^{(-)}(\vec{r}_{2}, t_{2}) E^{(+)}(\vec{r}_{2}, t_{2}) E^{(+)}(\vec{r}_{1}, t_{1}) \rangle =$$

$$s^{2} Tr[\hat{\rho} \cdot E^{(-)}(\vec{r}_{1}, t_{1}) E^{(-)}(\vec{r}_{2}, t_{2}) E^{(+)}(\vec{r}_{2}, t_{2}) E^{(+)}(\vec{r}_{1}, t_{1})]$$

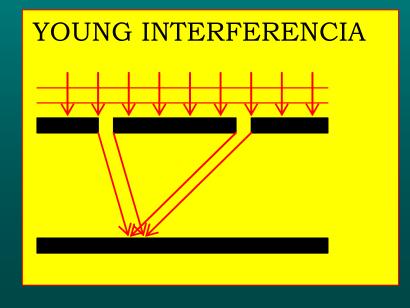
( n + m )-EDRENDŰ KVANTUM KOHERENCIA-FÜGGVÉNY

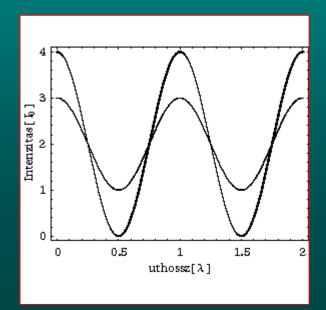
$$G^{(n,m)}(x_1,...,x_n;x_{n+1},...,x_{n+m}) \equiv \langle E^{(-)}(x_1)\cdots E^{(-)}(x_n)E^{(+)}(x_{n+1})\cdots E^{(+)}(x_{n+m}) \rangle$$
  $(x) \equiv (\vec{r},t)$ 

#### PÉLDA NORMÁLT KOHERENCIA-FÜGGVÉNYEKRE:

$$g^{(1,1)}(x_1;x_2) \equiv \frac{G^{(1,1)}(x_1;x_2)}{[G^{(1,1)}(x_1;x_1) \cdot G^{(1,1)}(x_2;x_2)]^{1/2}}$$

$$g^{(2,2)}(x_1, x_2; x_2, x_1) = \frac{G^{(2,2)}(x_1, x_2; x_2, x_1)}{G^{(1,1)}(x_1; x_1) \cdot G^{(1,1)}(x_2; x_2)}$$





#### **KIRCHHOFF**

$$E^{(+)}(\vec{r},t) = K_1 E^{(+)}(\vec{r}_1,t-t_1) + K_2 E^{(+)}(\vec{r}_2,t-t_2)$$

$$\langle E^{(-)}(\vec{r},t)E^{(+)}(\vec{r},t)\rangle = G^{(1,1)}(x,x) =$$

$$2 |K|^2 G^{(1,1)}(x_1,x_1) \{1 + |g^{(1,1)}(x_1,x_2)| \cos \varphi(x_1,x_2) \}$$

#### LÁTHATÓSÁG

$$l = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = l(x) = |g^{(1,1)}(x_1, x_2)|$$

ÉSZLELÉSI PONT  $(x) \equiv (\vec{r}, t)$ 

LYUKAK 
$$(x_1) \equiv (\vec{r}_1, t - |\vec{r} - \vec{r}_1|/c)$$
,  $(x_2) \equiv (\vec{r}_1, t - |\vec{r} - \vec{r}_2|/c)$ 

## Koherens állapotok, P-reprezentáció

HA 
$$G^{(n,m)} = V^*(x_1) \cdots V^*(x_n) V(x_{n+1}) \cdots V(x_{n+m}) \quad \forall n, m$$

AKKOR AZ EM TÉR TELJESEN KOHERENS: |g| = 1

ELÉGSÉGES FELTÉTEL :  $\hat{E}^{(+)}(x)|V\rangle = V(x)|V\rangle$ 

V(x) ~ 'ANALITIKUS SZIGNÁL' ( GÁBOR DÉNES, 1946 )

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \rightarrow |\alpha\rangle = \sum_{n=1}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle e^{-\frac{1}{2}|\alpha|^2} \qquad p_n = \frac{\lambda^n}{n!} e^{-\lambda}, \quad \lambda \equiv |\alpha|^2$$

 $[\psi_{\alpha}(x,t) = \langle x | \alpha \rangle_{t}$  SCHRÖDINGER (1926), MARKOV (1927) ]

P-REPREZENTÁCIÓ 
$$\hat{\rho} = \int d^2 \alpha \, |\alpha\rangle P(\alpha)\langle\alpha|$$

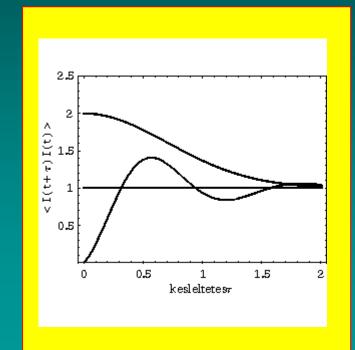
## Fotoncsomósodás, Fotonritkulás

$$P_n(t,t+T) = \int_0^\infty dv \frac{v^n}{n!} e^{-v} \cdot W(v),$$

$$W(v) = \int d^2 \{\alpha\} \delta(v - N_{\{\alpha\}\{\alpha\}}) P(\{\alpha\})$$

$$N_{\{\beta\}\{\alpha\}} = \int_{t}^{t+T} dt' \iint d\vec{r} d\vec{r}' \sum_{\mu\mu'} K_{\mu\mu'}(\vec{r}, \vec{r}')$$

$$\times E_{\mu}^{(cl)}(\{\beta\}, \vec{r}, t')^* E_{\mu'}^{(cl)}(\{\alpha\}, \vec{r}', t')$$



 $\langle I(t)I(t+ au) 
angle$ 

## Korreláció ~ Fluktuáció

#### INTENZITÁS KORRELÁCÓ ~ ENERGIA FLUKTUÁCIÓ

$$g^{(2,2)}(\tau) = \frac{\left\langle E^{(-)}(t)E^{(-)}(t+\tau)E^{(+)}(t+\tau)E^{(+)}(t)\right\rangle}{\left\langle E^{(-)}(t)E^{(+)}(t)\right\rangle^{2}} \to$$

$$g^{(2,2)}(0) = \frac{\langle \hat{a}^{+} \hat{a}^{+} \hat{a} \hat{a} \rangle}{\langle \hat{a}^{+} \hat{a} \rangle^{2}} = 1 + \frac{(\Delta n)^{2} - \overline{n}}{(\overline{n})^{2}} = 1 + \frac{(\Delta E_{1})^{2} - h \nu \overline{E}_{1}}{(\overline{E}_{1})^{2}}$$

$$g_{cl}^{(2,2)}(0) = 1 + \frac{(\Delta E_1)^2}{(\overline{E}_1)^2}$$

KOHERENS ÁLLAPOT

$$(\Delta n)^2 = n$$

$$(\Delta n)^2 = \overline{n}$$
  $g^{(2,2)}(0) = 1 = g_{cl}^{(2,2)}(0)$ 

$$(\Delta n)^2 = (\overline{n})^2 + \overline{n}$$

$$(\Delta n)^2 = (\bar{n})^2 + \bar{n}$$
  $g^{(2,2)}(0) = 2 = g_{cl}^{(2,2)}(0)$ 

$$\langle I(t+\tau)I(t)\rangle$$

## "Megtévesztő alakja"

$$g^{(2,2)}(\tau) = \frac{\left\langle E^{(-)}(t)E^{(-)}(t+\tau)E^{(+)}(t+\tau)E^{(+)}(t)\right\rangle}{\left\langle E^{(-)}(t)E^{(+)}(t)\right\rangle^{2}} \to$$

$$G_{12}^{(1)}(\tau) \equiv \left\langle E^{(-)}(t+\tau)E^{(+)}(t) \right\rangle$$

$$G_{12}^{(2)}(\tau) \equiv \left\langle E^{(-)}(t)E^{(-)}(t+\tau)E^{(+)}(t+\tau)E^{(+)}(t) \right\rangle$$

$$\langle I_1(t+\tau)I_2(t)\rangle = G_{12}^{(2)}(\tau) = I_1I_2 + |G_{12}^{(1)}(\tau)|^2$$

$$I_1 \equiv G_{11}^{(1)}(0) \equiv \left\langle E^{(-)}(\boldsymbol{r}_I, t) E^{(+)}(\boldsymbol{r}_I, t) \right\rangle$$

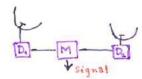
#### KÉTFOTON KORRELÁCIÓ? ENTANGLEMENT?

R. GLAUBER: Quantum Optics and Heavy Ion Physics.

(Quark Matter 2005, Budapest, 2005).

R. Hanbury Brown + R.Q. Twiss

Intensity interferometry



$$E(n t) = E^{(t)}(n t) + E^{(t)}(n t)$$
  
 $E^{(t)}(n t) \sim \{E^{(t)}(n t)\}^{*}$ 

Ordinary (Amplitude) interferometry

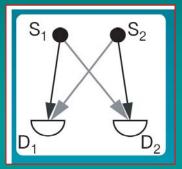
Intensity interferometry measures

Two-photon dilemma



UGO FANO: Quantum Theory of Interference Effects in the Mixing of Light from Phase-Independent Sources.

Am. J. Phys. 29, 539-545 (1961)



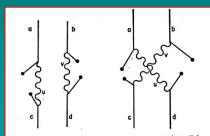


Fig. 1. Lowest-order diagrams representing light emission by a pair of atoms and its absorption by another pair of atoms. The heavy dots indicate ground-state lines.

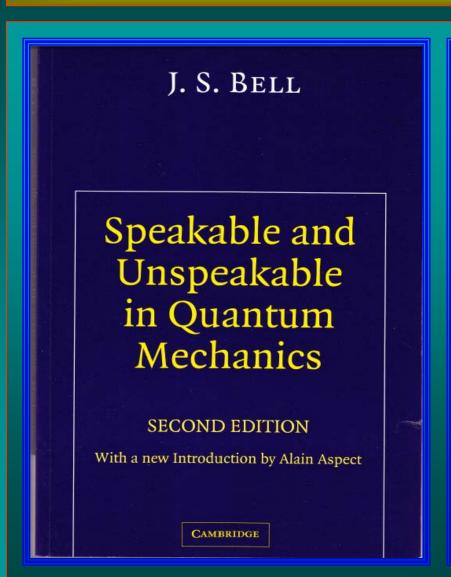
$$P_{c}(t) = \left\langle \left| D_{c,a}(t) + D_{c,b}(t) \right|^{2} \right\rangle$$

$$= \left\langle \left| D_{c,a}(t) \right|^{2} + \left| D_{c,b}(t) \right|^{2} \right\rangle$$

$$= P_{c,a}(t) + P_{c,b}(t)$$

$$P_{cd}(t) = \left\langle \left| D_{c,a}(t) D_{d,b}(t) + D_{d,a}(t) D_{c,b}(t) \right|^2 \right\rangle$$

## John Bell [ 1964 ]a



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## John Bell [ 1964 ]b

## 6 John Bell's legacy: questioning quantum mechanics is fruitful

Quantum mechanics was, and continues to be, revolutionary, primarily because it demands the introduction of radically new concepts to better describe the world. In addition we have argued that *conceptual* quantum revolutions in turn enable *technological* quantum revolutions.

John Bell started his activity in physics at a time when the first quantum revolution had been so successful that nobody would 'waste time' in considering questions about the very basic concepts at work in quantum mechanics. It took him a decade to have his questions taken seriously. For somebody who has observed reactions to his work on the EPR situation and entanglement, in the early 1970s, it is certainly amusing to see that an entry of the Physics and Astronomy Classification Scheme is now assigned to 'Bell inequalities' With his questions about entanglement, John Bell was able to clarify the Einstein—Bohr debate in an unanticipated manner, offering the opportunity to settle the question experimentally. His work, without a doubt, triggered the second quantum revolution, primarily based on the recognition of the extraordinary features of entanglement, and pursued with efforts to use entanglement for quantum information. In fact, it not only triggered the

## ELEMI FOTON-ANTI-KORRELÁCIÓK [KÖZELITŐ n-SZEKVENCIÁKBAN]

P. Grangier, G. Roger and A. Aspect, Experimental evidence for a photon anticorrelation effect on a beam splitter.... Europhysics Letters 1, 173-179 (1986)

A. Aspect and P. Grangier, Wave-particle duality for single photons. *Hyperfine Interactions* 37, 3-18 (1987)

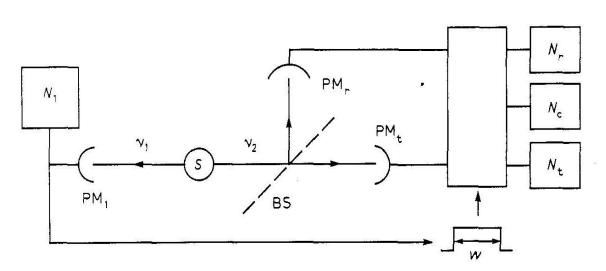


Fig. 1. – Triggered experiment. The detection of the first photon of the cascade produces a gate w, during which the photomultipliers  $PM_t$  and  $PM_r$  are active. The probabilities of detection during the gate are  $p_t = N_t/N_1$ ,  $p_r = N_r/N_1$  for singles, and  $p_c = N_c/N_1$  for coincidences.

## FOTON ANTIKORRELÁCÓ<sub>a</sub> [ *n-SZEKVENCIÁK BINOMIÁLIS SZÉRIÁI* ]

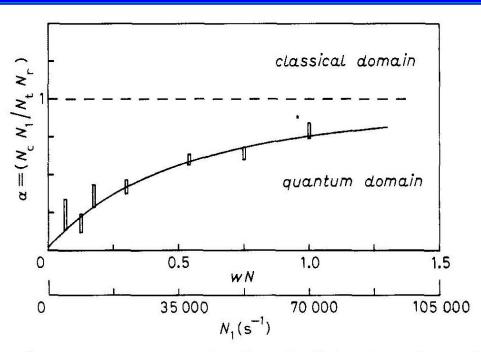


Fig. 2. – Anticorrelation parameter  $\alpha$  as a function of wN (number of cascades emitted during the gate) and of  $N_1$  (trigger rate). The indicated error bars are  $\pm$  one standard deviation. The full-line curve is the theoretical prediction from eq. (8). The inequality  $\alpha \ge 1$  characterizes the classical domain.

$$g^{(2,2)} = \frac{\left\langle \hat{a}_{1}^{+} \hat{a}_{1}^{+} \hat{a}_{1} \hat{a}_{1} \right\rangle}{\left\langle \hat{a}_{1}^{+} \hat{a}_{1} \right\rangle^{2}} = \frac{\left\langle n_{1} (n_{1} - 1) \right\rangle}{\left\langle n_{1} \right\rangle^{2}} \rightarrow 1 - \frac{1}{n_{1}}$$

$$\overline{\xi_{n} \cdot \eta_{n}} = 1 - \frac{1}{n}$$

#### EGYFOTONOS INTERFERENCIA I.

2. A. A

A. Aspect, P. Grangier / Wave-particle duality

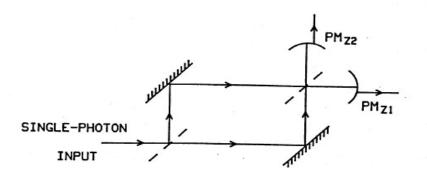


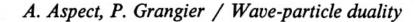
Fig. 5. Mach-Zehnder interferometer. The photomultipliers PM<sub>Z1</sub> and PM<sub>Z2</sub> are gated as in the previous experiments, so that the interferences are due to single photon wave packets. The path difference is controlled by moving the mirrors.

We have also demonstrated a source that produces single photon wave-packets, with a synchronized triggering signal. On a beam splitter, these single photon pulses exhibit a very clear anticorrelation, that is to say a behaviour characteristic of single photons, as predicted by quantum mechanics.

With such a source, it is thus possible to revisit the question of single photon interferences.

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## EGYFOTONOS INTERFERENCIA II.



13

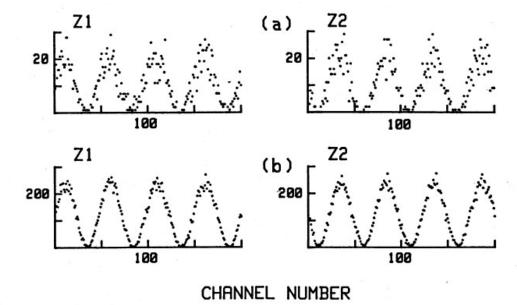
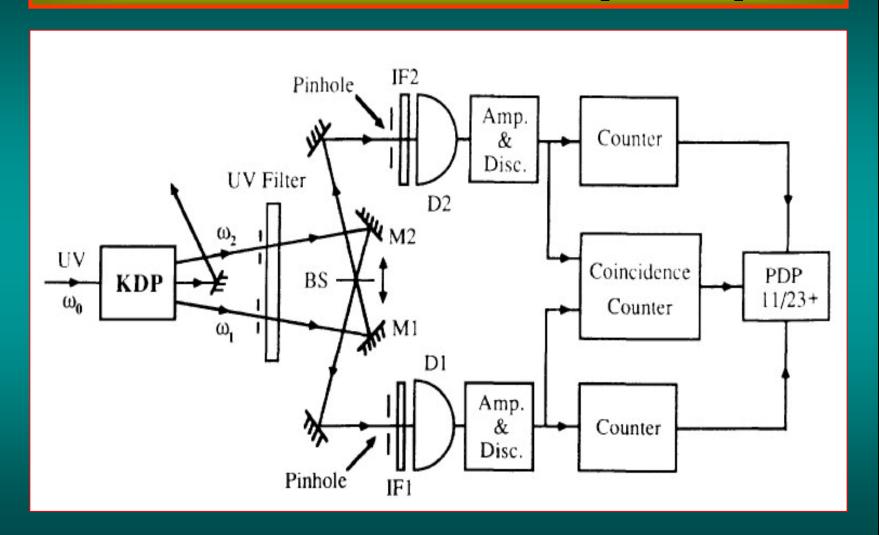
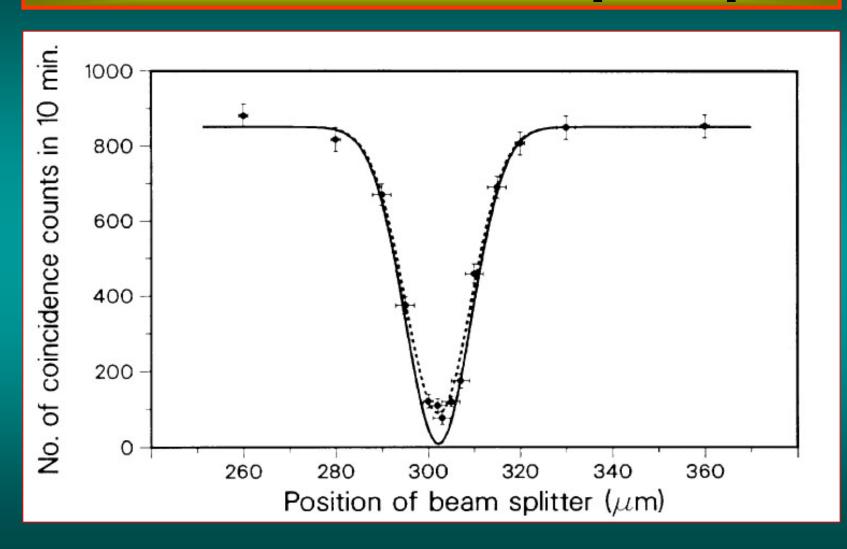


Fig. 6. Number of counts in outputs Z1 and Z2 as a function of the path difference  $\delta$  (one channel corresponds to  $\lambda/50$ ). (a) 1 s counting time per channel; (b) 15 s counting time per channel (compilation of 15 elementary sweeps). This experiment corresponds to an anticorrelation parameter  $\alpha = 0.18$ .

