1. Susy algebra on particle states

algebra

$$[\mathbf{L}_a{}^b, \mathbf{J}_c] = \delta_c^b \mathbf{J}_a - \frac{1}{2}\delta_a^b \mathbf{J}_c\,, \qquad [\mathbf{R}_\alpha{}^\beta, \mathbf{J}_\gamma] = \delta_\gamma^\beta \mathbf{J}_\alpha - \frac{1}{2}\delta_\alpha^\beta \mathbf{J}_\gamma\,,$$

$$[\mathbf{L}_a{}^b, \mathbf{J}^c] = -\delta_a^c \mathbf{J}^b + \frac{1}{2}\delta_a^b \mathbf{J}^c\,, \qquad [\mathbf{R}_\alpha{}^\beta, \mathbf{J}^\gamma] = -\delta_\alpha^\gamma \mathbf{J}^\beta + \frac{1}{2}\delta_\alpha^\beta \mathbf{J}^\gamma\,,$$

$$\{\mathbf{Q}_\alpha{}^a, \mathbf{Q}_b^{\dagger\beta}\} = \delta_b^a \mathbf{R}_\alpha{}^\beta + \delta_\alpha^\beta \mathbf{L}_b{}^a + \frac{1}{2}\delta_b^a \delta_\alpha^\beta \mathbf{H}\,,$$

$$\{\mathbf{Q}_\alpha{}^a, \mathbf{Q}_\beta{}^b\} = \epsilon_{\alpha\beta}\epsilon^{ab}\,\mathbf{C}\,, \qquad \{\mathbf{Q}_a^{\dagger\alpha}, \mathbf{Q}_b^{\dagger\beta}\} = \epsilon_{ab}\epsilon^{\alpha\beta}\,\mathbf{C}^\dagger\,.$$

acting on 1-particle state

$$\mathbf{L}_a{}^b|e_c\rangle = |e_M\rangle L_{ac}^{bM} = \delta_c^b|e_a\rangle - \frac{1}{2}\delta_a^b|e_c\rangle\,, \qquad \mathbf{R}_\alpha{}^\beta|e_\gamma\rangle = |e_M\rangle R_{\alpha\gamma}^{\beta M} = \delta_\gamma^\beta|e_\alpha\rangle - \frac{1}{2}\delta_\alpha^\beta|e_\gamma\rangle$$

$$\mathbf{Q}_\alpha{}^a|e_b\rangle = |e_M\rangle Q_{\alpha b}^{aM} = a\,\delta_b^a|e_\alpha\rangle\,, \qquad \mathbf{Q}_\alpha{}^a|e_\beta\rangle = |e_M\rangle Q_{\alpha\beta}^{aM} = b\,\epsilon_{\alpha\beta}\epsilon^{ab}|e_b\rangle$$

$$\mathbf{Q}_a^{\dagger\alpha}|e_\beta\rangle = |e_M\rangle \overline{Q}_{a\beta}^{\alpha M} = d\,\delta_\beta^\alpha|e_a\rangle\,, \qquad \mathbf{Q}_a^{\dagger\alpha}|e_b\rangle = |e_M\rangle \overline{Q}_{ab}^{\alpha M} = c\,\epsilon_{ab}\epsilon^{\alpha\beta}|e_\beta\rangle$$

① $$\{Q^a_\alpha, Q^{\dagger\beta}_b\}|e_c\rangle = Q^a_\alpha\, c\,\epsilon^{\beta\gamma}\epsilon_{bc}|e_\gamma\rangle + Q^{\dagger\beta}_b\, a\,\delta^a_c|e_\alpha\rangle$$

$$= bc\,\delta^\beta_\alpha(\delta^a_b|e_c\rangle - \delta^a_c|e_b\rangle) + ad\,\delta^a_c\delta^\beta_\alpha|e_b\rangle$$

$$\equiv (\delta^a_b R_\alpha{}^\beta + \delta_\alpha^\beta L^a_b + \tfrac{1}{2}\delta^a_b\delta^\beta_\alpha H)|e_c\rangle$$

$$= \delta^\beta_\alpha\,\delta^a_c|e_b\rangle - \tfrac{1}{2}\,\delta_\alpha^\beta\,\delta^a_b|e_c\rangle + \tfrac{H}{2}\delta^a_b\,\delta^\beta_\alpha|e_c\rangle$$

