The scenario of perfect fluid hydrodynamics

in A+A collisions at RHIC

T. Csörgő
with M. Csanád, R. Lacey et al.

- **Introduction:**
  - RHIC Scientists serve up "Perfect Liquid", April 18, 2005
  - BRAHMS, PHENIX, PHOBOS, STAR White Papers, NPA, 2005
  - AIP Top Physics Story 2005
- **Hydrodynamics and scaling of soft observables**
  - Exact (i.e. not numerical) integrals of fluid dynamics
    - non-relativistic and relativistic solutions
  - Evidence for hydrodynamic scaling in RHIC data
    - Scaling of slope parameters
    - Scaling of Bose-Einstein /HBT radii
    - Universal scaling of elliptic and higher order flows
- **Intermediate p_t region: breaking of the hydro scaling**

Discovering New Laws

"In general we look for a new law by the following process.
First we guess it.
Then we compare the consequences of the guess to see
what would be implied if this law that we guessed is right.
Then we compare the result of the computation to nature,
with experiment or experience, compare it directly with observation,
to see if it works.
If it disagrees with experiment it is wrong.

In that simple statement is the key to science.
It does not make any difference how beautiful your guess is.
It does not make any difference how smart you are,
who made the guess, or what his name is —
if it disagrees with experiment it is wrong."

/R.P. Feynman/

Phases of QCD Matter, EoS

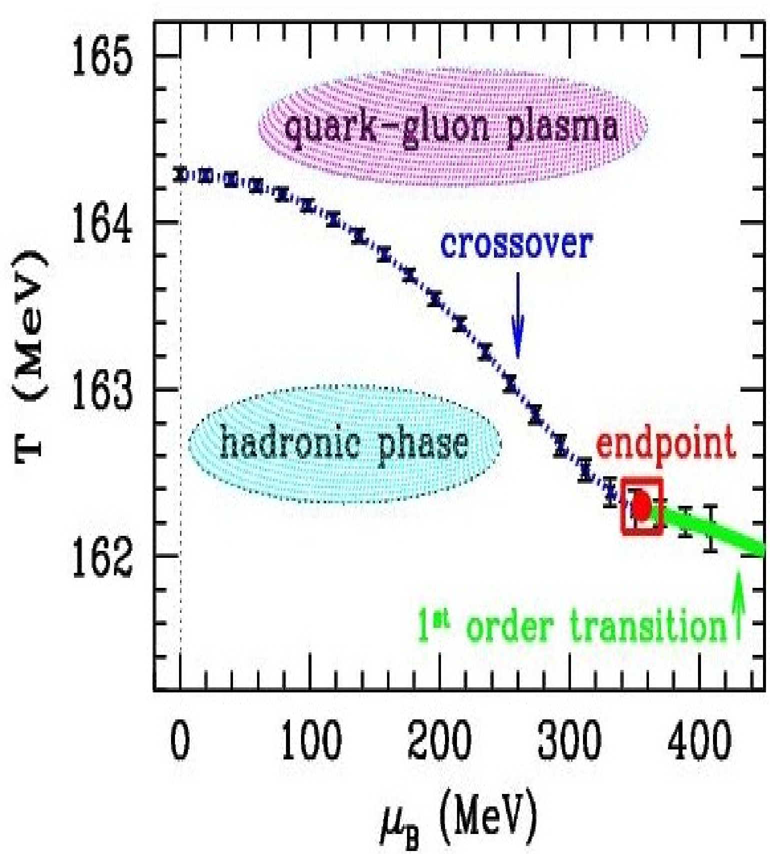
Quark Gluon Plasma

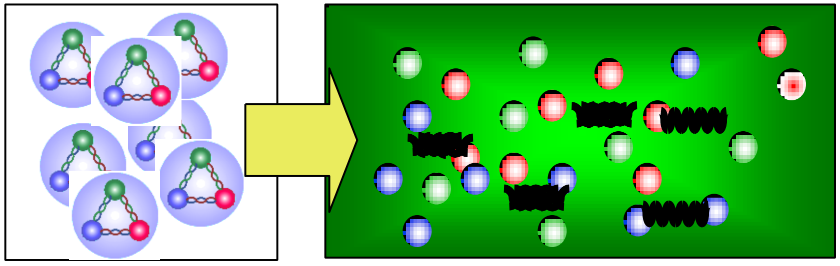
“Ionize” nucleons with heat

“Compress” them with density

New state(s?) of matter

Z. Fodor and S.D. Katz:
T_c = 164 ± 2 -> 189 ± 8 MeV, QM’05(?)
even at finite baryon density,
Cross over like transition.
(hep-lat/0106002, hep-lat/0402006)





Most recent result:

T_c=176±3MeV (~2 terakelvin)
(hep-ph/0511166)

Input for hydrodynamics

Nonrelativistic dynamics of perfect fluids

- **Equations of nonrelativistic hydro:**

$$\partial_t n + \nabla \cdot (n\mathbf{v}) = 0\,,$$
$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -(\nabla p)/(mn)\,,$$
$$\partial_t \epsilon + \nabla \cdot (\epsilon\mathbf{v}) = -p\nabla\cdot\mathbf{v}\,,$$

- **Not closed, EoS needed:**

$$p = nT\,, \qquad \epsilon = \kappa(T)nT,$$

- **Perfect fluid: definitions are equivalent, term used by PDG**

  # 1: no bulk and shear viscosities, and no heat conduction.

  # 2: energy-momentum tensor diagonal in the local rest frame.

- **ideal fluid: ambiguously defined term, discouraged**

  #1: keeps its volume, but conforms to the outline of its container

  #2: an inviscid fluid

Exact, ellipsoidal, nonrelativistic solutions

- **A new family of PARAMETRIC, exact, scale-invariant solutions**
  - T. Cs. Acta Phys. Polonica B37 (2006) 1001, hep-ph/0111139

$$
\begin{aligned}
n(t,\mathbf{r}) &= n_0 \frac{V_0}{V}\nu(s) \\
\mathbf{v}(t,\mathbf{r}) &= \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z \right) \\
T(t,\mathbf{r}) &= T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \mathcal{T}(s) \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0} \int_0^s \frac{du}{\mathcal{T}(u)} \right)
\end{aligned}
$$

$$
s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}
$$

$$
X\ddot X = Y \ddot Y = Z \ddot Z = \frac{T_i}{m} \left(\frac{V_0}{V}\right)^{1/\kappa}
$$

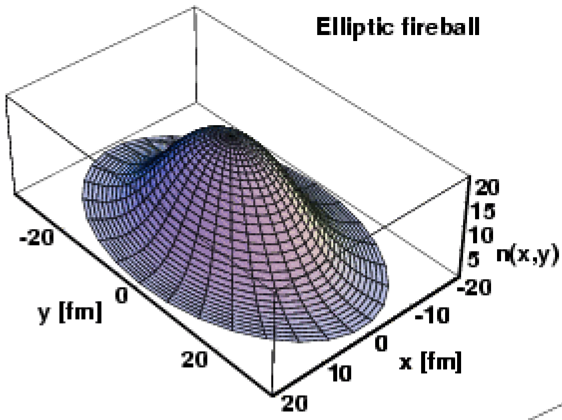
- **Temperature scaling function is arbitrary, e.g. homogeneous temperature $\Rightarrow$ Gaussian density**

ányi-Bondorf-Garpman profile:

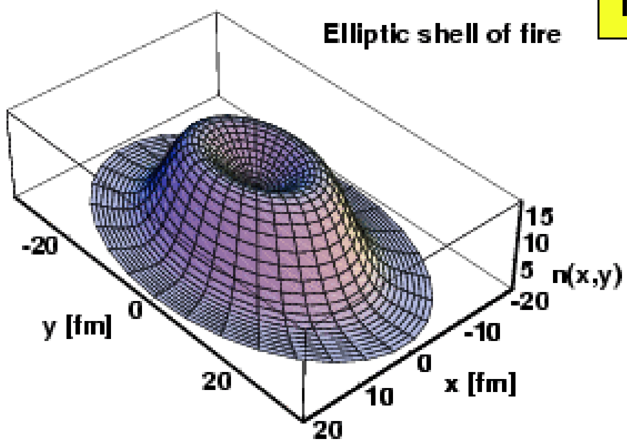
$$
\begin{aligned}
\mathcal{T}(s) &= \frac{1}{1+bs} \\
\nu(s) &= (1+bs)\exp\left[-\frac{T_i}{2T_0}(s + bs^2/2) \right]
\end{aligned}
$$

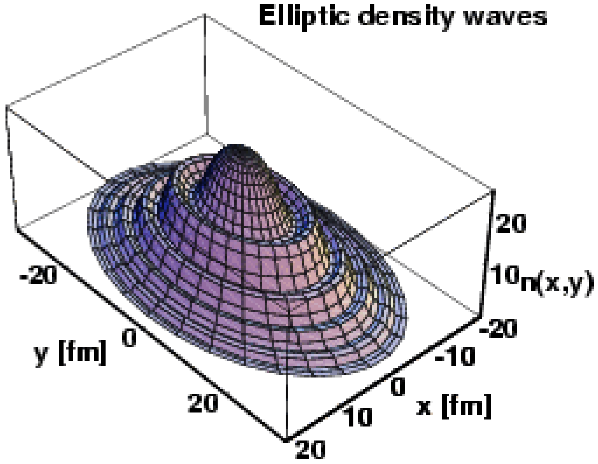
$$
\begin{aligned}
\mathcal{T}(s) &= (1-s)\,\Theta(1-s) \\
\nu(s) &= (1-s)^{\alpha}\,\Theta(1-s)
\end{aligned}
$$

Exact integrals of fluid dynamics



Examples of density profiles
- Fireball
- Ring of fire
- Embedded shells of fire
Exact integrals of hydro
No numerical viscosity