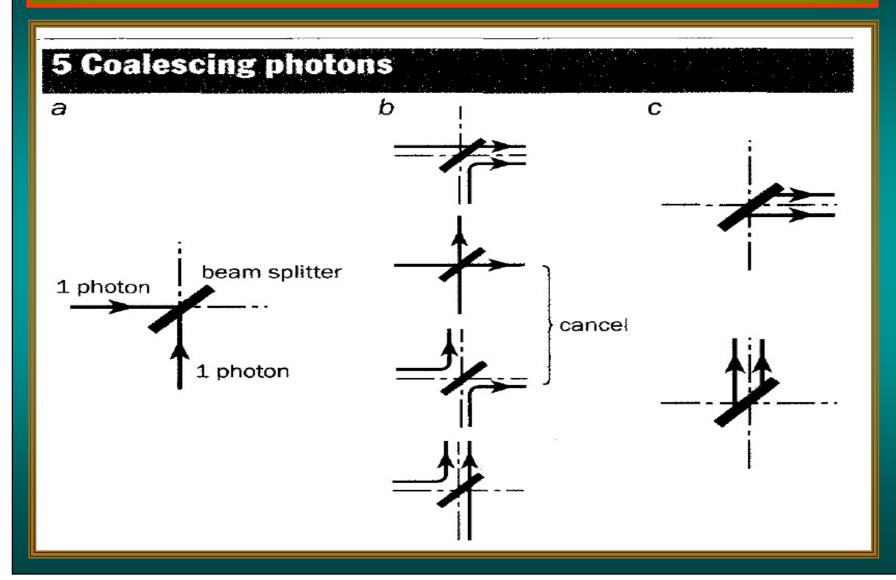
## HONG-OU-MANDEL DIP [ 1987 ]a



## HONG-OU-MANDEL DIP [ 1987 ]b



# A FOTONOK BOZONOK [ 2002 ]



# KÉT-RÉSZECSKÉS SZIMMETRIÁK I.

$$\begin{split} & \Psi^{(symmetric)}(\mathbf{r_1, r_2} \mid \mathbf{k_1} s_1, \mathbf{k_2} s_2) = \\ & \frac{1}{\sqrt{2}} \Big[ \Phi_{\mathbf{k_1} s_1}(\mathbf{r_1}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_2}) + \Phi_{\mathbf{k_1} s_1}(\mathbf{r_2}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_1}) \Big] \end{split}$$

$$\begin{split} & \Psi^{(asymmetric)}(\mathbf{r_1, r_2} \mid \mathbf{k_1} s_1, \mathbf{k_2} s_2) = \\ & \frac{1}{\sqrt{2}} \left[ \Phi_{\mathbf{k_1} s_1}(\mathbf{r_1}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_2}) - \Phi_{\mathbf{k_1} s_1}(\mathbf{r_2}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_1}) \right] \end{split}$$

# KÉT-RÉSZECSKÉS SZIMMETRIÁK II.

$$\Psi_{\mathbf{k_1k_2}}^{(-)}(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{\sqrt{2}} \left[ \phi_{\mathbf{k_1}}(\mathbf{r_1}) \phi_{\mathbf{k_2}}(\mathbf{r_2}) - \phi_{\mathbf{k_1}}(\mathbf{r_2}) \phi_{\mathbf{k_2}}(\mathbf{r_1}) \right]$$

$$\left| \times \left\{ \left| \uparrow \uparrow \right\rangle, \quad \left| \downarrow \downarrow \right\rangle, \quad \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle + \left| \downarrow \uparrow \right\rangle \right) \right\} \quad \Rightarrow \frac{3}{4}$$

$$\Psi_{\mathbf{k_1k_2}}^{(+)}(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{\sqrt{2}} \left[ \phi_{\mathbf{k_1}}(\mathbf{r_1}) \phi_{\mathbf{k_2}}(\mathbf{r_2}) + \phi_{\mathbf{k_1}}(\mathbf{r_2}) \phi_{\mathbf{k_2}}(\mathbf{r_1}) \right]$$

$$\left| \times \frac{1}{\sqrt{2}} \left( \uparrow \downarrow \right) - \left| \downarrow \uparrow \right\rangle \right) \implies \frac{1}{4}$$

#### Az Einstein - féle fénykvantumok. (Hogy is kezdődött?) I.

$$S_{\upsilon} - S_{\upsilon_0} = (E/\beta \nu) \log(\upsilon/\upsilon_0) = k \log \left[ (\upsilon/\upsilon_0)^{\frac{E}{k\beta \nu}} \right]$$

n pontszerű részecskéből álló ideális gázra:

$$S_{\upsilon} - S_{\upsilon_0} = k \log \left[ (\upsilon / \upsilon_0)^n \right]$$

E = nhv

- A) "Kis sűrűségű (a Wien-féle sugárzási képlet érvényességi tartományán belül) monokromatikus sugárzás hőelméleti szempontból úgy viselkedik, mintha  $R\beta v/N$  [ =  $k\beta v$  = hv] nagyságú, egymástól független energiakvantumokból állna."
- B) "Az itt kifejtésre kerülő felfogás szerint az egy pontból kiinduló fénysugarak szétterjedésénél az energia nem folytonosan egyre nagyobb és nagyobb térrészre oszlik el, hanem véges számú térbeli pontban lokalizált energiakvantumból áll, amelyek úgy mozognak, hogy nem bomlanak részekre, s csak mint egészek nyelődhetnek el vagy keletkezhetnek."

# <u>Advanced Information on the Nobel Prize in Physics 2005</u> (4 October 2005, Information Department, www.kva.se)

1.) "In contrast to a common misconception, there were no accurate data on photoemission of electrons at the time of Einstein's publication. Such results were provided after his work by several investigators, culminating in the convincing demonstrations by R. A. Millikan, which were quoted in the citation for his Nobel Prize in 1923. "A 2005-ös Nobel-díj információs anyagban ez a mondat egyerűen téves, ugyanis Lenard kvantitativen kimérte, hogy az energiaküszöb létezik. Einstein gondolatmenetét pontosan Lenard igen alapos mérései inicializálták. Erről híres cikkében így ír:

"Az a szokásos felfogás, mely szerint a fény energiája folytonosan oszlik el a fény átjárta térrészben, a fényelektromos jelenségek magyarázata kapcsán különösen nagy nehézségekre vezet amint azt Lenard úr úttörő munkájában kifejtette.7"

"Annak ellenőrzésére, hogy nagyságrendi egyezésben áll-e a levezetés a tapasztalattal,

... [válasszuk a következő paramétereket]. Ekkor [ a potenciálra ] P.10-7 = 4,3 Volt adódik, mely nagyságrendileg megegyezik Lenard úr eredményeivel.9 "

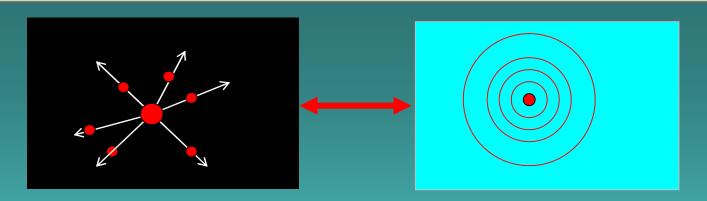
"Én magam úgy látom, hogy a fenti felfogás nem áll ellentmondásban a fényelektromos jelenség Lenard úr által megfigyelt tulajdonságaival. Ha a gerjesztő fény minden energiakvantuma az összes többitől függetlenül adja át az energiát az elektronoknak, akkor az elektronok sebességeloszlása – más szóval a gerjesztett katódsugárzás jellege – a gerjesztő fény intenzitásától független lesz; másrészt – egyébiránt azonos feltételek mellett – a testből kilépő elektronok száma a gerjesztő fény intenzitásával egyenesen arányos lesz.10 "

[1] A. Einstein: Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. Ann. der Phys. 17, 132-148 (1905) Ebben a cikkben fogalmazza meg Einstein a fénykvantumok hipotézisét. Érdekes, hogy Lenard ugyanebben az évben kapott Nobel-díjat.

2.) "Attosecond pulses can be formed if the equidistantly spaced high harmonics are phase-locked together, in a way analogous with the case of a mode-locked laser in the visible region." { as was first proposed by Farkas and Toth [2a] in 1992.} [ A 2005-ös Nobel-díj információs anyagban a mondatot így illett volna folytatni a történeti hűség kedvéért legalább.]

2a) Gy. Farkas, Cs. Tóth: Proposal for attosecond light pulse generation using multiple harmonic conversion processes in rare gases. Phys. Lett. **A168**, 447-450 (1992)

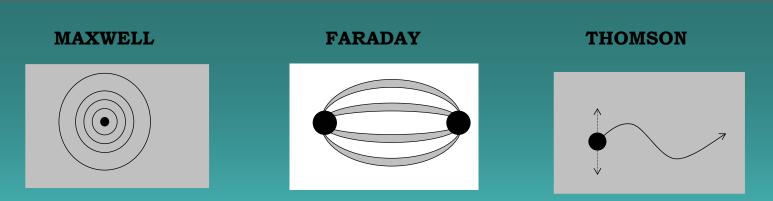
#### Az Einstein - féle fénykvantumok.(Hogy is kezdődött?) II.



#### Megjegyzések:

- 1) Érdekes, hogy tekintettel arra, hogy a Wien-formula már 1896-tól ismert volt a fenti gondolatmenetet követve, 9 évvel korábban következtetni lehetett volna a (kis sűrűségű) fekete sugárzás Einstein által kapott "szemcsésségére". A Planck-állandó felfedezése előtt azonban valószínűleg nem vetődött volna fel hogy numerikus becslést adjanak a kvantumok energiájára, s enélkül az egész megközelítés a levegőben lógott volna. Azért az is érdekes, hogy Planck a Wien-féle képlet exponensében szereplő β paraméterből már 1899-ben kiszámította a hatáskvantumot. Más: Ha a fekete sugárzás igazi részecske-gáz volna, akkor egy (1/3)-os faktor hiányozna a Placktörvény alapján kiszámolt állapotegyenletből. Ezt sem Einstein, sem a kortársak nem vették észre.
- 2) Fotoeffektus.  $h_V = A + E_{kin}$ . "Én magam úgy látom, hogy a fenti felfogás nem áll ellentmondásban a fényelektromos jelenség Lenard úr által megfigyelt tulajdonságaival. Ha a gerjesztő fény minden energiakvantuma az összes többitől függetlenül adja át az energiát az elektronoknak, akkor az elektronok sebességeloszlása másszóval a gerjesztett katódsugárzás jellege a gerjesztő fény intenzitásától független lesz; másrészt egyébiránt azonos feltételek mellett a testből kilépő elektronok száma a gerjesztő fény intenzitásával egyenesen arányos lesz." [Einstein (1905), 1922-es Nobel-díjánál a fotoeffektus magyarázatát emelik ki]

## A Maxwell-egyenletek megoldásainak értelmezése



"The difficulty of imagining a definite uniform limit of divisibility of matter will always be a philosophical obstacle to an atomic theory, so long as atoms are regarded discrete particles moving in empty space. But as soon as we take the next step in physical development, that of ceasing to regard space as mere empty geometrical continuity, the atomic constitution of matter ( each ultimate atom consisting of parts which are incapable of separate existence, as Lucretius held ) is raised to a natural and necessary consequence of the new stanpoint. We may even reverse the argument, and derive from the ascertained atomic constitution of matter a philosophical necessity for the assumption of a plenum, in which the ultimate atoms exist as the nuclei which determine its strains and motions."

[Larmor: Aether and Matter. (1900)]

A részecskék tehát az éter szinguláris pontjai lennének, a sugárzás és a forrás végülis egy entitás, s ezzel érthetővé válik egyben a kisugárzás 'mechanizmusa'.

#### "Hertz dipól" I. Euler-féle hullámfüggvény.

$$S = \frac{1}{r}\sin\kappa(r - ct)$$

#### Csillapítással:

$$S = \frac{1}{r} e^{+v(r-ct)} \sin \kappa (r-ct)$$

$$E_{x} = \frac{\partial^{2} S}{\partial x \partial z}$$

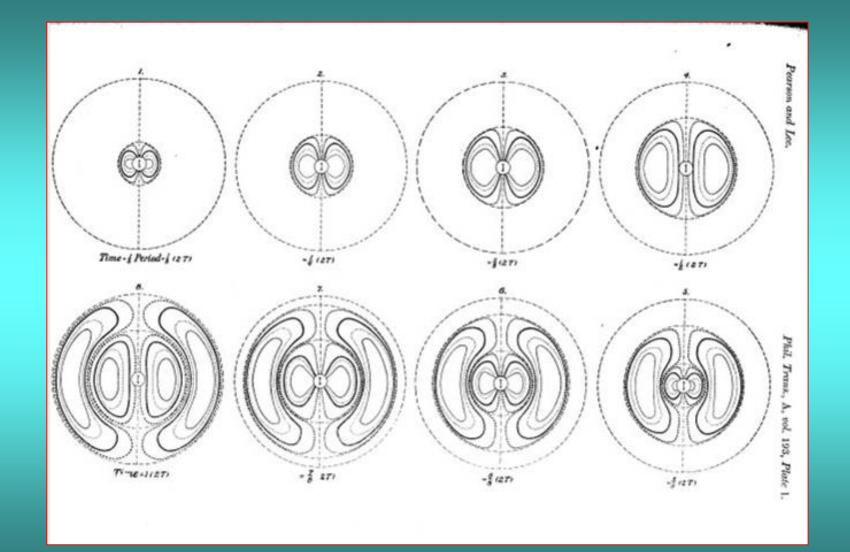
$$E_{x} = \frac{\partial^{2} S}{\partial x \partial z}$$

$$\boldsymbol{E} \cdot \boldsymbol{B} = 0$$

$$\mathbf{E} \cdot \mathbf{B} = 0$$
 
$$\mathbf{E}^2 - \mathbf{B}^2 = 0$$

"Hence at very great distance from the origin the field is practically a selfconjugate field and so the energy travels with a velocity very nearly equal to the velocity of light." (H. Bateman, 1915)

# SELÉNYI PÁL NAGYSZÖGŰ INTERFERENCIAKISÉRLETE [ 1911 ]



#### "Hertz dipól" I. Euler-féle hullámfüggvény.

$$S = \frac{1}{r}\sin\kappa(r - ct)$$

#### Csillapítással:

$$S = \frac{1}{r} e^{+v(r-ct)} \sin \kappa (r-ct)$$

$$E_{x} = \frac{\partial^{2} S}{\partial x \partial z}$$

$$E_{x} = \frac{\partial^{2} S}{\partial x \partial z}$$

$$\boldsymbol{E} \cdot \boldsymbol{B} = 0$$

$$\mathbf{E} \cdot \mathbf{B} = 0$$
 
$$\mathbf{E}^2 - \mathbf{B}^2 = 0$$

"Hence at very great distance from the origin the field is practically a selfconjugate field and so the energy travels with a velocity very nearly equal to the velocity of light." (H. Bateman, 1915)

# "TÜSUGÁRZÁS" (Needle Radiation). Bateman 1915

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FIELDS WITH MOVING SINGULAR CURVES

CH.

In Sir Joseph Thomson's theory of the Röntgen rays the kink in the tube of force becomes longer and longer as it recedes from the charge. A similar remark applies to the

oscillations of the thread attached to our point charge. This phenomenon may be due entirely to the fact that the tube of force and thread extend to infinity. If we suppose that the tube or thread does not extend to infinity but ends at some other point charge, the circumstances of the motion will be different. If in this case we treat the thread as a singular line of an electromagnetic field and suppose that it is given by an equation of the form

$$f(\alpha,\beta)=0$$

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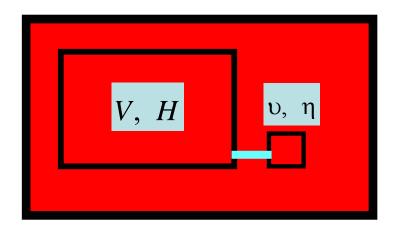
"T

# Einstein: fluktuációs formula a Planck-formula alapján; hullám-részecske kettősség

Beteszünk két egymással termodinamikailag kommunikáló dobozt V-t és v-t, egy termikus sugárzással kitöltött Hohlraum-ba, pillanatnyi energiájuk H és  $\eta$ . Az egyensúly beálltával, a homogenitás következtében  $H_0:\eta_0=V:v$ . Entrópiájuk S=klogW,  $dW=\exp(S/k)d\eta$ .  $S=\Sigma+\sigma$  és  $\eta=\eta_0+\epsilon$ , ahol  $\epsilon$  random eltérés  $\eta_0$ -tól. S-et  $\epsilon$ -ban másodrendig kifejtve kapjuk:

$$dW = const \times \exp\left\{-\frac{1}{2k} \left| \frac{d^2 \sigma}{d\eta^2} \right|_0 \varepsilon^2\right\}$$

$$\Delta \eta^2 \equiv \overline{(\eta - \eta_0)^2} = \overline{\varepsilon^2} = \left\{ \frac{1}{k} \left| \frac{d^2 \sigma}{d \eta^2} \right|_0 \right\}^{-1}$$



$$\Delta \eta^2 = h \nu \eta_0 + \frac{c^3}{8\pi v^2 dv} \cdot \frac{\eta_0^2}{v}$$

$$\frac{d^2\sigma}{d\eta^2} = \frac{1}{m_v} \cdot \frac{d^2S_1}{dU_1^2} = \frac{1}{m_v} \frac{a}{U_1(b+U_1)}$$

$$m_{v} = \frac{8\pi v^2}{c^3} \cdot dv \cdot v$$

# Újabb erdmények a fekete sugárzásról [S.V.]

Wave

$$\Delta E_{\eta}^2 = \overline{E}_{\eta}^2 / M_{\nu}$$

**GAUSS**:  $\eta = \xi + \zeta$ 

$$\Delta E_{\xi}^{2} = h \nu \overline{E}_{\xi} + \overline{E}_{\xi}^{2} / M_{\nu}$$
 PLANCK:  $\xi$ 

DARK: ζ

**Boson Wave-Particle** 

 $\Delta E_{\varsigma}^{2} = 2\overline{n}h \, v \cdot \overline{E}_{\varsigma} + \overline{E}_{\varsigma}^{2} / M_{v} - h \, v \cdot \overline{E}_{\varsigma}$ 

Dark Fluctuation (?)