$$\Longrightarrow \quad \boxed{ad - bc = 1}\,, \qquad bc = \frac{1}{2}(H-1)$$

$$\rightarrow \boxed{H = 2bc + 1 = ad + bc}$$

② $$\{Q^a_\alpha, Q^b_\beta\}|e_c\rangle = Q^a_\alpha(a\delta^b_c|e_\beta\rangle) + Q^b_\beta(a\,\delta^a_c|e_\alpha\rangle)$$

$$= ab\,\epsilon_{\alpha\beta}\epsilon^{ad}\delta^b_c|e_d\rangle + ab\,\delta^a_c\,\epsilon_{\beta\alpha}\epsilon^{bd}|e_d\rangle$$

$$\langle e_c|\{Q^a_\alpha, Q^b_\beta\}|e_c\rangle = ab\,\epsilon_{\alpha\beta}\,(\underbrace{\epsilon^{ac}\delta^b_c - \epsilon^{bc}\delta^a_c}_{\epsilon^{ab}}) \equiv C\,\epsilon_{\alpha\beta}\epsilon^{ab}$$

$$\boxed{C = ab}$$

①

$$S\cdot Q^c_\alpha|\phi^1_a\,\phi^2_b\rangle = a_1 e^{i\frac{p_2}{2}}\,\delta^c_a\, S|\psi^1_\alpha\,\phi^2_b\rangle + a_2\,\delta^c_b\, S|\phi^1_a\,\psi^2_\alpha\rangle$$

$$= a_1 e^{i\frac{p_2}{2}}\delta^c_a\left(K\,|\psi^2_\alpha\,\phi^1_b\rangle + L\,|\phi^2_b\,\psi^1_\alpha\rangle\right)$$

$$+ a_2\,\delta^c_b\,\left(G\,|\psi^2_\alpha\,\phi^1_a\rangle + H\,|\phi^2_a\,\psi^1_\alpha\rangle\right)$$

$$Q^c_\alpha\cdot S\,|\phi^1_a\,\phi^2_b\rangle = Q^c_\alpha\left(A^+\,|\phi^2_a\,\phi^1_b\rangle + A^-\,|\phi^2_b\,\phi^1_a\rangle + \frac{C}{2}\epsilon_{ab}\epsilon^{\alpha\beta}|\psi^2_\alpha\,\psi^1_\beta\rangle\right)$$

$$= A^+\left(a_2\, e^{i\frac{p_1}{2}}\,\delta^c_a\,|\psi^2_\alpha\,\phi^1_b\rangle + a_1\,\delta^c_b\,|\phi^2_a\,\psi^1_\alpha\rangle\right)$$

$$+ A^-\left(a_2\, e^{i\frac{p_1}{2}}\,\delta^c_b\,|\psi^2_\alpha\,\phi^1_a\rangle + a_1\,\delta^c_a\,|\phi^2_b\,\psi^1_\alpha\rangle\right)$$

$$-\frac{1}{2}C\left(\delta^c_a\delta^d_b - \delta^d_a\delta^c_b\right)\left(b_2\, e^{i\frac{p_1}{2}}\,|\phi^2_d\,\psi^1_\alpha\rangle + b_1\,|\psi^2_\alpha\,\phi^1_d\rangle\right)$$

$$\boxed{\begin{aligned}
a_1 e^{i\frac{p_2}{2}} K &= a_2 e^{i\frac{p_1}{2}} A^+ - \frac{1}{2} b_1 C \\
a_1 e^{i\frac{p_2}{2}} L &= a_1 A^- - \frac{1}{2} b_2 e^{i\frac{p_1}{2}} C \\
a_2 G &= a_2 e^{i\frac{p_1}{2}} A^- + \frac{1}{2} b_1 C \\
a_2 H &= a_1 A^+ + \frac{1}{2} b_2 e^{i\frac{p_1}{2}} C
\end{aligned}}$$

