Holography for condensed matter phyiscs

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AdS/CFT Correspondence and its Applications
Tihany, August 28 2009

Meta-slide: Why do I talk about this?

- I think it's interesting
- Maybe the best chance to get something *measurable* out of holography. . . (e.g. you don't need a heavy ion collider)
- very fresh
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- Quantum criticality
- Holographic duals for finite T, B, μ systems
- Application: transport coefficients (conductivities and Nernst effect)
- Holographic superconductors
- Conclusions: potential and limitations of the AdS/CFT approach

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- strongly correlated (e^--)systems; quantum criticality
  $\longrightarrow$ traditional approach
  (weakly coupled q.particles, classical order parameter)
  does NOT work
- there are **many** Hamiltonians!
  engineering $\longrightarrow$ experimental AdS/CFT?
- insights on both sides ("Which one is more fundamental?")

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Quantum criticality

phase transition at $T = 0$

$$\xi \sim (g - g_c)^{-\nu}\,,$$
$$\Delta \sim (g - g_c)^{\nu z}$$

At g_c:

$$t \to \lambda^z t\,,$$
$$x \to \lambda x$$

- Natural place to start: scale invariance, lack of q.particles
- $z = 1$: Lorentz-invariance and spec. conform trf's
- mostly 2+1 D (layered superconductors)

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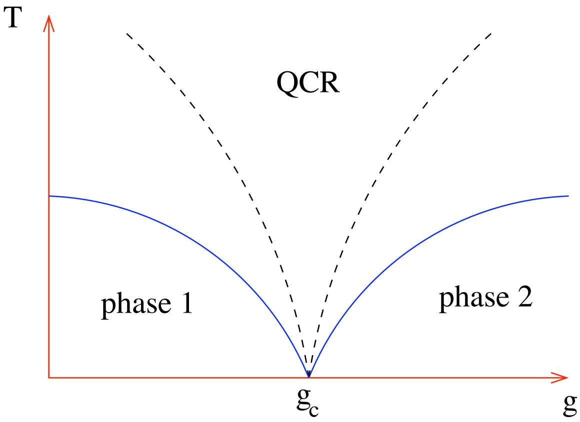
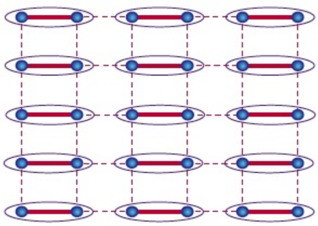
Phase diagram



Example: Insulating quantum magnet

$$H_{\mathrm{AF}} = \sum_{\langle ij \rangle} J_{ij} S_i \cdot S_j \,, \qquad J_{ij} = J \text{ or } J/g$$



$g \to 1:$ Néel-order, spin waves

$g \to \infty:$ spin-singlet dimers, triplons

$$S[\Phi] = \int d^3x \left((\partial\Phi)^2 + r\Phi^2 + u\left(\Phi^2\right)^2 \right)$$

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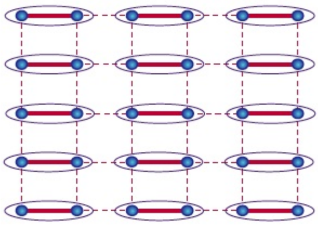

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Example: Bose–Hubbard model with integer filling

$$H_{\mathrm{BH}} = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + U\sum_i n_i\,(n_i - 1)$$

U(1) symmetry: $b_i \to e^{i\phi} b_i$

$$U/t \to \infty : \quad \text{decoupled sites}, \;\; < b_i >= 0, \;\; \textit{insulator}$$

$$U/t \to 0 : \quad < b_i > \neq 0, \;\; \textit{superfluid}$$

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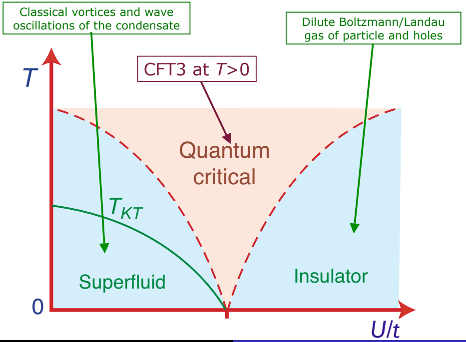
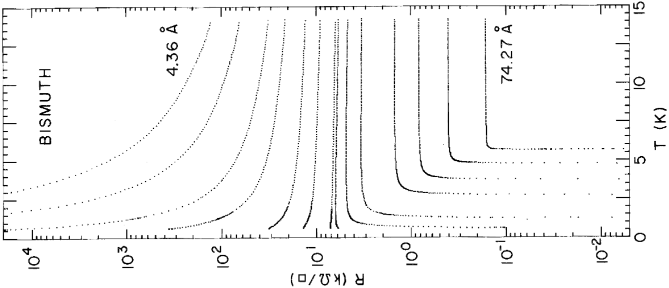