CLASSICAL (POISSON): x<sub>m</sub>

BINARY (Fermion): u<sub>s</sub>

$$(\Delta E_{\nu}^{(m)})^2 = mh \, \nu \cdot \overline{E}_{\nu}^{(m)}$$

$$(\Delta E_{\nu}^{(s)})^2 = 2^s h \nu \cdot \overline{E}_{\nu}^{(s)} - [\overline{E}_{\nu}^{(s)}]^2 / M_{\nu}$$

**Particle** 

Fermion Wave-Particle

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QUANTUM

A Study on Black-Body Radiation: Classical and Binary

Sándor Varró

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Received 11 November 2006

Abstract. The present study gives a detailed analysis of the thermal radiation based completely on classical random variables. It is shown that the energy of mode of a classical chaotic radiation field, following the continuous exponentia distribution as a classical chaotic described by the continuous exponential continuous expon

decomposed into a teger part is a disc is just the Planck-I ation. The fraction distribution, which scribes the observat Planck-Bose distrib ways. On one han sum of independen (more accurately l with s = 0, 1, 2, ...they serve as a un this way, the black namically indepen other hand, the Pla variables which des molecules, or photo the black-body rad independent gases of statistics. From th the Wien formula is shown that the cother hand, the bi

**Irreducible decomposition of Gaussian** distributions and the spectrum of black-body radiation

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#### EINSTEIN'S FLUCTUATION FORMULA. A HISTORICAL OVERVIEW

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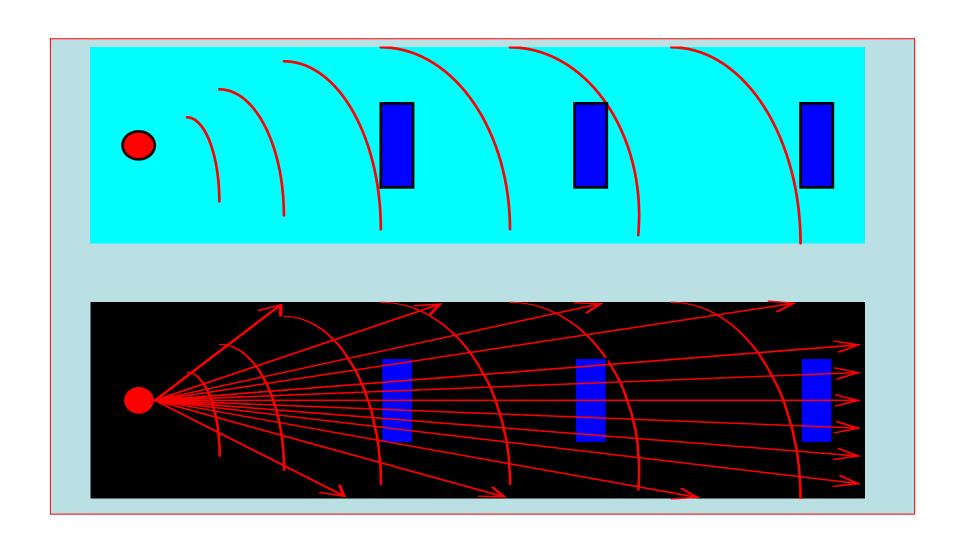
A historical overview is given on the basic results which appeared by the year 1926 concerning Einstein's fluctuation formula of black-body radiation, in the context of light-quanta and waveparticle duality. On the basis of the original publications - from Planck's derivation of the black-body spectrum and Einstein's introduction of the photons up to the results of Born, Heisenberg and Jordan on the quantization of a continuum - a comparative study is presented on the first lines of thoughts that led to the concept of quanta. The nature of the particle-like fluctuations and the wave-like fluctuations are analysed by using several approaches. With the help of classical probability theory, it is shown that the infinite divisibility of the Bose distribution leads to the new concept of classical "poissonian photo-multiplets" or to the "binary photo-multiplets" of fermionic character. As an application, Einstein's fluctuation formula is derived as a sum of fermion type fluctuations of the binary photo-multiplets.

Keywords: Black-body radiation; Einstein's fluctuation formula; wave-particle duality.

# Planck, Einstein, Ehrenfest és Poincaré

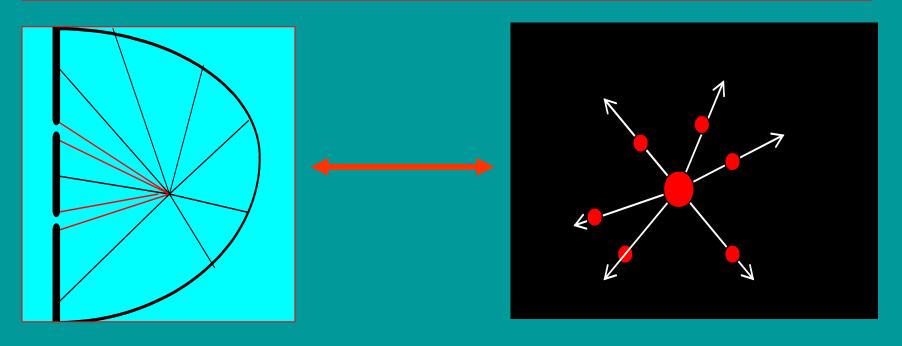
- (a) A v frekvenciájú oszcillátor lehetséges energiái 0, hv, 2hv,... Einstein: igen, Planck: igen
- (b) Ezek az energiaértékek egymástól független hv egységekből állnak össze Einstein: igen, Planck: nem tette fel ezt a levezetésnél
- (c) A fénykvantumok nemcsak mint az emisszió és abszorpció "atomjaiként" funkcionálnak, hanem léteznek az üres térben is Einstein: igen, Planck: nem
- 1911-ben Ehrenfest bebizonyítja hogy a kvantáltság szükséges és elégséges a Planck eloszlás érvényességéhez
- Poincaré, hazatérvén az I. Solvay Kongresszusról, más módszerrel szintén bebizonyítja ezt. 1912-ben publikálja. Ez győzi meg Jeans-t, Planck nagy ellenfelét a kvantumok jogos feltételézéről. Jeans által az angolszász kutatók Poincaré levezetését ismerik meg, Ehrenfest munkája továbbra is gyakorlatilag ismeretlen marad.
- A (b) ponthoz Ehrenfest: Ha az ɛ energiaelemek Planck-nál igazi részecskék lennének (ahogyan Einstein a fotonokat képzeli), akkor ezeknek nincs individualitásuk, és nem-klasszikus korrelációt mutatnak.

# A fotoelektron energiája, E = hv - A, nem függ a kiváltó fény erősségétől. Planck megjegyzése.



# Diffrakció, Kirchhoff-féle integrálegyenlet (+ Fénykvantumok ?)

$$\psi(\vec{x}) = -\frac{1}{4\pi} \iint_{S_1 + S_2} d\vec{f}' \cdot \left[ \vec{\nabla}' \psi + ik \left( 1 + \frac{i}{k \mid \vec{x} - \vec{x}' \mid} \right) \hat{R} \psi \right] \frac{e^{ik \mid \vec{x} - \vec{x}' \mid}}{\mid \vec{x} - \vec{x}' \mid}$$



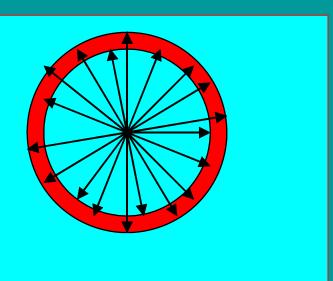
#### Megjegyzés a Planck-formula Bose-féle levezetéséhez

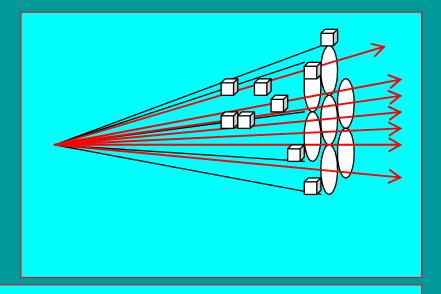
PHOTON ENERGY:  $E = h \nu$  FOTON MOMENTUM:  $\vec{p} = \frac{h \nu}{c} \vec{n}$ 

$$Z_{\nu}$$
 = NUMBER OF CELLS =

$$2 \cdot \frac{1}{h^3} \int \int dx dy dz dp_x dp_y dp_z = 2 \cdot \frac{1}{h^3} V p^2 dp \int d\Omega(\vec{n}) =$$

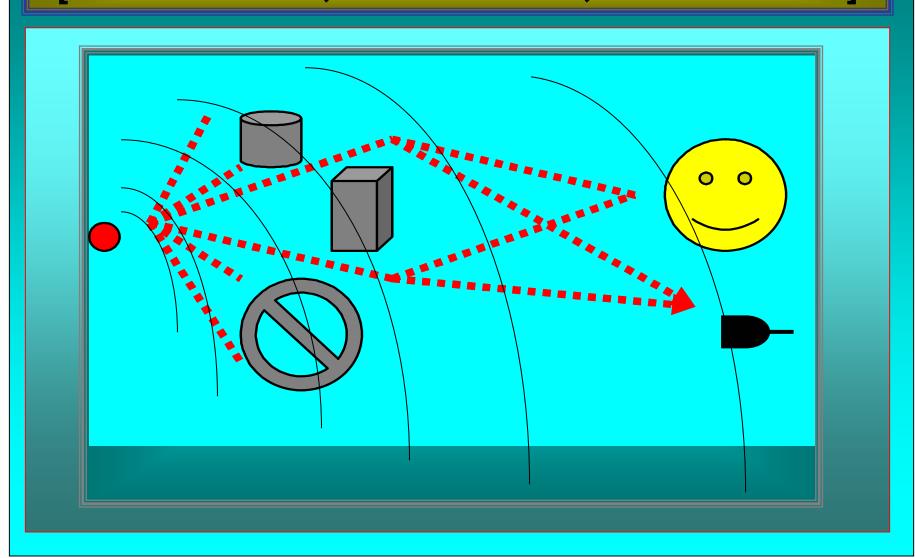
$$V \frac{8\pi v^2}{c^3} dv = M_v = \text{NUMBER OF MODES in } (v, v + dv)$$



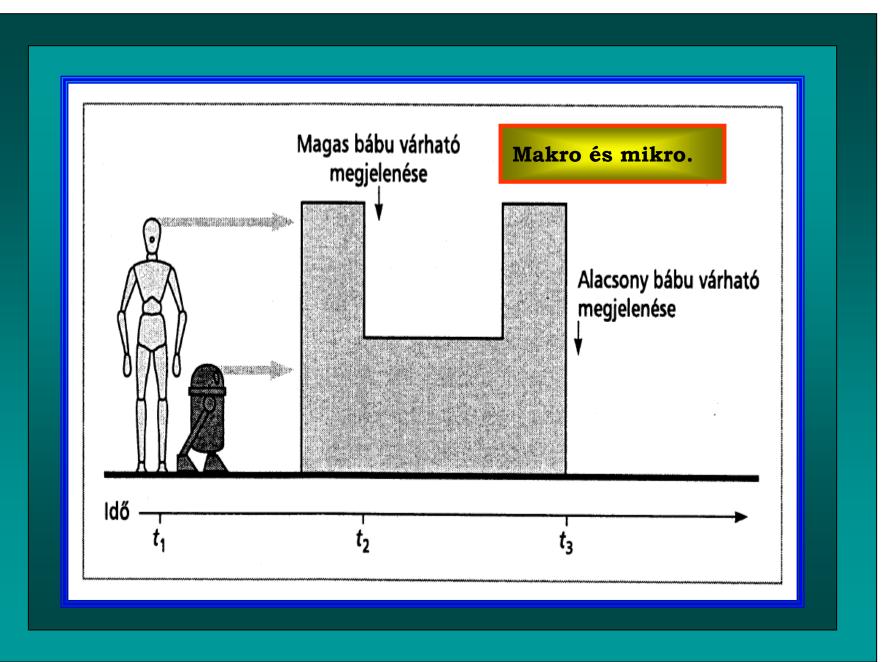


Mivel a teljes térszögre kiintegráltunk, egy energiaadagot két ellentétes irányban terjedő foton is kaphat. Nem véletlen tehát a pontszerű részecskék "misztikus" interferenciája, ugyanis ezek valójában egyáltalán nem lokálisak.

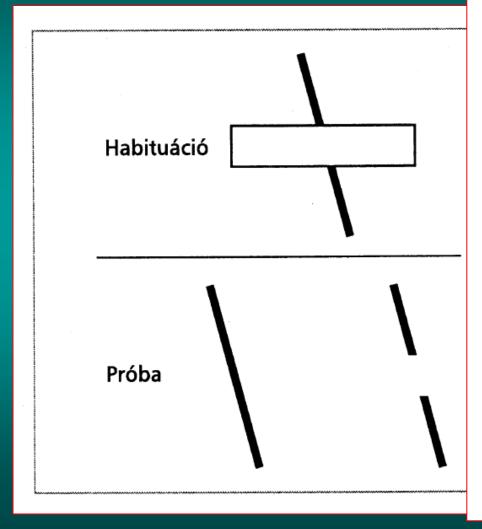
# "Gespensterfeld" [Bohr-Einstein viták] ["Ghost field", "Szellemtér", "Pilot wave"]

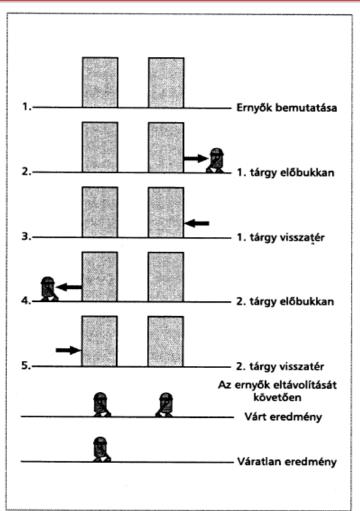


- "All diese 50 Jahre Brüten haben mich kein biβchen weiter gebracht zu der Frage was Lichtquanten sind. Heute glaubt jeder Lump, er wisse es – doch er irrt sich."
- "Teljes ötven év töprengése sem vitt egy kicsit sem közelebb a kérdés megválaszolásához: 'Mik a fénykvantumok?' Manapság minden gézengúz azt hiszi hogy tudja a választ, de téved."
- "A full fifty years of conscious brooding have not brought me any closer to the answer to the question: 'What is a light quantum?' Nowadays every Tom, Dick, and Harry thinks he knows it, but he deceives himself."



#### Koherencia, Korreláció II.





#### 5.11. ÁBRA

Kísérleti elrendezés annak kimutatására, hogy a csecsemők képesek a folytonosság-megszakítottság jelzéseit arra használni, hogy két különböző tárgy létezésére következtessenek **■** (Forrás: Spelke 1995)

## **PARADOXONOK**

Fizikai Szemle 1998/5. 149.0 SZÁZADVÉGI FELADAT: A KEZDET PARADOXONAINAK FELOLDÁSA Tisza László

az MIT emeritus professzora, az Eötvös Társulat tiszteleti tagja

..."Planck óvatos stratégiája alkalmas volt *h* értékének megállapítására, ám ahhoz, hogy magának a hatáskvantumnak jelentést lehessen tulajdonítani, egyenesen a kvantumjelenségekhez kellett fordulni. A kísérleti ismeretek akkori gyér voltát tekintve ez elképesztő feladat volt, melyet Einstein és Bohr tűzött ki. Ők elég bátrak voltak ahhoz, hogy a találgatásban vállalják a hibázás lehetőségét, s tudták, hogy hogyan leplezzék kétértelműséggel és paradoxonnal a tudatlanságot. Viszont azt is tudták, hogyan vonják ki a kétértelműségből az igazságot, és ez példaképpé tette őket: a bátorság és kétértelműség és a paradoxon már nincs helyén a részecskék leírásánál, miután azok rutinobjektumokká váltak laboratóriumok ezreiben. A "részecske" szó jelentésének kétértelműsége tette annyira csökönyössé az Einstein-Planck vitát is."...

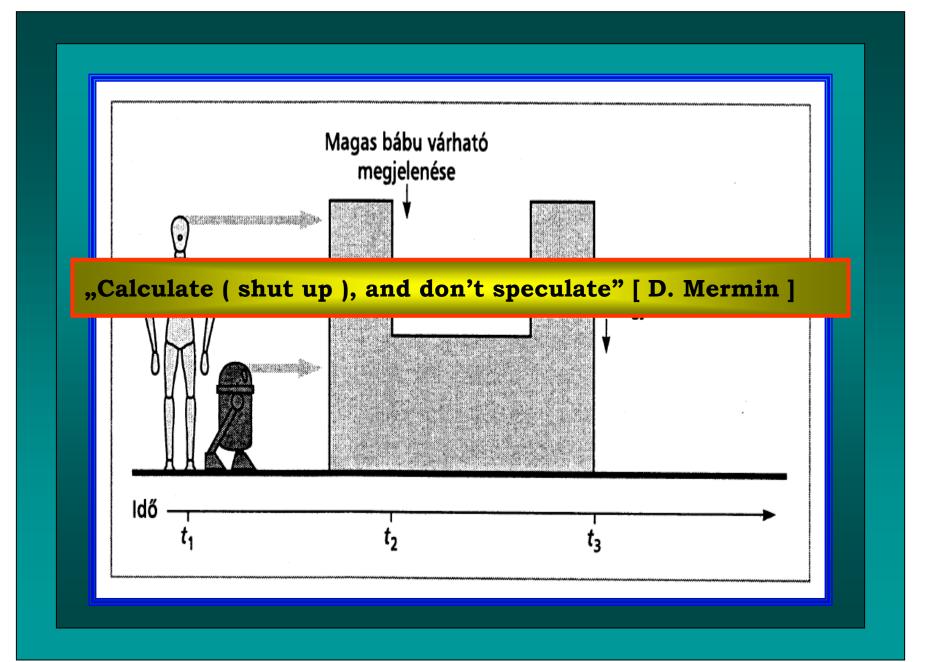
... "Sajnos, Einstein ezt [ mármint a 'Progresszív Egyesítést' ] nem tette meg mindig, 1905 márciusában írt egy cikket a fénykvantumról, ebben azt mondta, hogy a sugárzási rendszernek diszkrét struktúrával kell rendelkeznie. Ha itt is a Progresszív Egyesítést alkalmazta volna, akkor azt kellett volna mondania, hogy ha fenntartjuk a newtoni pontszerű részecske fogalmát, akkor baj lesz. Egy pont nem tud hullámozni. [ Ezt de Broglie is mindig hangsúlyozta. ] A hullámzás azt jelenti, hogy van egy véges koherenciatartomány. "...

"Lehet hogy nincs semmi gyakorlati jelentősége annak hogy tudjuk hogy a  $\pi$  transzcendens szám. Mégis, elviselhetetlen érzés lenne ha nem tudnánk."

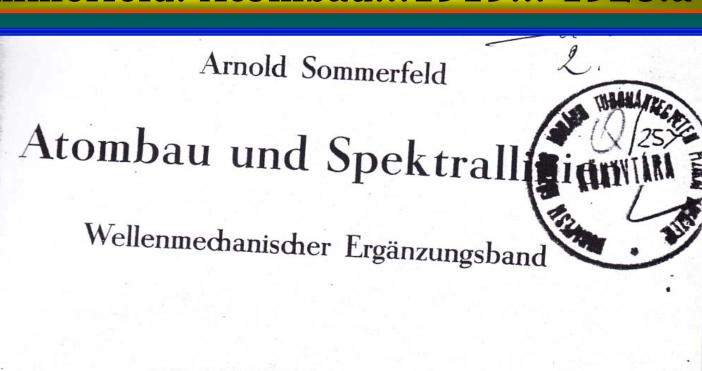
# Edward C. Titshmarsch

3.14159265358979323846264338327950288419716939937510 58209749445923078164062862089986280348253421170679 00056812714526356082778577134275778960917363717872 146844090122495343014654958537105079227968925 51870721134**999999**837297804995105973173281609631859 502445945534690830264252230825334468503526193<sup>-</sup> 18577805321712268066130019278766111959092164201989 3809525720106548586327886593615338182796823030<sup>-</sup>

82148086513282306647093844609550582231725359408128 48111745028410270193852110555964462294895493038196 44288109756659334461284756482337867831652712019091 45648566923460348610454326648213393607260249141273 72458700660631558817488152092096282925409171536436 78925903600113305305488204665213841469519415116094 33057270365759591953092186117381932611793105118548 07446237996274956735188575272489122793818301194912 98336733624406566430860213949463952247371907021798 60943702770539217176293176752384674818467669405132 42019956112129021960864034418159813629774771309960 51870721134**999999**837297804995105973173281609631859 71010003137838752886587533208381420617177669147303 59825349042875546873115956286388235378759375195778 03530185296899577362259941389124972177528347913151 55748572424541506959508295331168617278558890750983 81754637464939319255060400927701671139009848824012 85836160356370766010471018194295559619894676783744 94482553797747268471040475346462080466842590694912 93313677028989152104752162056966024058038150193511...



# Sommerfeld. Atombau...1919... 1928.a



A budapesti kir. m. tudomány-egyetem

A 643
elméleti physikai tanszergyűjtemény

Könyvtára.