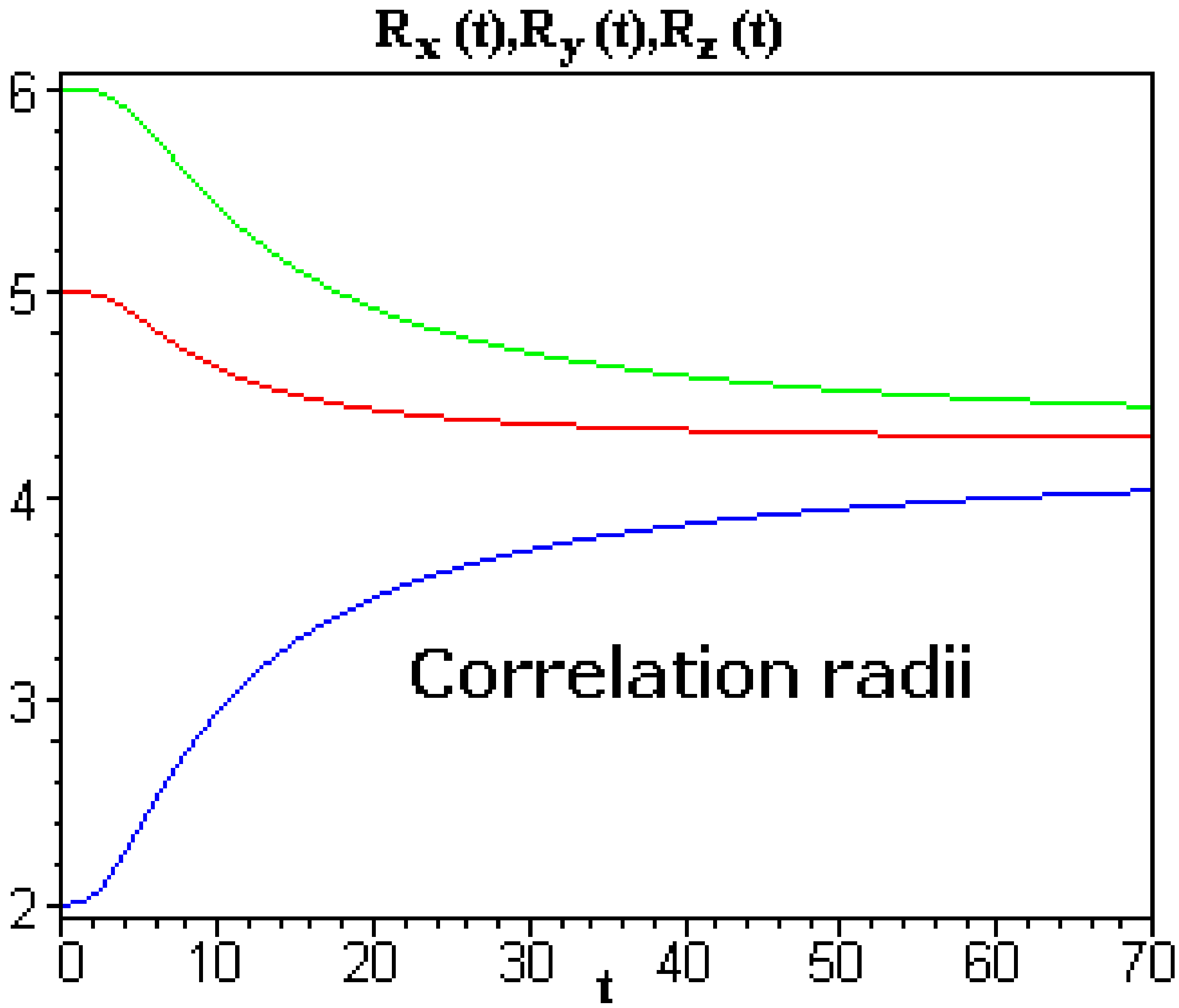




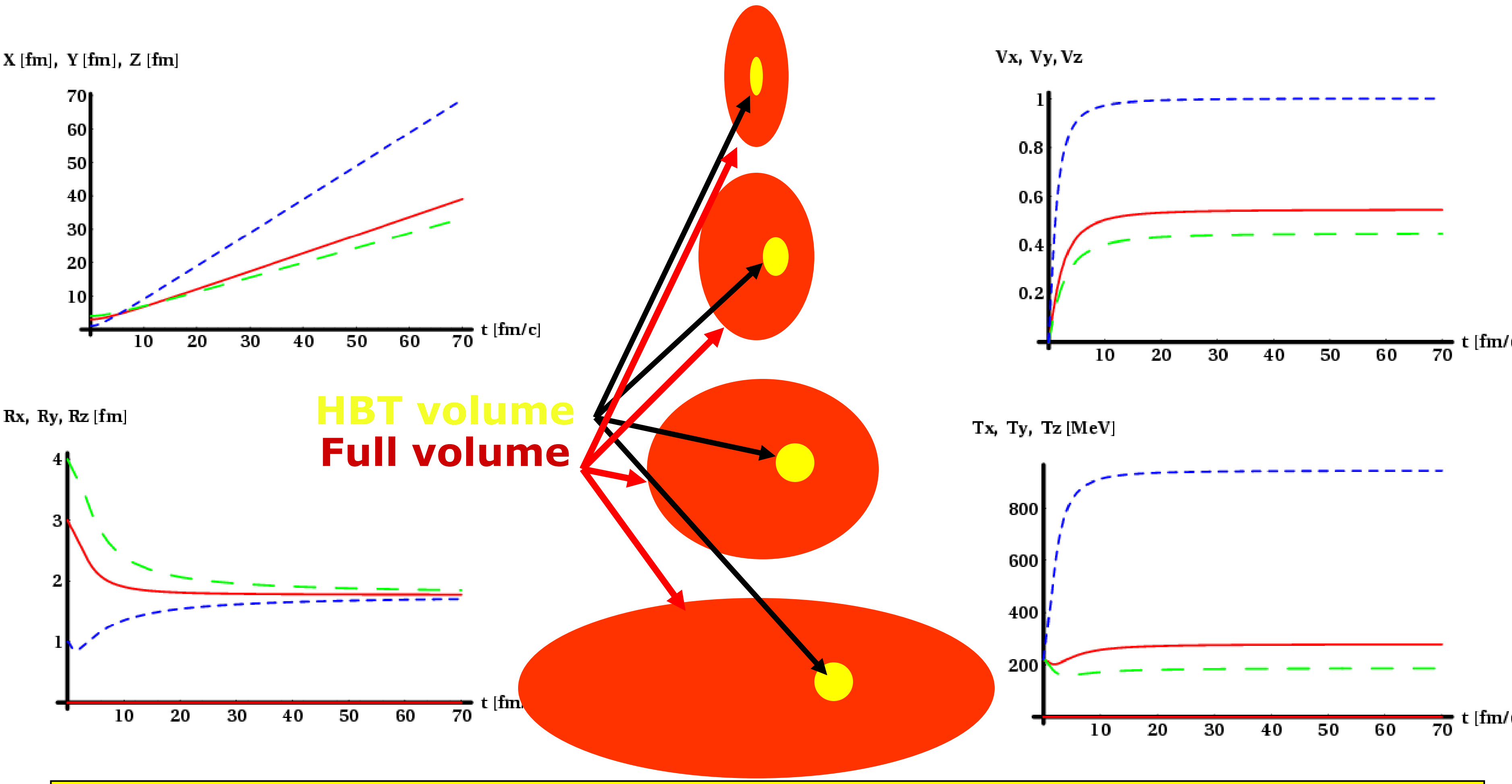
Time evolution of the scales (X,Y,Z)
follows a classic potential motion.
Scales at freeze out -> observables.
info on history <u>LOST</u>!
No go theorem - constraints
on initial conditions
(penetrating probels) indispensable.

Examples of exact hydro results

- **Propagate the hydro solution in time numerically:**

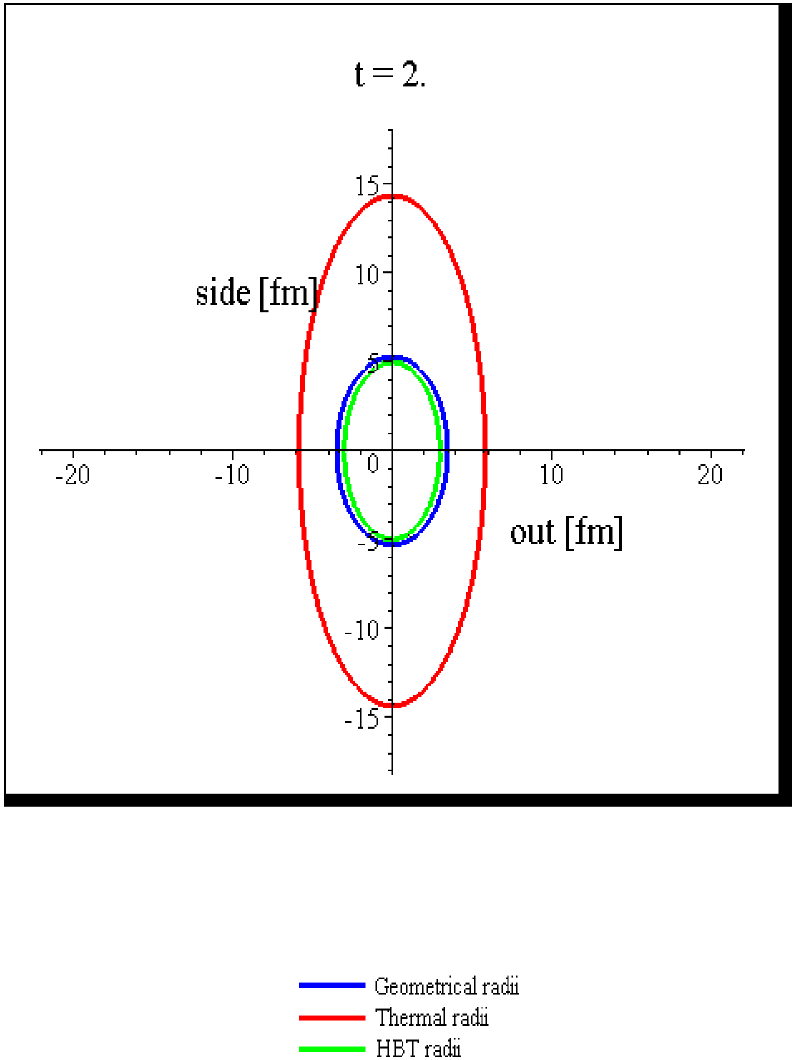


Solution of the "HBT puzzle"



Geometrical sizes keep on increasing. Expansion velocities tend to constants.
HBT radii R_x, R_y, R_z approach a direction independent constant.
Slope parameters tend to direction dependent constants.
General property, independent of initial conditions - a beautiful exact result.

Geometrical & thermal & HBT radii



3d analytic hydro: exact time evolution

geometrical size (fugacity ~ const)
Thermal sizes (velocity ~ const)
HBT sizes (phase-space density ~ const)

HBT dominated by the smaller of the geometrical and thermal scales

nucl-th/9408022, hep-ph/9409327
hep-ph/9509213, hep-ph/9503494

HBT radii approach a constant of time
HBT volume becomes spherical
HBT radii -> thermal ~ constant sizes

hep-ph/0108067, nucl-th/0206051
animation by Máté Csanád

Scaling predictions of fluid dynamics

$$T'_x = T_f + m\dot{X}_f^2\,,$$
$$T'_y = T_f + m\dot{Y}_f^2\,,$$
$$T'_z = T_f + m\dot{Z}_f^2\,.$$

- Slope parameters increase linearly with mass
- Elliptic flow is a universal function its variable w is proportional to transverse kinetic energy and depends on slope differences.

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

$$w = \frac{k_t^2}{4m}\left(\frac{1}{T'_y} - \frac{1}{T_x}\right),$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

Inverse of the HBT radii increase linearly with mass analysis shows that they are asymptotically the same

Relativistic correction: m -> m_t

hep-ph/0108067,
nucl-th/0206051

$$R'^{-2}_x = X_f^{-2}\left(1 + \frac{m}{T_f}\dot{X}_f^2\right),$$
$$R'^{-2}_y = Y_f^{-2}\left(1 + \frac{m}{T_f}\dot{Y}_f^2\right),$$
$$R'^{-2}_z = Z_f^{-2}\left(1 + \frac{m}{T_f}\dot{Z}_f^2\right).$$

Relativistic Perfect Fluids

- **Rel. hydrodynamics of perfect fluids is defined by:**

$$\partial_\mu (n u^\mu) = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p g^{\mu\nu}$$

- **A recent family of exact solutions: nucl-th/0306004**

$$u^\mu = \frac{x^\mu}{\tau}$$
$$n(t,\mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s)$$
$$p(t,\mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$
$$T(t,\mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)}$$

$$u_\nu u^\mu \partial_\mu p + (\epsilon + p) u^\mu \partial_\mu u_\nu - \partial_\nu p = 0\,,$$
$$u^\mu \partial_\mu T + \frac{1}{\kappa} T \partial_\mu u^\mu = 0\,.$$

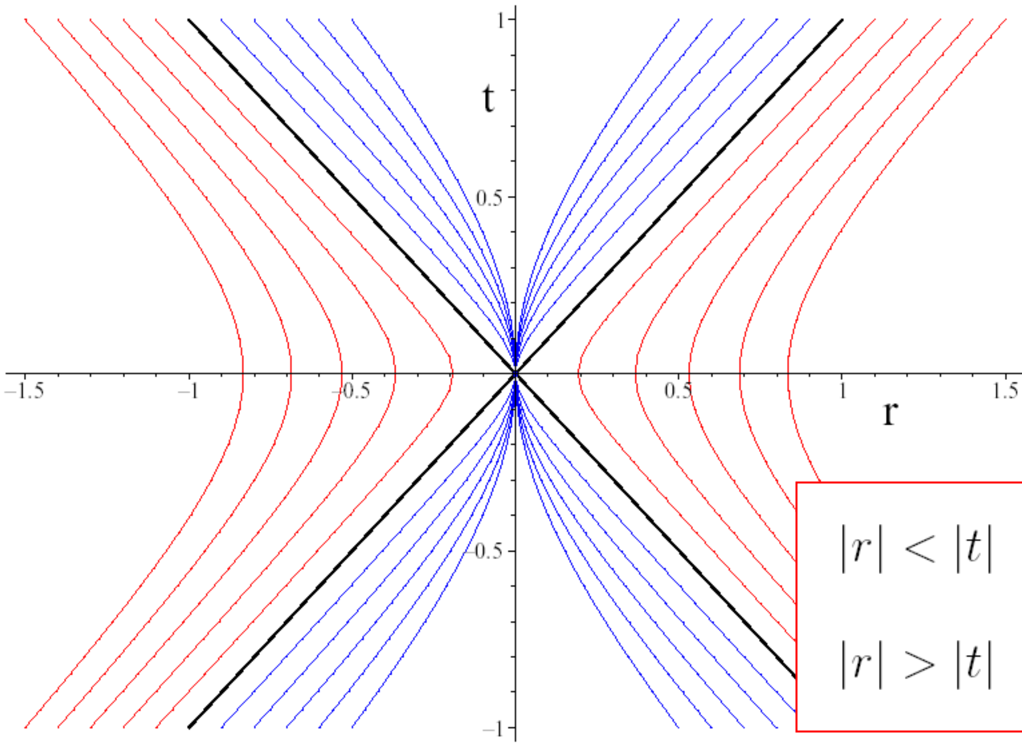
$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}\,,$$

$$\epsilon = mn + \kappa p\,,$$
$$p = nT.$$

- **Overcomes two shortcomings of Bjorken's solution:**
  - **Yields finite rapidity distribution, includes transverse flow**
- **Hubble flow ⇒ lack of acceleration.** $u^\mu \partial_\mu u_\nu = 0$
- **Accelerating, new rel. hydro solutions: nucl-th/0605070**

Solutions of Relativistic Perfect Fluids

- **A new family of exact solutions:**
  - **T. Cs, M. I. Nagy, M. Csanád: nucl-th/0605070**
- **Overcomes two shortcomings of Bjorken's solution:**
  - **Yields Finite Rapidity distribution, similarly to Landau's solution**
  - **Includes relativistic acceleration of the matter**
  - **Works in 1+1 and 1+3 spherically symmetric expansions**



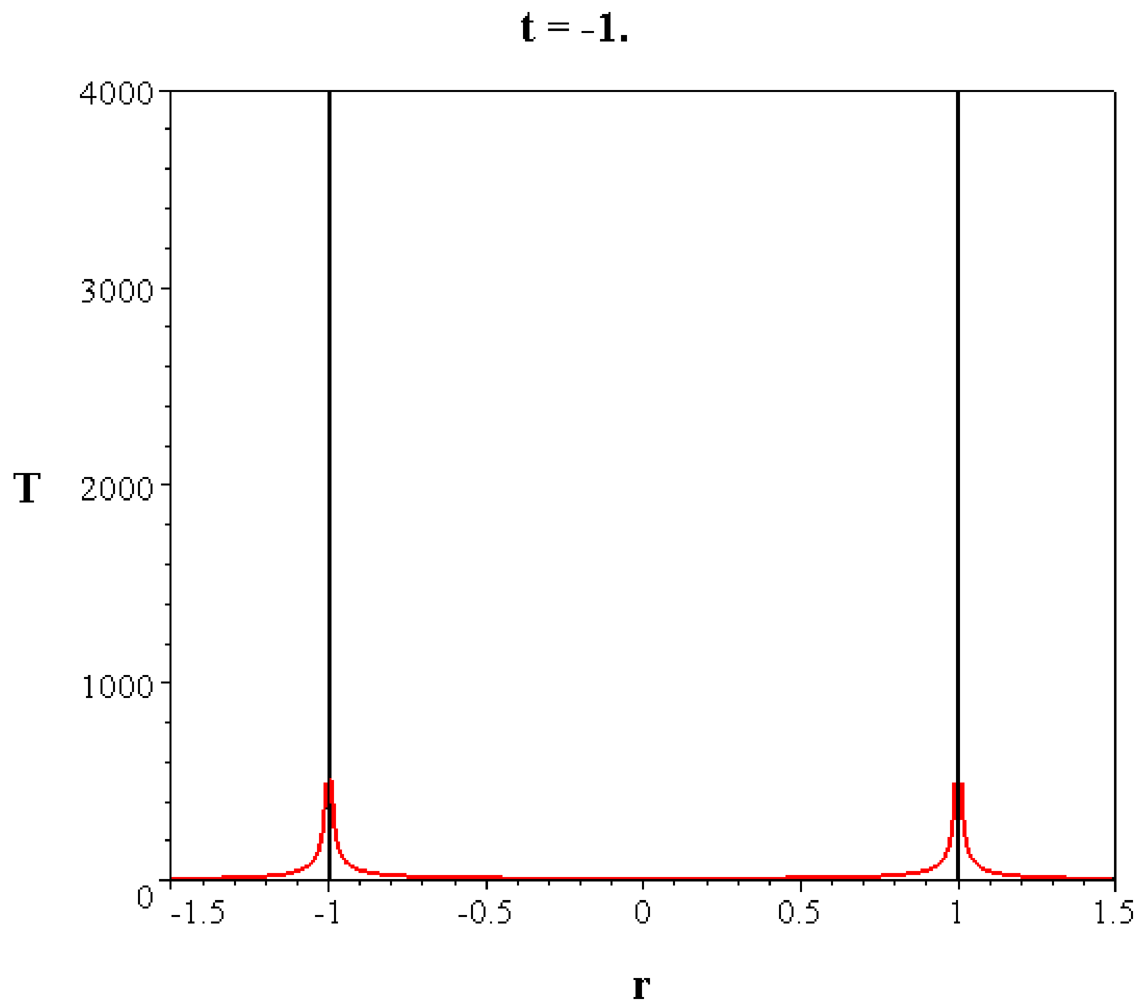
$$
\begin{aligned}
v &= \tanh \lambda\eta, \\
n &= n_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \nu(s), \\
T &= T_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \frac{1}{\nu(s)}.
\end{aligned}
$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = 0.$$

$$
\begin{aligned}
|r| < |t| \;&:\; s(\tau,\eta) = \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \sinh\left((\lambda-1)\,\eta\right), \\
|r| > |t| \;&:\; s(\tau,\eta) = \left(\frac{\tau_0}{\tau}\right)^{\lambda-1} \cosh\left((\lambda-1)\,\eta\right).
\end{aligned}
$$