QC in the real world



Nonconventional superconductors

- heavy fermion metals (Kondo lattice)
- high T_c superconductors (?)

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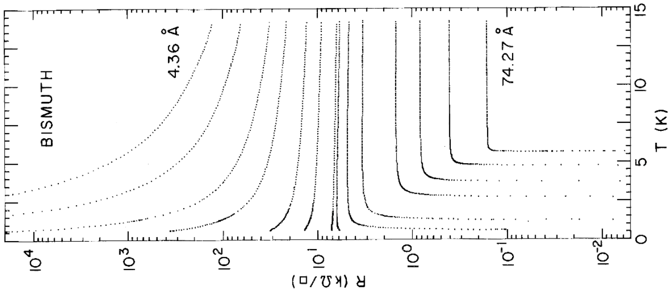

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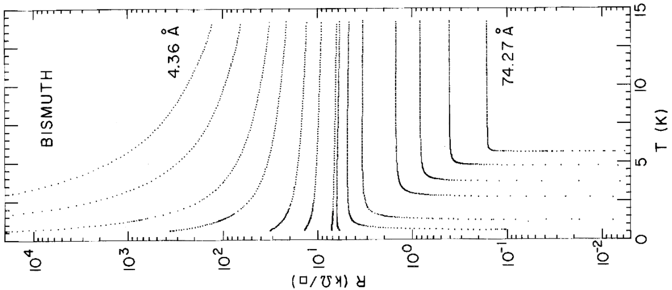

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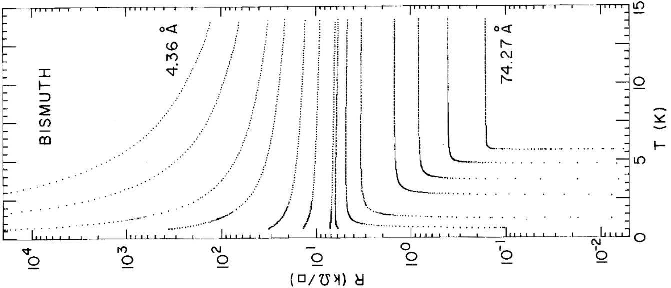

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Scale invariant geometry

- Realize the symmetries of the field theory as geometrical symmetries

$$ds^2 = L^2 \left(-\frac{dt^2}{r^{2z}} + \frac{dx^i dx^i}{r^2} + \frac{dr^2}{r^2} \right)$$

- The physics of our strongly coupled field theory in the large N limit is captured by classical dynamics about this background metric.
- For $z = 1$ AdS space: more symmetries and solution of

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^{d+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} \right)$$

- $z \neq 1$ also possible...

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Finite T at equilibrium

- μ, T break dilatation symmetry
- for large E scales it is restored
- energy scale is an extra dimension $\Rightarrow$ *Asymptotically* AdS!

Schwarzschild–AdS solution

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right)$$

$$f(r) = 1 - \left(\frac{r}{r_+}\right)^d$$

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Thermodynamics of the Schwarzschild–AdS black hole

- temperature: $T = \frac{d}{4\pi r_+}$
- Euclidean action: $S_E = -\frac{L^{d-1}}{2\kappa^2 r_+^d}\frac{V_{d-1}}{T} = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^{d-1}$
- free energy: $F = -T \log Z = TS_E[g_\star] = -\frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^d} V_{d-1} T^d$
- entropy $S = -\frac{\partial F}{\partial T} = \frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^{d-1}} V_{d-1} T^{d-1}$

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Finite chemical potential and magnetic field

