# Universal scaling of $v_2$ in Au+Au collisions

# and the Perfect Fluid at RHIC

**T. Csörgő**

**with M. Csanád, R. Lacey et al.**

- **Introduction:**
  - **Press release, BNL: RHIC Scientists serve up "Perfect Liquid", April 18, 2005 - "White Papers" in Nucl. Phys. A**
  - **New results at QM05 and at the 5th Budapest RHIC School**
- **Hydrodynamics and scaling of soft observables**
  - **Exact hydro results**
  - **Scaling of slope parameters**
  - **Bose-Einstein /HBT radii**
  - **the elliptic and higher order flows**
- **Intermediate pt region: breaking of the hydro scaling**

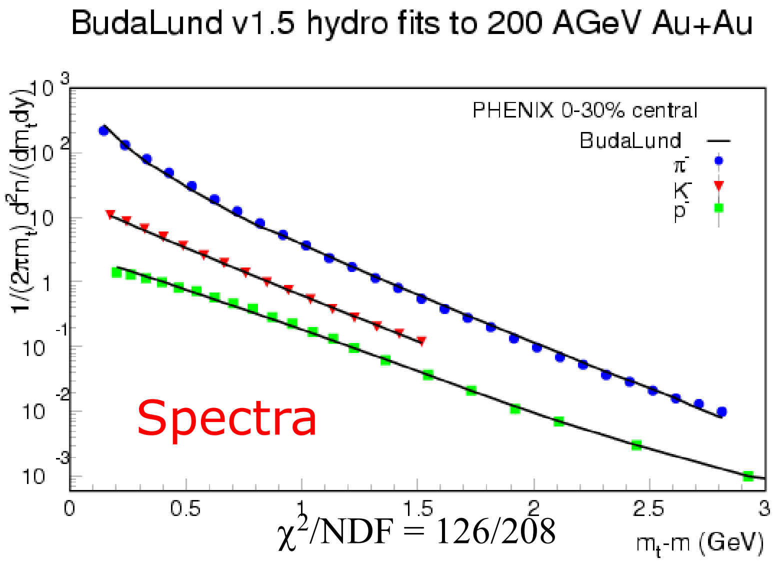
# Discovering New Laws

"In general we look for a new law by the following process.
First we guess it.
Then we compare the consequences of the guess to see
what would be implied if this law that we guessed is right.
Then we compare the result of the computation to nature,
with experiment or experience, compare it directly with observation,
to see if it works.
If it disagrees with experiment it is wrong.

In that simple statement is the key to science.
It does not make any difference how beautiful your guess is.
It does not make any difference how smart you are,
who made the guess, or what his name is —
if it disagrees with experiment it is wrong.

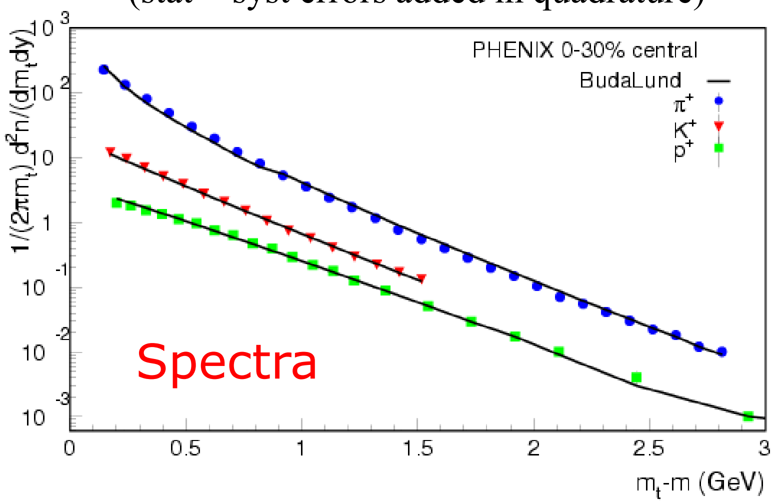
/R.P. Feynman/"

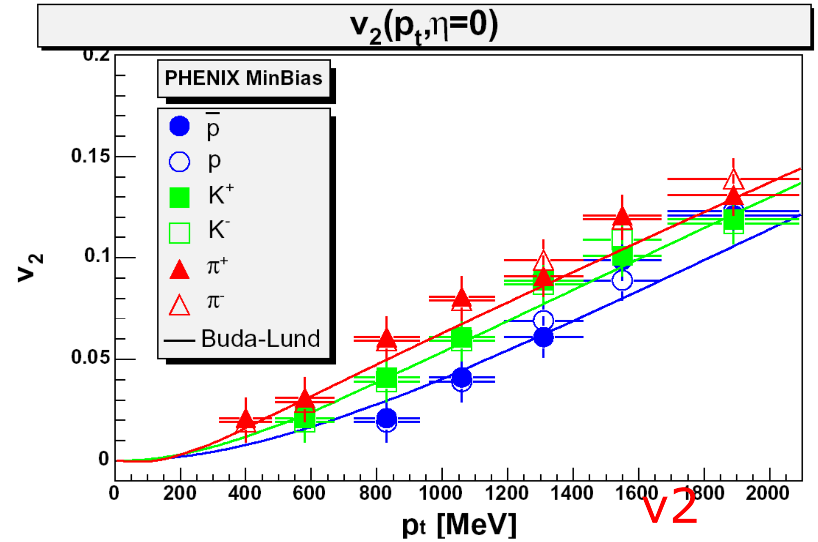
# Buda-Lund hydro and Au+Au@RHIC

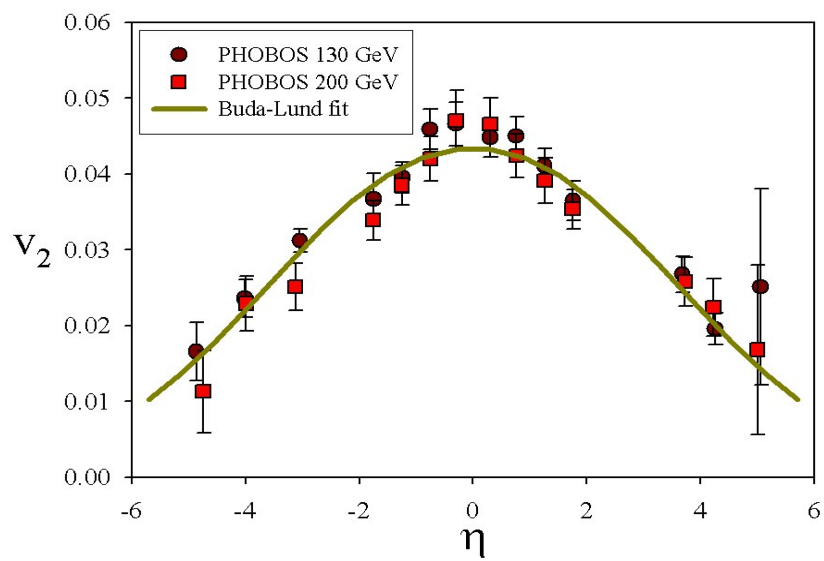



$\chi^2/NDF = 126/208$

(stat + syst errors added in quadrature)



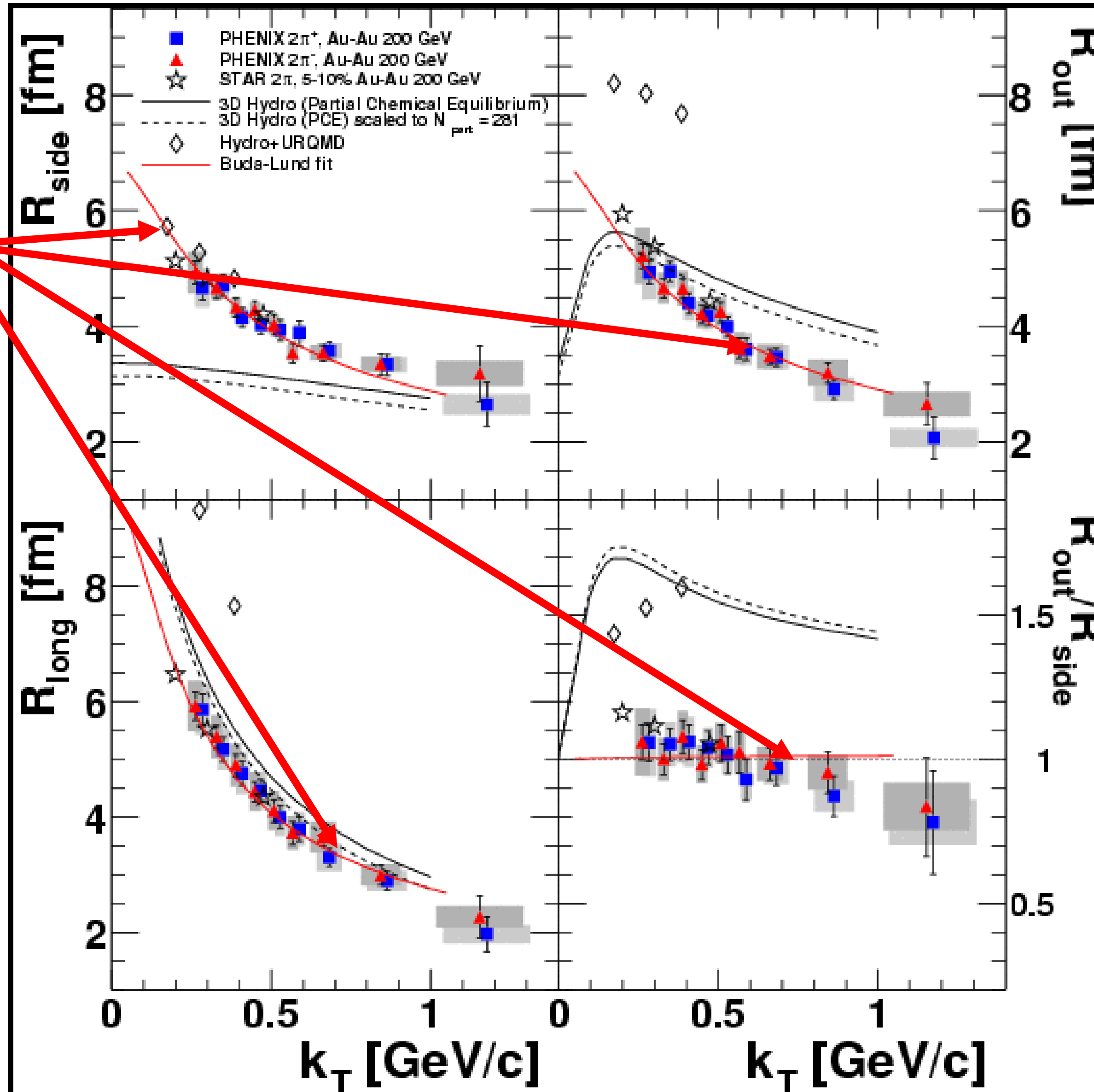




nucl-th/0311102, nucl-th/0207016, nucl-th/0403074

# Femptoscopy signal of supercooled QGP

Buda-Lund hydro fit indicates
- scaling of HBT radii
- sudden hadronization
- a hint for supercooled QGP.