#### A "KVANTUMOPTIKA" SZÓ ELSŐ MEGJELENÉSE

#### Zur Quantenoptik.

Von Gregor Wentzel in München.

(Eingegangen am 2. Februar 1924.)

Seit Einsteins Ableitung des Planckschen Strahlungsgesetzes pflegt man in der Quantenstatistik den Emissions- und Absorptionsvorgängen gewisse Wahrscheinlichkeiten zuzuschreiben, ohne aber über diese nähere Angaben zu machen. Wir wollen hier einen allgemeinen Angatz für gelehe Wahrscheinlichkeiten zuzuschlagen der gesiehen.

scl

sät Qu

w



WENTZEL [ 1924 ]: Teljesen korrekt pályaintegrálos megfogalmazás ( Feynman előtt 25 évvel ).

cine quantenmanige Dentung der weitentneoretischen Dichtphase.

 $\S$  1. Die Phase. Betrachten wir den Weg eines Lichtstrahls vom emittierenden Atomsystem E bis zum absorbierenden Atomsystem A. Für die Wellentheorie der Interferenz ist wesentlich die Phase:

$$\varphi = \int_{T}^{A} \frac{ds}{\lambda} = \frac{\nu}{c} \int_{T}^{A} n \, ds \tag{1}$$

 $(\nu = \text{Frequenz}, \lambda = \text{Wellenlänge}, n = \text{Brechungsindex}, ds = \text{Wegelement}).$  Wir behaupten, daß die Phase  $\varphi$  quantenmäßig als eine rein mechanische Größe gedeutet werden kann.

Als die wichtigste Grundlage der Quantentheorie darf man wohl den Satz ansprechen, daß ein atomares System nicht strahlen kann, solange es sich in mechanischen Zuständen befindet, d. h. daß Einund Ausstrahlung immer mit unmechanischen "Übergängen" verbunden sind. Aber nicht nur der Emissions- und Absorptionsakt wird unmechanisch sein; auch längs seines ganzen Weges wird das Licht in den Atomen des durchsetzten Mediums dauernd unmechanische



#### A kontinuum kvantálása: Born, Heisenberg és Jordan [1926]

Transverse oscillations in a plane of a string ( $c^2$ =tension/mass density):

$$\frac{1}{c^2} \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}, \quad y(0,t) = y(L,t) = 0$$

Bernoulli solution:

$$y(x,t) = \sum_{n=1}^{\infty} (\hbar/\omega_n)^{1/2} \hat{q}_n(t) f_n(x)$$

$$\hat{q}_n(t) = (\hat{a}_n e^{-i\omega_n t} + \hat{a}_n^+ e^{+i\omega_n t}) / 2^{1/2}$$

$$f_n(x) = (2/\sigma L)^{1/2} \sin k_n x$$

$$f_n(x) = (2/\sigma L)^{1/2} \sin k_n x$$

The average of a matrix is a diagonal matrix with the same diagonal elements as that of the original matrix.  $\rightarrow$  kinematic effect.

$$\overline{\Delta^{2}} \sim (h\nu)^{2} \overline{(\hat{a}_{\nu}^{+} \hat{a}_{\nu}^{-} + 1/2)(\hat{a}_{\nu'}^{+} \hat{a}_{\nu'}^{-} + 1/2)} - (h\nu/2)^{2}$$

$$\rightarrow h\nu \cdot \overline{E}_{\nu} + \overline{E}_{\nu}^{2}$$

#### A Kvantumelektrodinamika kialakulása

- Wentzel: Zur Quantenoptik; Pályaintegrál (1924) "The third Way of Quantum Mechanics is the forgotten First"
- Born, Heisenberg & Jordan ("drei Männer Arbeit", 1926)
- Dirac : Spontán emisszió (1927), Dirac-egyenlet (1928)
- Jordan & Wigner : Fermionok (1928)
- Heisenberg & Pauli : Kovariáns QED (1929)
- Heisenberg (1934), Dirac (1934), Weisskopf (1936) : Párkeltés, töltésfluktuációk
- Pauli (1940): Spin és Statisztika
- Lamb & Retherford, Bethe (1947), Kroll (1949)
- Tomonaga (1946-), Schwinger (1948-), Feynman (1949-),
   Dyson (1949-): Kovariáns QED, S-mátrix, korrekciók...
- Pauli & Villars (1949), Källen (1953): Regularizáció...

Selected Papers on

QUANTUM

ELECTRODYNAMICS

Edited by Julian Schwinger
Professor of Physics, Harvard University

# QED. Dirac [ 1927 ]...

34 selected papers by
Foley Kusch Fermi
Heisenberg Dyson We
Lamb Nordsieck
Oppenheimer Re
Källen Pauli Bethe
Klein Schwinger
Feynman Krol

The Quantum Theory of the Emission and Absorption of Radiation.

By P. A. M. DIRAC, St. John's College, Cambridge, and Institute for Theoretical Physics, Copenhagen.

(Communicated by N. Bohr, For. Mem. R.S.—Received February 2, 1927.)

#### § 1. Introduction and Summary.

The new quantum theory, based on the assumption that the dynamical variables do not obey the commutative law of multiplication, has by now been developed sufficiently to form a fairly complete theory of dynamics. One can treat mathematically the problem of any dynamical system composed of a number of particles with instantaneous forces acting between them, provided it is describable by a Hamiltonian function, and one can interpret the mathematics physically by a quite definite general method. On the other hand, hardly anything has been done up to the present on quantum electrodynamics. The questions of the correct treatment of a system in which the forces are propagated with the velocity of light instead of instantaneously, of the production of an electromagnetic field by a moving electron, and of the reaction of this field on the electron have not yet been touched. In addition, there is a serious difficulty in making the theory satisfy all the requirements of the restricted

#### A sugárzási tér kvantumelmélete. Fermi I.

JANUARY, 1932

REVIEWS OF MODERN PHYSICS

**VOLUME 4** 

#### **QUANTUM THEORY OF RADIATION\***

By Enrico Fermi University of Rome, Italy

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## Sommerfeld. Atombau...1919... 1928.c

- 1900 Aachen, Königliche Technische Hochschule
  Alkalmazott mechanika, Hidrodinamika (később dokt.: W.Heisenberg, L. Hopf)
- 1906 München, Institute für Theoretische Physik igazgató 1938-ig, nyugdíjba meneteléig.

Nobel-díjas doktoranduszok: W. Heisenberg, W. Pauli, P. Debye, H. Bethe.

Híres doktoranduszok: W. Heitler, R. Peierls, G. Wentzel, A. Landé, (L. Brillouin),

P. Ewald, H. Fröhlich, L. Hopf, A. Kratzer, O. Laporte, W. Lenz, K. Meissner,

Posztgraduális: Nobel-díjasok L. Pauling, I. Rabi,

W. Allis, E. Condon, E. Kemble, K. Herzfeld, P. Morse,

H. Robertson,

Habilitáció: W. Kossel, M. von Laue (Nobel-dij), W. Rubinowicz

- 1942-1951 Theoretische Physik I V.
  - Pl. 1919 ATOMBAU UND SPEKTRALLINIEN
    - + Wellentheoretische Ergänzungsband ("Bible of atomic theory")

Dr. Bethe:

Thank you very much Dr. Segrè. I think I can see from the applause that you all enjoyed the personal flavor of this talk as much as I did.

Segrè has mentioned the puzzle that was posed by the activities induced in uranium by neutrons, and you all know that this puzzle found its solution in the discovery of fission by Hahn and Strassmann in late 1938. You also know that the political situation which Segrè mentioned and which looked bad in 1935 became increasingly bad in the ensuing years; Italy came under the domination of Nazi Germany and Fermi, like Segrè

before him, decided to leave Italy for a more hospitable country. You know that Fermi received the Nobel prize of 1938 for the research in neutron physics which you have just heard, and you know that having received the Nobel prize in Sweden, he then took the wrong boat—instead of the boat to Italy he took that to America. We were most fortunate to have him come and work with us here in this country in early 1939, and much of history would have been different if he had not come. Just at the same time that Fermi came to this country, came the news of fission and this news led to very spectacular developments about which you will now hear from Dr. Zinn of the Argonne National Laboratory.

## A MÁSODIK VILÁGHÁBORÚ

REVIEWS OF MODERN PHYSICS

VOLUME 27, NUMBER 3

JULY, 1955

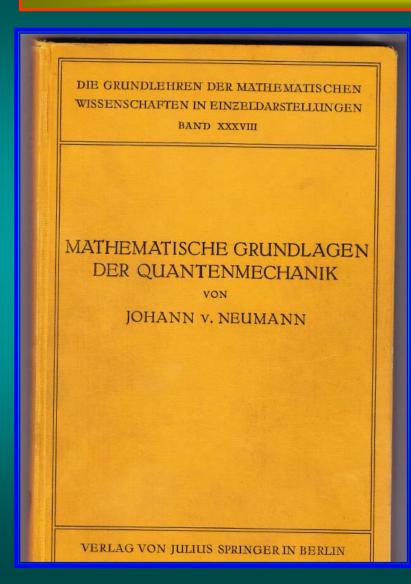
### Fermi and Atomic Energy

WALTER H. ZINN

Argonne National Laboratory, Lemont, Illinois

WE are assembled here today to honor the memory of a great scientist, and a cherished friend. This tribute would be paid to him even if nuclear physics had not brought about the discoveries and events of the was transferred entirely to the Los Alamos Laboratory in New Mexico. In July and August of 1945, the atomic bombs were detonated, thus bringing to a close Fermi's immediate participation in the development of weapons.

## Neumann János és a Hilbert-tér. 1932.



DIE GRUNDLEHREN DER

#### MATHEMATISCHEN WISSENSCHAFTEN

IN EINZELDARSTELLUNGEN MIT BESONDERER BERÜCKSICHTIGUNG DER ANWENDUNGSGEBIETE

GEMEINSAM MIT

W. BLASCHKE M. BORN HAMBURG

GÖTTINGEN

C. RUNGE† GÖTTINGEN

HERAUSGEGEBEN VON

R. COURANT

GÖTTINGEN

BAND XXXVIII

MATHEMATISCHE GRUNDLAGEN DER QUANTENMECHANIK

VON

JOHANN V. NEUMANN



BERLIN VERLAG VON JULIUS SPRINGER

# Neumann János és a Hilbert-tér. 1955a.

# MATHEMATICAL FOUNDATIONS OF QUANTUM MECHANICS

By JOHN VON NEUMANN

translated from the German edition by ROBERT T. BEYER





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### Neumann János és a Hilbert-tér. 1955b.

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#### TRANSLATOR'S PREFACE

This translation follows closely the text of the original German edition. The translated manuscript has been carefully revised by the author so that the ideas expressed in this volume are his rather than those of the translator, and any deviations from the original text are also due to the author.

The translator wishes to express his deep gratitude to Professor von Neumann for his very considerable efforts in the rendering of the ideas of the original volume into a translation which would convey the same meanings.

Robert T. Beyer

Providence, R. I. December, 1949

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### Neumann János és a Hilbert-tér. 1964a.

DIE GRUNDLEHREN
DER MATHEMATISCHEN WISSENSCHAFTEN
BAND XXXVIII

MATHEMATISCHE GRUNDLAGEN DER QUANTENMECHANIK

JOHANN V. NEUMANN

ИОГАНН фон НЕЙМАН

### МАТЕМАТИЧЕСКИЕ ОСНОВЫ КВАНТОВОЙ МЕХАНИКИ

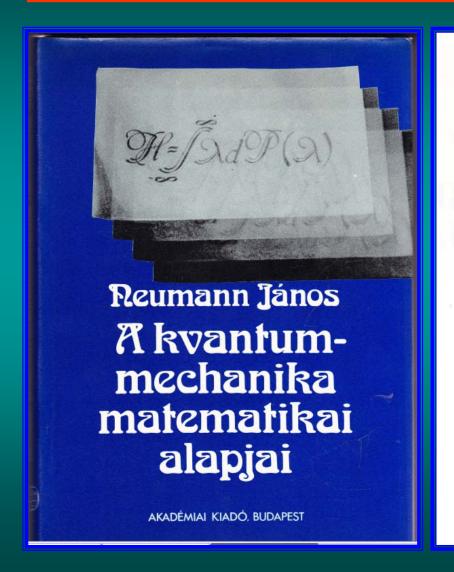
Перевод с немецкого М. К. ПОЛИВАНОВА и Б. М. СТЕПАНОВА

> Под редакцией акад. Н. Н. БОГОЛЮБОВА

BERLIN VERLAG VON JULIUS SPRINGER 1932

ИЗДАТЕЛЬСТВО «НАУКА»

### Neumann János és a Hilbert-tér. 1980a.



Az eredeti mű adatai

ИОГАНН ФОН НЕЙМАН
МАТЕМАТИЧЕСКИЕ ОСНОВЫ КВАНТОВОЙ МЕХАНИКИ
ИЗДАТЕЛЬСТВО «НАУКА»
МОСКВА 1964

Fordította SEBESTYÉN ÁKOS

a fizikai tudományok kandidátusa

Ellenőrző szerkesztő ABONYI IVÁN

a fizikai tudományok kandidátusa

### Kolmogorov [ 1933 ]a

### ERGEBNISSE DER MATHEMATIK UND IHRER GRENZGEBIETE

HERAUSGEGEBEN VON DER SCHRIFTLEITUNG DES "ZENTRALBLATT FOR MATHEMATIK"

#### GRUNDBEGRIFFE DER WAHRSCHEINLICHKEITS-RECHNUNG

A. KOLMOGOROFF





BERLIN VERLAG VON JULIUS SPRINGER

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### Kolmogorov [ 1956 ]

#### **FOUNDATIONS**

OF THE

#### THEORY OF PROBABILITY

A. N. KOLMOGOROV

Second English Edition

TRANSLATION EDITED BY NATHAN MORRISON

WITH AN ADDED BIBLIOGRAPHY BY
A. T. BHARUCHA-REID
UNIVERSITY OF OREGON

CHELSEA PUBLISHING COMPANY NEW YORK

1956

#### **EDITOR'S NOTE**

In the preparation of this English translation of Professor Kolmogorov's fundamental work, the original German monograph Grundbegriffe der Wahrscheinlichkeitrechnung which appeared in the Ergebnisse Der Mathematik in 1933, and also a Russian translation by G. M. Bavli published in 1936 have been used.

It is a pleasure to acknowledge the invaluable assistance of two friends and former colleagues, Mrs. Ida Rhodes and Mr. D. V. Varley, and also of my niece, Gizella Gross.

Thanks are also due to Mr. Roy Kuebler who made available for comparison purposes his independent English translation of the original German monograph.

Nathan Morrison

### DOOB: STOCHASTIC PROCESSES [1953]a

Stochastic Processes

> J. L. DOOB Professor of Mathematics University of Illinois

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### DOOB: STOCHASTIC PROCESSES [1953]b

Although it would be absurd to write a book on stochastic processes which does not assume a considerable background in probability on the part of the reader, there is unfortunately as yet no single text which can be used as a standard reference. To compensate somewhat for this

"Although it would be absurd to write a book on stochastic processes which does not assume a considerable background in probability on the part of the reader, there is unfortunately as yet no single text which can be used as a standard reference."

Probability is simply a branch of measure theory, with its own special emphasis and field of application, and no attempt has been made to sugar-coat this fact. Using various ingenious devices, one can drop

"There has been no compromise with the mathematics of probability.

Probability is simply a branch of measure theory, with its own special emphasis and field of application, and no attemp has been made to sugar-coat this fact."

law of large numbers) rigorously. However, such a treatment is no longer necessary and results in a spurious simplification of some parts of the subject, and a genuine distortion of all of it.

There is probably no mathematical subject which shares with probability the features that on the one hand many of its most elementary

### FELLER: "PROBABILITY THEORY" [1950-70]a

An Introduction to Probability Theory and Its Applications

WILLIAM FELLER (1906-1970)

Eugene Higgins Professor of Mathematics

Princeton University

Preface to the revised printing

VOLUME I

IN CONTRAST TO THE FIRST EDITION, THE THIRD WAS MARRED BY A disturbing number of errata. In the present revised printing all discovered errata are corrected. Moreover, some formulations have been improved and hints to problems added where this could be done without resetting type. I am grateful to my publisher for permitting these costly changes which should make for much better readability.

Almost all changes were suggested either by Professor R. E. Machol and Dr. J. Croft working together in Chicago. Ill., or else by Lt. Col. Preben Kühl (now retired) of the Royal Danish Army. They have read the book with unusual care and understanding, and I have greatly profited by the ensuing enjoyable correspondence.

THIRD EDIT

Revised Printing

Princeton, N.J. June, 1970

### GÁBOR D.: "FÉNY ÉS INFORMÁCIÓ" [1950-66]

It is of some interest to put the expression for the unit cell in a form in which it applies to all kinds of particles, to electrons for instance as well as to photons. The unit cell for light is defined by

$$[2] \cdot 2 \frac{\mathrm{d}S}{\lambda^2} \,\mathrm{d}\Omega \,\mathrm{d}\nu \,\mathrm{d}t = 1. \tag{1}$$

The first factor [2] relates to the polarization, this is peculiar to light. The second 2 stands for the "spatial phase", i.e. for the fact that for every spatial period one can distinguish a sine and a cosine component. We have dropped the factor (2), considering the "time phase" as unobservable.

We now put this into a more general form by making use of Einstein's equation

$$E = hv$$
 (2)

where E is the energy of the particle, and of de Broglie's relation

$$p = \frac{h}{\lambda}$$
 (3)

where p is its momentum. This gives the general definition of the cell

$$[2] \frac{2p^2}{h^3} dS d\Omega dE dt = 1.$$
 (4)

In the case of light p = hr/c = E/c, and we can write (4) in the form

$$\frac{4}{h^3c^2} dS d\Omega E^2 dE dt = 1$$
 (5)

while in the case of slow electrons  $p^2 = (mv)^2 = 2mE$ , and the cell is

$$\frac{4m}{h^3} dS d\Omega E dE dt = 1.$$
 (6)

There is a fundamental difference between light optics and electron optics, where by Pauli's exclusion principle the maximum occupation of a cell is one or two. It will be shown in a moment that two antiparallel electrons in a cell in a free beam is an extremely unlikely case; it is questionable whether it can occur at all. Two electrons in a cell with opposite spins could not be distinguished from a particle

IV

#### LIGHT AND INFORMATION †

BY

D. GABOR

Imperial College, London

is the substance of a Ritchie lecture, delivered by the author 51 at the University of Edinburgh. The contents of the lecture to a wider audience through the distribution of a limited number d notes, which have since become widely quoted in the literature. en often expressed that a permanent record of the lecture should lly available. We are glad to be able to meet this wish.

### GÁBOR D.: FIZIKAI INFORMÁCIÓELMÉLET [1950]

CIII. Communication Theory and Physics.

By D. Gabor, Imperial College, London\*.

[Received August 10, 1950.]

#### SUMMARY.

The electromagnetic signals used in communication are subject to the general laws of radiation. One obtains a complete representation of a signal by dividing the time-frequency plane into cells of unit area and associating with every cell a "ladder" of distinguishable steps in signal intensity. The steps are determined by Einstein's law of energy fluctuation, involving both waves and photons.

This representation, however, gives only one datum per cell, viz., the energy, while in the classical description one has two data: an amplitude and a phase. It is shown in the second part of the paper that both descriptions are practically equivalent in the long-wave region, or for strong signals, as they contain approximately the same number of independent, distinguishable data, but the classical description is always a little less complete than the quantum description. In the best possible experimental analysis by an electronic device the number of distinguishable steps in the measurement of amplitude and phase is only the fourth root of the number of photons. Thus it takes a hundred million photons per cell in order to define amplitude and phase to one per cent each.