- Additional structure: global $U(1)$
- gauged $U(1)$ in the bulk:
  subgroup of large gauge trf's act non-trivially on the boundary $\longrightarrow$ gives the global symmetry group for the FT
- Einstein–Maxwell theory

$$S = \int \mathrm{d}^{d+1}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F^2\right]$$

for $d = 3$ consistent truncation of M-theory on $AdS_4 \times X^7$

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for $d = 3$ B does not break isotropy $\longrightarrow$
dyonic Reissner–Nordström–AdS black hole

$$f(r) = 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4 ,$$

$$A = \mu\left[1 - \frac{r}{r_+}\right] dt + Bx\, dy$$

$\gamma^2 = \frac{(d-1)g^2L^2}{(d-2)\kappa^2}$

- $T = \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right)$
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for $d = 3$ B does not break isotropy $\longrightarrow$
dyonic Reissner–Nordström–AdS black hole

$$f(r) = 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4,$$

$$A = \mu\left[1 - \frac{r}{r_+}\right] dt + Bx\, dy$$

$\gamma^2 = \frac{(d-1)g^2L^2}{(d-2)\kappa^2}$

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Operators, expectation values

$$Z_{\text{bulk}}[\phi \to \delta\phi_{(0)}] = \langle \exp\left(i \int \mathrm{d}^d x \sqrt{-g_{(0)}}\,\delta\phi_{(0)}\mathcal{O} \right) \rangle_{\text{F.T.}}$$

If $(d-2)/2 < \Delta = \dim[\mathcal{O}] < d$ scalar, then ϕ near the boundary

$$\phi(r) = \left(\frac{r}{L}\right)^{d-\Delta} \phi_{(0)} + \left(\frac{r}{L}\right)^{\Delta} \phi_{(1)} + \dots \quad \text{as} \quad r \to 0\,,$$

$$(Lm)^2 = \Delta(\Delta - d)$$

$$\langle \mathcal{O} \rangle = -i\frac{\delta Z_{\text{bulk}}[\phi_{(0)}]}{\delta\phi_{(0)}} \overset{N\to\infty}{=} \frac{\delta S[\phi_{(0)}]}{\delta\phi_{(0)}} = \frac{2\Delta - d}{L}\phi_{(1)}$$

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Linear response

Next step: time and space-dependent perturbations
Broken time reversal symmetry $\Leftrightarrow$ irreversibility of falling into a BH

$$\delta\langle\mathcal{O}_A\rangle(\omega,k) = G^R_{\mathcal{O}_A\mathcal{O}_B}(\omega,k)\delta\phi_{B(0)}(\omega,k)$$

So

$$G^R_{\mathcal{O}_A\mathcal{O}_B} = \left.\frac{\delta\langle\mathcal{O}_A\rangle}{\delta\phi_{B(0)}}\right|_{\delta\phi=0} = \frac{2\Delta_A - d}{L}\frac{\delta\phi_{A(1)}}{\delta\phi_{B(0)}}$$

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Thermal and electric conductivities ($B = 0$)

Generalised Ohm's Law (nonzero net charge):

$$\begin{pmatrix} \langle J_x \rangle \\ \langle Q_x \rangle \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix},$$

where the heat current is $Q_x = T_{tx} - \mu J_x$

$$\sigma(\omega) = \frac{-iG^R_{J_x J_x}(\omega)}{\omega} = \frac{-1}{g^2 L} \frac{i}{\omega} \frac{\delta A_{x(1)}}{\delta A_{x(0)}},$$

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Electric conductivity for $B = 0$ from AdS/CFT

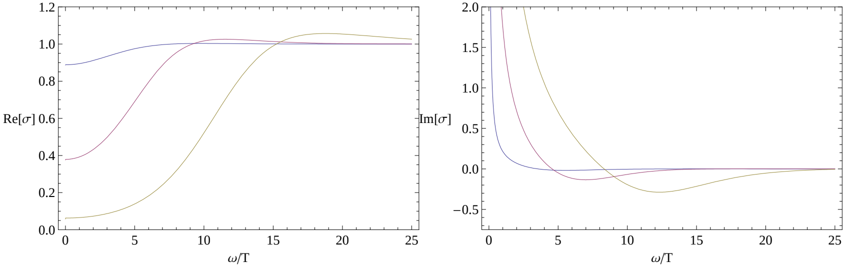

Figure: The real and imaginary parts of the electrical conductivity computed via AdS/CFT. The conductivity is shown as a function of frequency. Different curves correspond to different values of the chemical potential at fixed temperature. The gap becomes deeper at larger chemical potential.

