PARTICLE REPRESENTATIONS

MASSLESS Q^a_1, Q^a_2 ; $Q^a_2 = 0$ $\quad Z^{ab} = 0$

2^N DIM CLIFFORD ALGEBRA

HELICITY ≤ 1	$N=1$ gauge	$N=1$ chiral	$N=2$ gauge	$N=2$ hyper	$N=3$ gauge	$N=4$ gauge
1	1	0	1	0	1	1
1/2	1	1	2	2	3+1	4
0	0	1+1	1+1	4	3+3	6
−1/2	1	1	2	2	1+3	4
−1	1	0	1	0	1	1
TOTAL #	2×2	2×2	2×4	8	2×8	16

spectrum coincide, FT same

MASSIVE Q^a_1, Q^a_2, Z^{ab} central charge $\{Q^{\hat a}_{\alpha\pm}, Q^{b}_{\beta\pm}\} = \delta^{\hat a}_{b}\,\delta^{\beta}_{\alpha}\,(M \pm Z_{\hat a})$

ZUMINO DECOMPOSITION

BPS bound $M \geq |Z_{\hat a}|$ $\quad \hat a = 1..[\frac{N}{2}]$

$\rightarrow \quad 2^{2(N-r_a)}$; $M = |Z_a|$; r_a

SPIN ≤ 1	$N=1$ gauge	$N=1$ chiral	$N=2$ gauge	$N=2$ BPS gauge	$N=2$ BPS hyper	$N=4$ BPS gauge
$1^{\times 3}$	1	0	1	1	0	1
$1/2^{\times 2}$	2	1	4	2	2	4
$0^{\times 1}$	1	2	5	1	4	5
Total #	8	4	16	8	8	16

$M = |Z_{\hat a}|$ $\quad \hat a = 1..r_{\hat a}$ $\quad \sim \underset{\text{Clifford alg. assoc.}}{SO(4N-4r_a)} \sim \frac{1}{2^{r_a}}$ BPS dim. 2^{2N-2r_a}

SUSY LAGRANGIANS

Contain spin-$(0, \frac{1}{2}, 1)$ + special relations (masses, couplings)

Local Lagrangians $\left.\begin{array}{l} B - \partial\partial \\ F - \partial \end{array}\right\} \Rightarrow$ RENORMALIZABLE.

Simplest $\mathcal{N}=1$ gauge multiplet (A_μ, λ_α)

$$\mathcal{L} = -\frac{1}{2g^2} \mathrm{tr}\, F_{\mu\nu} F^{\mu\nu} + \frac{\Theta_I}{8\pi^2} \mathrm{tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{i}{2} \mathrm{tr}\, \bar{\lambda} \bar{\sigma}^\mu D_\mu \lambda$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu]$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

$$D_\mu = \partial_\mu + i [A_\mu, \lambda]$$

$\mathcal{N}=1$ SUSY INVARIANT

ξ — spin-$\frac{1}{2}$ GRASSMAN PARAMETER

$$\{Q, \bar{Q}\} \to [\xi Q, \bar{\xi} \bar{Q}] = 2 \xi \sigma^m \bar{\xi} P_m \quad ; \xi Q = \xi_\alpha Q^\alpha$$

$$\delta_\xi A_\mu = i \bar{\xi} \bar{\sigma}_\mu \lambda - i \bar{\lambda} \bar{\sigma}_\mu \xi$$

$$\delta_\xi \lambda = \sigma^{\mu\nu} F_{\mu\nu} \xi \quad \to$$

$m \lambda\lambda$ — Majoranna mass term in $\mathcal{L}$ would break susy.

WITH SCALAR FIELDS MORE COMPLICATED

~ GENERALLY SUPERFIELD METHOD

MANIFESTLY SUSY INVARIANT!

WESS-ZUMINO MODEL (1974)

FREE CHIRAL (χ) MULTIPLET (A, ψ)

$$\delta_\xi A = \sqrt{2}\,\xi\psi$$

$$\delta_\xi \psi = \underbrace{i\sqrt{2}\,\sigma^m \bar{\xi}\,\partial_m A}_{\text{CLOSES ON-SHELL!}} + \left(\sqrt{2}\,\xi F\right)$$

$$\delta_\xi F = i\sqrt{2}\,\bar{\xi}\bar{\sigma}^m \partial_m \psi \qquad (\text{! TOTAL DERIVATIVE})$$

F auxiliary field, needed to off-shell closing the algebra coeff's defined

$$S_{WZ} = \int d^4x \left(\partial^m A^* \partial_m A + i\,\psi^+ \bar{\sigma}^m \partial_m \psi\right) + \int d^4x\; F^* F$$

$[F] = 2$

EQ'N OF MOTION $\quad F = 0,\ F^* = 0$

DEGREES OF FREEDOM:

ON-SHELL	A, A^* - 2	ψ - 2 POLARIZATION
OFF-SHELL	A, A^* - 2	4 ψ - 2 COMPLEX WETL F.
	F - 2	

MUST BE A COMPLEX SCALAR!