Hadrons with $T>T_c$ escape-
a hint also for cross-over transition

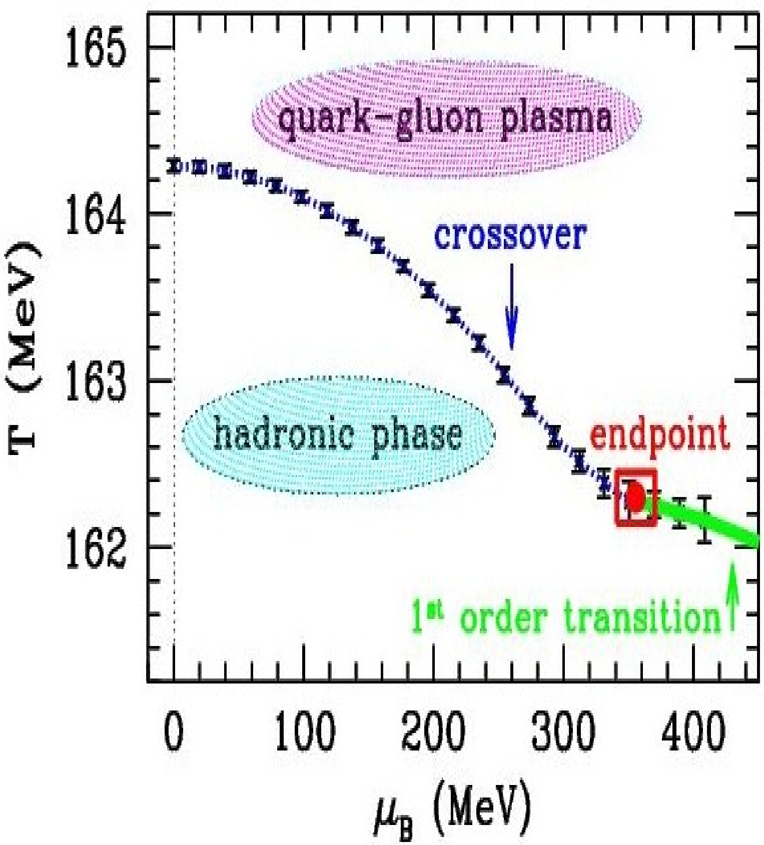
# Phases of QCD Matter, EoS

**Quark Gluon Plasma**

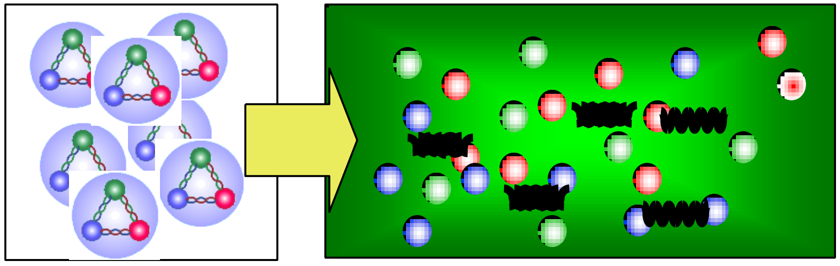
**“Ionize” nucleons with heat**

**“Compress” them with density**

**New state(s?) of matter**




**Z. Fodor and S.D. Katz:**
**$T_c$ = 164 ± 2 -> 189 ± 8 MeV, QM’05(?)**
**even at finite baryon density,**
**Cross over like transition.**
**(hep-lat/0106002, hep-lat/0402006)**



**For most recent results**

**S. D. Katz, http://qm2005.kfki.hu/**

**lattice QCD -> Equations of State**

**Input for hydrodynamics**

# Nonrelativistic hydrodynamics

- **Equations of nonrelativistic hydro:**

$$\begin{aligned} \partial_t n + \nabla(n\mathbf{v}) &= 0 \\ \partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} &= -(\nabla p)/(mn) \\ \partial_t \epsilon + \nabla(\epsilon\mathbf{v}) &= -p\nabla\mathbf{v} \end{aligned}$$

- **Not closed, EoS needed:**

$$\begin{aligned} \epsilon &= \kappa p \\ p &= nT \end{aligned}$$

- **Perfect fluid: no viscosity and heat conductivity**
- **We use the following scaling variable:**

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

- **X, Y and Z are characteristic scales, depend on (proper-) time**

# Exact nonrelativistic solutions

- **A general group of scale-invariant solutions (hep-ph/0111139):**

$$
\begin{aligned}
n(t,\mathbf{r}) &= n_0 \frac{V_0}{V}\nu(s) \\
\mathbf{v}(t,\mathbf{r}) &= \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z\right) \\
T(t,\mathbf{r}) &= T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \mathcal{T}(s) \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0}\int_0^s \frac{du}{\mathcal{T}(u)}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{r}' &= (r_x \frac{X_0}{X}, r_y \frac{Y_0}{Y}, r_z \frac{Z_0}{Z}) \\
n\,(t,\mathbf{r}) &= n(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ}\right) \\
v_x\,(t,\mathbf{r}) &= v_x(t_0,\mathbf{r}') \frac{\dot X}{\dot X_0}, \ldots \\
T\,(t,\mathbf{r}) &= T(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ}\right)^{1/\kappa}
\end{aligned}
$$

- **This is a PARAMETRIC but exact solution, if the scales fulfill:**

T. Csörgő, Acta Phys. Polonica B37 (2006) 1001

**□Tale of 1001 nights"**

$$
X\ddot X = Y\ddot Y = Z\ddot Z = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa}
$$

- **Temperature scaling function is arbitrary,**
  **e.g. Constant temperature ⇒ Gaussian density**

**Buda-Lund profiles:**

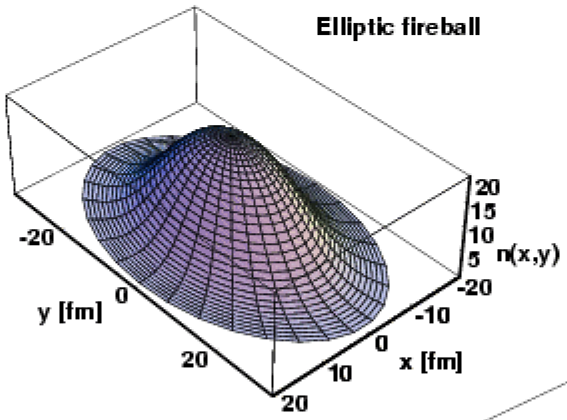
$$
\begin{aligned}
\mathcal{T}(s) &= \frac{1}{1+bs} \\
\nu(s) &= (1+bs)\exp\left[-\frac{T_i}{2T_0}(s + bs^2/2)\right]
\end{aligned}
$$

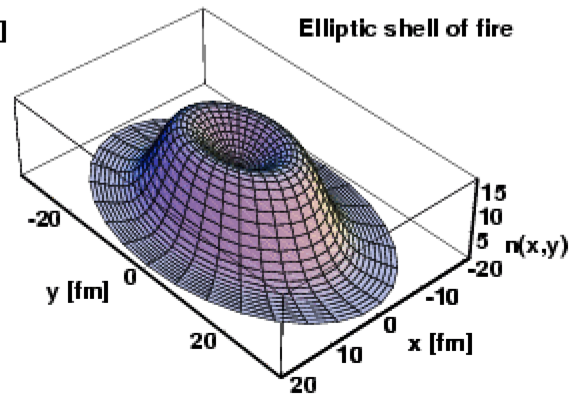
**Zimányi-Bondorf-Garpman profiles:**

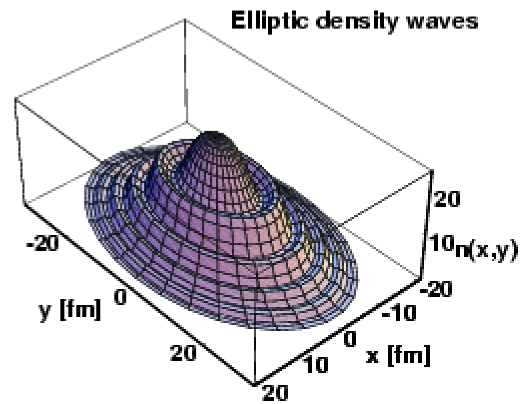
$$
\begin{aligned}
\mathcal{T}(s) &= (1-s)\,\Theta(1-s) \\
\nu(s) &= (1-s)^\alpha\,\Theta(1-s)
\end{aligned}
$$

# Some new solutions of hydro

- Non-relativistic as well
as relativistic generalizations
- 1d, 3d axial, 3d ellipsoidal
- arbitrary temperature
profile functions
- Excellent for illustration
- Good tool for students
- Asymptotic solutions vs.
  collisionless Boltzmann gas




**Time evolution of the scales follows**
**a classical motion!**
**Scale parameters determine observables - info on history <u>LOST</u>!**

# Friedmann eq. of heavy ion physics

- **Scale invariant solutions of fireball hydro, hep-ph/0111139:**

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa}$$

- **From global energy conservation -> "Friedmann equation"**

$$\frac{\partial}{\partial t}\int d^3r(\varepsilon + \frac{nmv^2}{2}) = 0$$

$$\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 + 3\frac{T_0}{m}\left(\frac{V_0}{V}\right)^{2/3} = A = const.$$

$$R^2(t) = X^2(t) + Y^2(t) + Z^2(t) \; = \; A(t-t_0)^2 + B(t-t_0) + C,$$

$$H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{3}{2}T_0\left(\frac{X_0Y_0Z_0}{XYZ}\right)^{2/3} \qquad (P_x, P_y, P_z) = m(\dot{X}, \dot{Y}, \dot{Z})$$