#### RICE, DAVENPORT & ROOT, WOODWARD [1940-50...]

PROBABILITY AND INFORMATION
THEORY, WITH APPLICATIONS
TO RADAR

VÉLETLEN JELEK ÉS ZAJ.
Woodward: idő-frekvencia
"Wigner-függvény", vagyis
Woodward-függvény.

By

P. M. WOODWARD, B.A.

Principal Scientific Officer, Telecommuni Research Establishment, Ministry of S

#### AUTHOR'S PREFACE

The first two chapters of this short monograph are concerned with established mathematical techniques rather than with fresh ideas. They provide the code in which so much of the mathematical theory of electronics and radar is nowadays expressed. Information theory is the latest extension of this code, and I hope that it will not be considered improper that I have tried in Chapter 3 to summarise so much of C. E. Shannon's original work, which already exists in book-form (*The Mathematical Theory of Communication*, by Claude Shannon and Warren Weaver). The account which is given in Chapter 3 may perhaps spur the reader who has not studied the original literature into doing so.

Chapters 4 and 5 deal with some of the fascinating problems, which have been discussed so often in recent years, of detecting signals in noise. The present approach was suggested to me by Shannon's work on communication theory and is based on inverse probability; it is my opinion that of all statistical methods, this one comes closest to expressing intuitive notions in the precise language

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#### "JELANALIZIS FORRADALOM" [ ~ 1980...90...]a

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 36, NO. 5, SEPTEMBER 1990

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# The Wavelet Transform, Time-Frequency Localization and Signal Analysis

INGRID DAUBECHIES, MEMBER, IEEE

Abstract —Two different procedures are studied by which a frequency analysis of a time-dependent signal can be effected, locally in time. The first procedure is the short-time or windowed Fourier transform, the second is the "wavelet transform," in which high frequency components are studied with sharper time resolution than low frequency components. The similarities and the differences between these two methods are discussed. For both schemes a detailed study is made of the reconstruction method and its stability, as a function of the chosen time-frequency density. Finally the notion of "time-frequency localization" is made precise, within this framework, by two localization theorems.

#### I. Introduction

A. The Windowed Fourier Transform and Coherent States

IN SIGNAL ANALYSIS one often encounters the socalled short-time Fourier transform, or windowed tion (see [2], [3]; we shall come back to this in Section II-C-1). The Gabor functions have been used in many different settings in signal analysis, either in discrete lattices (with  $\omega_0 \cdot t_0 < 2\pi$  for stable reconstruction) or in the continuous form described next. In many of these applications their usefulness stems from their time-frequency localization properties (see e.g., [4]).

Whatever the choice for g (Gaussian, supported on an interval, etc.), it is interesting to know to which extent the coefficients  $c_{mn}(f)$  of (1.1) define the function f. This is one of the main issues of this paper.

The coefficients  $c_{mn}(f)$  in (1.1) can also be viewed as inner products of the signal f to be analyzed with a discrete lattice of coherent states. Let us clarify this statement. By "coherent states" we understand here the

#### "JELANALIZIS FORRADALOM" [ ~ 1980...90...]b

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Communications in Mathematical Physics

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# A Block Spin Construction of Ondelettes\* Part II: The QFT Connection

Guy Battle\*\*

Mathematics Department, Cornell University, Ithaca, NY 14853, USA

Abstract. We apply the Lemarié basis of ondelettes to the Battle-Federbush cluster expansion for the  $\phi_3^4$  quantum field theory. Since there is no infrared problem for this model, we also show how the large-scale ondelettes can be thrown away and replaced by unit-scale functions. Finally, we apply the block spin machine of Part I to the construction of exponentially localized ondelettes orthogonal with respect to the free, massless action of the scalar field.

# RÉNYI ALFRÉD: VALÓSZINŰSÉGSZÁMITÁS. "Bell-egyenlőtlenségek".a

[A "Pitowsky-tétel" (1989) Rényi könyvének egyik feledataként (1962), 12. §. 1. feladat.]

#### Rénvi Alfréd

akadémikus, egyetemi tanár

### VALÓSZÍNŰSÉG-SZÁMÍTÁS

az AB, ill. a B halmaz elemei számának a hányadosával egyenlő.\*

Természetesen felvetődik a kérdés, hogy a P(A|B) feltételes valószínűségek kapcsolatban vannak-e a relatív gyakoriságokkal, tehát hogy az általánosított elméletnek van-e gyakorisági interpretációja.

A válasz erre a kérdésre igenlő és igen egyszerű. A P(A|B) feltételes valószínűség az általánosított elméletben is (éppúgy, mint a Kolmogorov-elméletben) úgy interpretálható, mint az a szám, amely körül A-nak a B feltétel melletti feltételes relatív gyakorisága ingadozik. Tehát az általánosított elméletnek a valósághoz való viszonya elvileg ugyanaz, mint a Kolmogorov-féle elméleté.

#### 12. §. FELADATOK

1. Bizonyítsuk be, hogy ahhoz, hogy adott  $p_1$ ,  $p_2$ ,  $p_{12}$  valós számokhoz található legyen olyan A és B esemény, amelyekre  $P(A)=p_1$ ,  $P(B)=p_2$ ,  $P(AB)=p_{12}$ , a következő négy egyenlőtlenség fennállása szükséges és elégséges:

$$(1) 1 - p_1 - p_2 + p_{12} \ge 0,$$

$$(2) p_1 - p_{12} \ge 0,$$

$$(3) p_2 - p_{12} \ge 0,$$

$$(4) p_{12} \geq 0.$$

ÚTMUTATÁS. Az (1)—(4) egyenlőtlenségek jobb oldalán  $\bar{A}\bar{B}$ ,  $\bar{A}\bar{B}$ ,  $\bar{A}\bar{B}$  és  $\bar{A}\bar{B}$  valószínűsége áll; ezeknek természetesen nemnegatívaknak kell lenniük, tehát a feltételek szükségesek. Elégségességük

# RÉNYI ALFRÉD: VALÓSZINŰSÉGSZÁMITÁS. "Bell-egyenlőtlenségek". b

12. §.]

Feladatok

75

a következőképpen látható be: (1)-(4)-ből nyilván

 $0 \leq p_{12} \leq p_1 \leq p_1 + p_2 - p_{12} \leq 1,$ 

és hasonlóképpen

 $0 \le p_{12} \le p_2 \le p_1 + p_2 - p_{12} \le 1$ .

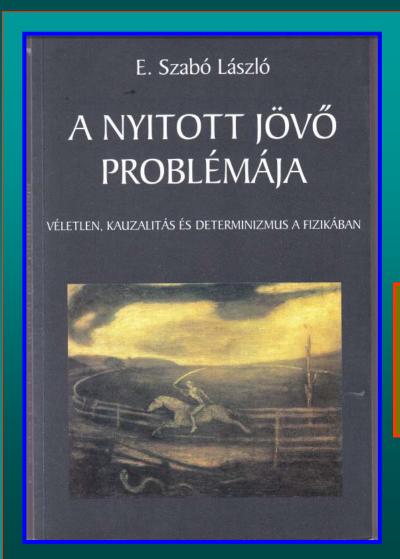
A p<sub>1</sub>, p<sub>2</sub>, p<sub>12</sub> számok tehát nemnegatívak és egynél nem nagyobbak.

Legyen egy P véletlenszerűen választott pont az I=(0,1) intervallumban egyenletes eloszlású, azaz annak valószínűsége, hogy P az I valamely részintervallumába esik, legyen egyenlő az illető intervallum hosszával. Legyen A az az esemény, hogy a pont a  $0 < x < p_1$  intervallumba esik, B pedig az, hogy a  $p_1 - p_{12} < x < p_1 + p_2 - p_{12}$  intervallumba esik. Ekkor nyilvánvalóan  $P(A) = p_1$ ,  $P(B) = p_2$ ,  $P(AB) = p_{12}$ .

- 2. Általánosítsuk az 1. feladat állítását n eseményre (n=3, 4, ...).
- 3. Vizsgáljuk meg, hogyan egyszerűsíthetők a 2. feladat feltételei, ha feltesszük, hogy  $p_{i_1 i_2 \dots i_k} = \mathbf{P}(A_{i_1} A_{i_2} \dots A_{i_k})$   $(1 \le i_1 < i_2 < \dots < i_k \le n)$  csak k-tól függ  $(k=1, 2, \dots, n-1)$ .
- 4. Hogyan egyszerűsíthetők a 2. feladat feltevései, ha feltesszük, hogy minden  $i_1, i_2, ..., i_k$ -ra (k=2, 3, ..., n)

$$p_{i_1 i_2 \dots i_k} = p_{i_1} p_{i_2} \dots p_{i_k}$$
?

### KLASSZIKUS VALÓSZINŰSÉGSZÁMITÁS. II. "Egyenlőtlenségek", "Kolmogorovi cenzúra".



4.2. A Pitowsky-tétel 71

Legyen  $\{E_i\}_{i=1,2,...,n}$  egy egységpartíció, tehát

$$\bigvee_{i=1}^{n} E_{i} = 1, \quad (\forall i \neq j) [E_{i} \wedge E_{j} = \emptyset]$$

Ekkor tetszőleges A ∈ Σ-ra fennáll, hogy

$$p(A) = \sum_{i=1}^{n} p(A|E_i) p(E_i) \qquad (4.9)$$

#### 4.2. A Pitowsky-tétel

63. Halasszuk el továbbra is annak tisztázását, hogy mit jelent pontosan egy esemény valószínűsége, vagyis, hogy miként rendelünk eseményekhez valószínűségnek nevezett számokat. Mindeddig úgy tűnik, hogy ez a hozzárendelés tetszőleges lehet, vagyis az eseményekhez tetszőleges, nulla és egy közé eső számokat rendelhetünk. Most azt fogjuk megvizsgálni, hogy milyen feltételeket kell ezeknek a számoknak kielégíteniük ahhoz, hogy a szóban forgó események és a hozzájuk rendelt "valószínűségek" reprezen-

"A kérdés tehát az, hogy milyen feltételek mellett létezik ilyen reprezentáció [ mármint mikor van a korrelációs mérés eredményének esemény háttere, á la Boole ]. Érdekes, hogy ezzel a kézenfekvő problémával egészen a nyolcvanas évek közepéig nem foglalkoztak."

> modell, s benne az eseményalgebrának olyan  $X_1, X_2, ..., X_n \in \Sigma$  elemei, melyekre fennáll, hogy

$$p_i = \mu(X_i)$$
  $i = 1, 2, ..., n$   
 $p_{ij} = \mu(X_i \land X_j)$   $(i, j) \in S$  (4.11)

# Fortschritte der Physik

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# Progress of Physics

#### Correlations in single-photon experiments

#### Sándor Varró

Research Institute for Solid State Physics and Optics, PO Box 49, 525 Budapest, Hungary

Received 6 September 2007, accepted 8 September 2007 Published online 21 December 2007

Key words Single-photon experiments, correlations, bunching, anti-bunching PACS 42.50 Ar, 42.50 Ct, 42.50 Dv, 42.50 Xa

Correlations of detection events in two photodetectors placed at the opposite sides of a beam splitter are studied in the frame of classical probability theory. It is assumed that there is always only one photon present

### The role of self-coherence in correlations of bosons and fermions in linear counting experiments.

Notes on the wave-particle duality

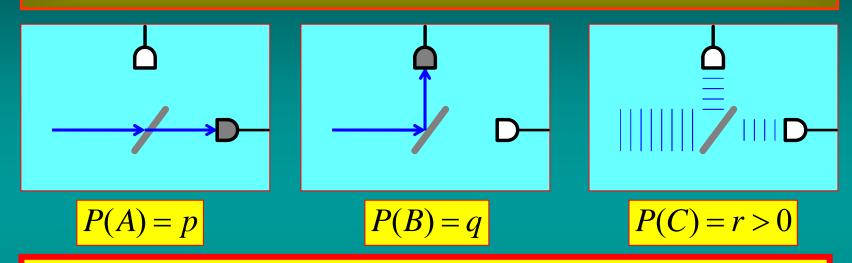
#### Sándor Varró

Research Institute for Solid State Physics and Optics of the Hungarian Academy of Sciences H-1525 Budapest, P. O. Box 49, Hungary, E-mail: varro@mail.kfki.hu

Abstract. Correlations of detection events in two detectors are studied in case of single-quantum excitations of the measuring apparatus. On the basis of classical probability theory and fundamental conservation laws, a general formula is derived for the two-point correlation functions for both bosons and fermions. The results obtained coincide with that derivable from quantum theory which uses quantized field amplitudes. By applying both the particle and the wave picture at the same time, the phenomena of photon bunching and antibunching, photon anticorrelation and fermion antibunching measured in beam experiments are interpreted in the frame of an intuitively clear description.

**Keywords:** Hanbury Brown and Twiss effect, photon bunching, fermion antibunching, Photon anticorrelation, wave-particle duality.

### ELEMI EGYKVANTUMOS MÉRÉSI AKTUS; HÁRMAS [TERNÁRIS] KIMENETEL



A, B és C egymást kölcsönösen kizárják és egy teljes eseményrendszert alkotnak. Ugyanakkor nem függetlenek.

$$A \cap B = O \mid B \cap C = O \mid A \cap C = O$$

$$A \cup B \cup C = I$$

$$P(A \cap B) = 0 \neq P(A) \cdot P(B)$$
  $P(A) + P(B) + P(C) = p + q + r = 1$ 

[ S. V.: Correlation in single-photon experiments. Fortschritte der Physik 56, 91-102 (2008) ]

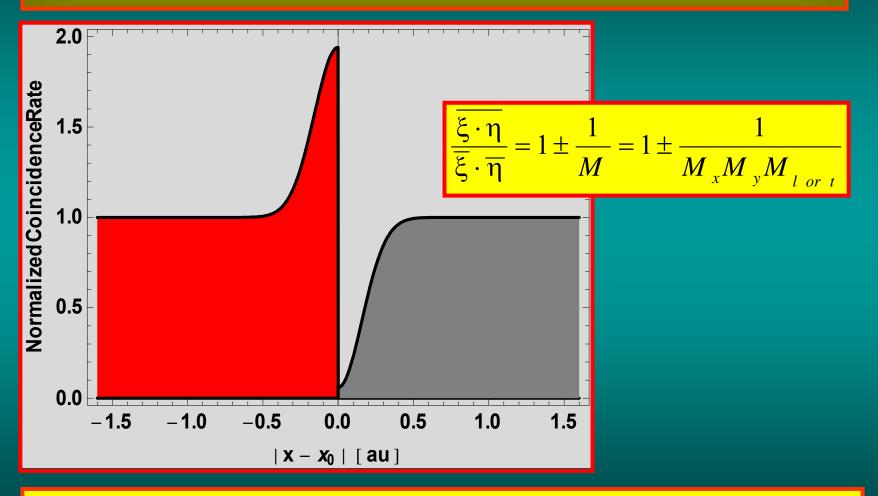
### N-SZEKVENCIÁK, SOROZATOK, MÓDUSOK

$$w_{mk}(n) \equiv P(\xi_n = m, \eta_n = k) = \frac{n!}{m! k! (n - m - k)!} p^m q^k r^{n - m - k}$$

$$P(\xi = m, \eta = k) = \sum_{n=0}^{\infty} W_n P(\xi_n = m, \eta_n = k), \quad with \quad \sum_{n=0}^{\infty} W_n = 1$$

$$W_n \equiv W_n(M) = W_n[M(R_1, T_1; R_2, T_2 \mid BS, D)]$$

### BOZON-KORRELÁCIÓK ÉS FERMION-KORRELÁCIÓK ÖSSZEHASONLITÁSA



S. Varró: Correlations in single-quantum experiments. A note on wave-particle duality. 40th Physics of Quantum Electronics, 2010, 3-7 January, Snowbird (Utah) USA

# "CLASSICAL" INTERPRETATION OF THE EXPERIMENTS OF ASPECT AND COWORKERS

#### SPACE FOR TABLE 1.

Reflected singles n <sub>2r</sub>	Transmitted singles n <sub>2t</sub>	Expected(1) coincidences	Expected(2) coincidences	Measured coincidences	Calculated(1) coincidences	Calculated(2) coincidences
2940	3876	25.5 <sup>#</sup>	2*	6	3 <sup>#</sup> (0.24)*	3 <sup>#</sup> (0.25)*
78260	95840	50.8	49	9	11	11
91908	124912	64.1	64	23	19	20
241920	326400	204	202	86	86	88
409200	535920	456	455	273	273	273
399840	519960	492	492	314	341	337
257400	344880	367	367	291	282	275

Table 1. Gives a comparison of the experimental data of Aspect and Grangier [58] on single-photon anti-correlation with the theoretical results quoted by the authors and with that of the present work. The seven raws correspond to the total number of coincidence gates  $n_g = N_1 T = 10^3 \times \{5664, 152564, 179080, 391680, 481800, 422520, 241560\}$  during which the number of counts were registered. We have calculated these numbers on the basis of the experimental data given by the authors, namely, we have taken for the trigger rates  $N_1 = \{4720, 8870, 12100, 20400, 36500, 50300, 67100\}$  sec<sup>-1</sup>, and for the gate durations  $T = \{1200, 17200, 14800, 19200, 13200, 8400, 3600\}$  sec, as have been given by the authors in the first and fourth columns in Table 3 in their paper. In the present table, in the first and the second columns, the calculated number of reflected photons,  $n_{2r} = N_{2r}T$ , and the calculated number of transmitted photons,  $n_{2t} = N_{2t}T$ , are shown, respectively. The numerical values of the fluxes  $N_1$ ,  $N_{2r}$ ,  $N_{2t}$  and the durations of the data acquisition T have been taken from Table 3 of the original reference [58]. In the third column (with heading "Expected(1) coincidences") the calculated number of accidental coincidences  $N_{2r}N_{2t}T/N_1$  are shown, as has been given in Ref [58]. These would be

### The experiment of Aspect et al.(1986)

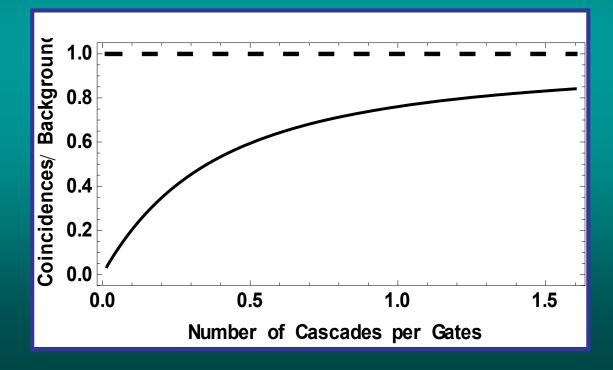
$$W_n(M) = \binom{M}{n} (\overline{n})^n (1 - \overline{n})^{M-n} \qquad \boxed{0 < \overline{n} \le 1}$$

$$K = \frac{\overline{\xi \cdot \eta}}{\overline{\xi} \cdot \overline{\eta}} = 1 - \frac{1}{M}$$

$$M = \left[ \frac{1}{x} - \frac{1}{2} \left( \frac{1}{x} \right)^2 (1 - e^{-2x}) \right]^{-1}$$

$$x \equiv \frac{2T}{\tau_c} \times \delta = \frac{4T}{\tau_s} \times \delta$$

$$\delta = Nw \times (1 - e^{-w/\tau_s})$$



 $\langle I(t)I(t+ au) 
angle$ 

### Korreláció ~ Fluktuáció

#### INTENZITÁS KORRELÁCÓ ~ ENERGIA FLUKTUÁCIÓ

$$g^{(2,2)}(\tau) = \frac{\left\langle E^{(-)}(t)E^{(-)}(t+\tau)E^{(+)}(t+\tau)E^{(+)}(t)\right\rangle}{\left\langle E^{(-)}(t)E^{(+)}(t)\right\rangle^{2}} \to$$

$$g^{(2,2)}(0) = \frac{\langle \hat{a}^{+} \hat{a}^{+} \hat{a} \hat{a} \rangle}{\langle \hat{a}^{+} \hat{a} \rangle^{2}} = 1 + \frac{(\Delta n)^{2} - \overline{n}}{(\overline{n})^{2}} = 1 + \frac{(\Delta E_{1})^{2} - h \nu \overline{E}_{1}}{(\overline{E}_{1})^{2}}$$

$$g_{cl}^{(2,2)}(0) = 1 + \frac{(\Delta E_1)^2}{(\overline{E}_1)^2}$$

KOHERENS ÁLLAPOT

$$(\Delta n)^2 = n$$

$$(\Delta n)^2 = \overline{n}$$
  $g^{(2,2)}(0) = 1 = g_{cl}^{(2,2)}(0)$ 

$$(\Delta n)^2 = (\overline{n})^2 + \overline{n}$$

$$(\Delta n)^2 = (\bar{n})^2 + \bar{n}$$
  $g^{(2,2)}(0) = 2 = g_{cl}^{(2,2)}(0)$ 

# "SINGLE-PHOTON WAVEFRONT-SPLITTING" [ Jacques et al. 2005 ]

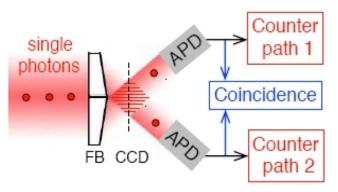


Fig. 1. Wavefront-splitting set-up based on a Fresnel's (FB). APDs are avalanche silicon photodiodes opera photon counting regime. An intensified CCD camera line) records interference fringes in the overlapping rethe two deviated wavefronts. When the CCD is remove then possible to demonstrate the single photon behave recording the time coincidences events between the two channels of the interferometer.