Electric conductivity for $B = 0$ in graphene

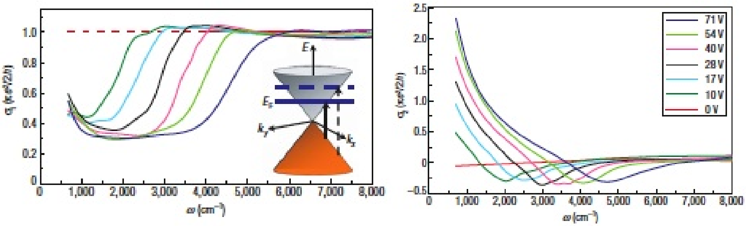

Figure: Experimental plots of the real and imaginary parts of the electrical conductivity in graphene as a function of frequency. The different curves correspond to different values of the gate voltage.

Conductivities with $B \neq 0$

- $B \neq 0 \;\Rightarrow$ off-diagonal elements: $\sigma_\pm = \sigma_{xy} \pm i\sigma_{xx}$ etc.
- Ward identities

$$\pm\alpha_\pm T\omega = (B \mp \mu\omega)\sigma_\pm - \rho\,,$$
$$\pm\bar{\kappa}_\pm T\omega = \left(\frac{B}{\omega} \mp \mu\right)\alpha_\pm T\omega - sT + mB$$

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Here

$$\sigma_Q = \frac{(sT)^2}{(\epsilon + P)^2}\frac{1}{g^2}$$ AdS/CFT!

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- $\mu = 0, \; B = 0:$ $\sigma_{xx} = 1/g^2$ frequency-independent!
- DC limit ($\omega = 0$) : $\sigma_{xx} = 0, \; \sigma_{xy} = \frac{\rho}{B}$ Hall conductivity
- $B = 0$ limit : $\sigma_{xx} = \sigma_Q + \frac{\rho^2}{(\epsilon+P)} \frac{i}{\omega}, \; \sigma_{xy} = 0$
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Beyond hydro: large B

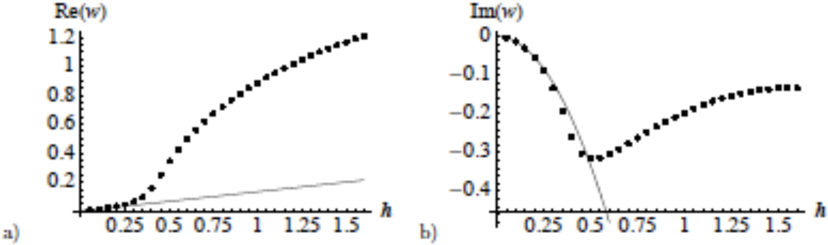

Figure: Location of the cyclotron pole in the retarded Green's function in the complex frequency plane as a function of the dimensionless magnetic field and at a fixed charge density.

Beyond hydro: S-duality

$$B \to \rho\,, \quad \rho \to -B\,, \quad \sigma_Q \to \frac{1}{\sigma_Q} \quad \Longrightarrow \quad \sigma_+(\omega) \to \frac{-1}{\sigma_+(\omega)}$$

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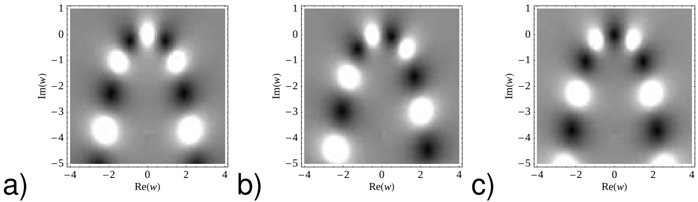

Figure: A density plot of $|\sigma_+|$ as a function of complex ω. a) $h = 0$ and $q = 1$, b) $h = q = 1/\sqrt{2}$, c) $h = 1$ and $q = 0$.

The Nernst effect

- constant voltage in response to a temperature gradient with no current
- $\theta = -\sigma^{-1} \cdot \alpha$
- in typical metal: vanishing
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- $\Rightarrow$ effective d.o.f. are not particles/holes? (vortices?)