# Hamiltonian motion in heavy ion physics

- **Direction dependent Hubble flow**

$$H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{3}{2}T_0\left(\frac{X_0Y_0Z_0}{XYZ}\right)^{2/3}$$

$$\begin{aligned} v_x(t,\mathbf{r}) &= \frac{\dot{X}(t)}{X(t)} r_x, \\ v_y(t,\mathbf{r}) &= \frac{\dot{Y}(t)}{Y(t)} r_y, \\ v_z(t,\mathbf{r}) &= \frac{\dot{Z}(t)}{Z(t)} r_z. \end{aligned}$$

$$(P_x, P_y, P_z) = m(\dot{X}, \dot{Y}, \dot{Z})$$

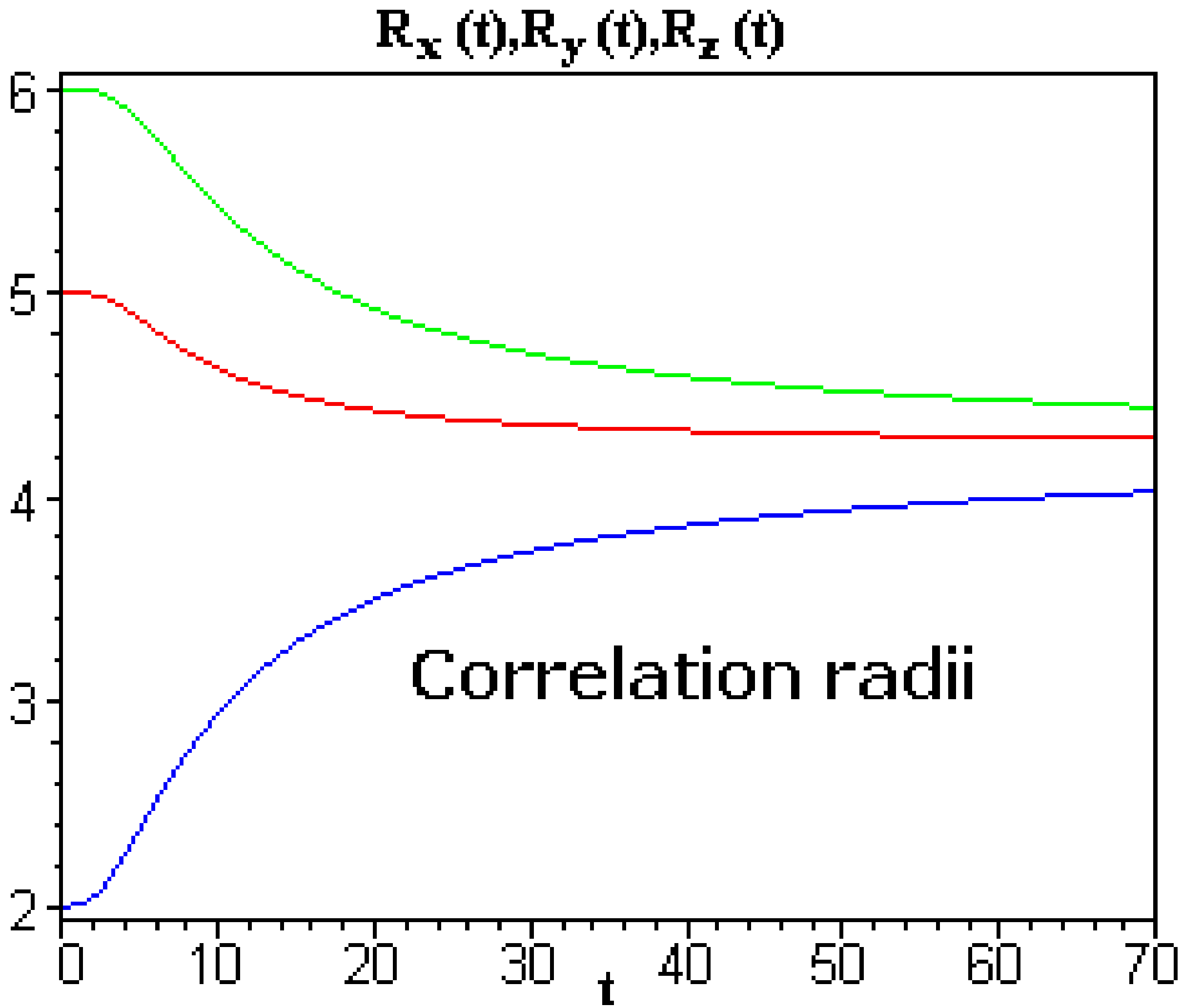
$$T(t) = T_0\left(\frac{V_0}{V(t)}\right)^{2/3},$$

$$n(t,\mathbf{r}) = n_0 \frac{V_0}{V(t)} \exp\left(-\frac{r_x^2}{2X(t)^2} - \frac{r_y^2}{2Y(t)^2} - \frac{r_z^2}{2Z(t)^2}\right),$$

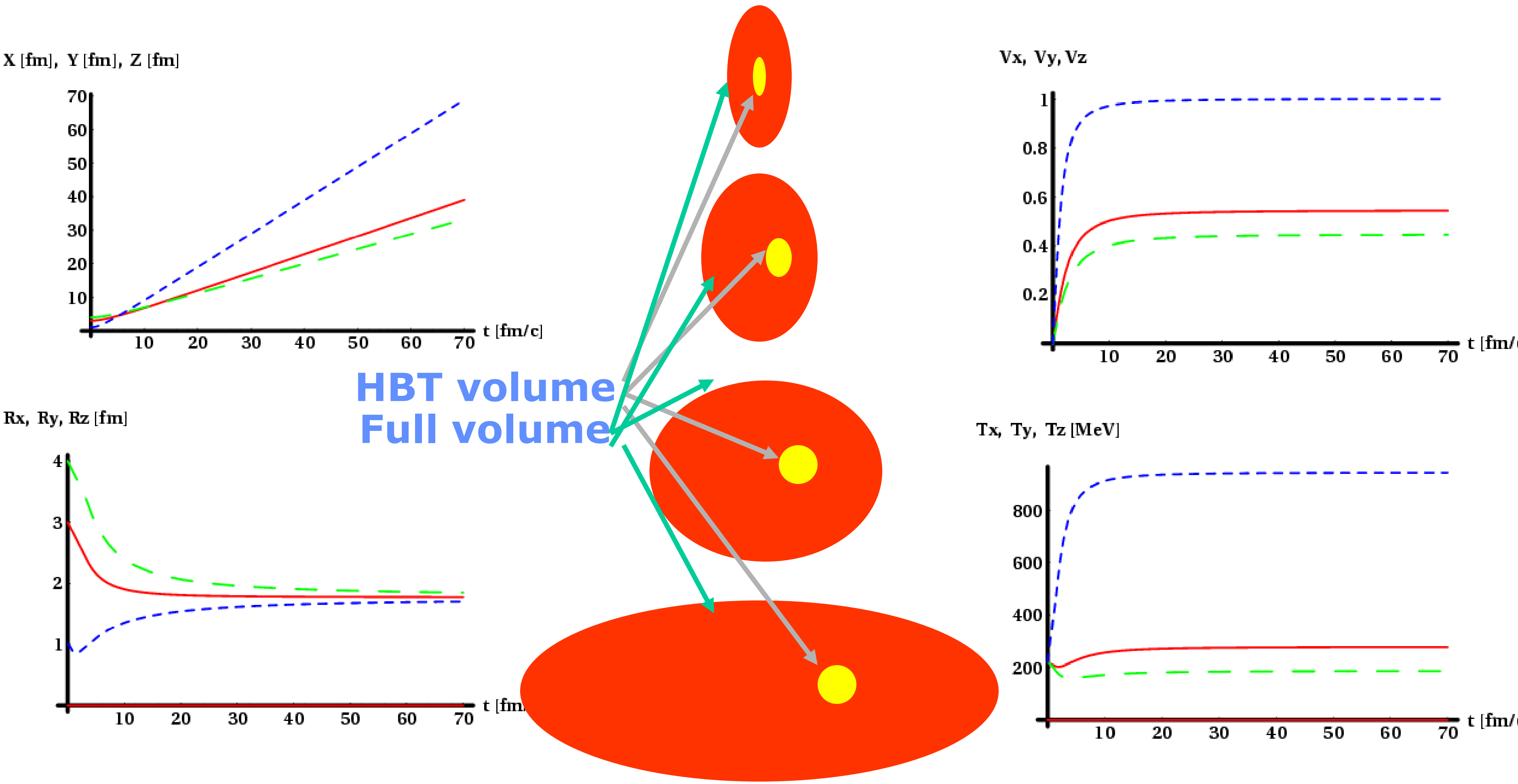
- **Late t -> v = H r, where H = 1/t. Spherical symmetry: R=X=Y=Z**
- **2/3 in general: $c_s^2$, if $T_0 < 0$, and $c_s^2 = 1/3$ -> Friedmann**

# Examples of exact hydro results

- **Propagate the hydro solution in time numerically:**

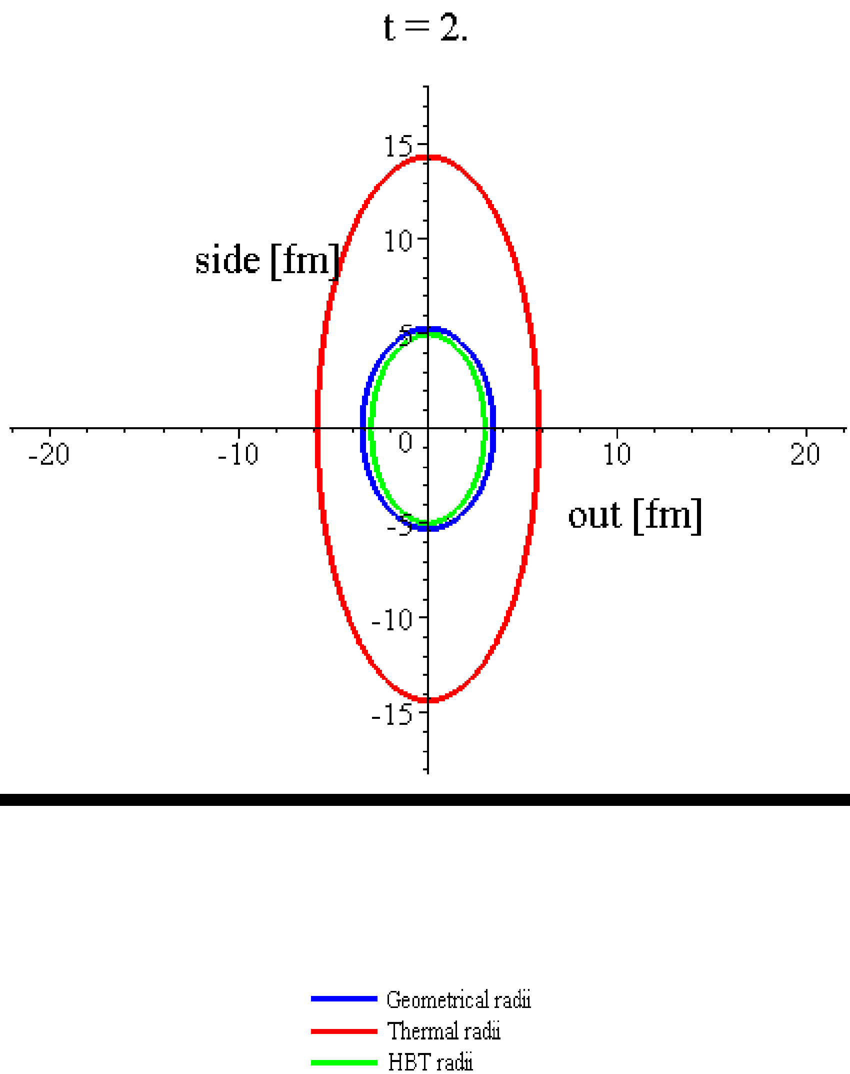


# The RHIC horizont problem




**Geometrical sizes increase, velocities tend to a constant. Slope parameters and radii freeze out and Rx, Ry, Rz -> const! General property, analytic result!**

# Geometrical & thermal & HBT radii




3d analytic hydro: exact time evolution (!!)

geometrical size (fugacity ~ const)

Thermal sizes (velocity ~ const)

HBT sizes (phase-space density ~ const)

HBT dominated by the smaller of the geometrical and thermal scales

nucl-th/9408022, hep-ph/9409327
hep-ph/9509213, hep-ph/9503494

HBT radii approach a const(t) (!!!)