(a)	) Faint	laser	pu	lses
-----	---------	-------	----	------

Counting time (s)	$N_1$	$N_2$	$N_C$	$\alpha$
4.780	49448	50552	269	1.180
4.891	49451	50449	212	0.937
4.823	49204	50796	211	0.934
4.869	49489	50511	196	0.875
4.799	49377	50623	223	0.981
4.846	49211	50789	221	0.982
4.797	49042	50958	232	1.021
4.735	49492	50508	248	1.077
4.790	49505	50495	248	1.090
4.826	49229	50771	219	0.970

#### (b) Single-photon pulses

Counting time (s)	$N_1$	$N_2$	$N_C$	$\alpha$
5.138	49135	50865	28	0.132
5.190	49041	50959	23	0.109
5.166	49097	50903	23	0.109
5.173	49007	50996	28	0.133
5.166	48783	51217	29	0.137
5.167	48951	51049	31	0.147
5.169	49156	50844	30	0.142
5.204	49149	50851	32	0.152
5.179	49023	50977	26	0.124
5.170	48783	51217	26	0.123

# "SINGLE-PHOTON WAVEFRONT-SPLITTING" [ Jacques et al. 2005 ]

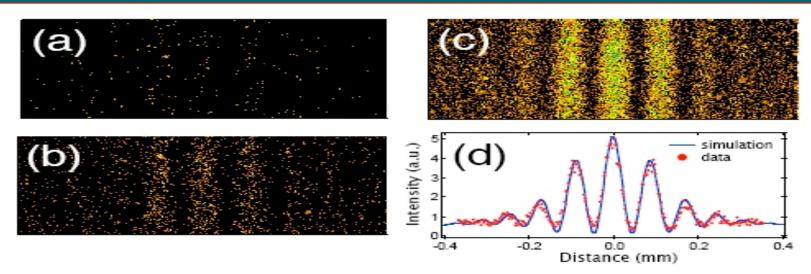


Fig. 4. Observation of the interference pattern expanded by the eyepiece and recorded by the intensified CCD camera. Image (a) (resp. (b) and (c)) is made of 272 photocounts (resp. 2240 and 19773) corresponding to an exposure duration of 20 s (resp. 200 s and 2000 s). Graph (d) displays the resulting interference fringes obtained by binning columns of CCD image (c) and fit of this interference pattern using coherent beam propagation in the Fresnel diffraction regime, and taking into account the finite temporal coherence due to the broad spectral emission of the NV colour centre. A visibility of 94% can be associated to the central fringe.

## BOZON- és FERMION-KORRELÁCIÓK 1995- ... 'HBT-RENESZÁNSZ'

- **ELEKTRONOK FÉLVEZETŐKBEN**
- RÖNTGENNYALÁBOK △I △I KORRELÁCIÓJA
- **'GHOST IMAGING'**
- **BOSE KONDENZÁTUMOK**
- NEUTRONOK

### A TEKINTÉLY SZEREPE A FIZIKÁBAN I.

### I. INTERNATIONAL CONFERENCE ON THE PHYSICAL INTERPRETATION OF RELATIVITY THEORY (1990, Imperial College of London)

... "Contributors should note that the starting point of the conference programme is the acceptance of the accuracy and excellence of Relativity Theory, so that questions raised are directed towards examining the philosophical, historical, and methodological aspects of the formal structure (mathematical theory), and the implications which these several interpretations have for the physical theories, listed under the specialist sections. Therefore polemical 'anti-Einstein' and 'anti-Relativity' papers will not be accepted for inclusion in the programme."...

### III. INTERNATIONAL CONFERENCE ON THE PHYSICAL INTERPRETATION OF RELATIVITY THEORY (1994, Imperial College of London)

... "So, I am authorized to invite you formally to participate in this conference. But we have a problem: outspoken opposition to the establishment is not welcome. However an intelligent critisism presented in moderate terms will be tolerated – and if you can promise that the <u>style</u> of your presentation will not be offensive to the orthodox, I can promise you that you will not be alone with your heresies! Is this acceptable to you?"...

[ G. Galetzky und P. Maquardt: Requiem für die Spezielle Relativität (Haag und Herchen Verlag Gmbh, Franfurt am Main, 1997), p. 19. ]

### A TEKINTÉLY SZEREPE A FIZIKÁBAN II.

Studies in History and Philosophy of Modern Physics 40 (2009) 280-289



Contents lists available at ScienceDirect

### Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb



### Quantum dissidents: Research on the foundations of quantum theory circa 1970

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Universidade Federal da Bahia, Instituto de Fisica, Campus de Ondina, 40210340 Salvador, Bahia, Brazil

#### ARTICLE INFO

Article history: Received 19 February 2009 Received in revised form 5 September 2009

Keywords: History of quantum mechanics Quantum dissidents Scientific controversies Zeh

#### ABSTRACT

This paper makes a collective biographical profile of a sample of physicists who were protagonists in the research on the foundations of quantum physics circa 1970. We study the cases of Zeh, Bell, Clauser, Shimony, Wigner, Rosenfeld, d'Espagnat, Selleri, and DeWitt, analyzing their training and early career, achievements, qualms with quantum mechanics, motivations for such research, professional obstacles, attitude towards the Copenhagen interpretation, and success and failures. Except for Rosenfeld, they were all dissidents, fighting against the dominant attitude among physicists at the time according to which foundational issues had already been solved by the founding fathers of the discipline. Theirs is a story of success as the foundations of quantum mechanics finally entered the physics mainstream despite the fact that their expectations of breaking down quantum mechanics were not fulfilled.

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### A TEKINTÉLY SZEREPE A FIZIKÁBAN III.

<sup>&</sup>lt;sup>2</sup> H. D. Zeh, 2008, interview with Fábio Freitas.

<sup>&</sup>lt;sup>3</sup> Ich mach es zu einer Lebensregel, so weit vermeidlich auf keinen Zeh zu treten, aber der Empfang eines von einem gewissen Dr. Zeh aus Ihrem Institut verfassten preprint veranlasst mich von dieser Regel abzuweichen. Ich habe allen Grund anzunehmen, dass ein solches Konzentrat wildesten Unsinnes nicht mit Ihrem Segen in die Welt verbreitet ist, und ich glaube Ihnen von Dienst zu sein, indem ich Ihre Aufmerksamkeit auf dieses Unglück richte."L. Rosenfeld to J. H. D. Jensen, 14 February 1968. Next Jensen tried to attenuate Rosenfeld's reaction, while fearing for its consequences: "I hope, that he does not quite have his reputation ruined," Jensen to Rosenfeld, 1 March 1968. Rosenfeld then considered Zeh's case in a "somewhat favorable light" though still considering Zeh's paper "more like a possession claim of a monopoly of highest wisdom" than "as an invitation to a factual discussion," Rosenfeld to Jensen, 6 March 1968. The affair occupied three more letters between Rosenfeld and Jensen: Jensen to Rosenfeld, 10 April 1968; 9 May 1968; Rosenfeld to Jensen, 25 April 1968. Rosenfeld Papers, Niels Bohr Archive, Copenhagen. I am indebted to Anja Jacobsen and Felicity Pors for recovering these letters and Christian Joas for the German translation.

<sup>&</sup>lt;sup>4</sup> H. D. Zeh, 2008, interview.

<sup>&</sup>lt;sup>5</sup> About Zeh's ulterior reflections on quantum mechanics, see (Camilleri, 2009).

<sup>&</sup>lt;sup>6</sup> H. D. Zeh to J. A. Wheeler, 30 October 1980, Wheeler Papers, Series II, Box Wo-Ze, folder Zeh, American Philosophical Society, Philadelphia.

### A TEKINTÉLY SZEREPE A FIZIKÁBAN IV.

Foundations of Physics, Vol. 1, No. 1, 1970

# On the Interpretation of Measurement in Quantum Theory

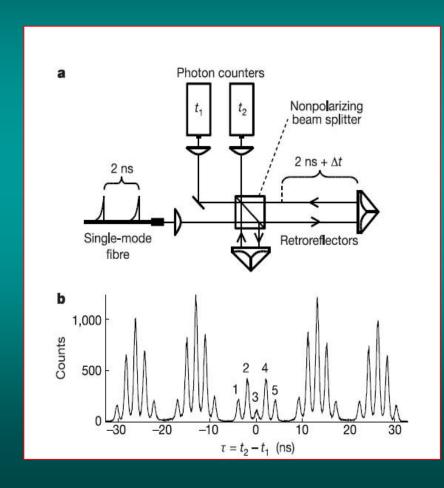
H. D. Zeh

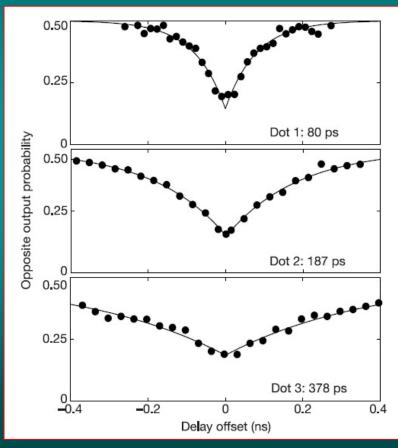
Institut für Theoretische Physik, Universität Heidelberg, Heidelberg, Germany

Received September 19, 1969

It is demonstrated that neither the arguments leading to inconsistencies in the description of quantum-mechanical measurement nor those "explaining" the process of measurement by means of thermodynamical statistics are valid. Instead, it is argued that the probability interpretation is compatible with an objective interpretation of the wave function.

### "OPPOSITE OUTPUT EXPERIMENTS" [ 2002 ].





### FOTON ANTIKORRELÁCIÓ<sub>b</sub> n-SZEKVENCIÁK BINOMIÁLIS SZÉRIÁI

APPLIED PHYSICS LETTERS

VOLUME 78, NUMBER 17

23 APRIL 2001

### Single quantum dots emit single photons at a time: Antibunching experiments

Valéry Zwiller<sup>a)</sup>
Solid State Physics, Lund University, SE-22100 Lund, Sweden

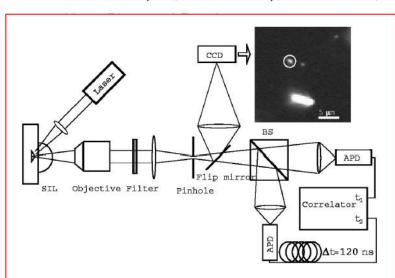


FIG. 1. A microscope objective is used to collect the quantum dot luminescence through a glass hemisphere. The collected light is subsequently filtered by a bandpass filter and a pinhole, and correlations are measured using a Hanbury–Brown and Twiss interferometer. A flip mirror can be used to image the quantum dots (inset). The white circle in the inset indicates the pinhole size, selecting a spot with diameter of around 3  $\mu$ m.

, SE-16440 Kista, Sweden

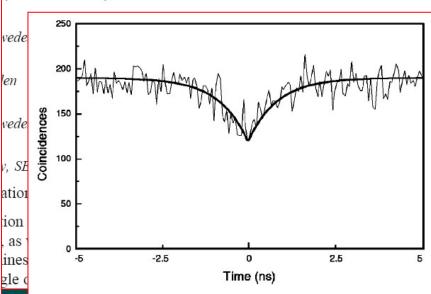
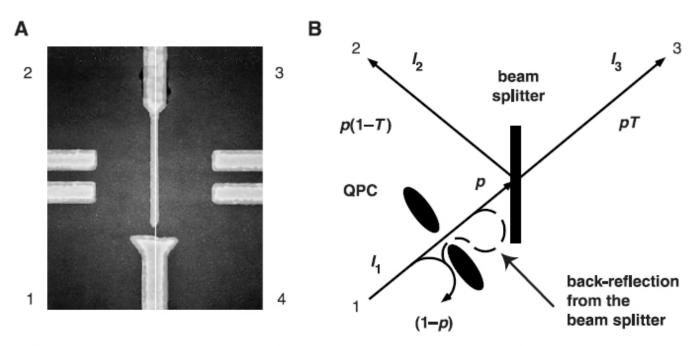


FIG. 3. Correlations obtained under cw excitation following 7.5 h of integration. The fit was obtained with a single exponential function with  $\tau = 0.74 \pm 0.11$  ns.

### **OLIVER** et al. [1999]



**Fig. 1.** (**A**) Scanning electron micrograph of an electron beam splitter device fabricated on a GaAs two-dimensional electron gas system. Schottky gates define the input quantum point contact (QPC) (port 1). Schottky gates and an etched trench (top center) define the output QPCs (ports 2 and 3). Port 4 is pinched off. The beam splitter is realized by the 40-nm finger (middle center). (**B**) Transmission and reflection model of the input QPC (port 1) and the beam splitter.  $I_1$  is the input current.  $I_2$  and  $I_3$  are the output currents. The probability p that electrons reach the beam splitter is the normalized conductance of the input QPC (port 1), accounting for the partial transmission through the QPC and the back-reflection from the beam splitter. The beam splitter has a transmission probability T. Prob $(1 \rightarrow 3) = pT$ . Prob $(1 \rightarrow 2) = p(1 - T)$ .

### 'HANBURY BROWN – TWISS ANTIKORRELÁCIÓ' ÉSZLELÉSE SZABAD ELEKTRONOKRA (2002)

#### letters to nature

#### Observation of Hanbury Brown– Twiss anticorrelations for free electrons

Harald Kiesel, Andreas Renz & Franz Hasselbach

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Fluctuations in the counting rate of photons originating from uncorrelated point sources become, within the coherently illuminated area, slightly enhanced compared to a random sequence of classical particles. This phenomenon, known in astronomy as the Hanbury Brown-Twiss effect1-5, is a consequence of quantum interference between two indistinguishable photons and Bose-Einstein statistics<sup>6</sup>. The latter require that the composite bosonic wavefunction is a symmetric superposition of the two possible paths. For fermions, the corresponding two-particle wavefunction is antisymmetric: this excludes overlapping wave trains, which are forbidden by the Pauli exclusion principle. Here we use an electron field emitter to coherently illuminate two detectors, and find anticorrelations in the arrival times of the free electrons. The particle beam has low degeneracy (about 10<sup>-4</sup> electrons per cell in phase space); as such, our experiment represents the fermionic twin of the Hanbury Brown-Twiss effect for photons.

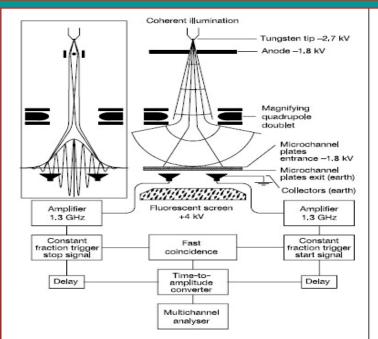


Figure 1 Electron optical set-up (top) and fast coincidence electronics (bottom) to measure electron anticorrelations. The quadrupoles produce an elliptically shaped beam of coherent electrons (schematically shown in the pictograms of Fig. 3). For geometrical reasons, fewer coherent electrons miss the collectors than for isotropic magnification. This greatly reduces the measuring time  $T_{\rm M}$ . The parts of the spherical cones emerging from the cathode represent single coherence volumina. Between the electron source and the quadrupole a biprism (inset) is inserted temporarily to check the coherence of illumination of the collectors. The very short electron avalanches (rise time and width of about 0.5 ns) leaving the channel plates are transferred coaxially from the collectors via microwave amplifiers (bandwidth 1.3 GHz) to modified constant fraction trigger modules that extract timing signals with low variance of transit time resulting in a very good time resolution. A fast coincidence circuit preselects events within a time window of  $\pm$  3 ns and opens the gate of a time-to-amplitude converter. The time spectra with a resolution of 26 ps are accumulated by a multichannel analyser. Capacitive crosstalk between the collectors was well below 1% and did not cause spurious coincidences.

### HANBURY BROWN – TWISS KORRELÁCIÓ RÖNTGEN-SUGARAKKAL. I.

PHYSICAL REVIEW A 69, 023813 (2004)

#### Intensity interferometry for the study of x-ray coherence

M. Yabashi\*
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K. Tamasaku and T. Ishikawa<sup>†</sup> SPring-8/RIKEN, Mikazuki, Hyogo 679-5148, Japan (Received 30 September 2003; published 26 February 2004)

Intensity interferometry has been performed for the study of x-ray coherence. A high-resolution monochromator at E = 14.41 keV was developed for enhancing the interference signal. Transverse coherence profiles of undulator radiation were evaluated from measurements of mode numbers. The obtained coherence length in vertical, which is perpendicular to the scattering plane of the monochromator, was proportional to the distance from the light source, as is expected from the Van Cittert–Zernike theorem. Vertical emittances of the storage ring were determined from the measured coherence lengths. Degradation of transverse coherence with phase object was measured and analyzed based on the propagation law of mutual intensity.

$$\frac{\overline{\xi \cdot \eta}}{\overline{\xi} \cdot \overline{\eta}} = 1 \pm \frac{1}{M} = 1 \pm \frac{1}{M_{x} M_{y} M_{l \text{ or } t}}$$

### HANBURY BROWN - TWISS KORRELÁCIÓ RÖNTGEN-SUGARAKKAL. II.

INTENSITY INTERFEROMETRY FOR THE STUDY OF ...

PHYSICAL REVIEW A 69, 023813 (2004)

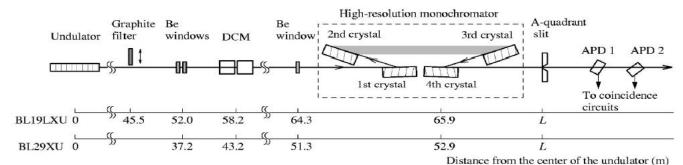


FIG. 1. Top view of the experimental setup. Undulator radiation ( $\Delta E/E \ge 10^{-3}$ ) is monochromatized with the double-crystal monochromator (DCM) using cryogenically cooled Si (111) [40] in  $\Delta E/E \sim 10^{-4}$ , and with the high-resolution monochromator (HRM) in  $\Delta E/E$  $\sim 10^{-8}$ . The slit placed after the HRM adjusts beam size to the two avalanche photodiodes (APDs). The distances from the center of the undulator to the components for two beam lines are indicated below the figure, while those to the slit (L) appear in the text.