INTERACTING WZ MODEL

BOSONIC PART $\quad [A] = 1 \quad [F] = 2$

$$\int d^4x \left[\partial^m A^* \partial_m A + F^* F \right] - \int d^4x \left(m A F + g A^2 F + h.c. \right)$$

F AUXILIARY FIELD

$$\frac{\delta \mathcal{L}}{\delta F} = F^* - m A - g A^2 \qquad F^* = W(A)$$

$$\rightarrow \int d^4x \left[\partial^m A^* \partial_m A + \underbrace{V(A, A^*)}_{\text{scalar potential}} \right]$$

$$V(A, A^*) = |F|^2 = \left(m A^* + g A^{*2} \right) \left(m A + g A^2 \right)$$

positive definit.

$\mathcal{N}=4$ SUPER YANG-MILLS

1 gauge multiplet $\quad \left(A_\mu, \lambda^a_\alpha, X^i\right)$

$SU_R(4) \quad 1 \quad 4 \quad 6$: rank-2 antisymm. tensor.

$\mathcal{L}$ UNIQUE

$$\mathcal{L} = \mathrm{tr}\Big\{ -\frac{1}{2g^2} F_{\mu\nu}F^{\mu\nu} + \frac{\Theta_I}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} - \sum_a \lambda^a \bar{\sigma}^\mu D_\mu \lambda_a - \sum_i D_\mu X^i D^\mu X^i +$$

$$+ \sum_{a,b,i} g\, C^{ab}_i \lambda_a [X^i, \lambda_b] + \sum_{a,b,i} g\, \bar{C}_{iab} [X^i, \bar{\lambda}_b] + \frac{g^2}{2}\sum_{i,j} [X^i, X^j]^2 \Big\}$$

C^{ab}_i, C_{iab} ~ RELATED TO CLIFFORD DIRAC MATRICES FOR $SO_R(6) \sim SU(4)_R$

$\mathcal{N}=4$ POINCARE SUSY INVARIANT, TRAFO'S

$$\delta X^i = C^{iab} \lambda_{\alpha b}$$

$$\delta \lambda_b = F^+_{\mu\nu} (\sigma^{\mu\nu})^\alpha{}_\beta\, \delta^a_b + [X^i, X^j]\, \epsilon_{\alpha\beta} (C_{ij})^a{}_b$$

$$\delta \bar{\lambda}^b_{\dot\beta} = C^{ab}_i\, \bar{\sigma}^\mu_{\alpha\dot\beta} D_\mu X^i$$

$$\delta A_\mu = (\sigma_\mu)_\alpha{}^{\dot\beta}\, \bar{\lambda}^\alpha_{\dot\beta}$$

~ bilinears of Clifford Dirac matrices

CLASSICALLY $\mathcal{L}$ IS SCALE INVARIANT $\quad [A_\mu] = [X^i] = 1 \quad [\lambda_a] = \frac{3}{2}$, $\quad [g] = 0 \quad [\Theta_I] = 0$

ALL TERMS ARE DIM-4!

$\mathcal{N}=4$ SYM CONT'D

IN RELATIVISTIC QFT,

SCALE INVARIANCE + POINCARE COMBINE INTO

$\rightsquigarrow$ $SO(2,4) \sim SU(2,2)$

CONFORMAL SYMMETRY GROUP

$\mathcal{N}=4$ POINCARE + CONFORMAL INV

$\rightsquigarrow$ SUPERCONFORMAL GROUP $SU(2,2|4)$

REMARK:
- PERTURBATIVE QUANTIZATION $\mathcal{N}=4$ SYM no UV DIV'S IN THE CORRELATION FUNCTIONS OF CANONICAL FIELDS.
- INSTANTONS $\rightarrow$ FINITE CONTRIBUTION

$\rightsquigarrow$ BELEIVED $\mathcal{N}=4$ SYM UV FINITE!

$\Rightarrow$ RG β- FUNCTION $\beta \equiv 0$

THEORY FULLY SCALE INVARIANT AT QUANTUM LEVEL

$SU(2,2|4)$ $\longrightarrow$ FULLY QUATUM SYMMETRY!

SUPER CONF. GROUP