HBT volume -> spherical

HBT radii -> thermal, constant lengths!!

hep-ph/0108067, nucl-th/0206051

<-- Thanks to Máté Csanád for animation

# Relativistic Perfect Fluids

- **Rel. hydrodynamics of perfect fluids is defined by:**

$$\partial_\mu (n u^\mu) = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

- **A recent family of exact solutions: (nucl-th/0306004):**

$$u^\mu = \frac{x^\mu}{\tau}$$
$$n(t, \mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s)$$
$$p(t, \mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$
$$T(t, \mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)}$$

- **Overcomes two shortcomings of Bjorken's solution:**
  - **Rapidity distribution**
  - **Transverse flow**

$$u^\mu \partial_\mu u_\nu = 0$$

- **Hubble flow ⇒ lack of acceleration.**
- **Accelerating exact rel. solutions: in preparation /Marci/**

# Hubble from numerical rel. hydro

Assume net barion-free,
approx. boost invariant case
Rel. Euler equation
Entropy conservation
4 independent eqs, 5 variables

$$u^\mu \partial_\mu (T u^\nu) = \partial^\nu T,$$

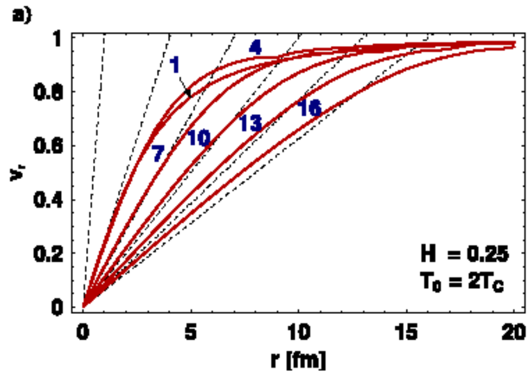
$$\partial_\mu (\sigma u^\mu) = 0,$$

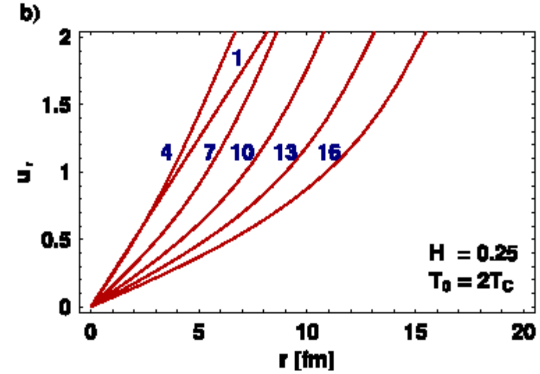
$$d\varepsilon = T d\sigma, \qquad dP = \sigma dT, \qquad w = \varepsilon + P = T\sigma,$$

Closed by thermodynamical relationships.
key quantity:
temperature dependent speed of sound
can be taken from lattice QCD

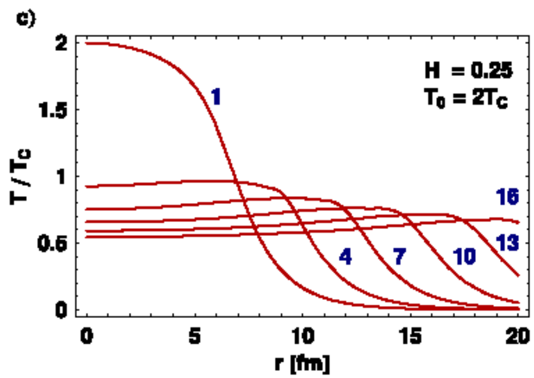
$$c_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{\sigma}{T}\frac{\partial T}{\partial \sigma}$$

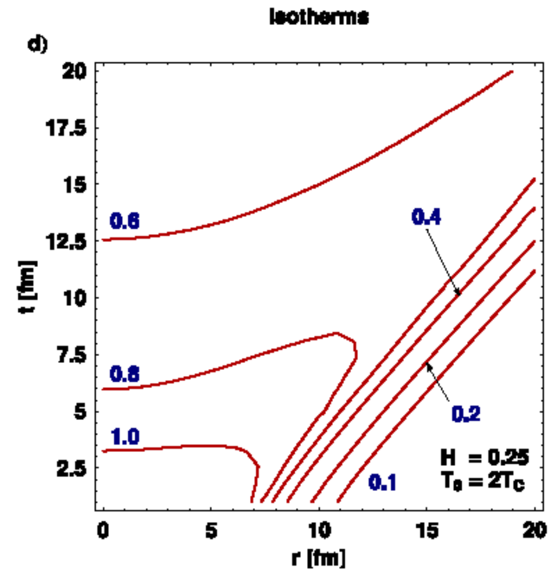
# Some num. rel. hydro solutions



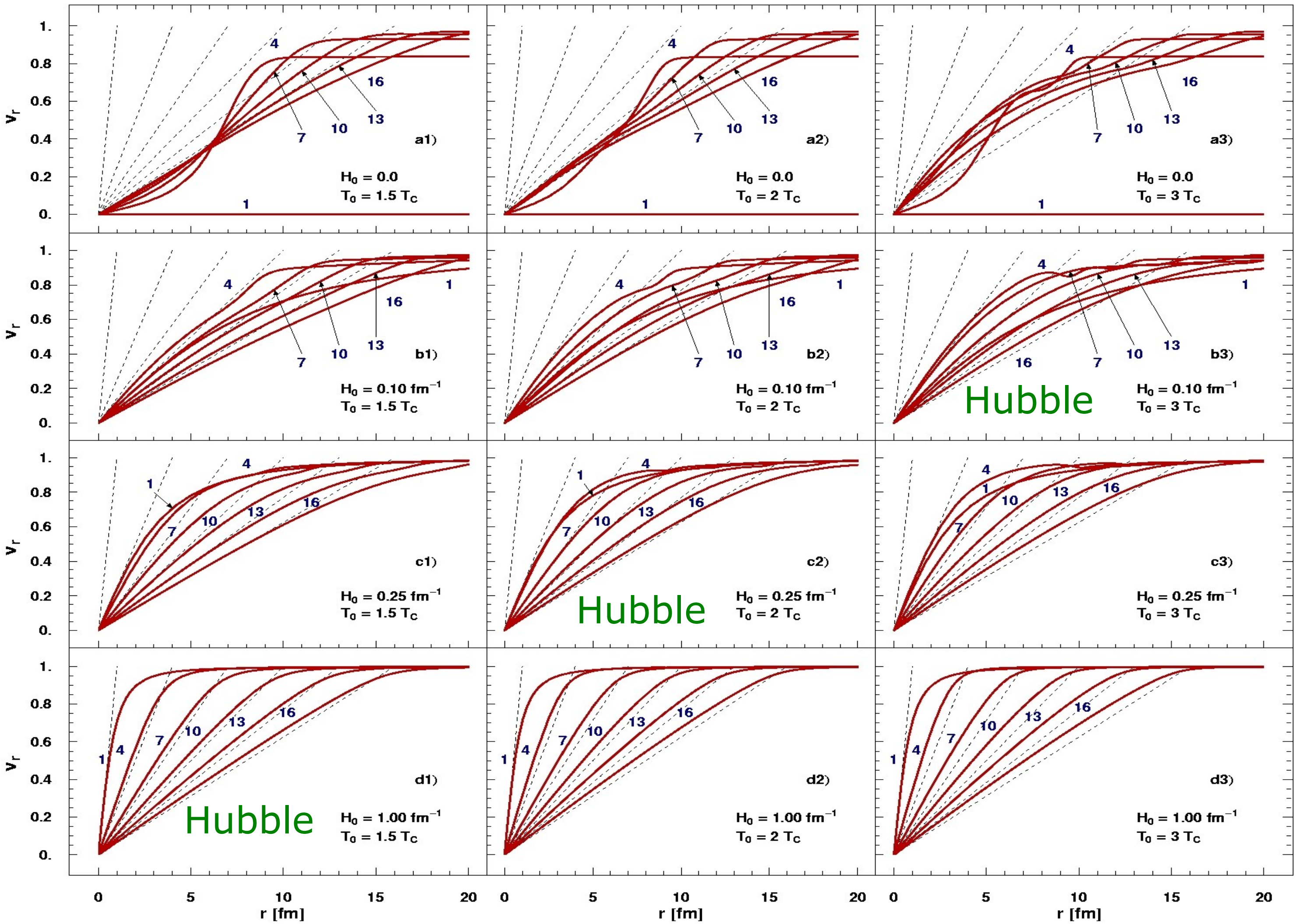

M. Chojnacki, W. Florkowski, T. Cs,
nucl-th/0410036
lattice QCD EOS ($\mu_B$=0)
$T_0(r)$ ~ initial entropy (Glauber)
$H_0$ ~ initial Hubble flow




Support the quick development of the Hubble flow
and the Blast-wave, Buda-Lund and Cracow etc models

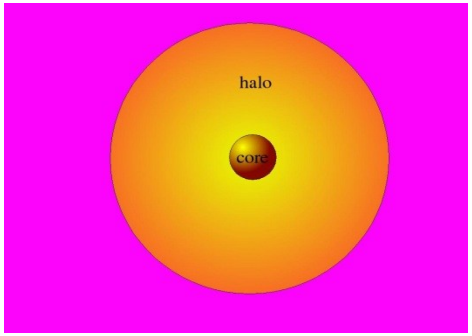
# Effects of pre-equilibrium flow



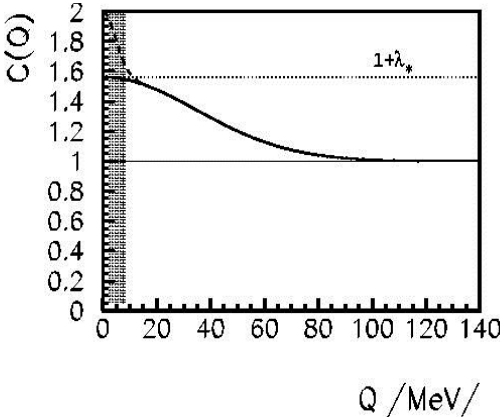

Initial temperature gradient and initial flow have to be co-varied to get Hubble in a sufficiently short time. $H_0 > 0$

# Principles for Buda-Lund hydro model

- Analytic expressions for all the observables
- 3d expansion, local thermal equilibrium, symmetry
- Goes back to known hydro solutions in: nonrel, Bjorken, and Hubble limits - but smoothly extrapolates in between