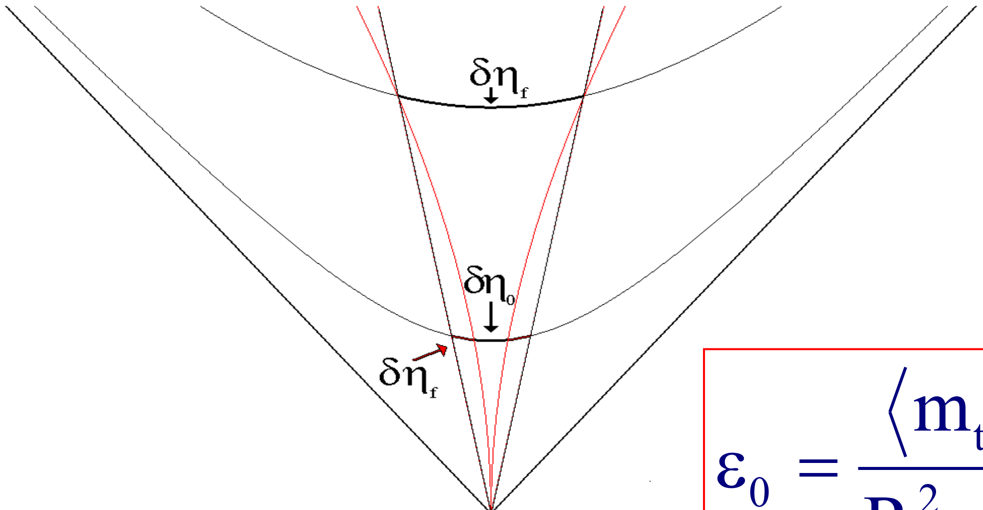
Animation of the new exact solution

nucl-th/0605070
dimensionless
λ = 2
1+1 d
both internal
and external
looks like A+A



nucl-th/0605070: advanced estimate of ε_0

- **Width of dn/dy distribution is due to acceleration:**
  - **acceleration yields longitudinal explosion, thus**
  - **Bjorken estimate underestimates the initial energy density**



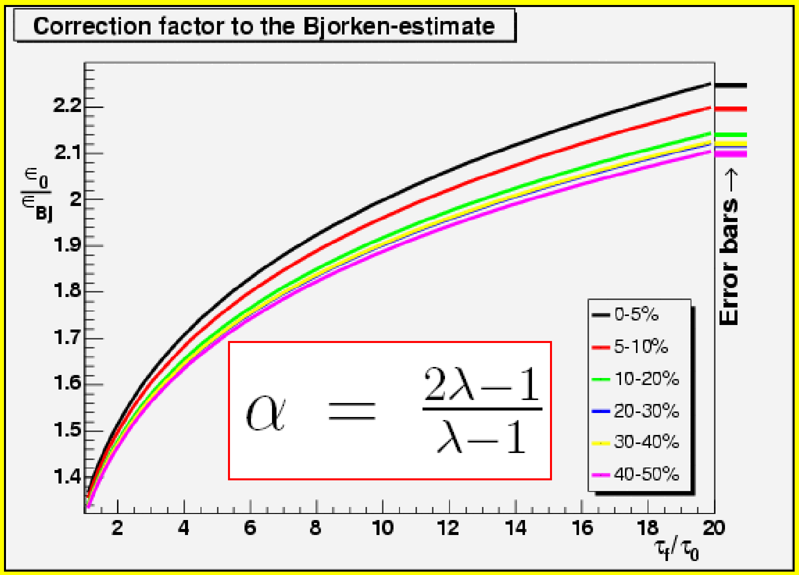
$$v = \tanh \lambda\eta,$$
$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \nu(s),$$
$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda} \frac{1}{\nu(s)}.$$

$$\varepsilon_0 = \frac{\langle m_t \rangle}{R^2 \pi \tau_0} \frac{dn}{d\eta_0} = \varepsilon_{Bj} \frac{dy}{d\eta_f} \frac{d\eta_f}{d\eta_0}$$

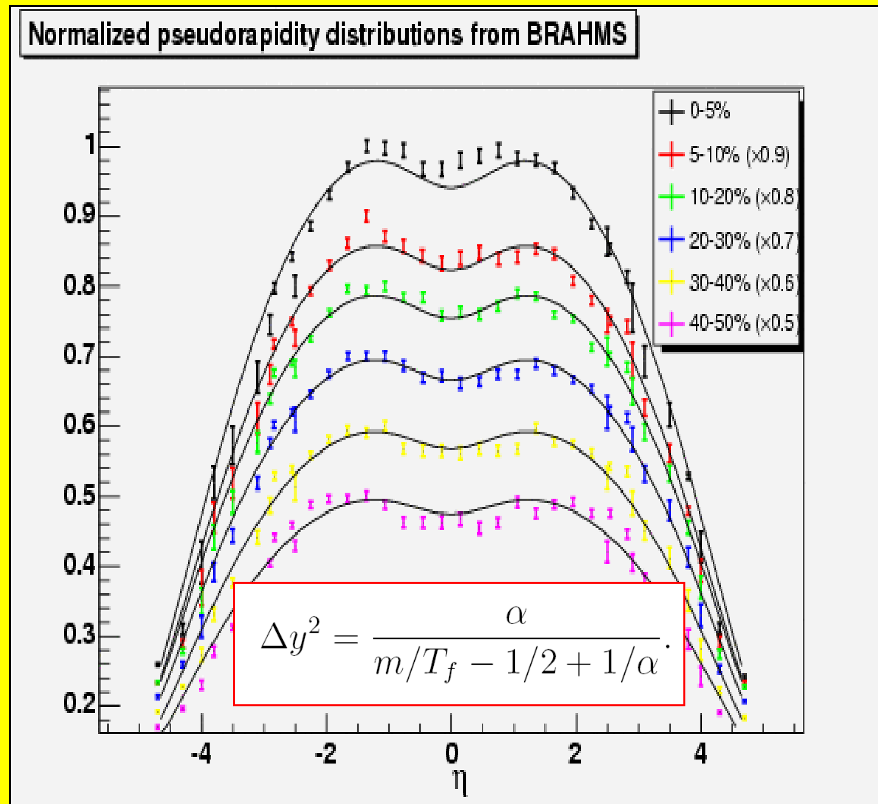
$$\frac{\varepsilon_0}{\varepsilon_{Bj}} = \frac{\alpha}{\alpha - 2} \left(\frac{\tau_f}{\tau_0}\right)^{\frac{1}{\alpha-2}} = (2\lambda - 1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}$$

nucl-th/0605070: advanced estimate of ε_0

- **Let us fit BRAHMS dn/deta data**
- **dn/deta width yields a correction factor of ~ 2.0 - 2.2**
- **Yields inital energy density of $\varepsilon \sim$ 10- 30 GeV/fm^3**
  - **a correction factor > 1 @ SPS!**

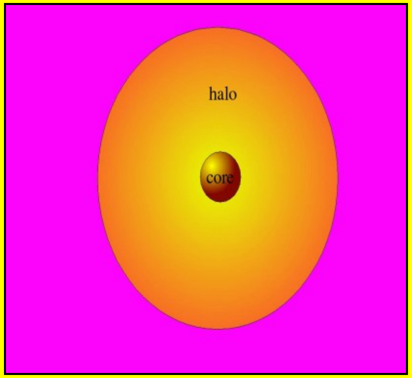


$$v = \tanh \lambda\eta,$$

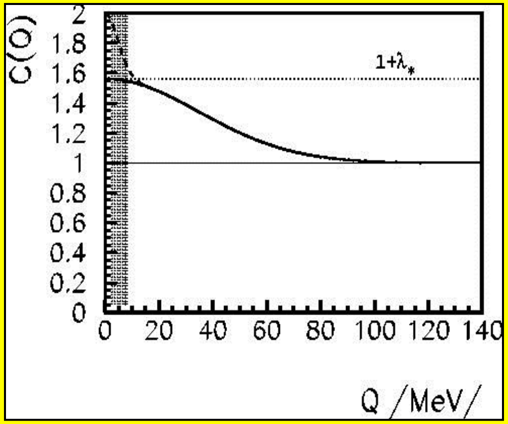


Principles for Buda-Lund hydro model

- **Analytic expressions for all the observables**
- **3d expansion, local thermal equilibrium, symmetry**
- **Goes back to known exact hydro solutions:**
  - **nonrel, Bjorken, and Hubble limits, 1+3 d ellipsoids**
  - **but phenomenology, extrapolation for unsolved cases**



- **Separation of the Core and the Halo**
  - **Core: perfect fluid dynamical evolution**
  - **Halo: decay products of long-lived resonances**

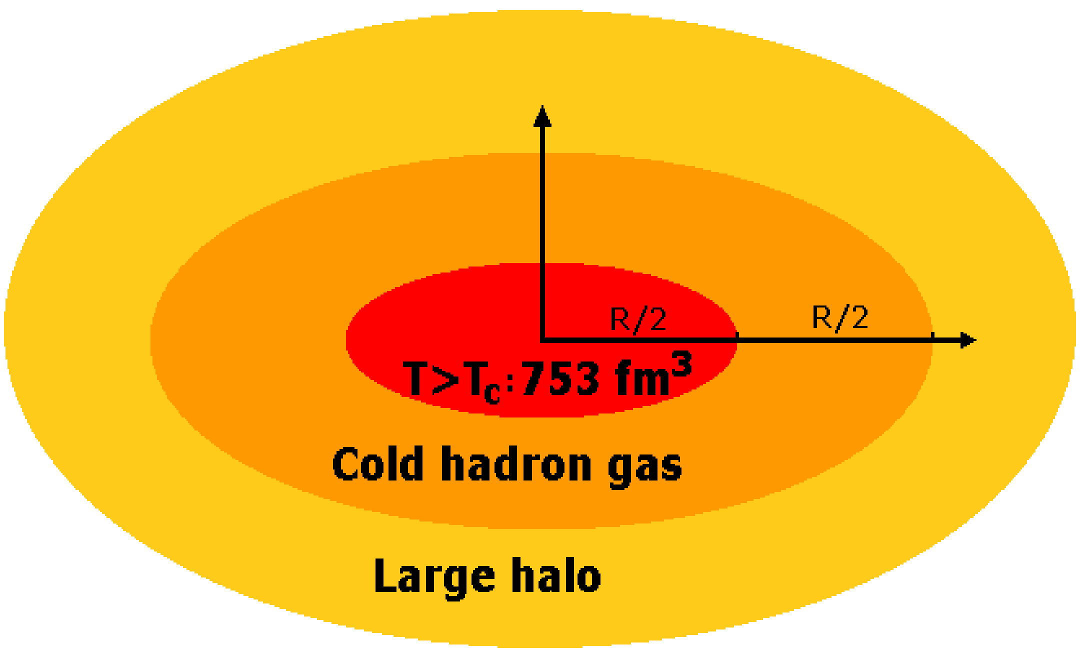


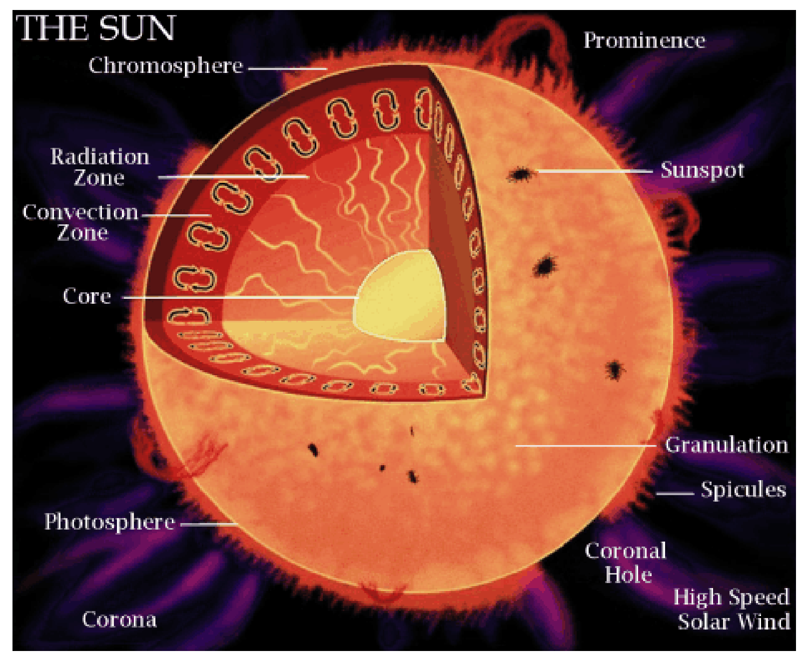
- **Missing links: phenomenology needed**
  - **search for accelerating ellipsoidal exact solutions**
  - **first accelerating solution found, nucl-th/0605070**

A useful analogy

Fireball at RHIC ⇔ our Sun

• Core	⇔	Sun
• Halo	⇔	Solar wind
• $T_{0,RHIC}$ ~ 210 MeV	⇔	$T_{0,SUN}$ ~ 16 million K
• $T_{surface,RHIC}$ ~ 100 MeV	⇔	$T_{surface,SUN}$ ~6000 K





Buda-Lund hydro model

The general form of the emission function:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

Calculation of observables with core-halo correction:

$$\mathrm{N}_1(p) = \frac{1}{\sqrt{\lambda_*}} \int d^4x S_c(p,x)$$

$$C(Q,p) = 1 + \left| \frac{\tilde{S}(Q,p)}{\tilde{S}(0,p)} \right|^2 = 1 + \lambda_* \left| \frac{\tilde{S}_c(Q,p)}{\tilde{S}_c(0,p)} \right|^2$$