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  $\Rightarrow$ need a charged scalar field in the bulk!
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$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{d(d-1)}{L^2}\right) - \frac{1}{4g^2}F^2 - |\nabla\phi - iqA\phi|^2 - m^2|\phi|^2 - V(|\phi|)$$

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- bulk scalar turns on continuously: instability of the BH under perturbations of the scalar field
- two effects:
  - effective mass from interaction with the EM filed: $m_{\text{eff}}^2 = m^2 + g^{tt} A_t^2 = m^2 - \frac{r^2}{f} \frac{q^2}{L^2} (1 - r/r_+)^2$
  - near-horizon AdS_2 throat
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- Breitenlohner–Freedman bound $\longrightarrow$

$$q^2 \gamma^2 \geq 3 + 2\Delta(\Delta - 3)$$

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- $(mL)^2 = \Delta(\Delta - 3)\,, \qquad \gamma^2 = 2g^2L^2/\kappa^2$

Holographic superconductors

- bulk scalar turns on continuously: instability of the BH under perturbations of the scalar field
- two effects:
  - effective mass from interaction with the EM filed: $m_{\text{eff}}^2 = m^2 + g^{tt} A_t^2 = m^2 - \frac{r^2}{f}\frac{q^2}{L^2}(1 - r/r_+)^2$
  - near-horizon AdS_2 throat
- (don't need a ϕ^4 term!)
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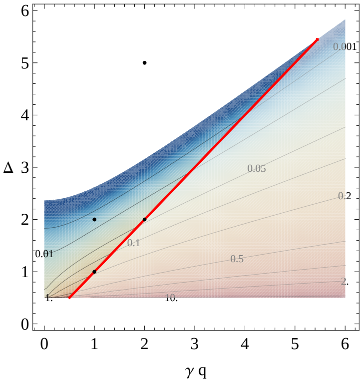

Figure: The critical temperature T_c as a function of charge and dimension. Contours are labeled by values of $\gamma T_c/\mu$. The bottom boundary of the plot is the unitarity bound at which T_c diverges.

Superconducting phase

in the superconducting phase the background is a BH with scalar hair

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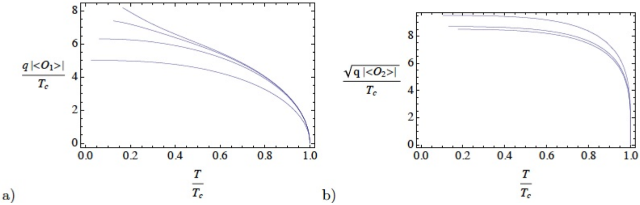

Figure: The condensate as a function of the temperature for the case $\Delta = 1$ (left) and $\Delta = 2$ (right). In curve (a), from bottom to top, $\gamma q = 1, 3, 6, 12$. In curve (b), from top to bottom, $\gamma q = 3, 6, 12$.

Conductivity

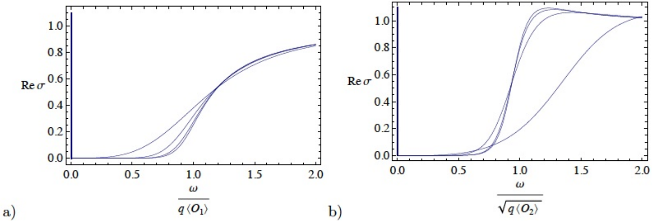

Figure: The real (dissipative) part of the electrical conductivity at low temperature in the presence of a $\Delta = 1$ and $\Delta = 2$ condensate. The curves with steeper slope correspond to larger γq.

Conductivity

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- delta at the origin ($\omega = 0$)
- gap at low frequencies $\omega < \omega_g$

for normal SC:	$\omega_g/T_c \approx 3.5$
for high T_c cuprates:	$\omega_g/T_c \approx 4 - 7$
holographic SC (N-independent!):	$\omega_g/T_c \approx 8$! (in the probe limit)

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Conclusions: potential and limitations

Limitations:

- theories that have weakly curved geometrical duals are quite different from FT's in cond-mat (like SUSY, large N)
- the most precise experimental probes directly measure the electron densities
  $\leftrightarrow$ expectation values of 'bare' or 'UV' operators
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  test and refine generic expectations
  (real time transport boils down to solving ODE's!)

  first calculable example of hydrodynamic to collisionless transition
  very useful for the hydrodynamic calculations
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