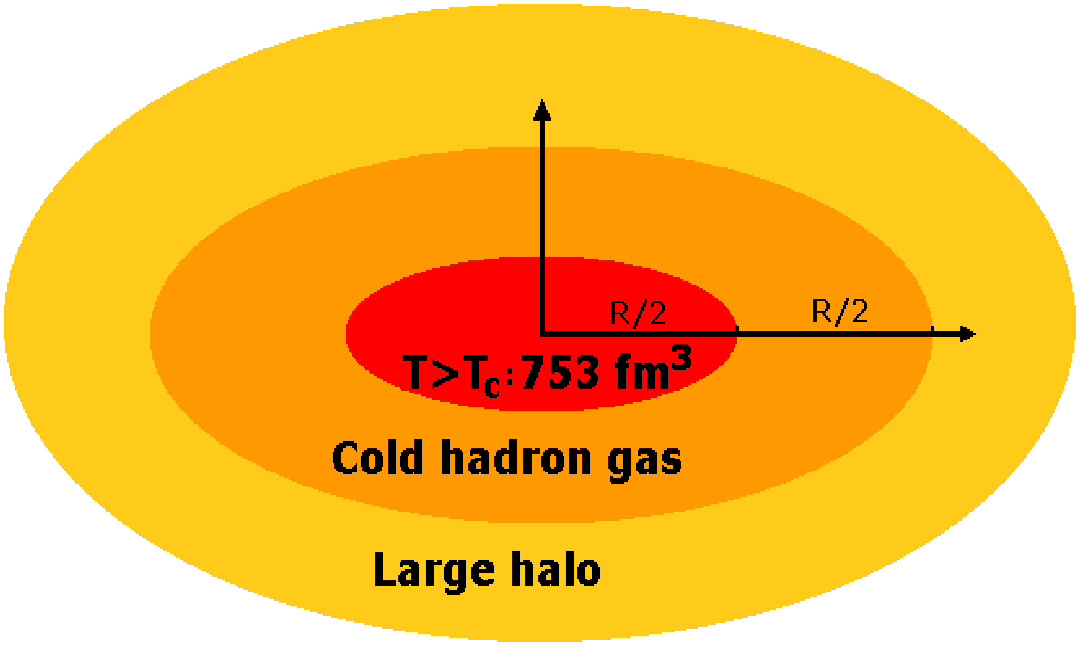

- Separation of the Core and the Halo
  - Core: hydrodynamic evolution
  - Halo: decay products of long-lived resonances
- Missing link: accelerating simple solutions of rel. hydro
  - Yu. Karpenko, M. Nagy

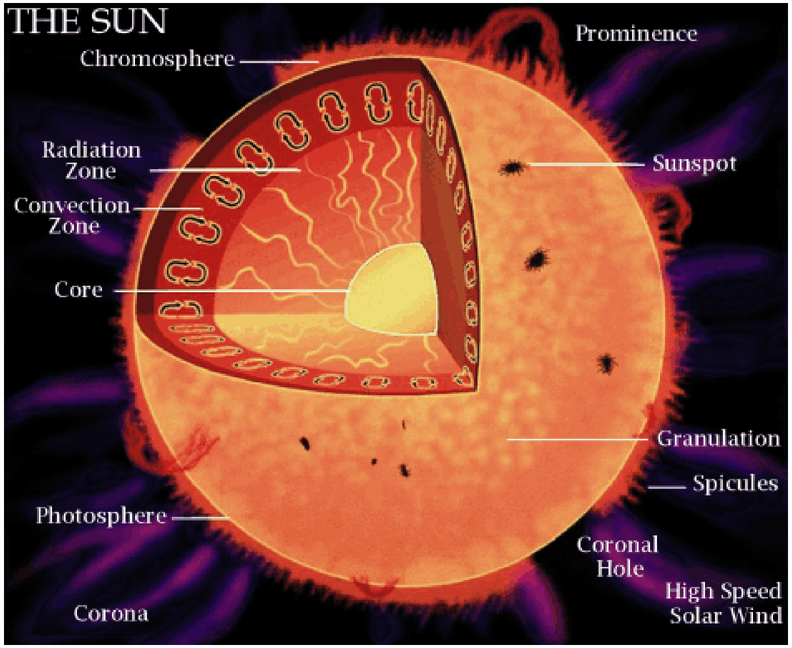


# A useful analogy

## Fireball at RHIC ⇔ our Sun

- Core ⇔ Sun
- Halo ⇔ Solar wind
- $T_{0,RHIC}$ ~ 210 MeV ⇔ $T_{0,SUN}$ ~ 16 million K
- $T_{surface,RHIC}$ ~ 100 MeV ⇔ $T_{surface,SUN}$ ~6000 K



# Buda-Lund hydro model

**The general form of the emission function:**

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

**Calculation of observables with core-halo correction:**

$$\mathrm{N}_1(p) = \frac{1}{\sqrt{\lambda_*}} \int d^4x S_c(p,x)$$

$$C(Q,p) = 1 + \left|\frac{\tilde{S}(Q,p)}{\tilde{S}(0,p)}\right|^2 = 1 + \lambda_* \left|\frac{\tilde{S}_c(Q,p)}{\tilde{S}_c(0,p)}\right|^2$$

**Assuming special shapes for the flux, temperature, chemical potential and flow:**

# Buda-Lund hydro model

**Invariant single particle spectrum:**

$$N_1 = \frac{d^2 n}{2\pi m_t dm_t dy} = \frac{g}{(2\pi)^3} \overline{E}\,\overline{V}\,\overline{C} \frac{1}{exp\left(\frac{p^\mu u_\mu(x_s) - \mu(x_s)}{T(x_s)}\right) + s_q}$$

**Invariant Buda-Lund correlation function:**
**oscillating, non-Gaussian prefactor!**

$$C_2(k_1,k_2) = 1 + \lambda_* \Omega(Q_{||}) \exp\left(-Q_{||}^2 R_{||}^2 - Q_{=}^2 R_{=}^2 - Q_{\perp}^2 R_{\perp}^2\right)$$

**Non-invariant Bertsch-Pratt parameterization,**
**in a Gaussian approximation:**

$$C_2(k_1,k_2) = 1 + \lambda_* \exp\left(-Q_o^2 R_o^2 - Q_s^2 R_s^2 - Q_l^2 R_l^2 - 2Q_{os}^2 R_o R_s\right)$$

**Non-Gaussian BL form ⟶ Gaussian BP *approximation*:**

$$R_{||,\Omega}^2 = R_{||}^2 \left(1 + \frac{\overline{\Delta\eta}^2}{\overline{\eta}}\right)$$

# The generalized Buda-Lund model

- **The original model was for axial symmetry only, central coll.**
- **In the most general hydrodynamical form:**

  **'Inspired by' nonrelativistic 3d hydrodynamical solutions:**

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

- **Have to assume special shapes:**
  - **Generalized Cooper-Frye prefactor:**

$$p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x \qquad H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - **Four-velocity distribution:**

$$u^\mu = (\gamma,\, \sinh\eta_x,\, \sinh\eta_y,\, \sinh\eta_z)$$

  - **Temperature:**

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s}\, s\right)\left(1 + \frac{T_0 - T_e}{T_e}\, \frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - **Fugacity:**

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

# Some analytic results

- **Distribution widths**

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2}+\frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2}+\frac{\dot{X}_i^2}{X_i^2}\right) \qquad a^2 = \frac{T_0 - T_s}{T_s} = \left\langle \frac{\Delta T}{T} \right\rangle_r$$

- **Slopes, effective temperatures**

$$T_{eff} = \frac{1}{2}\left(\frac{1}{T_{*,x}}+\frac{1}{T_{*,y}}\right) \qquad T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

- **Flow coefficients**

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$

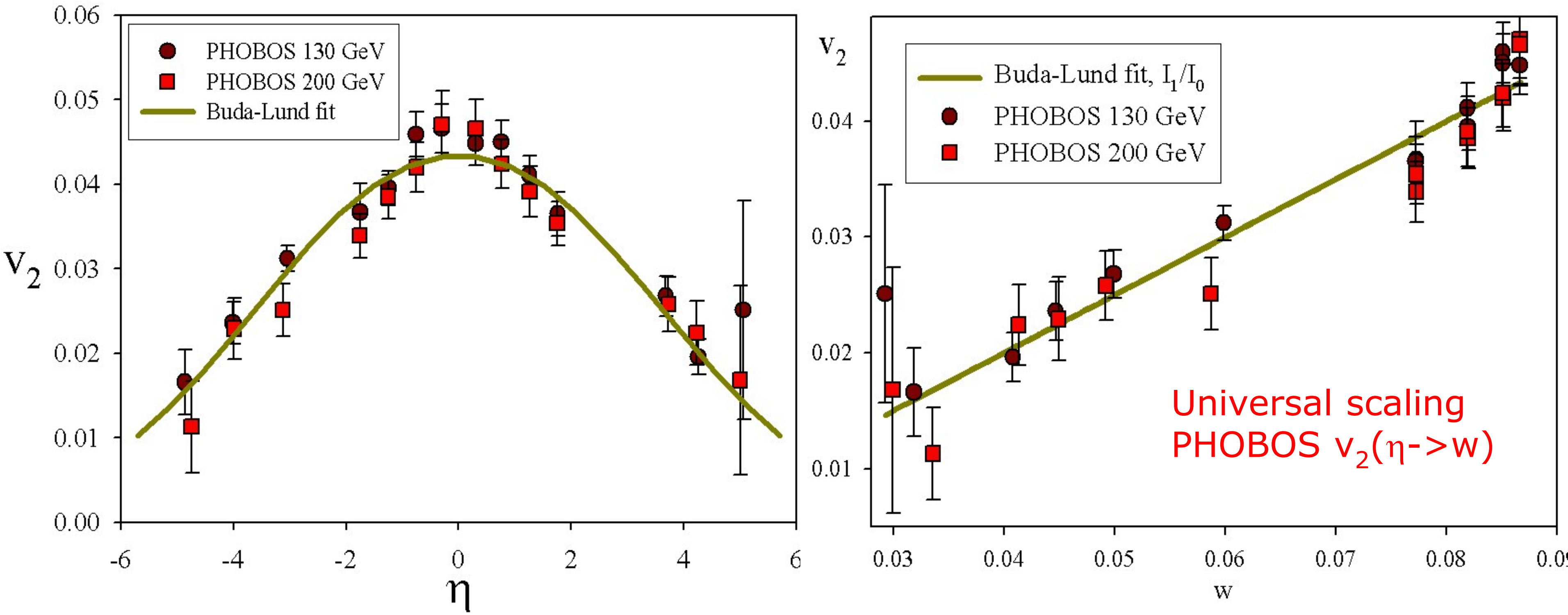
$$v_{2n+1} = 0$$

**w ~ Energy x (slope difference)**

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}}-\frac{1}{T_{*,x}}\right)$$

$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

# Confirmation




see nucl-th/0310040 and nucl-th/0403074,
R. Lacey@QM2005/ISMD 2005
A. Ster @ QM2005.