Assuming profiles for

flux, temperature, chemical potential and flow

Buda-Lund hydro model

Invariant single particle spectrum:

$$N_1 = \frac{d^2n}{2\pi m_t dm_t dy} = \frac{g}{(2\pi)^3}\overline{E}\,\overline{V}\,\overline{C}\,\frac{1}{exp\left(\frac{p^\mu u_\mu(x_s)-\mu(x_s)}{T(x_s)}\right)+s_q}$$

**Invariant Buda-Lund correlation function:
oscillating, non-Gaussian prefactor!**

$$C_2(k_1,k_2) = 1 + \lambda_* \Omega(Q_{||}) \exp\left(-Q_{||}^2 R_{||}^2 - Q_{=}^2 R_{=}^2 - Q_{\perp}^2 R_{\perp}^2\right)$$

**Non-invariant Bertsch-Pratt parameterization,
in a Gaussian approximation:**

$$C_2(k_1,k_2) = 1 + \lambda_* \exp\left(-Q_o^2 R_o^2 - Q_s^2 R_s^2 - Q_l^2 R_l^2 - 2Q_{os}^2 R_o R_s\right)$$

Non-Gaussian BL form ⟶ Gaussian BP *approximation*:

$$R_{||,\Omega}^2 = R_{||}^2\left(1 + \frac{\overline{\Delta\eta}^2}{\overline{\eta}}\right)$$

The generalized Buda-Lund model

- **The original model was for axial symmetry only, central coll.**
- **In its general hydrodynamical form:**

  Based on 3d relativistic and non-rel solutions of perfect fluid dynamics:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

- **Have to assume special shapes:**
  - **Generalized Cooper-Frye prefactor:**

$$p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x$$

$$H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - **Four-velocity distribution:**

$$u^\mu = (\gamma, \sinh\eta_x, \sinh\eta_y, \sinh\eta_z)$$

  - **Temperature:**

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s} s\right)\left(1 + \frac{T_0 - T_e}{T_e}\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - **Fugacity:**

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

Scaling predictions of fluid dynamics

$$T'_x = T_f + m\dot{X}_f^2 \ ,$$
$$T'_y = T_f + m\dot{Y}_f^2 \ ,$$
$$T'_z = T_f + m\dot{Z}_f^2 \ .$$

- Slope parameters increase linearly with mass
- Elliptic flow is a universal function its variable w is proportional to transverse kinetic energy and depends on slope differences.

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

$$w = \frac{k_t^2}{4m}\left(\frac{1}{T'_y} - \frac{1}{T_x}\right),$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

Inverse of the HBT radii increase linearly with mass analysis shows that they are asymptotically the same

Relativistic correction: m -> m_t

hep-ph/0108067,
nucl-th/0206051

$$R'^{-2}_x = X_f^{-2}\left(1 + \frac{m}{T_f}\dot{X}_f^2\right),$$
$$R'^{-2}_y = Y_f^{-2}\left(1 + \frac{m}{T_f}\dot{Y}_f^2\right),$$
$$R'^{-2}_z = Z_f^{-2}\left(1 + \frac{m}{T_f}\dot{Z}_f^2\right).$$

Scaling predictions of Buda-Lund

$$T_x = T_0 + \overline{m}_t \dot{X}^2 \frac{T_0}{T_0 + \overline{m}_t a^2},$$

$$\overline{m}_t = m_t \cosh(\eta_s - y).$$

- Slope parameters increase linearly with transverse mass
- Elliptic flow is same universal function.
- Scaling variable w is prop. to generalized transv. kinetic energy and depends on effective slope diffs.

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

Inverse of the HBT radii increase linearly with mass analysis shows that they are asymptotically the same

Relativistic correction: m -> m_t

hep-ph/0108067,
nucl-th/0206051

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s, p)}{B(x_s, p) + s_q}\left(\frac{1}{X_i^2} + \frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2} + \frac{\dot{X}_i^2}{X_i^2}\right)$$

Some analytic Buda-Lund results

HBT radii widths

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2}+\frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2}+\frac{\dot{X}_i^2}{X_i^2}\right)$$

$$a^2 = \frac{T_0 - T_s}{T_s} = \left\langle \frac{\Delta T}{T} \right\rangle_r$$

Slopes, effective temperatures

$$\overline{m}_t = m_t \cosh(\eta_s - y).$$

$$\frac{1}{T_*} = \frac{1}{2}\left(\frac{1}{T_x}+\frac{1}{T_y}\right).$$

$$T_x = T_0 + \overline{m}_t \dot{X}^2 \frac{T_0}{T_0 + \overline{m}_t a^2},$$

Flow coefficients

$$\varepsilon = \frac{T_x - T_y}{T_x + T_y}.$$

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$

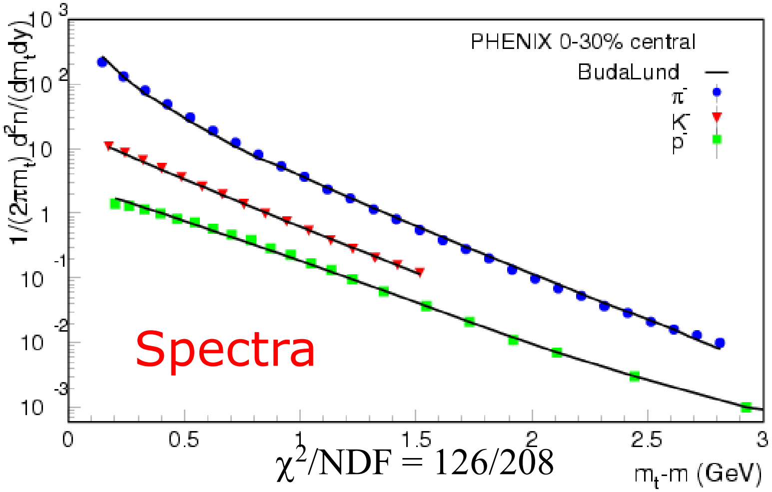
$$v_{2n+1} = 0$$

$$w = \frac{E_K}{2T_*}\varepsilon$$

$$E_K = \frac{p_t^2}{2\overline{m}_t}$$

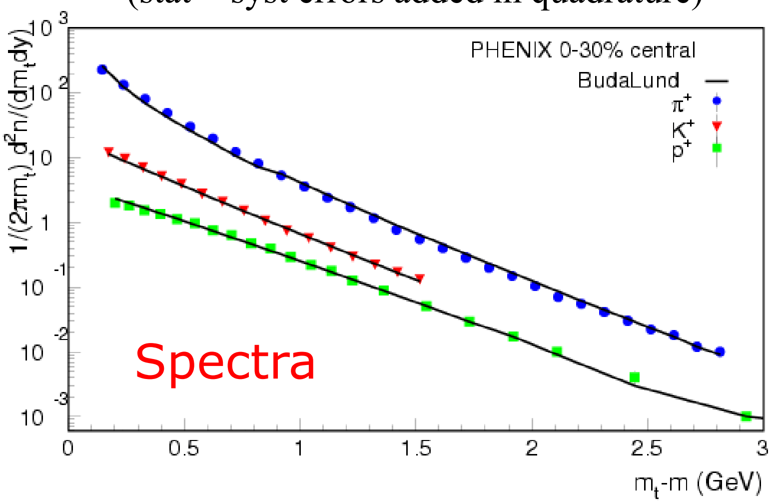
Buda-Lund hydro and Au+Au@RHIC

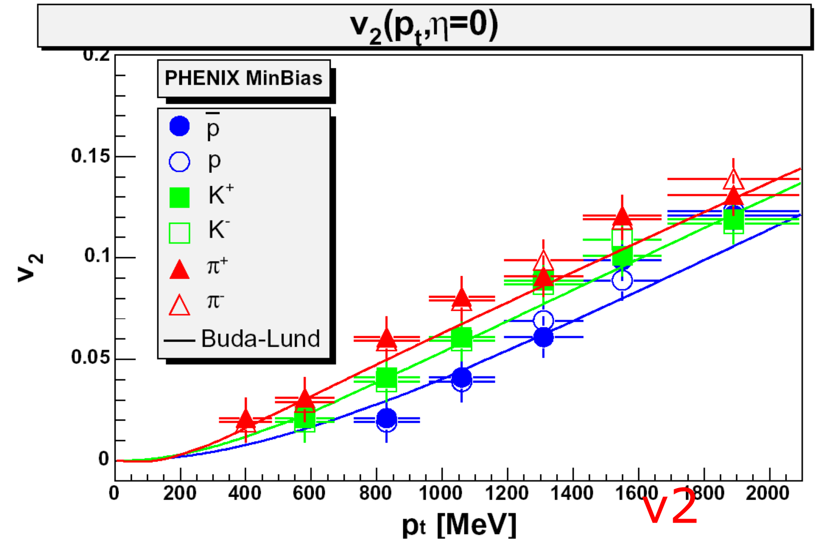
BudaLund v1.5 hydro fits to 200 AGeV Au+Au

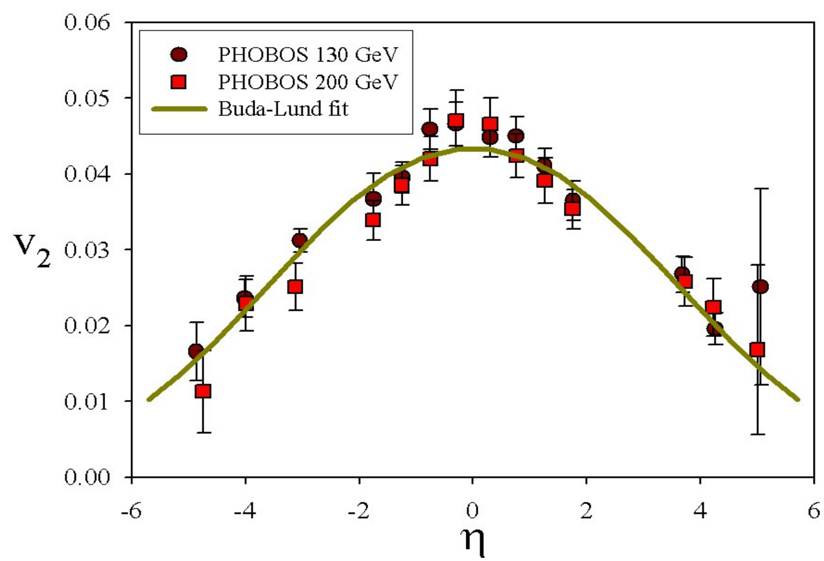


$\chi^2/NDF = 126/208$

(stat + syst errors added in quadrature)







nucl-th/0311102, nucl-th/0207016, nucl-th/0403074

Femptoscopy signal of sudden hadronization

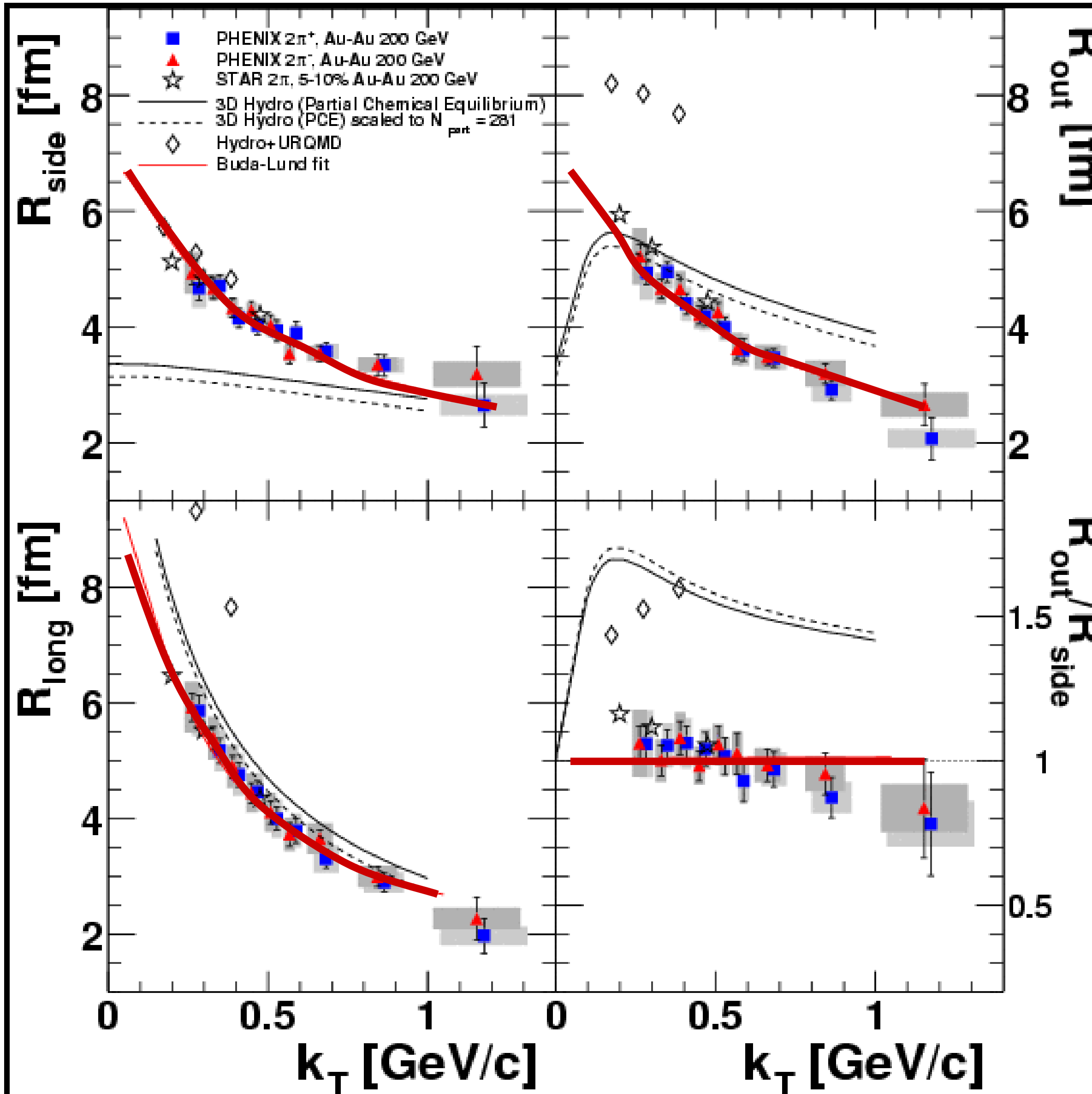
Buda-Lund hydro fit indicates hydro predicted (1994-96) scaling of HBT radii