#### INTENSITY INTERFEROMETRY FOR THE STUDY OF ...

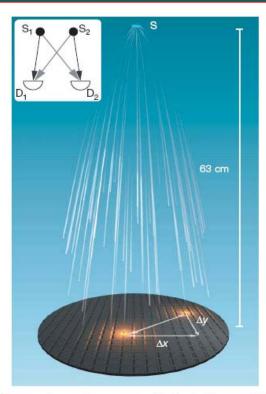
PHYSICAL REVIEW A 69, 023813 (2004)

TABLE I. Sample characters and experimental conditions for refractive contrast imaging.

Sample no.	Material	Purity (%)	Roughness (Ra, μm)	Thickness (µm)	X-ray wavelength (nm)	Sample-to-camera distance (m)
$\overline{A}$	Graphite <sup>a</sup>			100×9	0.086	1.8
В	Be (powder foil)	98.5	>1	250	0.10	1.5
C	Be (ingot foil)	99.8	0.1	250	0.10	1.5

<sup>a</sup>Density of 1 g/cm<sup>3</sup>.

### BOZON-KORRELÁCIÓK [ He<sup>4</sup> ] FERMION-KORRELÁCIÓK [ He<sup>3</sup> ]



**Figure 1** | **The experimental set-up.** A cold cloud of metastable helium atoms is released at the switch-off of a magnetic trap. The cloud expands and falls under the effect of gravity onto a time-resolved and position-sensitive detector (microchannel plate and delay-line anode) that detects single atoms. The horizontal components of the pair separation  $\Delta r$  are denoted  $\Delta x$  and  $\Delta y$ . The inset shows conceptually the two 2-particle amplitudes (in black or grey) that interfere to give bunching or antibunching:  $S_1$  and  $S_2$  refer to the initial positions of two identical atoms jointly detected at  $D_1$  and  $D_2$ .

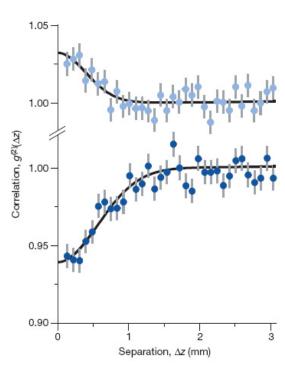
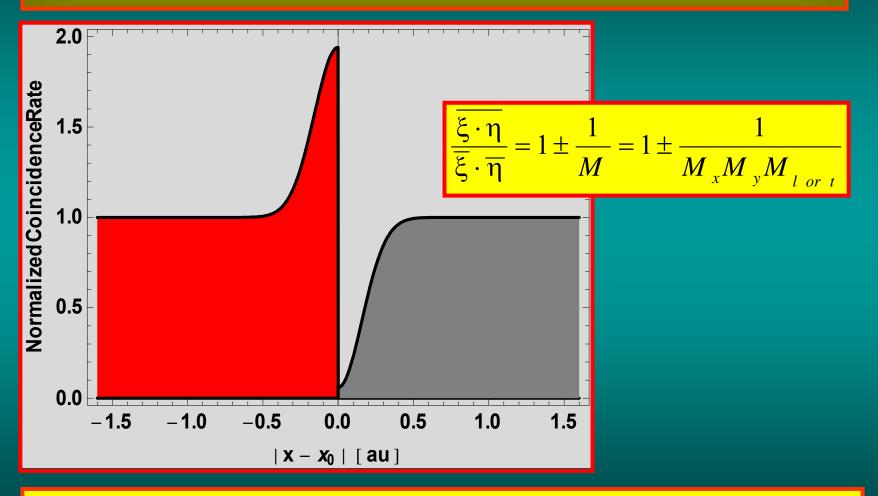


Figure 2 | Normalized correlation functions for  $^4\text{He}^*$  (bosons) in the upper plot, and  $^3\text{He}^*$  (fermions) in the lower plot. Both functions are measured at the same cloud temperature  $(0.5~\mu\text{K})$ , and with identical trap parameters. Error bars correspond to the square root of the number of pairs in each bin. The line is a fit to a gaussian function. The bosons show a bunching effect, and the fermions show antibunching. The correlation length for  $^3\text{He}^*$  is expected to be 33% larger than that for  $^4\text{He}^*$  owing to the smaller mass. We find 1/e values for the correlation lengths of  $0.75\pm0.07~\text{mm}$  and  $0.56\pm0.08~\text{mm}$  for fermions and bosons, respectively.

## BOZON-KORRELÁCIÓK ÉS FERMION-KORRELÁCIÓK ÖSSZEHASONLITÁSA



S. Varró: Correlations in single-quantum experiments. A note on wave-particle duality. 40th Physics of Quantum Electronics, 2010, 3-7 January, Snowbird (Utah) USA

### 'NEUTRON ANTI-BUNCHING'

M. Iannuzzi, A. Orecchini, F. Sacchetti, P. Facchi and S. Pascazio: Direct experimental observation of free-fermion antibunching. *Phys. Rev. Lett.* 96, 080402 (2006)

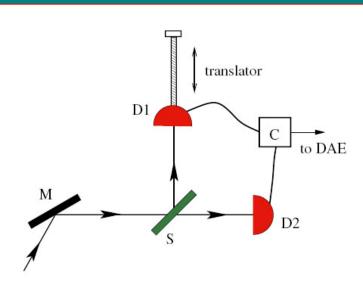
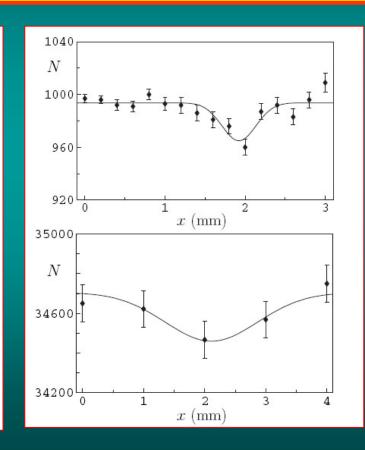


FIG. 1 (color online). Schematic drawing of the experimental setup: M, monochromator; S, beam splitter; D1 and D2, detectors; C, coincidence counter; DAE, Data Acquisition Electronics. The two detectors can be positioned at the same distance from S, and one of them can be moved across this distance. The collimators are not shown.



"Dear Colleague,

you published recently a paper ( Prog.Phys. 56 (2008) 91) which closely relates to a frequently discussed paper in neutron optics on anti-bunching (M. lannuzzi et all. PRL 96 (2006) 080402). Since we are interested in advanced neutron optics this experiment would be quite substantial but most colleagues do not believe on these results. Thus we are organizing a Mini-workshop on this topic (6.-8. March 2008). In the attachment I send you some more information and would like to invite you to this rather informal Workshop. We can finance your travel and accommodation costs.

I look forward to meeting you in Vienna

**Best regards** 

**Helmut Rauch**"

# The role of self-coherence in correlations of bosons and fermions in linear counting experiments.

Notes on the wave-particle duality

#### Sándor Varró

Research Institute for Solid State Physics and Optics of the Hungarian Academy of Sciences H-1525 Budapest, P. O. Box 49, Hungary, E-mail: varro@mail.kfki.hu

Abstract. Correlations of detection events in two detectors are studied in case of single-quantum excitations of the measuring apparatus. On the basis of classical probability theory and fundamental conservation laws, a general formula is derived for the two-point correlation functions for both bosons and fermions. The results obtained coincide with that derivable from quantum theory which uses quantized field amplitudes. By applying both the particle and the wave picture at the same time, the phenomena of photon bunching and antibunching, photon anticorrelation and fermion antibunching measured in beam experiments are interpreted in the frame of an intuitively clear description.

**Keywords:** Hanbury Brown and Twiss effect, photon bunching, fermion antibunching, Photon anticorrelation, wave-particle duality.

### KÉT-RÉSZECSKÉS SZIMMETRIÁK I.

$$\begin{split} & \Psi^{(symmetric)}(\mathbf{r_1, r_2} \mid \mathbf{k_1} s_1, \mathbf{k_2} s_2) = \\ & \frac{1}{\sqrt{2}} \Big[ \Phi_{\mathbf{k_1} s_1}(\mathbf{r_1}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_2}) + \Phi_{\mathbf{k_1} s_1}(\mathbf{r_2}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_1}) \Big] \end{split}$$

$$\begin{split} & \Psi^{(asymmetric)}(\mathbf{r_1, r_2} \mid \mathbf{k_1} s_1, \mathbf{k_2} s_2) = \\ & \frac{1}{\sqrt{2}} \left[ \Phi_{\mathbf{k_1} s_1}(\mathbf{r_1}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_2}) - \Phi_{\mathbf{k_1} s_1}(\mathbf{r_2}) \Phi_{\mathbf{k_2} s_2}(\mathbf{r_1}) \right] \end{split}$$

# KÉT-RÉSZECSKÉS SZIMMETRIÁK II.

$$\Psi_{\mathbf{k_1k_2}}^{(-)}(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{\sqrt{2}} \Big[ \phi_{\mathbf{k_1}}(\mathbf{r_1}) \phi_{\mathbf{k_2}}(\mathbf{r_2}) - \phi_{\mathbf{k_1}}(\mathbf{r_2}) \phi_{\mathbf{k_2}}(\mathbf{r_1}) \Big]$$

$$\left| \times \left\{ \left| \uparrow \uparrow \right\rangle, \quad \left| \downarrow \downarrow \right\rangle, \quad \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle + \left| \downarrow \uparrow \right\rangle \right) \right\} \quad \Rightarrow \frac{3}{4}$$

$$\Psi_{\mathbf{k_1k_2}}^{(+)}(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{\sqrt{2}} \left[ \phi_{\mathbf{k_1}}(\mathbf{r_1}) \phi_{\mathbf{k_2}}(\mathbf{r_2}) + \phi_{\mathbf{k_1}}(\mathbf{r_2}) \phi_{\mathbf{k_2}}(\mathbf{r_1}) \right]$$

$$\left| \times \frac{1}{\sqrt{2}} \left( \uparrow \downarrow \right) - \left| \downarrow \uparrow \right\rangle \right) \implies \frac{1}{4}$$

### EGY KONCEPCIONÁLISAN HELYTELEN ( de ~jó eredményre vezető ) INTERPRETÁCIÓ.

Detektor válaszfüggvény:

Párkorrelációs függvény:

$$R(t) = \exp(-t^2 / 2\tau_D^2)$$

$$R(t) = \exp(-t^2/2\tau_D^2)$$
  $C(t) = 1 - (1/2) \exp(-t^2/2\tau_c^2)$ 

$$-\frac{3}{4} + \frac{1}{4} = -\frac{1}{2}$$

$$\frac{N(t)}{N_{bg}} = [R * C](t) \approx 1 - \frac{1}{2} \frac{\tau_c}{\tau_D} \exp\left(-\frac{t^2}{2\tau_D^2}\right)$$

# NEUTRON ANTIBUNCHING I. Boffi & Caglioti [ 1971 ]a

IL NUOVO CIMENTO

Vol. 3B, N. 2

11 Giugno 1971

# Further Remarks on the Coherence Properties of a Thermal Neutron Beam (\*).

#### S. Boffi

Istituto di Fisica dell'Università - Pavia Istituto Nazionale di Fisica Nucleare - Gruppo di Pavia

#### G. CAGLIOTI

Centro Studi Nucleari Enrico Fermi del Politecnico - Milano Comitato Nazionale per l'Energia Nucleare - Ispra (Varese)

(ricevuto il 14 Dicembre 1970)

Summary. — The possibility of obtaining experimental evidence for antibunching effects due to the Fermi-Dirac statistics of the neutron is discussed and anticipated.

### **NEUTRON ANTIBUNCHING I.** Boffi & Caglioti [ 1971 ]b

(11) 
$$p_c(\tau|\tau+t)\Delta\tau = B(0)\Sigma \frac{R_B}{4\pi D^2} \left\{ 1 + \varepsilon g \frac{t_c}{t} g_{12}(0) \right\} \Delta\tau.$$

Therefore, while for classical particles the conditional probability is  $C \cdot \Delta \tau$ , for bosons (fermions) a bunching (antibunching) effect is present, whose size is due both to spatial and time coherence of the beam through the factors  $g_{12}(0)$  and  $t_c$ , respectively.

In practice, i into the other. short resolving ti gate the function general case of a

(12)

We conclude that to this encouraging prediction some experimental efforts should follow.

\* \* \*

We are indebted to Prof. F. T. Arecchi for a critical discussion. One of us (S.B.) is grateful to the Comitato Nazionale per l'Energia Nucleare for having made this collaboration possible.

#### RIASSUNTO

Si discute e si anticipa la possibilità di ottenere prove sperimentali per gli effetti contrari all'aggruppamento dovuti alla statistica di Fermi-Dirac del neutrone.

### NEUTRON ANTIBUNCHING II. Silverman [ 1988 ]a

Volume 132, number 4

PHYSICS LETTERS A

3 October 1988

### ON THE FEASIBILITY OF A NEUTRON HANBURY BROWN-TWISS EXPERIMENT WITH GRAVITATIONALLY-INDUCED PHASE SHIFT

#### M.P. SILVERMAN

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Received 15 March 1988; revised manuscript received 11 May 1988; accepted for publication 8 July 1988 Communicated by J.P. Vigier

It is shown that a gravitational potential difference between the two components of a split fermion beam in a Mach-Zehnder type interferometer can influence the fermion second-order, one-point correlation function and hence the conditional probability of fermion arrivals at one output port. Manifestation of the effect requires use of two statistically-inequivalent input beams. The configuration is a particle analogue of the optical Hanbury Brown-Twiss experiments. Contrary to previous report of a signal-to-noise performance orders of magnitude higher than that characterising the cross-correlation of particle fluctuations in two beams, it is shown that both types of measurements are characterised by comparable signal-to-noise expressions. The implications for observation of neutron antibunching are discussed.

# NEUTRON ANTIBUNCHING II. Silverman [ 1988 ]b

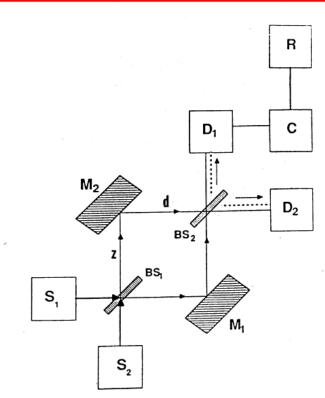


Fig. 1. Schematic diagram of a four-port fermion interferometer with beam splitters  $BS_1$  and  $BS_2$ , mirrors  $M_1$  and  $M_2$ , detectors  $D_1$  and  $D_2$ , time-correlator/counter C, and recording device R. The two horizontal beam paths of length d have vertical separation z. The conditional probability calculated in the text is for arrival of neutrons at  $D_1$ .

claim is correct, then the goal of a hybrid Hanbury Brown-Twiss/Colella-Overhauser-Werner experiment with neutrons would become possible providing that gravity can influence the second-order onepoint correlation function of fermions (in the interferometer of fig. 1).

In this Letter I show that gravity can indeed in-

Consider, for example, two input neutron beams with mean de Broglie wavelength  $\lambda_0 = 4.48$  Å and respective resolutions  $(\Delta\lambda)_1/\lambda_0 = 0.01$ ,  $(\Delta\lambda)_2/\lambda_0 = 0.02$  (corresponding coherence times  $T_1 = 8.0 \times 10^{-12}$  s,  $T_2 = 4.0 \times 10^{-12}$  s) yielding a count rate of 20 neutrons/s at a detector with resolution time of 1 ns [16,17]. It follows from eqs. (7) and (9a), (9b) that the fermionic contribution to the second-order correlation function can be written as

$$\langle \langle g_{11}^{(2)}(t) \rangle \rangle - 1 = -2.55 \times 10^{-3} [1 + 0.35 \cos(\theta) + 0.041 \cos(2\theta)].$$
 (11)

With disregard of gravity, the threshold (S/N=1) counting time to observe antibunching of neutrons with characteristic coherence time  $T_c \approx 5 \times 10^{-12}$  s by means of auto-correlation with delay time  $t \approx 5$  ns and counting window  $T \approx 1$  ns is from eq. (10c)  $T_{\text{tot}} \approx 2.5 \times 10^{12}$  s or  $\approx 8 \times 10^4$  years. The analysis of ref. [12] would have predicted a total counting time of  $\approx 14$  h.

# AZONBAN; EXTRÉM KICSI DEGENERÁCIÓS PARAMÉTER; δ ~10<sup>-14</sup>

" Dear Professor Varró,

The neutron intensity is indeed rather low. When we consider the phase space density (degeneracy parameter) it is 10E-14 and that means that there is on the average always only one neutron in the apparatus, the next one is still in the Uranium nucleus of the reactor fuel.

Best regards, Helmut Rauch "

"All the performed experiments belong to the regime of self-interference because the phase-space density of any neutron beam is extremely low (10<sup>-14</sup>) and nearly every case when a neutron passes through the interferometer the next neutron is still in a uranium nucleus of the reactor fuel."

[ H. Rauch, J. Sumhammer, M. Zawisky and E. Jericha: Low-contrast and low-counting-rate measurements in neutron interferometry.

Phys. Rev. A 42, 3726-3732 (1990) ]

### MEGFONTOLÁSOK A FÁZISTÉRRE I.c

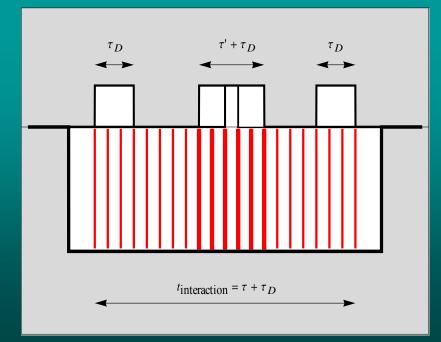
### [ RELEVÁNS '(TÉRIDŐ-)MÓDUSOKRÓL' ÁLTALÁBAN ]

A releváns módusok száma egyrészt a hullám-részecske nyaláb térbeli koherenciájától, polarizációs tulajdonságától ÉS a tényleges detektálási időtől ( holtidő), mintavételi frekvenciától, kapuzástól. Spektrális kereszt-tisztaság esetén:

$$\frac{\overline{\xi \cdot \eta}}{\overline{\xi} \cdot \overline{\eta}} = 1 \pm \frac{1}{M} = 1 \pm \frac{1}{M_x M_y M_{l \ or \ t}}$$

$$M \to \frac{2M}{1 + Pol}$$

$$M_{t} = \sqrt{1 + \frac{\tau_{D}^{2}}{\tau_{c}^{2}}}$$



$$M_{x} = \left\{ \frac{\pi^{1/2} \sigma_{x}}{w_{x}} \operatorname{erf}\left(\frac{w_{x}}{\sigma_{x}}\right) - \frac{\sigma_{x}^{2}}{w_{x}^{2}} \left[ 1 - \exp\left(-\frac{w_{x}^{2}}{\sigma_{x}^{2}}\right) \right] \right\}^{-1}$$

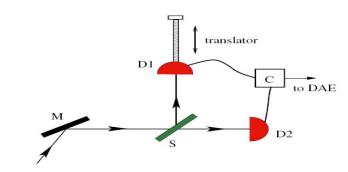
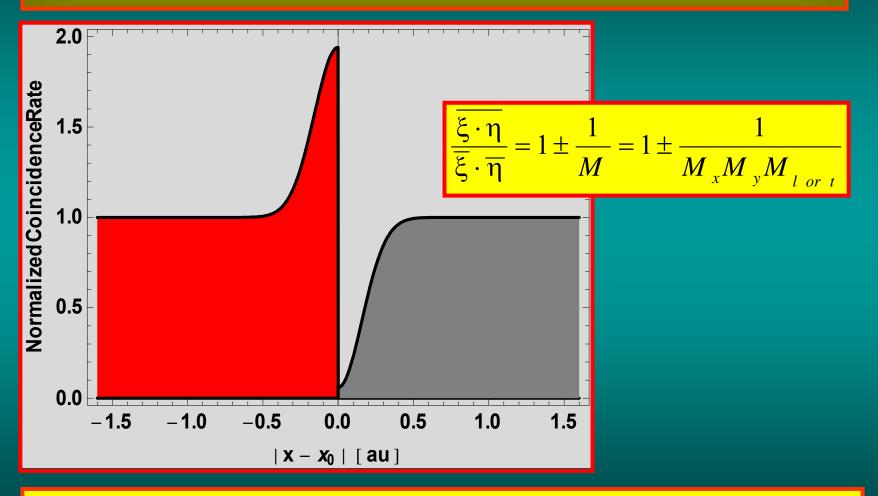


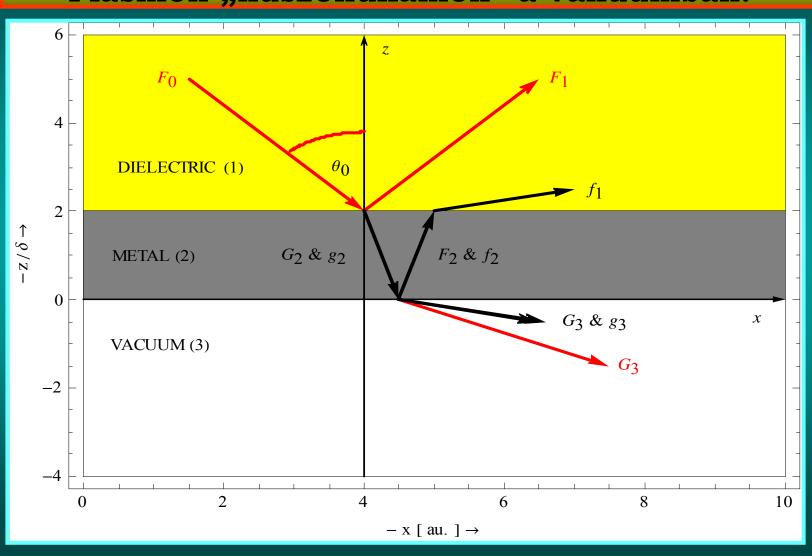
FIG. 1 (color online). Schematic drawing of the experimental setup: M, monochromator; S, beam splitter; D1 and D2, detectors; C, coincidence counter; DAE, Data Acquisition Electronics. The two detectors can be positioned at the same distance from S, and one of them can be moved across this distance. The collimators are not shown.