# Slope parameters

**Buda-Lund rel. hydro formula:**

$$T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

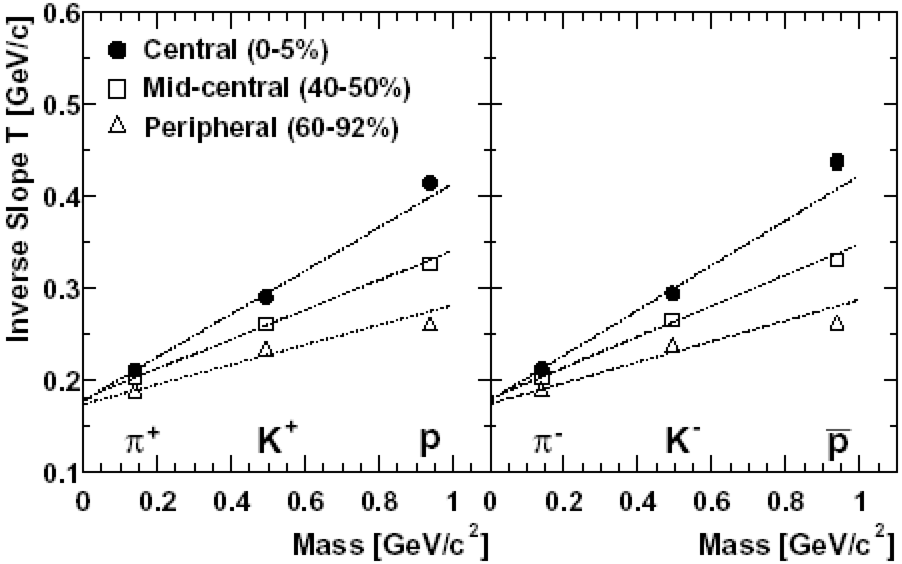
- **Exact non-rel. hydro solution:**

same, but $m_t$ -> m, a-> 0

$$T'_x = T_f + m\dot{X}_f^2 \; ,$$

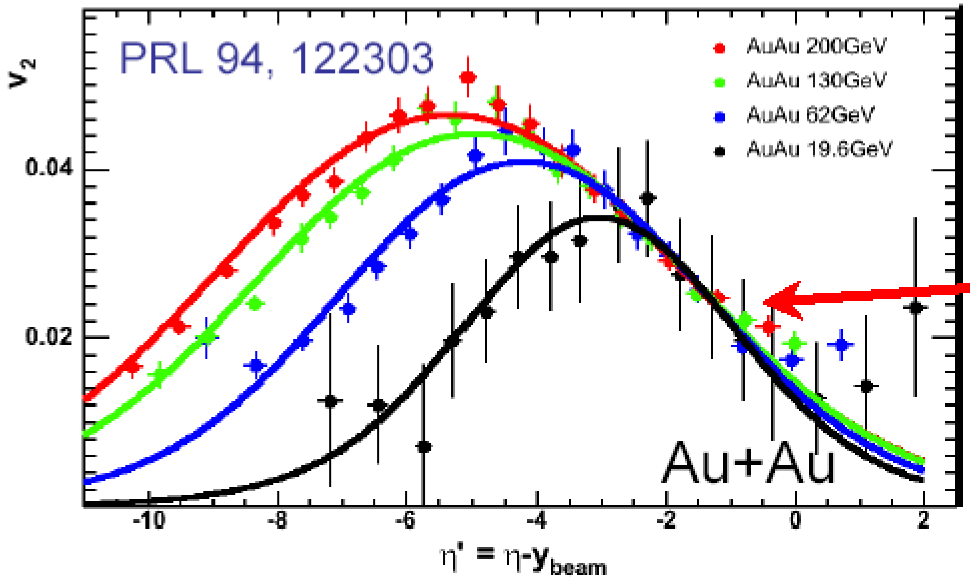
$$T'_y = T_f + m\dot{Y}_f^2 \; ,$$

**Experimental test: PHENIX, STAR**



# Elliptic flow, limits

## Extended longitudinal scaling: $v_2$




A surprising scaling!

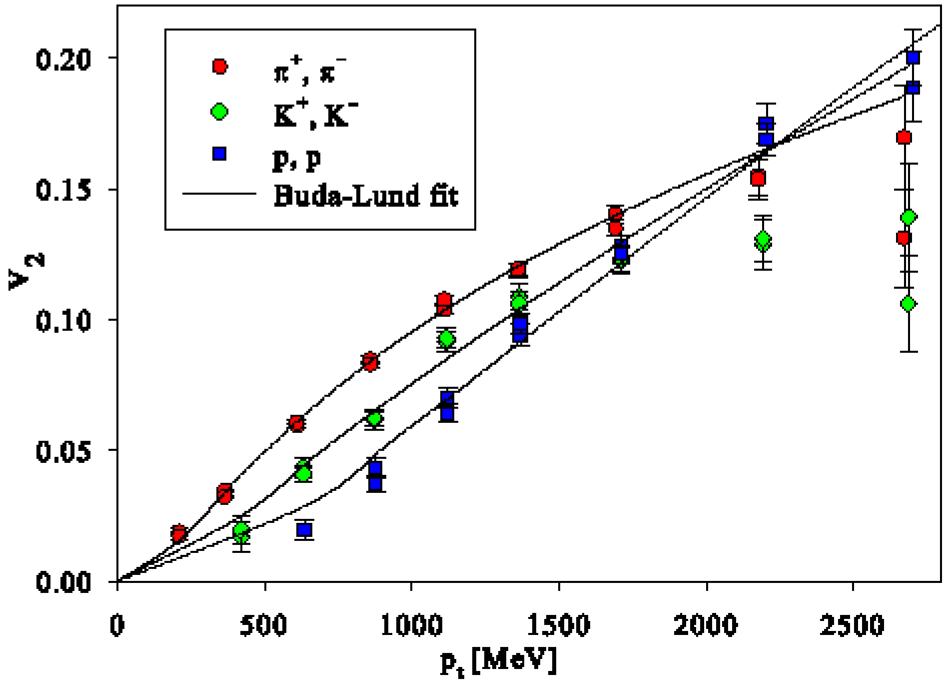
Not an initial state effect

nucl-th/0505019

Scaling reproduced by the Buda-Lund parametrization of the emitting source.