T. Cs, L.P. Csernai hep-ph/9406365

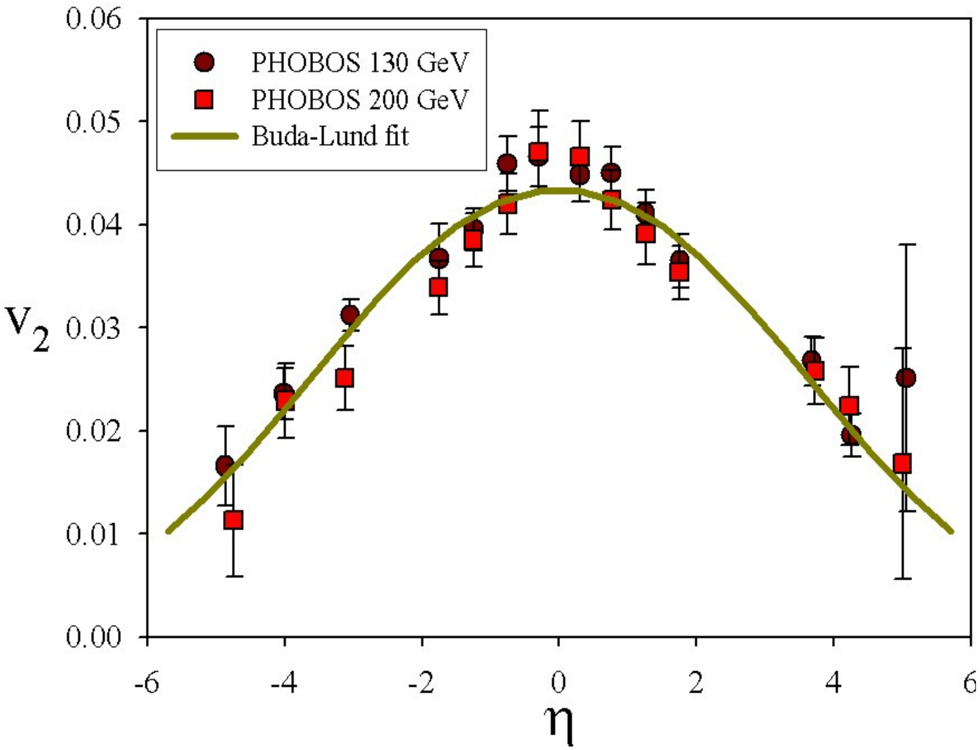
T. Cs, B. Lörstad hep-ph/9509213

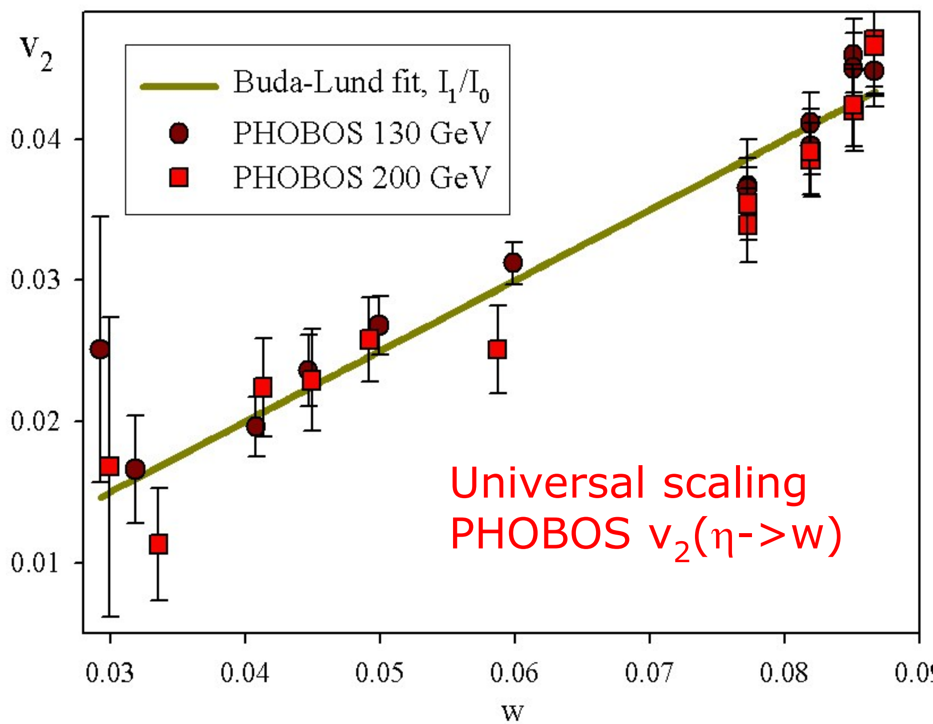
Hadrons with $T>T_c$: a hint for cross-over

M. Csanád, T. Cs, B. Lörstad and A. Ster, nucl-th/0403074



Confirmation





see nucl-th/0310040 and nucl-th/0403074,
R. Lacey@QM2005/ISMD 2005
A. Ster @ QM2005.

Slope parameters

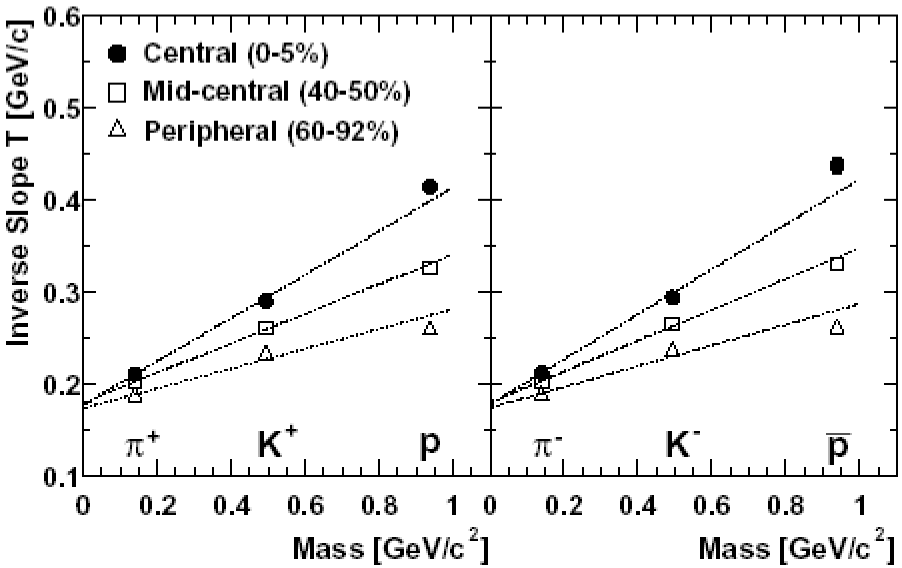
Buda-Lund rel. hydro formula:

$$T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

- **Exact non-rel. hydro solution:**

$$T'_x = T_f + m\dot{X}_f^2 \;,$$
$$T'_y = T_f + m\dot{Y}_f^2 \;,$$
$$T'_z = T_f + m\dot{Z}_f^2 \;.$$

Experimental test: PHENIX, STAR



Similar scaling of Bose-Einstein/HBT radii

$1/R^2_{eff} = 1/R^2_{geom} + 1/R^2_{therm}$
and $1/R^2_{therm} \sim m_t$

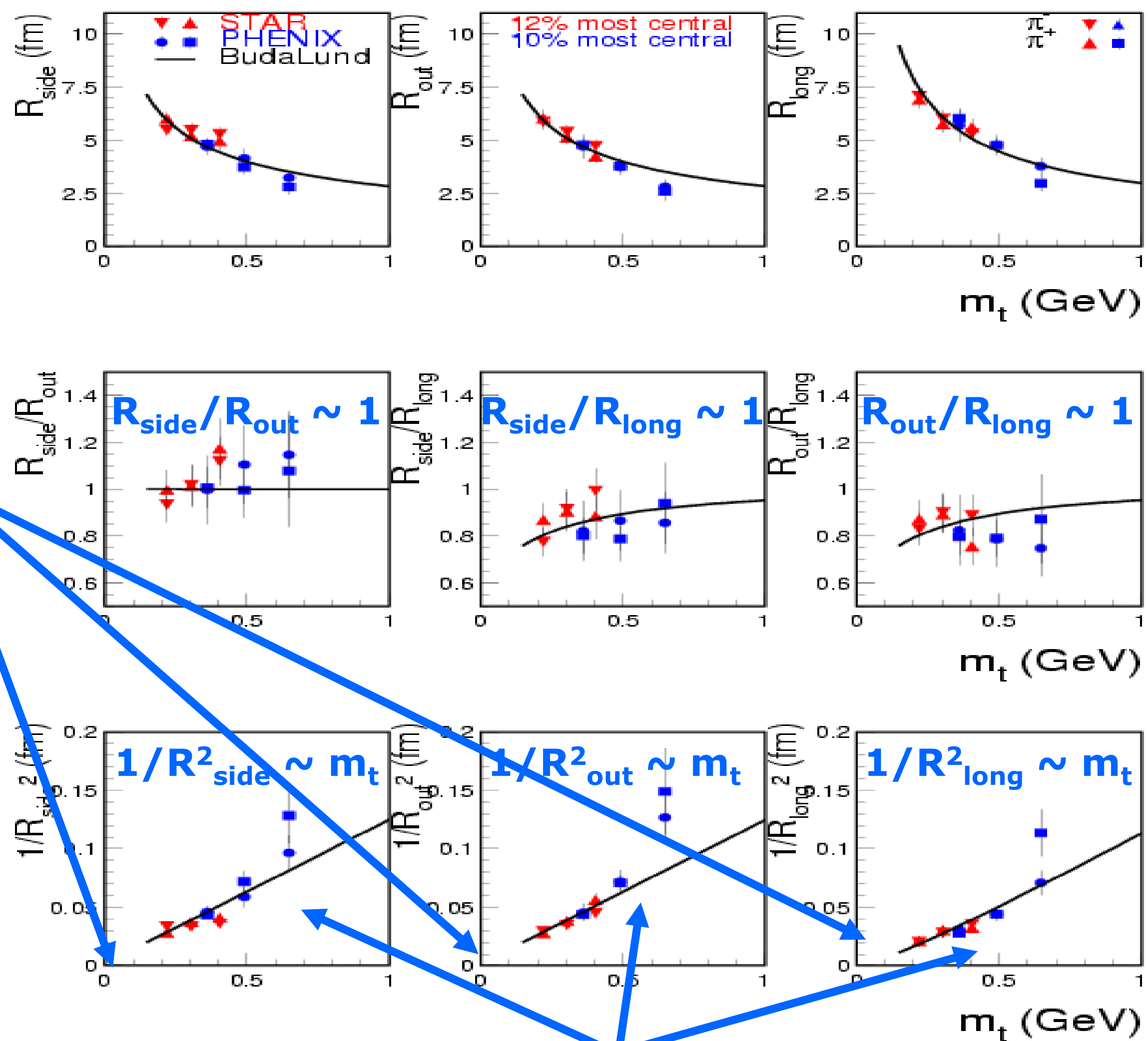
↓

intercept is nearly 0,
indicating $1/R_G^2 \sim 0$,

thus $\mu(x)/T(x)$ = const!

reason for success of
thermal models @ RHIC!

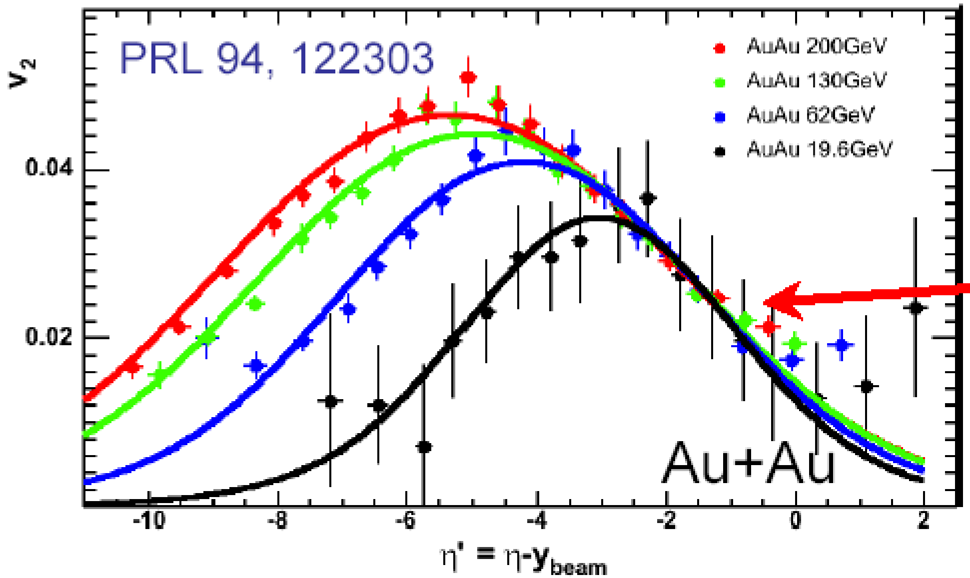
BudaLund hydro fits to 130 AGeV Au+Au



same slopes ~ fully developed, **3d Hubble flow**

Elliptic flow, limits

Extended longitudinal scaling: v_2



A surprising scaling!