② Do the same for the other 2-particle states

$$
\begin{aligned}
1 &= \prod_{j=1}^{K_4} \frac{x_{4,j}^+}{x_{4,j}^-}\,, \\
1 &= \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} + \frac{i}{2}\eta_1}{u_{1,k} - u_{2,j} - \frac{i}{2}\eta_1} \prod_{j=1}^{K_4} \frac{1 - g^2/2x_{1,k}x_{4,j}^{+\eta_1}}{1 - g^2/2x_{1,k}x_{4,j}^{-\eta_1}}\,, \\
1 &= \prod_{\substack{j=1\\ j\neq k}}^{K_2} \frac{u_{2,k} - u_{2,j} - i\eta_1}{u_{2,k} - u_{2,j} + i\eta_1} \prod_{j=1}^{K_3} \frac{u_{2,k} - u_{3,j} + \frac{i}{2}\eta_1}{u_{2,k} - u_{3,j} - \frac{i}{2}\eta_1} \prod_{j=1}^{K_1} \frac{u_{2,k} - u_{1,j} + \frac{i}{2}\eta_1}{u_{2,k} - u_{1,j} - \frac{i}{2}\eta_1}\,, \\
1 &= \prod_{j=1}^{K_2} \frac{u_{3,k} - u_{2,j} + \frac{i}{2}\eta_1}{u_{3,k} - u_{2,j} - \frac{i}{2}\eta_1} \prod_{j=1}^{K_4} \frac{x_{3,k} - x_{4,j}^{+\eta_1}}{x_{3,k} - x_{4,j}^{-\eta_1}}\,, \\
1 &= \left(\frac{x_{4,k}^-}{x_{4,k}^+}\right)^L \prod_{\substack{j=1\\ j\neq k}}^{K_4} \left(\frac{x_{4,k}^{+\eta_1} - x_{4,j}^{-\eta_1}}{x_{4,k}^{-\eta_2} - x_{4,j}^{+\eta_2}} \frac{1 - g^2/2x_{4,k}^+ x_{4,j}^-}{1 - g^2/2x_{4,k}^- x_{4,j}^+} \sigma^2(x_{4,k}, x_{4,j}) \right) \\
&\qquad \times \prod_{j=1}^{K_1} \frac{1 - g^2/2x_{4,k}^{-\eta_1} x_{1,j}}{1 - g^2/2x_{4,k}^{+\eta_1} x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^{-\eta_1} - x_{3,j}}{x_{4,k}^{+\eta_1} - x_{3,j}} \prod_{j=1}^{K_5} \frac{x_{4,k}^{-\eta_2} - x_{5,j}}{x_{4,k}^{+\eta_2} - x_{5,j}} \prod_{j=1}^{K_7} \frac{1 - g^2/2x_{4,k}^{-\eta_2} x_{7,j}}{1 - g^2/2x_{4,k}^{+\eta_2} x_{7,j}}\,, \\
1 &= \prod_{j=1}^{K_6} \frac{u_{5,k} - u_{6,j} + \frac{i}{2}\eta_2}{u_{5,k} - u_{6,j} - \frac{i}{2}\eta_2} \prod_{j=1}^{K_4} \frac{x_{5,k} - x_{4,j}^{+\eta_2}}{x_{5,k} - x_{4,j}^{-\eta_2}}\,, \\
1 &= \prod_{\substack{j=1\\ j\neq k}}^{K_6} \frac{u_{6,k} - u_{6,j} - i\eta_2}{u_{6,k} - u_{6,j} + i\eta_2} \prod_{j=1}^{K_5} \frac{u_{6,k} - u_{5,j} + \frac{i}{2}\eta_2}{u_{6,k} - u_{5,j} - \frac{i}{2}\eta_2} \prod_{j=1}^{K_7} \frac{u_{6,k} - u_{7,j} + \frac{i}{2}\eta_2}{u_{6,k} - u_{7,j} - \frac{i}{2}\eta_2}\,, \\
1 &= \prod_{j=1}^{K_6} \frac{u_{7,k} - u_{6,j} + \frac{i}{2}\eta_2}{u_{7,k} - u_{6,j} - \frac{i}{2}\eta_2} \prod_{j=1}^{K_4} \frac{1 - g^2/2x_{7,k}x_{4,j}^{+\eta_2}}{1 - g^2/2x_{7,k}x_{4,j}^{-\eta_2}}\,,
\end{aligned}
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