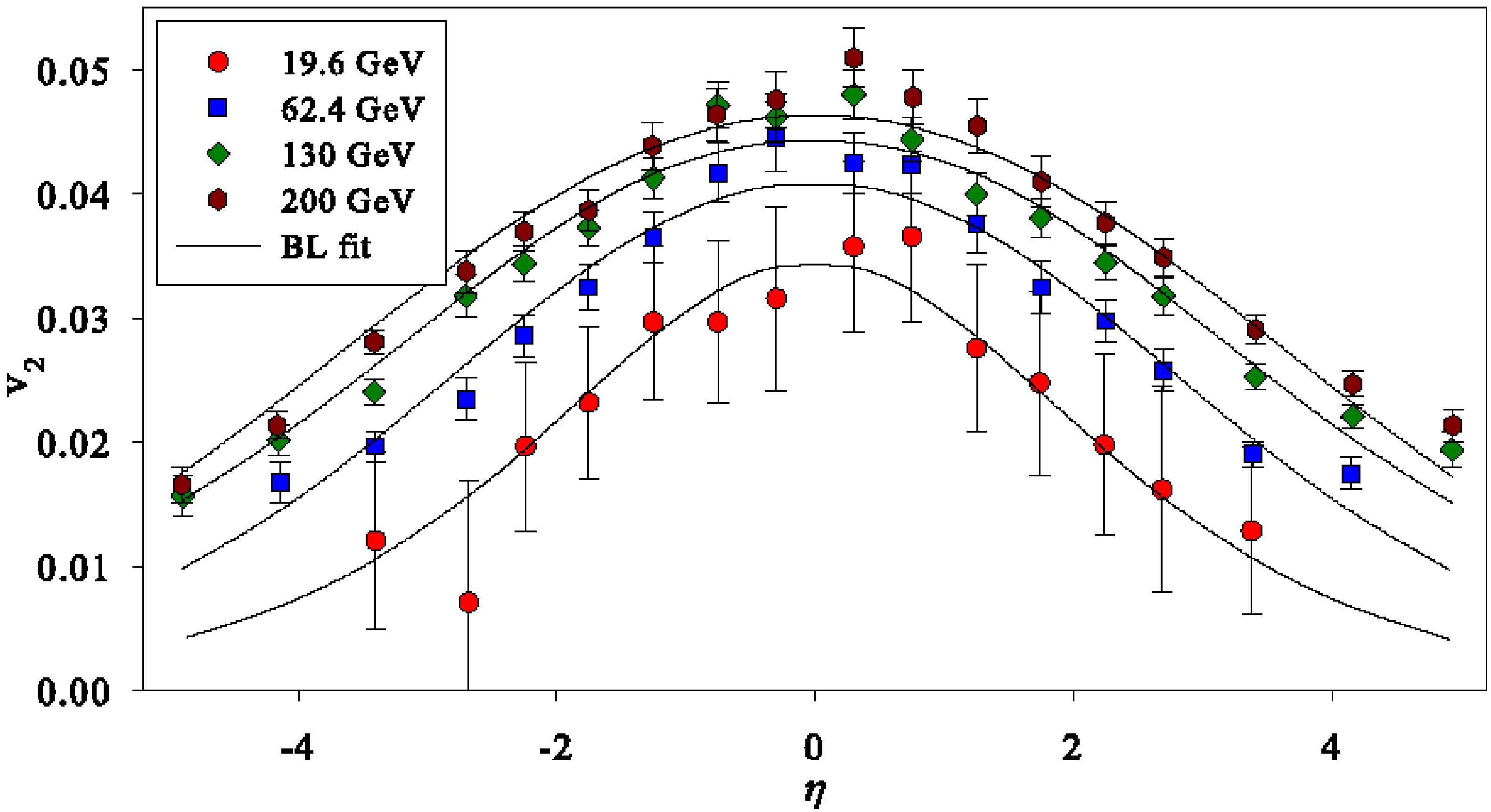
G. Veres, PHOBOS data, proc QM2005

# Elliptic flow, PHENIX data



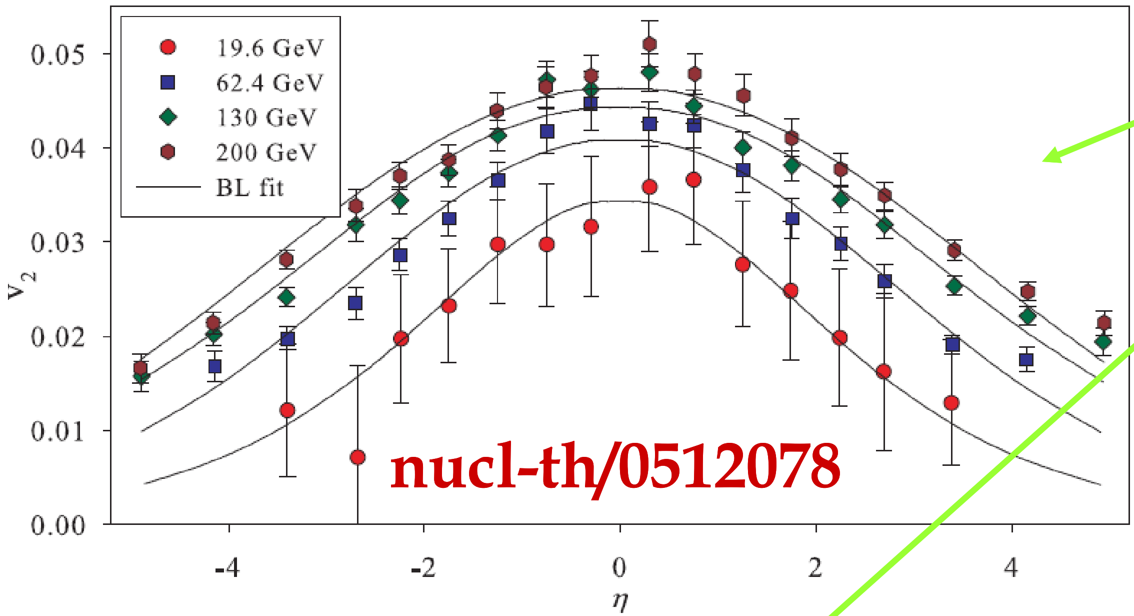

M. Csanád et al. nucl-th/0512078

# Elliptic flow, PHOBOS data

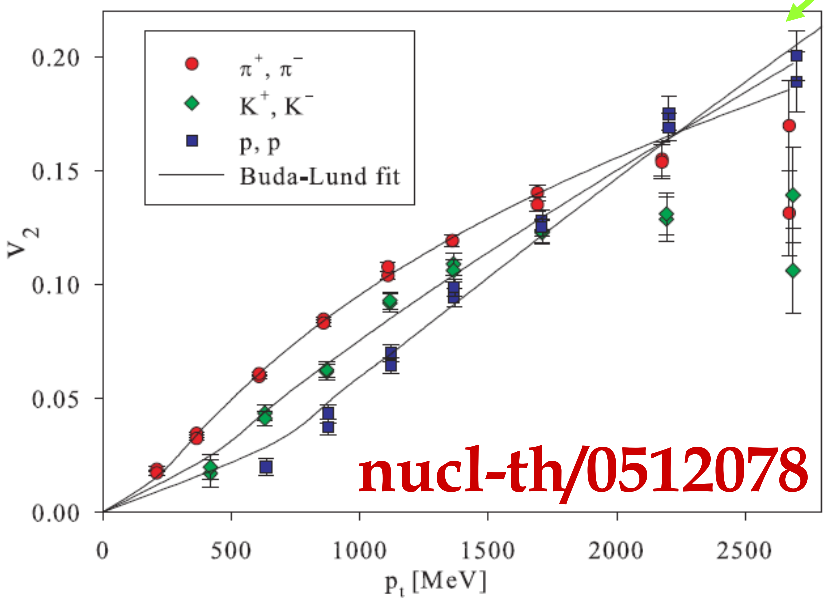


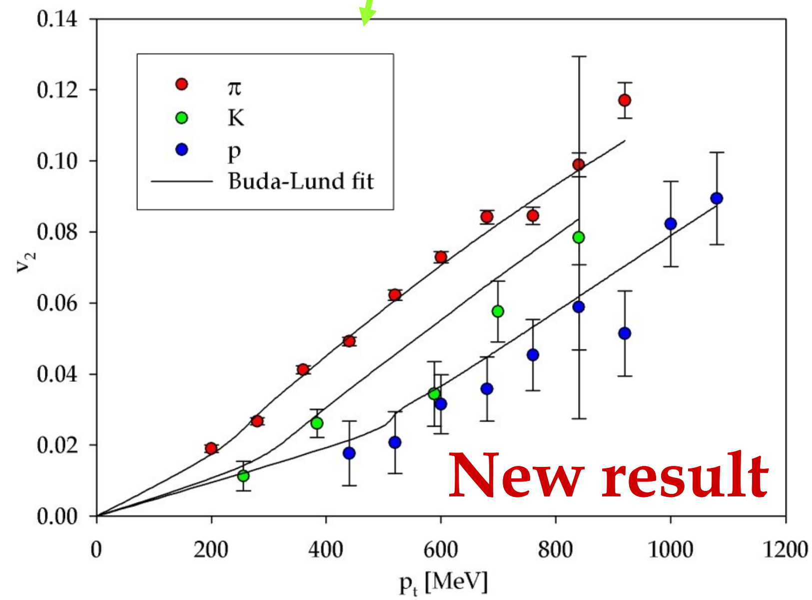

M. Csanád et al. nucl-th/0512078

# Fits to RHIC Au+Au data




- PHOBOS
  - 0-40%
- PHENIX
  - 200GeV, 0-92%
- STAR (new!)
  - 200GeV, 20-60%



# Elliptic flows

**Buda-Lund rel. hydro formula nucl-th/1003040 (2003!):**

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}} - \frac{1}{T_{*,x}}\right)$$

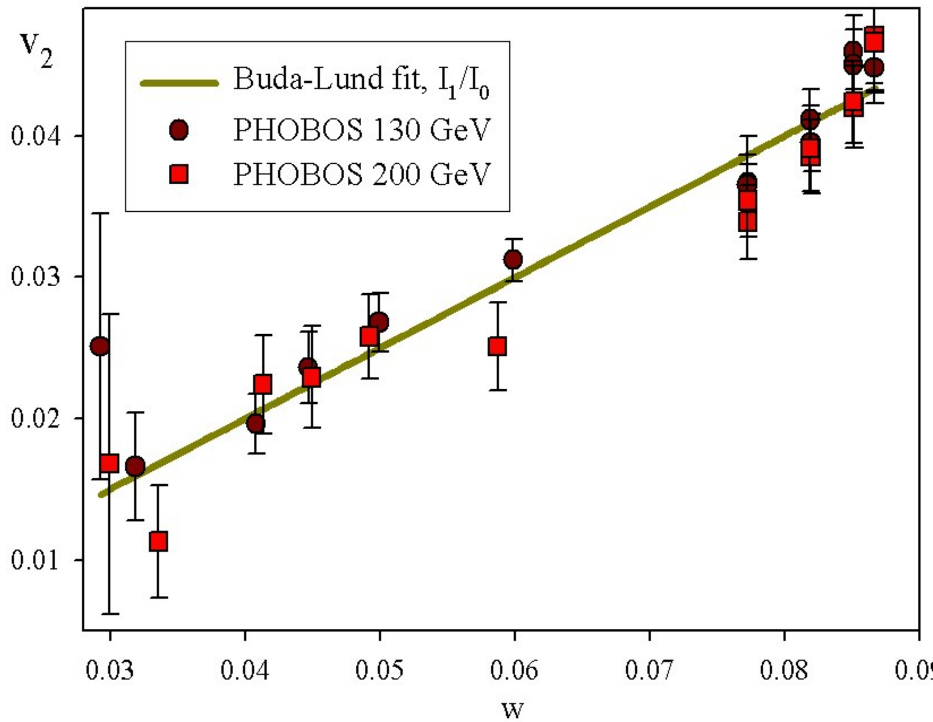
$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

**Experimental test (on PHOBOS data, PRL 2005!)**

**Exact non-relativistic result:**

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

**same, but $\underline{m_t}$ -> m**



# Hydro predicts scaling

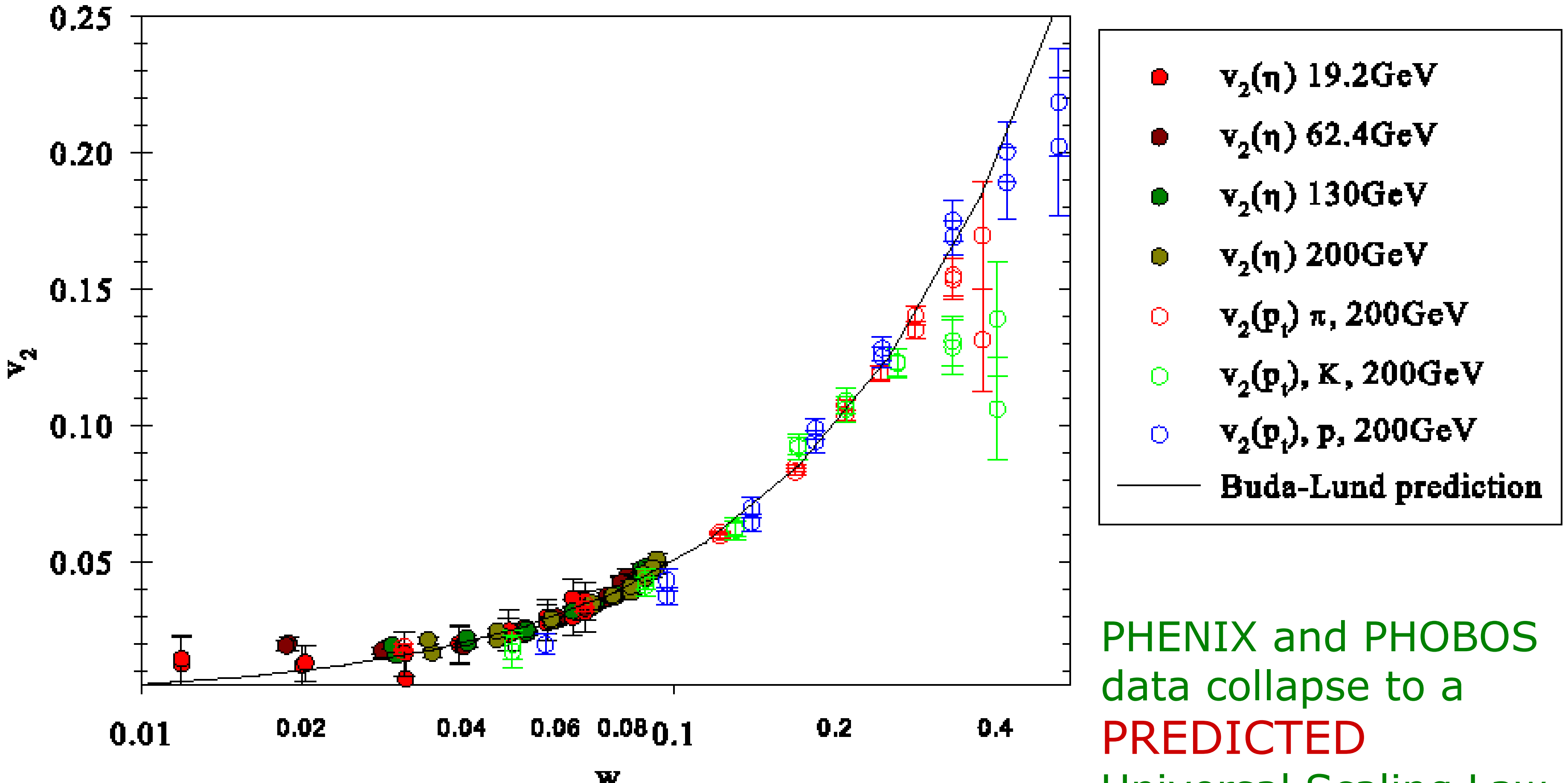
- **Scaling variable** $$w = \frac{p_t^{\,2}}{4m_t}\left(\frac{1}{T_{*,y}} - \frac{1}{T_{*,x}}\right)$$