Not an initial state effect

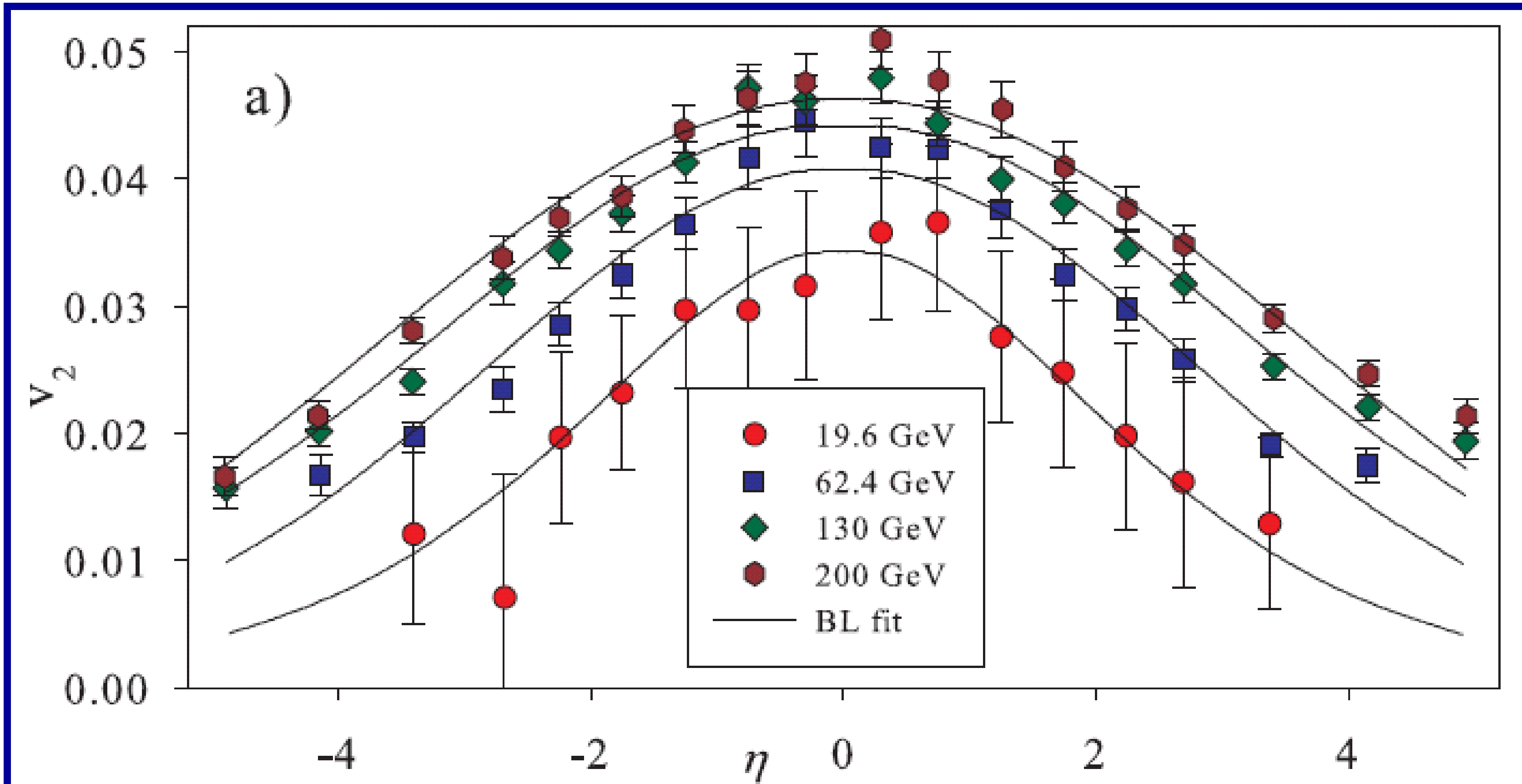
nucl-th/0505019

Scaling reproduced by the Buda-Lund parametrization of the emitting source.

G. Veres, PHOBOS data, proc QM2005

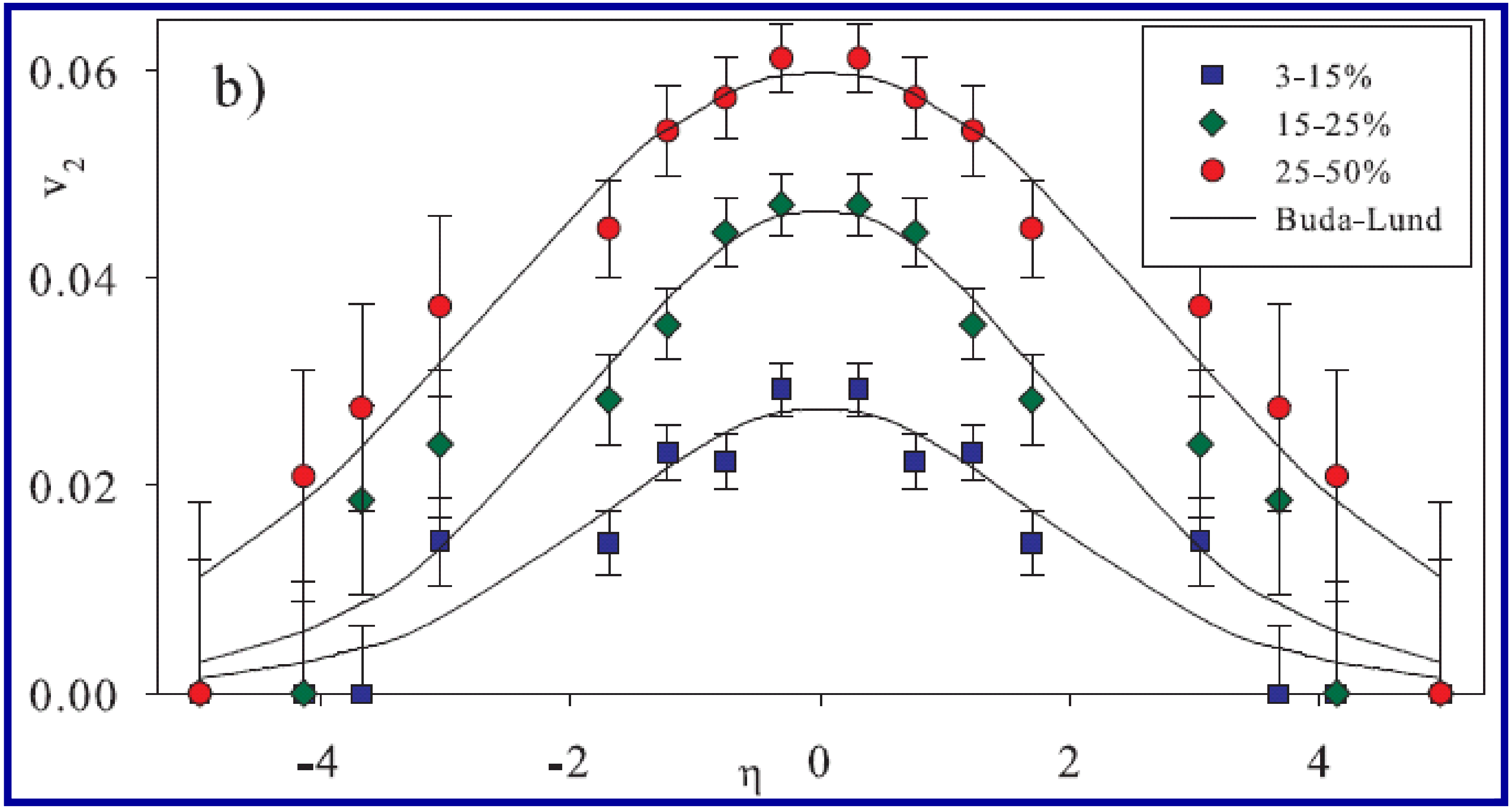
Universal scaling and $\sqrt{s}$ dependence

PHOBOS, nucl-ex/0406021



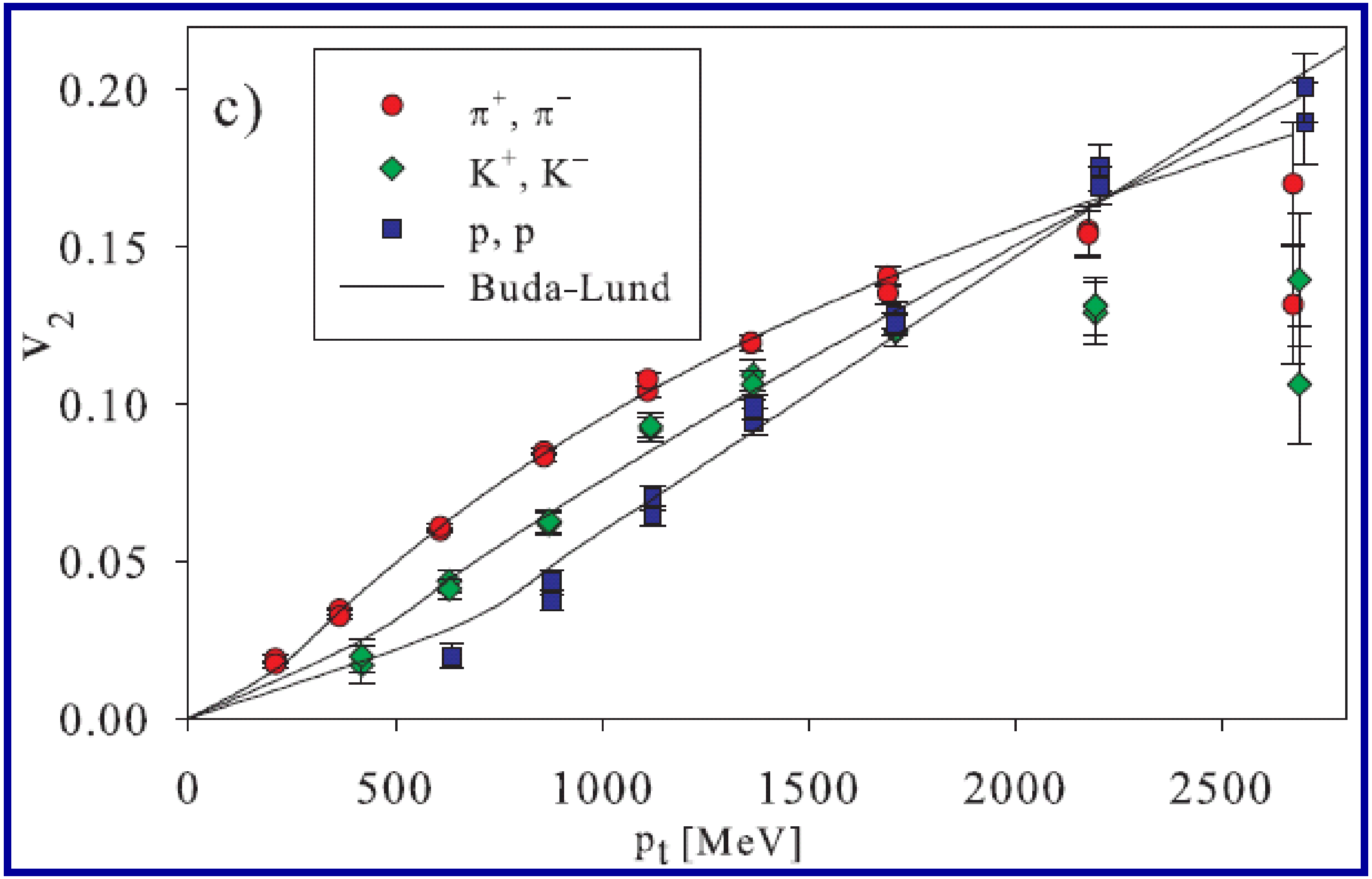
Universal scaling and v_2(centrality,η)