## BOZON-KORRELÁCIÓK ÉS FERMION-KORRELÁCIÓK ÖSSZEHASONLITÁSA

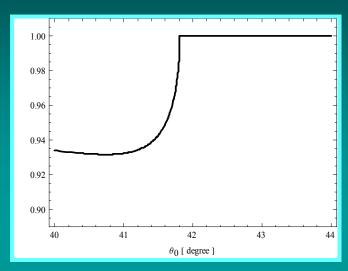


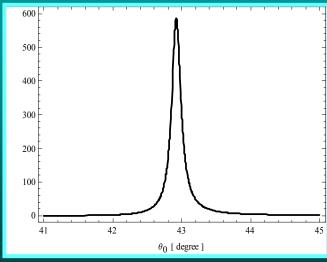
S. Varró: Correlations in single-quantum experiments. A note on wave-particle duality. 40th Physics of Quantum Electronics, 2010, 3-7 January, Snowbird (Utah) USA

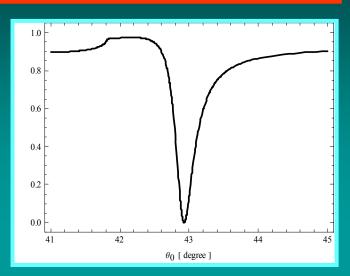
### EVANESZCENS TEREK. I. Plasmon "kúszóhullámok" a vákuumban.

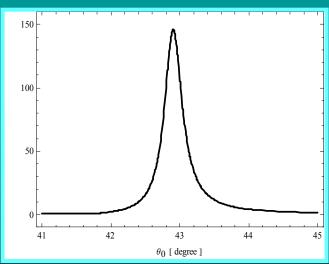


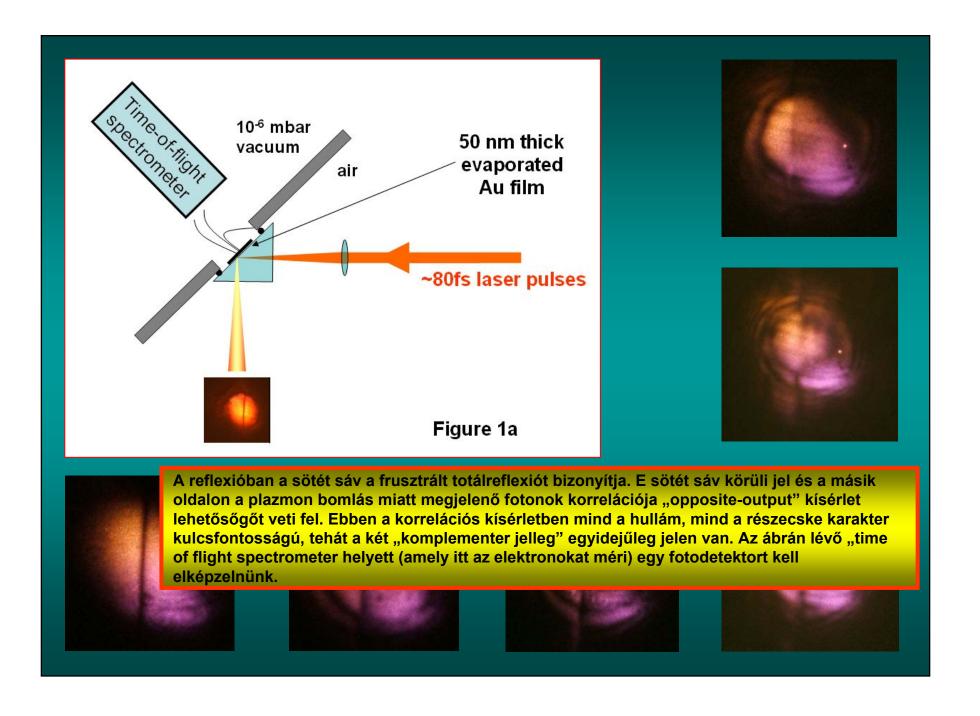
## EVANESZCENS TEREK.II. "TUNNELEZÉS"



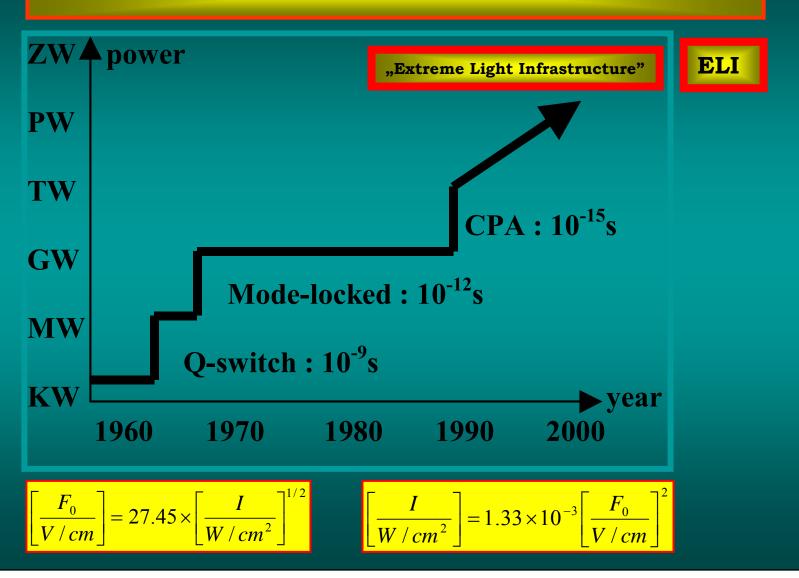








### **■ Maximum Laser Intensities**



### **■ Critical Fields in QED I.**

$$w \sim \exp\left[-\int_{0}^{2mc^{2}/eF} dx \sqrt{2m(2mc^{2} - eFx)}\right] rate/cm^{3} \sim 10^{50} \xi^{2} e^{\frac{-8}{3\xi}} pairs/\sec/cm^{3}$$

$$eE_{cr}\lambda_{C} = mc^{2} = eE_{cr}(\hbar/mc)$$

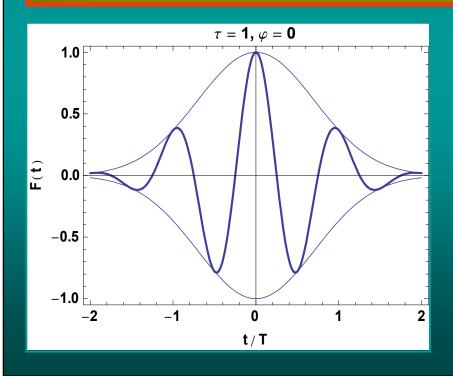
$$E_{cr} = \frac{m^2 c^3}{e\hbar} \sim 1.3 \times 10^{16} V / cm$$

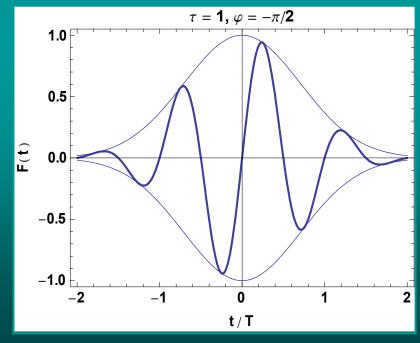
A vákuumban egyáltalán terjedni tudó nagyintenzitású lézertér intenzitásának határa ~ 10<sup>28</sup> W/cm<sup>2</sup>. Ezen túl egy hullámhosszon belül átkonvrtálódik az energiája elektron-pozitron párokká.

# ■ 'Carrier-Envelope Phase Difference' CEPD I. Field Stength

$$\mathbf{E}(\mathbf{r},t) = \mathbf{e}F_0 f(t - \mathbf{n} \cdot \mathbf{r}/c) \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \varphi_0)$$

Érdekes kérdés a pár-ciklusos fényimpulzusok "kvantumfázisa".





Laser and Particle Beams (2007), 25, 379–390. Printed in the USA. Copyright © 2007 Cambridge University Press 0263-0346/07 \$20.00 DOI: 10.1017/S0263034607000250

#### Linear and nonlinear absolute phase effects in interactions of ulrashort laser pulses with a metal nano-layer or with a thin

plasma layer

Laser and Particle Beams (2008), 26, 9–19. Printed in the USA. Copyright © 2008 Cambridge University Press 0263-0346/08 \$20.00 doi: 10.1017/S0263034608000037

#### S. VARRÓ

Research Institute for Solid 3 (RECEIVED 14 December 200

#### Abstract

It has been shown that in non-oscillatory wakefield field strength, and the de incoming pulse. When we metal surface or a gas is scheme can serve as a relativistic laser intensiti prepulse, which is follow kinematics lead to the apare downshifted due to t intensity-dependent frequattention has also been p It is also shown that the magnitude forming a qua has obtained.

Keywords: Carrier-envel plasma interactions

# Attosecond electron pulses from interference of above-threshold de Broglie waves

Varró, S., Intensity and phase effects... [ Chapter in the book "Laser Pulses" ]

X

1

#### S. VARRÓ AND GY. FARKAS

Research Institute for Solid State Physics and Op (Received 2 May 2007; Accepted 27 August 200

#### Abstract

It is shown that the above-threshold electron interfering to yield attosecond electron properties are superposition of high harmonics generated model is based on the Floquet analysis of generated by the incoming laser field of lon the propagation of attosecond de Broglie we propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape. The clessurface is largely degraded due to the propagate without changing shape.

**Keywords:** Charged particle beam sources; mechanics; Strong-field excitation; Surface

# Intensity Effects and Absolute Phase Effects in Nonlinear Laser-Matter Interactions. Attosecond Pulse Generation and Probing

Sándor Varró

Research Institute for Solid State Physics and Optics of the Hungarian Academy of Sciences Hungary

#### 1. Introduction

The generation of short pulses of electromagnetic radiations rely in many cases on nonlinear interactions, and, on the other hand, the resulting sources may have very large intensities, and can induce high-order nonlinear processes. Of course, these two faces of light-matter interactions are intimately connected, but they can usually be treated separately, depending on the emphasize put on one side or on the other. The subject of the present chapter belongs to the physics of "strong-field phenomena" taking place in laser-matter interactions, and it partly concerns quantum optics, too. A brief overview of theoretical methods for treating nonlinear light-matter interaction is given, and some characteristic examples of extreme short-pulse generation are presented. We review the basic classical and quantum approaches in simple terms, by possibly avoiding involved mathematical derivations. We

### **New Journal of Physics**

The open-access journal for physic

#### Entangled photon-electron states and the number-phase minimum uncertainty states of the photon field

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New Journal of Physics 10 (2008) Received 27 December 2007 Published 20 May 2008 Online at http://www.njp.org/ doi:10.1088/1367-2630/10/5/053028

Abstract. The exact analytic solu system consisting of a free electror are used for studying the physical i uncertainty states. The states of product of the photon number an 1 49-61) cosine operator have be 9 339-46). However, these state constructions without any physic fundamental interaction in quantur a free electron with a mode of the o to the generation of the mention that from the entangled photon-eld number state, due to the interactio minimum-uncertainty states of Jacl coordinate representation, the phy of these states is the joint proba of an electron and of a definite probabilities in these states preser they depend on the location in spa of the entanglement entropies deri from the electron's density operate

pstop353533 on 10 June 2010

PHYSICA SCRIPTA

Phys. Scr. 82 (2010) 000000 (8pp)

UNCORRECTED PROOF

# **Entangled states and entropy remnants** of a photon–electron system

#### S Varró

IOP PUBLISHING

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Online at stacks.iop.org/PhysScr/82/000000

#### Abstract

In the present paper, an example of entanglement between two different kinds of interacting particles, photons and electrons is analysed. The initial-value problem of the Schrödinger equation is solved non-perturbatively for the system of a free electron interacting with a quantized mode of the electromagnetic radiation. Wave packets of the dressed states so obtained are constructed in order to describe the spatio-temporal separation of the subsystems before and after the interaction. The joint probability amplitudes are calculated for the detection of the electron at some space—time location and the detection of a definite number of photons. The analytical study of the time evolution of entanglement between the initially separated electron wave packet and the radiation mode leads to the conclusion that in general there are non-vanishing entropy remnants in the subsystems after the interaction. On the basis of the simple model to be presented here, the calculated values of the entropy remnants crucially depend on the character of the assumed switching-on and -off of the interaction.

PACS numbers: 03.65.Ud, 42.50.Hz, 03.65.-w, 42.50.Dv

(Some figures in this article are in colour only in the electronic version.)

#### Pár-ciklusos fényimpulzusok "kvantumfázisa".

(see e.g. Varró (2008c)). By introducing the *formal* polar decomposition of the quantized amplitudes,  $a=E(a^{\dagger}a)^{1/2}$ ,  $a=(a^{\dagger}a)^{1/2}E^{\dagger}$ , we define

$$C = (E + E^{\dagger})/2, \quad S = (E - E^{\dagger})/2i, \quad \Delta C^2 = \langle (C - \langle C \rangle)^2 \rangle, \quad \Delta S^2 = \langle (S - \langle S \rangle)^2 \rangle, \tag{26}$$

where the variances  $\Delta C^2$  and  $\Delta S^2$  characterize the phase uncertainties. In Fig. 7 we have plotted the sum of this variances as a function of frequency in a Gaussian few-cycle pulse (which may result from high-harmonic generation, resulting in a quantum state like  $|\Phi(t)\rangle$  above in Eq. (22)).

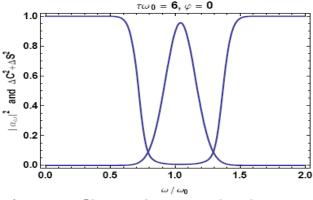
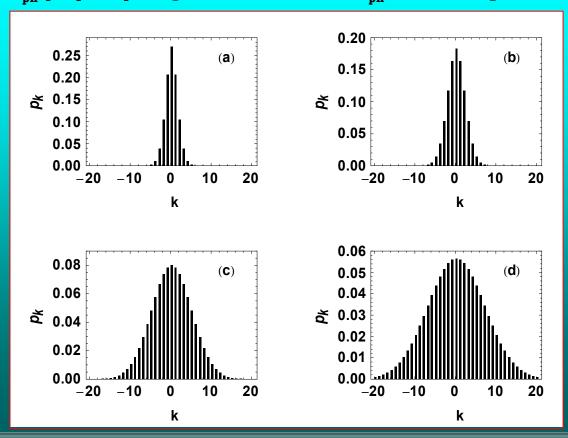


Figure 7. Shows the normalized quantum amplitude distribution  $|\alpha_{\omega}|^2$  of a 3–cycle femtosecond pulse and the dependencece of the quantum phase uncertainty  $\Delta C^2 + \Delta S^2$  associated to each spectral components with normalized frequency  $\omega/\omega_0$ . The pulse has been represented by a continuous Gaussian multimode coherent state, like that given in Eq. (22). The spectrum is peaked around  $\omega_0$ , where the quantum phase uncertainty has its minimum (it is essentially very close to zero).

#### S. V.: Intensity and phase effects... (2010)

### ÖSSZEFONÓDÁS, FOTONSZÁMELOSZLÁS NAGY INTENZITÁSÚ FÉNY-ELEKTRON KÖLCSÖNHATÁSNÁL I.

True photon number distributions  $\{p_k\} = I_k(q) \exp(-q)$  calculated from the reduced density operator  $P = Tr'\{|\psi><\psi|\}$  for different intensities, where  $q = 10^{-18} \times [I/E_{ph}^{\ 2}] \times [\lambda/w]^2$ . E.g. if  $\lambda/w \sim 10^4$  and  $E_{ph} \sim 1$ , then  $q = 10^{-10} \times I$ .



Varro\_CEWQO2009\_Paper\_PhysicaScripta



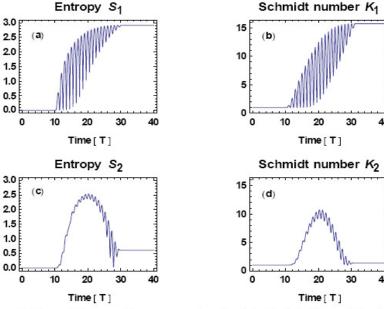


Figure 2. Shows the time-evolution of the von Neumann entropies (a):  $S_1$ , (c):  $S_2$  and the Schmidt numbers (b):  $K_1$ , (d):  $K_2$  of the photon number distribution given by Eqs. (26), and (27), respectively, in the special case when  $\vec{p}_0=0$ , and for the numerical value  $q_0=2$ . The subscripts refer to the switching function used in the calculation. In the defining equation of q, Eq. (23), we have set  $\mu(0)=\mu_0=10^{-3}$  and  $\lambda/4\pi w=10^3$ . This means that for the optical radiation we have taken the wavelength  $\lambda=10^{-4}$  cm and the intensity  $I=10^{12}W/cm^2$ . For the initial width of the electron wave packet  $w\approx 10^{-8}$  cm, i.e. one Ångstöm has been assumed. The time scales in this figure are the same as that in Fig. 1. As is seen in these figures, in each cases of the two different switching functions there are 'entropy remnants' left in the subsystems after the interaction was swithed – off.

## "AZ ÓKORI GÖRÖGÖK"

"KÉTSZER NEM LÉPHETSZ UGYANABBA A FOLYÓBA."

[ HERAKLEITOSZ ]

"EGYSZER SEM LÉPHETSZ UGYANABBA A FOLYÓBA."

[SZOFISTÁK]