- **For every type of measurement:** $$v_2 = \frac{I_1(w)}{I_0(w)}$$

- **Elliptic flow depends on every physical parameter only through** $w$

- **Scaling curve** $I_1 / I_0$ **?**

# Universal $v_2$ scaling predicted in 2003

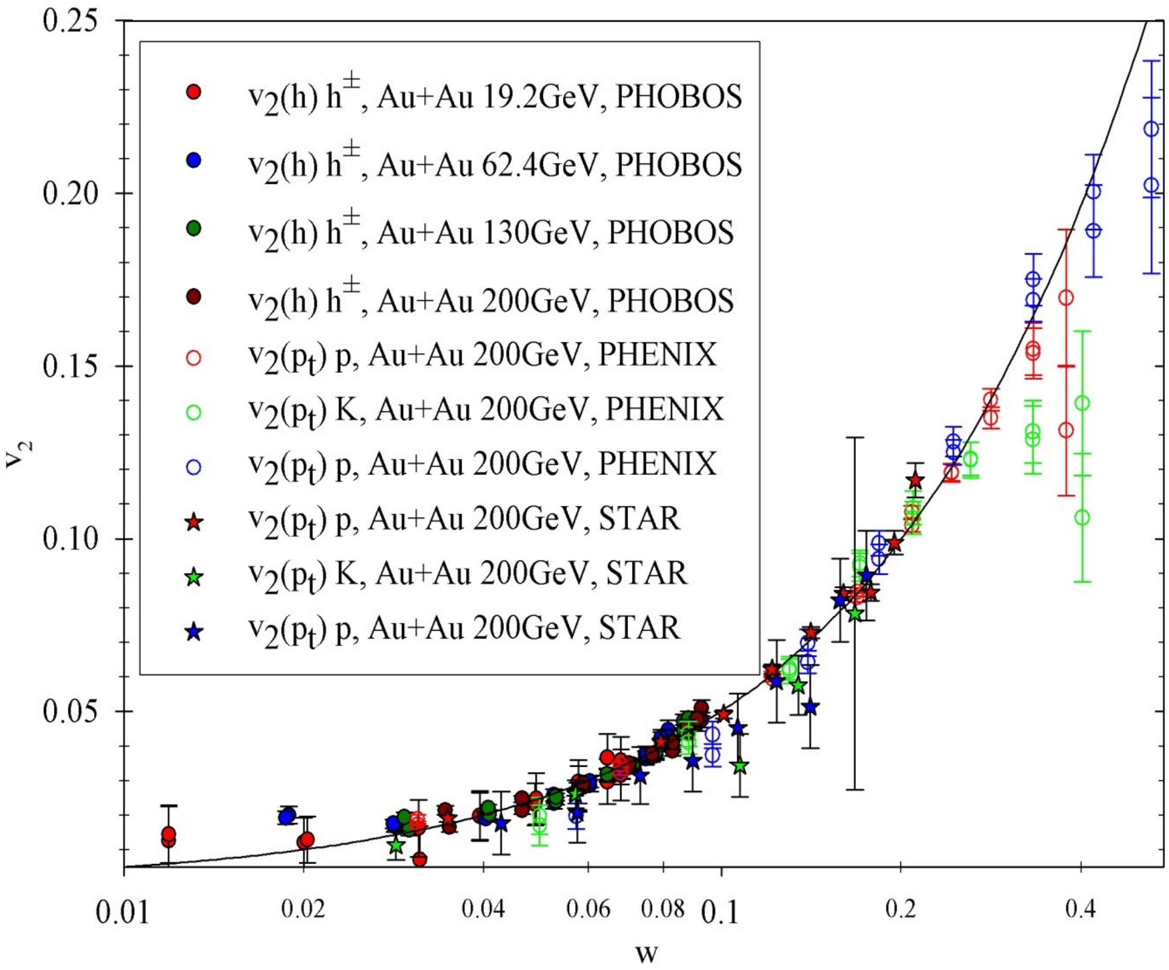



PHENIX and PHOBOS data collapse to a PREDICTED Universal Scaling Law of perfect fluid hydro

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

# Universal scaling

- **Scale parameter w**




**The perfect fluid extends from very small to very large rapidities at RHIC**

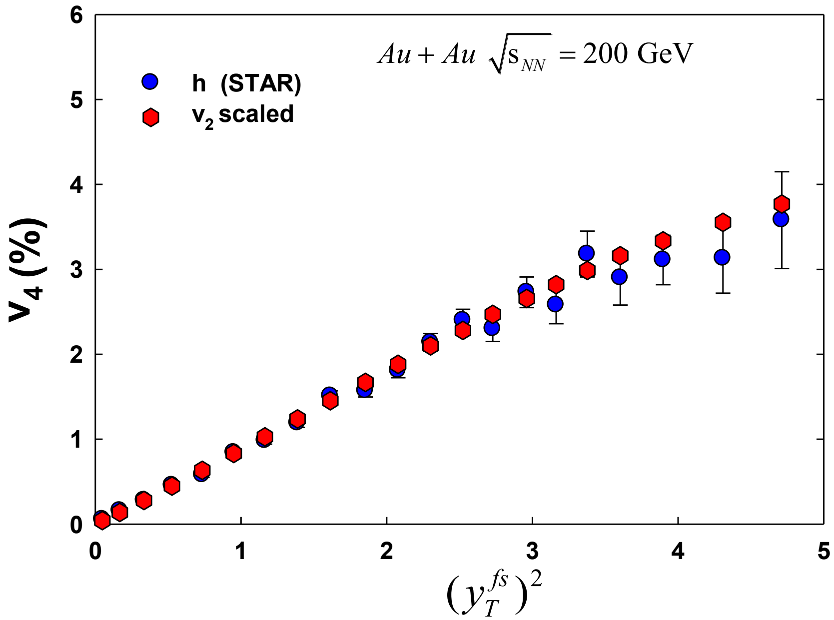
# Higher flow coefficients

**Buda-Lund rel. hydro formula:**

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$
$$v_{2n+1} = 0$$

**Exact non-relativistic result:**

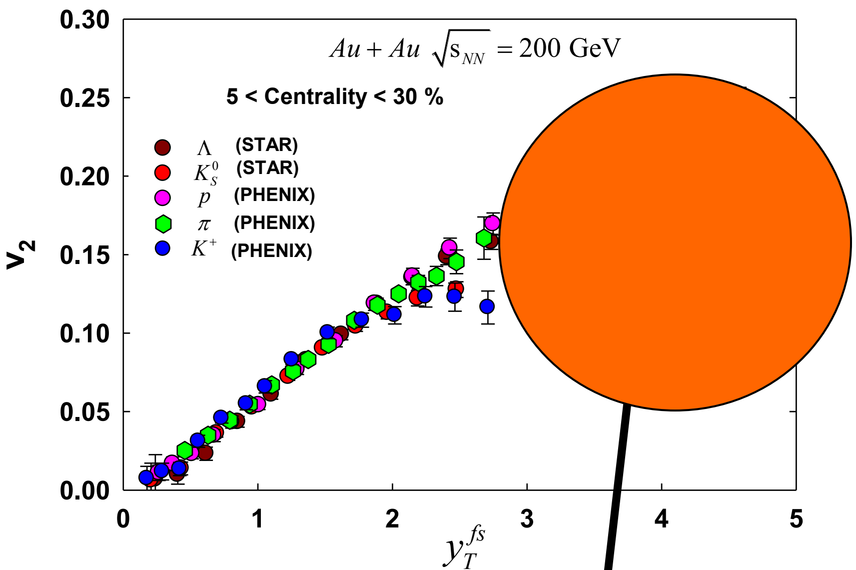
$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$
$$v_{2n+1} = 0$$

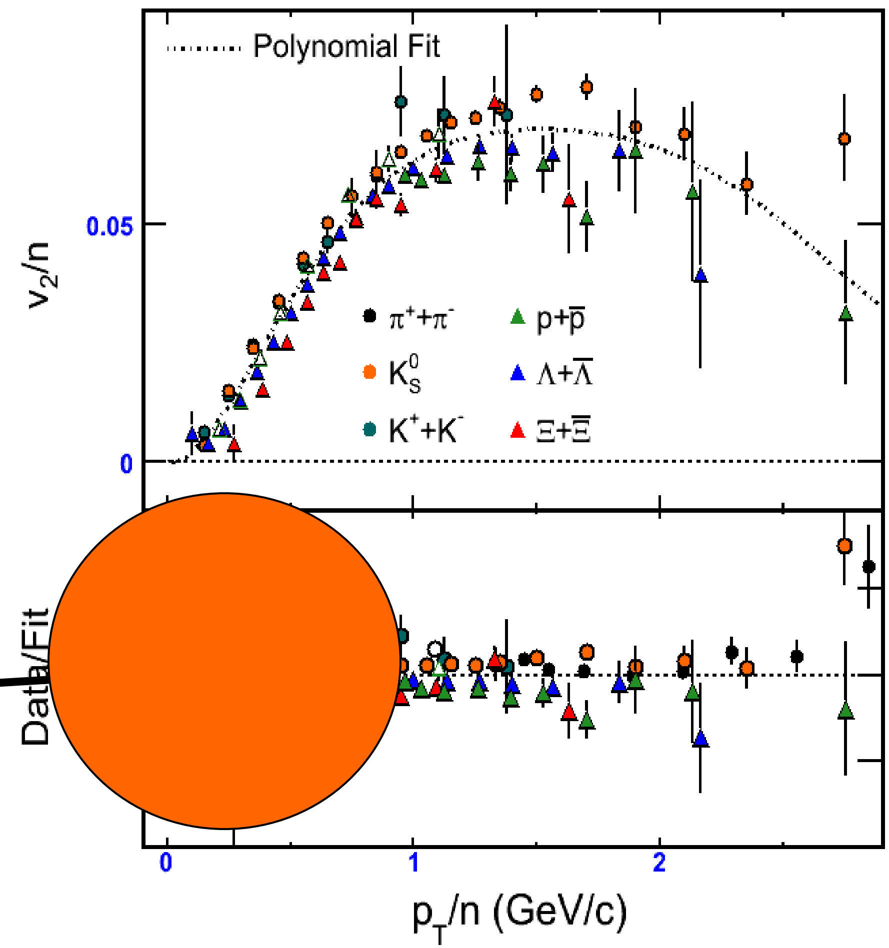



$$v_4 = \frac{v_2^2}{2} + k \times y_T^4$$

R. Lacey, Proc. QM 2005

# Scaling and scaling violations




Universal hydro scaling breaks where quark number scaling sets in, pt ~ 1-2 GeV
Fluid of QUARKS!!

R. Lacey and M. Oldenburg Proc. QM 2005

# Scaling laws from hydro

## Exact non-rel. and Buda-Lund rel.

Single particle spectra
Slope
Rapidity width
Elliptic flow
Higher harmonics
HBT radius parameters
asHBT

Au+Au data at RHIC satisfy the scaling laws that were predicted by the Buda-Lund hydro model.

v2(y,pt, ...) is mapped already to a universal scaling function

-> compelling evidence for a perfect fluid at RHIC

scaling breaks between 1-2 GeV, where quark number scaling sets in.

# Summary

## Universal scaling of $v_2$ is observed

**Au+Au data at RHIC satisfy the**
**UNIVERSAL scaling laws**
**predicted in 2003 by the Buda-Lund hydro model,**
**based on exact solutions of**
**PERFECF FLUID hydrodynamics**

**quantitative evidence for a perfect fluid in Au+Au at RHIC**

**scaling breaks in pt at ~ 1.5 GeV,**
**in rapidity at ~ ymax - 0.5**
**Search for establishing the domain of applicability started.**
**Further tests with STAR and BRAHMS data.**