PHOBOS, nucl-ex/0407012



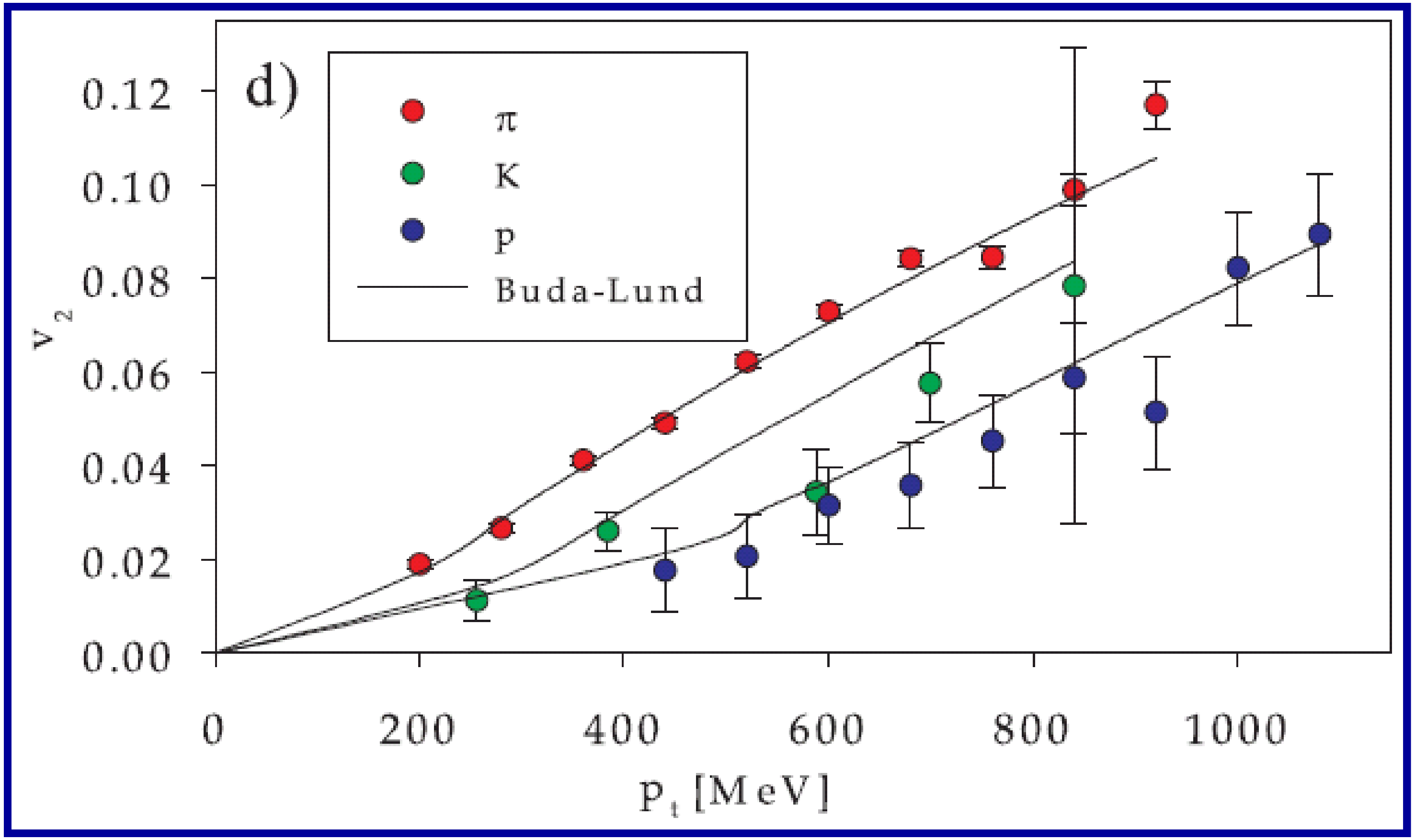
Universal v2 scaling and PID dependence

PHENIX, nucl-ex/0305013

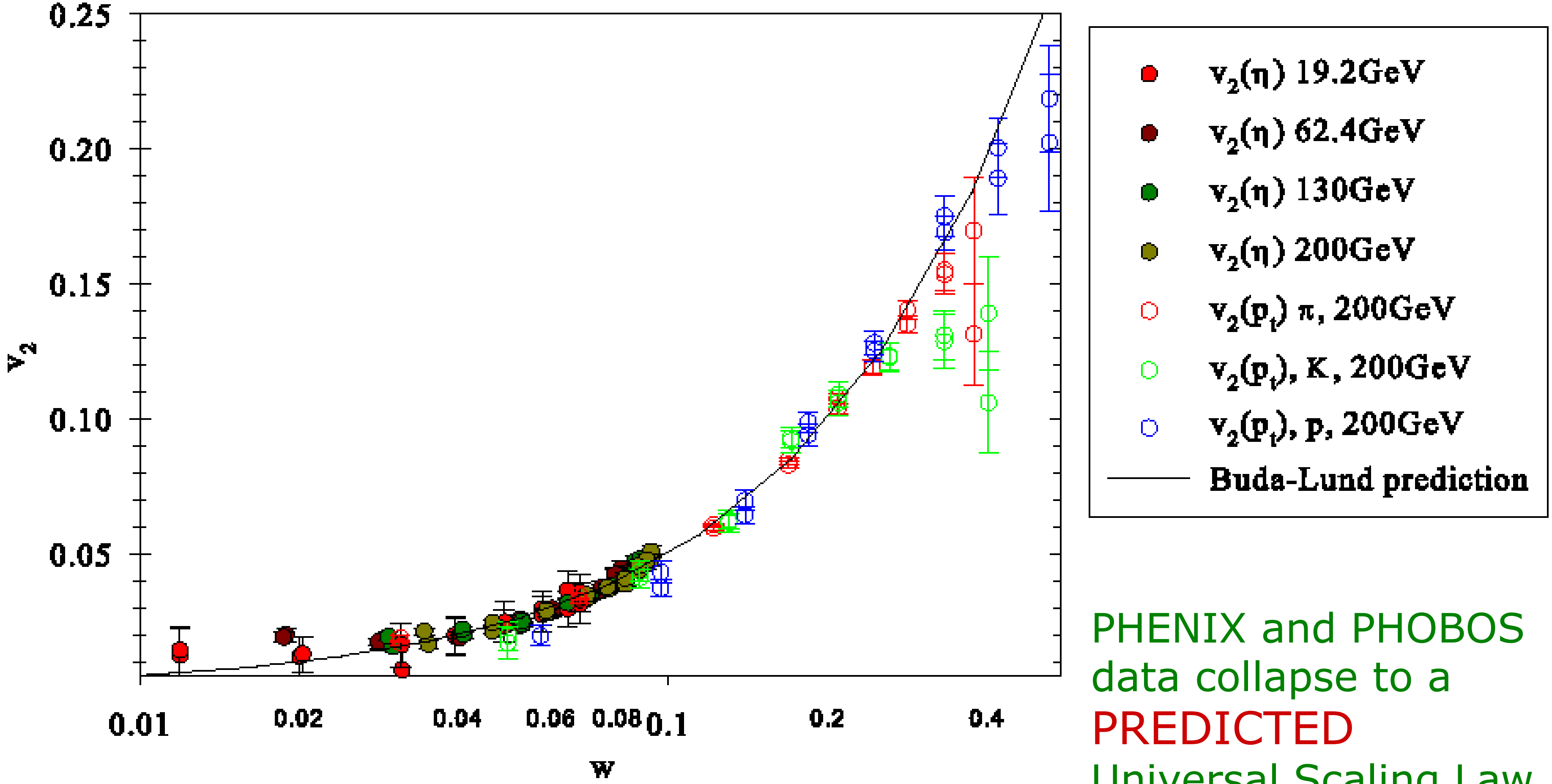


Universal scaling and fine structure of v2

STAR, nucl-ex/0409033



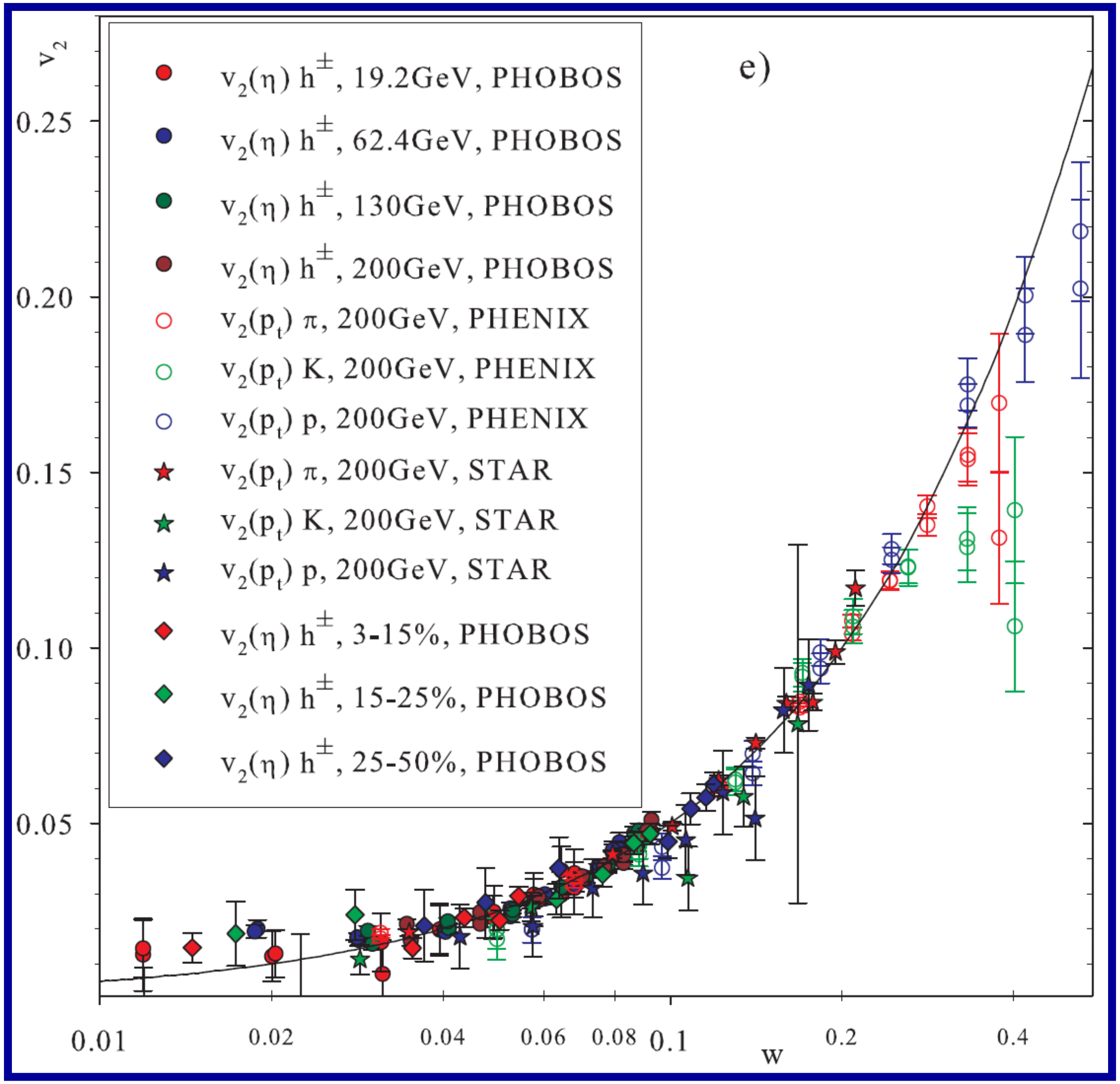
Universal v_2 scaling predicted in 2003



PHENIX and PHOBOS data collapse to a PREDICTED Universal Scaling Law of perfect fluid hydro

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

Summary on universal scaling of v_2

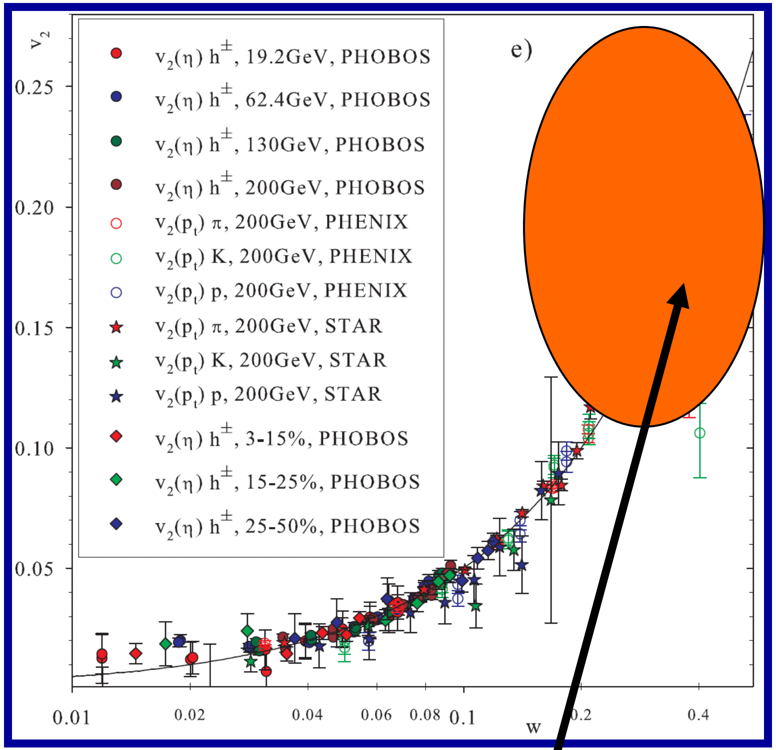


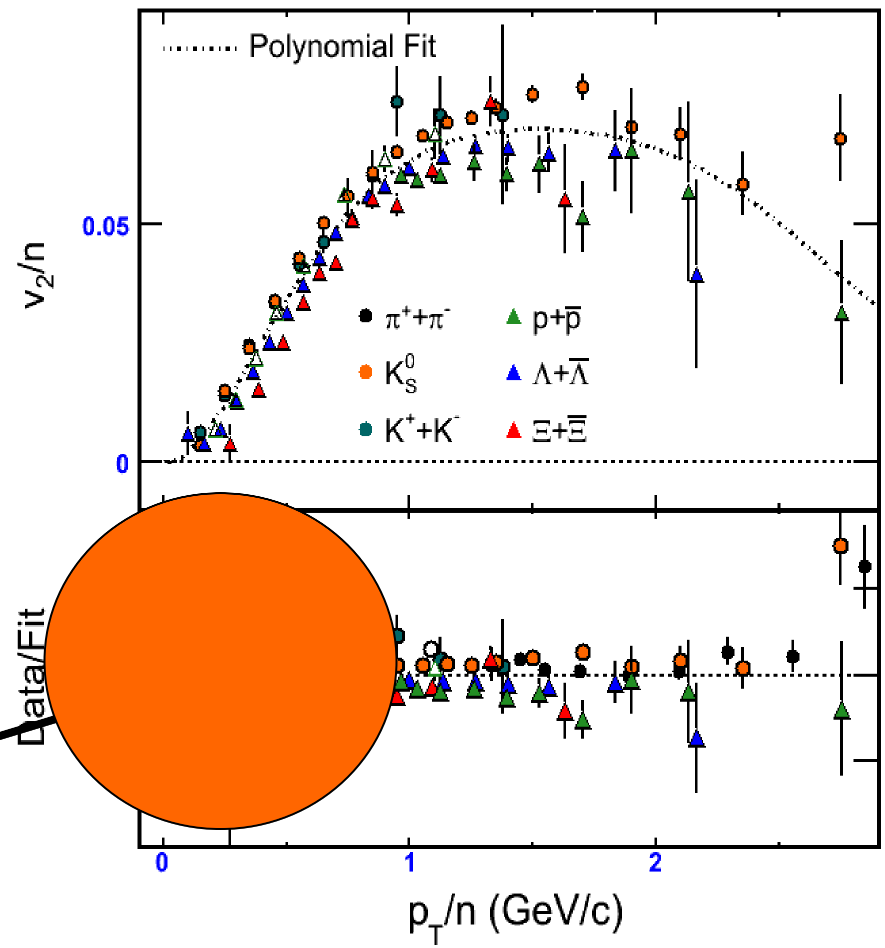
Black scaling law: Theoretical prediction from analytic perfect fluid hydrodynamics:

$$v_2 = \frac{I_1(w)}{I_0(w)}$$

hep-ph/0108067, nucl-th/0310040

Scaling and scaling violations





Universal hydro scaling breaks where quark number scaling sets in, pt ~ 1-2 GeV
Fluid of QUARKS!!

R. Lacey and M. Oldenburg
Proc. QM 2005

Summary

Au+Au elliptic flow data at RHIC satisfy the
UNIVERSAL scaling laws
predicted
(2001, 2003)
by the (Buda-Lund) hydro model,
based on exact solutions of
PERFECF FLUID hydrodynamics

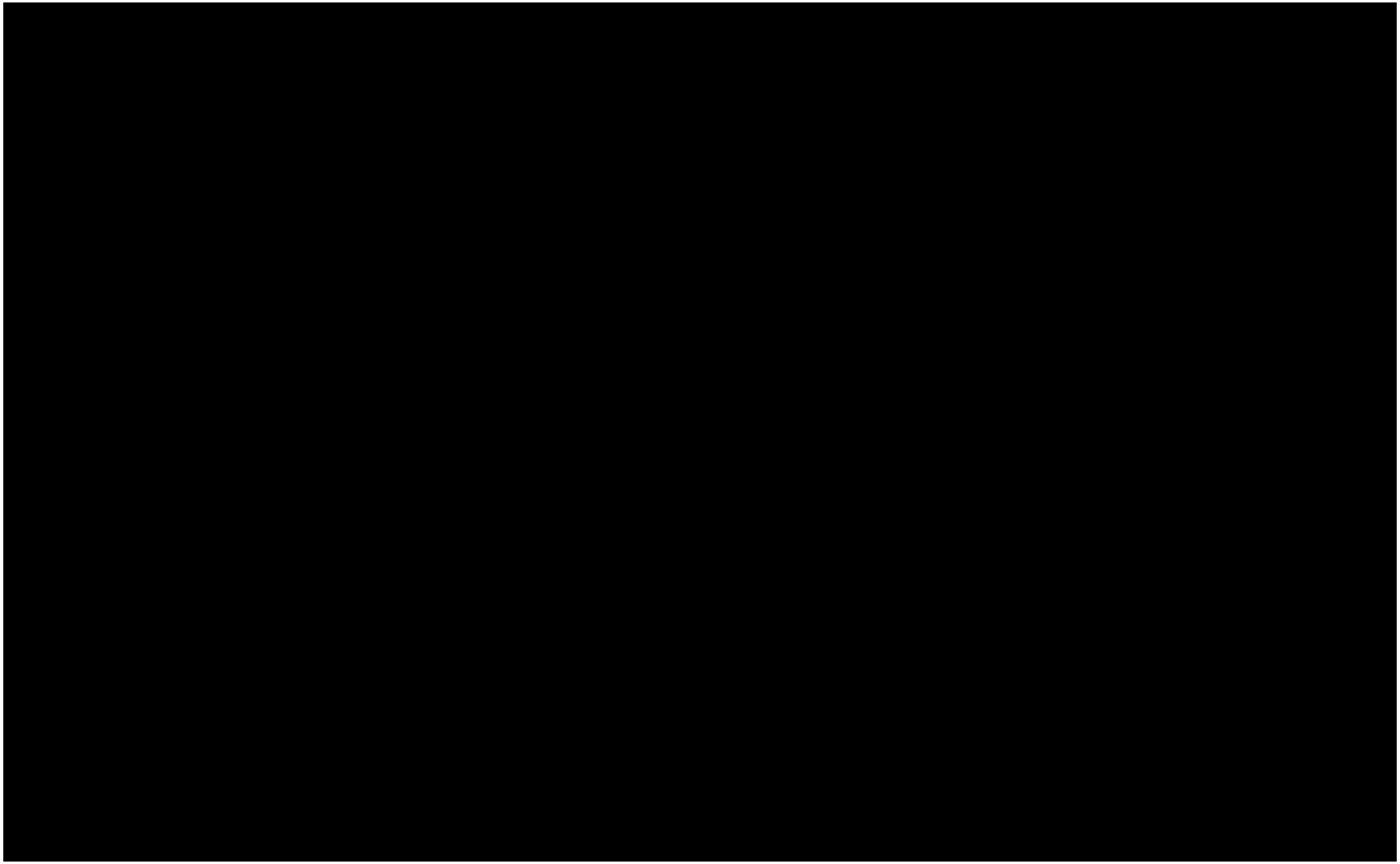
quantitative evidence for a perfect fluid in Au+Au at RHIC

scaling breaks in p_t at ~ 1.5 GeV,
in rapidity at ~|y| > y_{may} - 0.5
Search for establishing the domain of applicability started.
Scaling of HBT radii and spectra:
first tests passed, further tests going on.

Conclusion

Thank you for your attention !

Backup Slides from now on



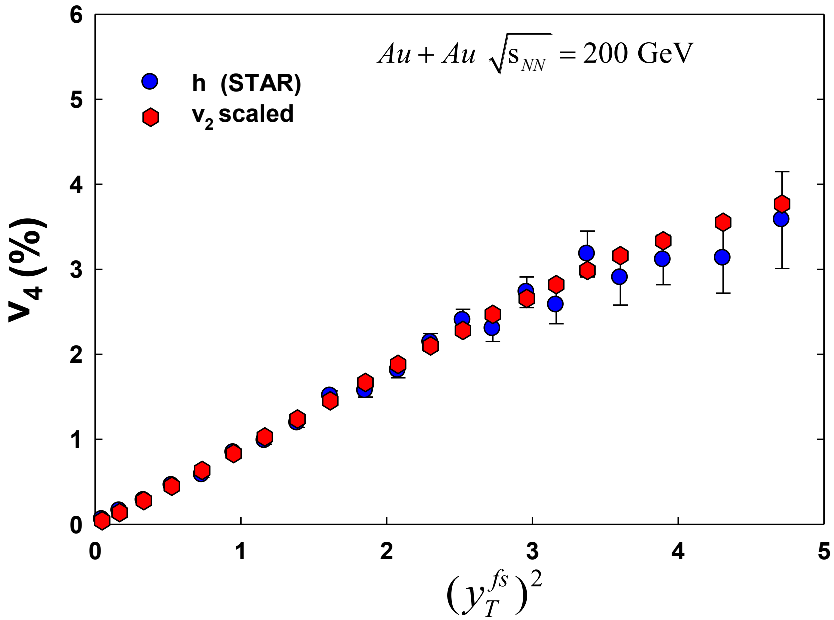
Higher flow coefficients

Buda-Lund rel. hydro formula:

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$
$$v_{2n+1} = 0$$

Exact non-relativistic result:

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$
$$v_{2n+1} = 0$$



$$v_4 = \frac{v_2^2}{2} + k \times y_T^4$$

R. Lacey, Proc. QM 2005

Hamiltonian motion in heavy ion physics

- **Direction dependent Hubble flow**

$$(P_x, P_y, P_z) = m(\dot{X}, \dot{Y}, \dot{Z})$$

$$H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{3}{2}T_0 \left(\frac{X_0 Y_0 Z_0}{XYZ}\right)^{2/3}$$

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa}$$

$$\begin{aligned} v_x(t,\mathbf{r}) &= \frac{\dot{X}(t)}{X(t)} r_x, \\ v_y(t,\mathbf{r}) &= \frac{\dot{Y}(t)}{Y(t)} r_y, \\ v_z(t,\mathbf{r}) &= \frac{\dot{Z}(t)}{Z(t)} r_z. \end{aligned}$$

$$T(t) = T_0 \left(\frac{V_0}{V(t)}\right)^{2/3},$$

$$n(t,\mathbf{r}) = n_0 \frac{V_0}{V(t)} \exp\left(-\frac{r_x^2}{2X(t)^2} - \frac{r_y^2}{2Y(t)^2} - \frac{r_z^2}{2Z(t)^2}\right),$$

- **Late t -> v = H r, where H = 1/t. Spherical symmetry: R=X=Y=Z**
- **2/3 in general: c_s^2, if $T_0 < 0$, and $c_s^2 = 1/3$ -> Friedmann**

Friedmann eq. of heavy ion physics

- **Scale invariant solutions of fireball hydro, hep-ph/0111139:**

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa}$$

- **From global energy conservation -> "Friedmann equation"**

$$\frac{\partial}{\partial t}\int d^3r(\varepsilon + \frac{nmv^2}{2}) = 0$$

$$\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 + 3\frac{T_0}{m}\left(\frac{V_0}{V}\right)^{2/3} = A = const.$$

$$R^2(t) = X^2(t) + Y^2(t) + Z^2(t) = A(t-t_0)^2 + B(t-t_0) + C,$$

$$H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{3}{2}T_0\left(\frac{X_0Y_0Z_0}{XYZ}\right)^{2/3} \qquad (P_x, P_y, P_z) = m(\dot{X}, \dot{Y}, \dot{Z})$$

Scaling laws from hydro

Exact non-rel. and Buda-Lund rel.

Single particle spectra
Slope
Rapidity width
Elliptic flow
Higher harmonics
HBT radius parameters
asHBT

Au+Au data at RHIC satisfy the scaling laws that were predicted by the Buda-Lund hydro model.

v2(y,pt, ...) is mapped already to a universal scaling function

-> compelling evidence for a perfect fluid at RHIC

scaling breaks between 1-2 GeV, where quark number scaling sets in.

Hubble from numerical rel. hydro

Assume net barion-free,
approx. boost invariant case
Rel. Euler equation
Entropy conservation
4 independent eqs, 5 variables

$$u^\mu \partial_\mu (T u^\nu) = \partial^\nu T,$$

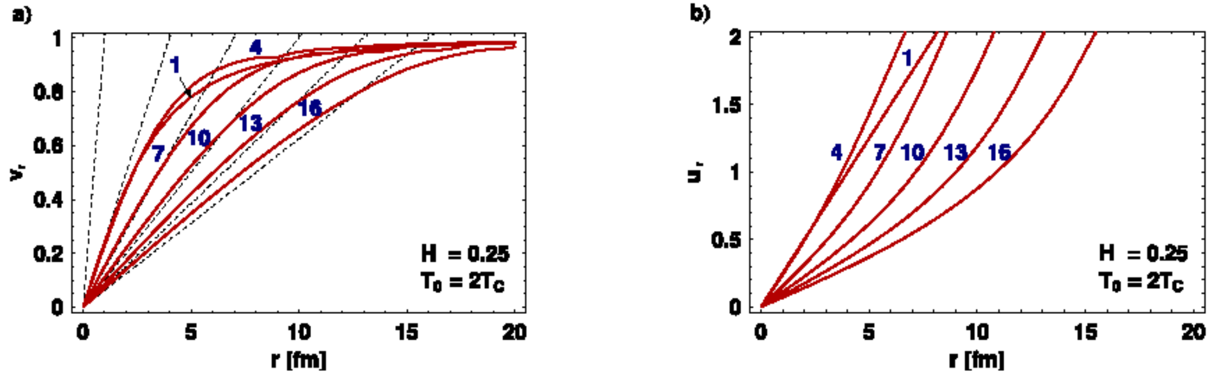
$$\partial_\mu (\sigma u^\mu) = 0,$$

$$d\varepsilon = T d\sigma, \qquad dP = \sigma dT, \qquad w = \varepsilon + P = T\sigma,$$

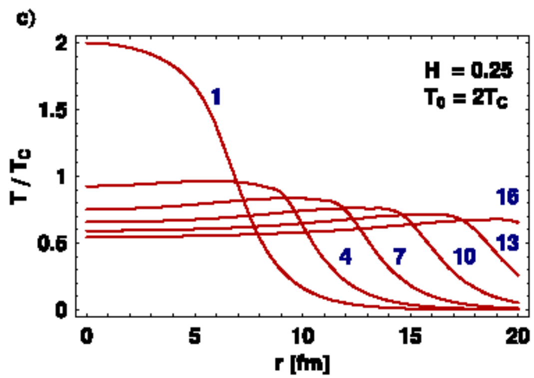
Closed by thermodynamical relationships.
key quantity:
temperature dependent speed of sound
can be taken from lattice QCD

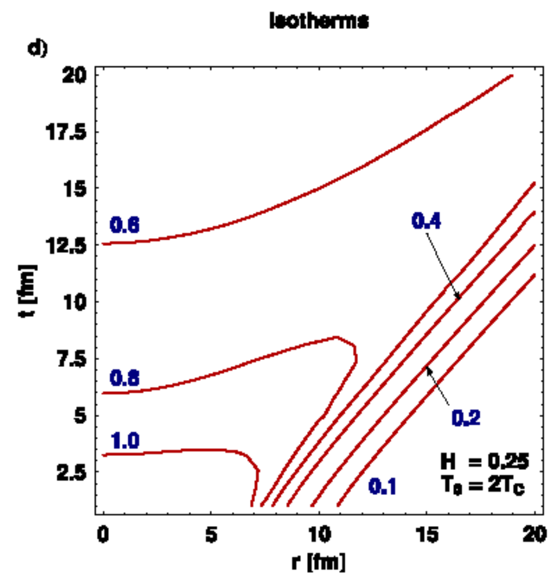
$$c_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{\sigma}{T} \frac{\partial T}{\partial \sigma}$$

Some num. rel. hydro solutions



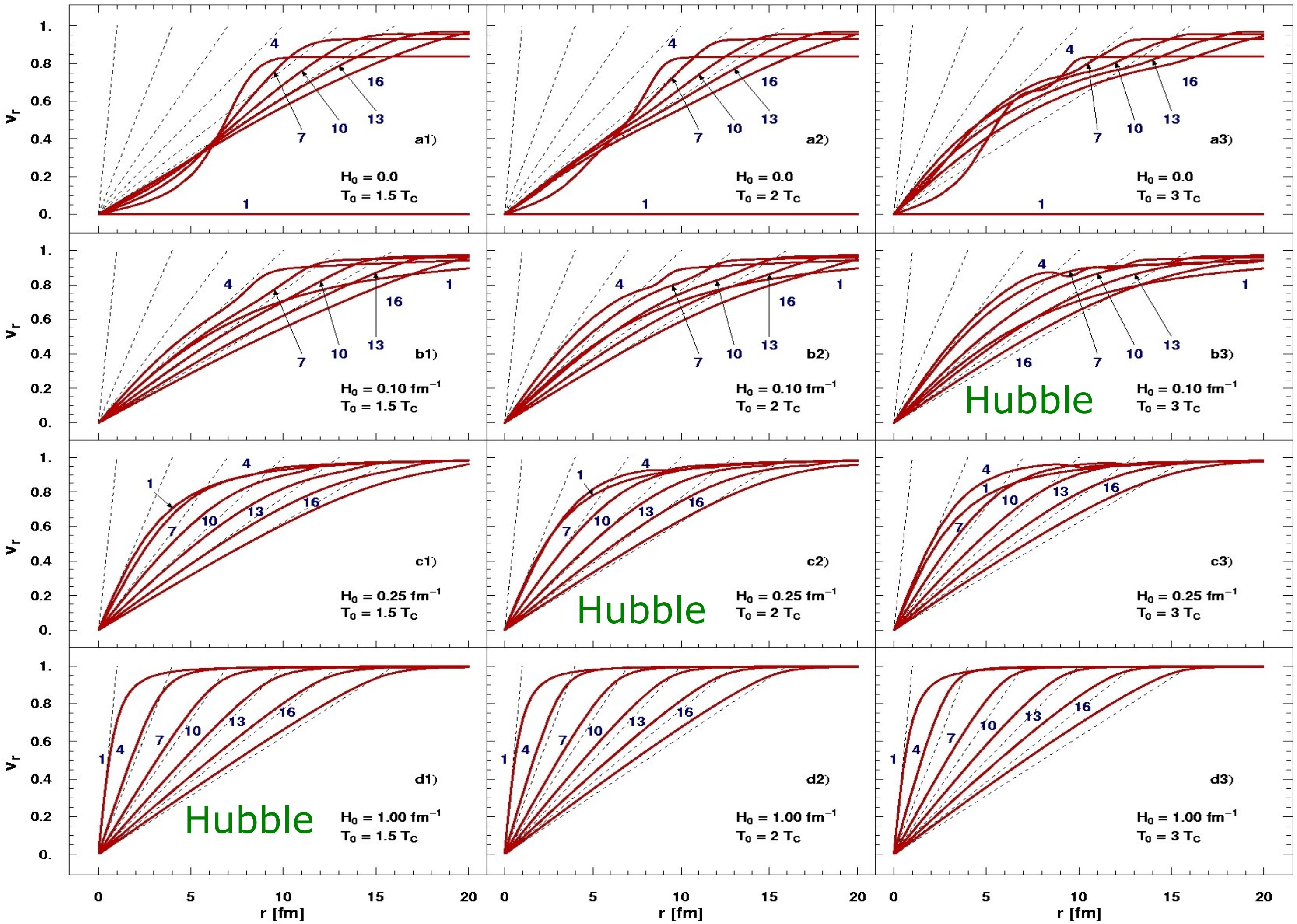
M. Chojnacki, W. Florkowski, T. Cs,
nucl-th/0410036
lattice QCD EOS (μ_B=0)
$T_0(r)$ ~ initial entropy (Glauber)
H_0 ~ initial Hubble flow





Support the quick development of the Hubble flow
and the Blast-wave, Buda-Lund and Cracow etc models

Effects of pre-equilibrium flow



Initial temperature gradient and initial flow have to be co-varied to get Hubble in a sufficiently short time. $H_0 > 0$