Universal scaling of v_2 in Au+Au collisions

and the Perfect Fluid at RHIC

T. Csörgő

with M. Csanád, R. Lacey et al.

- **Introduction:**
  - Press release, BNL: RHIC Scientists serve up "Perfect Liquid", April 18, 2005 - "White Papers" in Nucl. Phys. A
  - New results at QM05 and at the 5th Budapest RHIC School
- **Hydrodynamics and scaling of soft observables**
  - Exact hydro results
  - Scaling of slope parameters
  - Bose-Einstein /HBT radii
  - the elliptic and higher order flows
- **Intermediate pt region: breaking of the hydro scaling**

Discovering New Laws

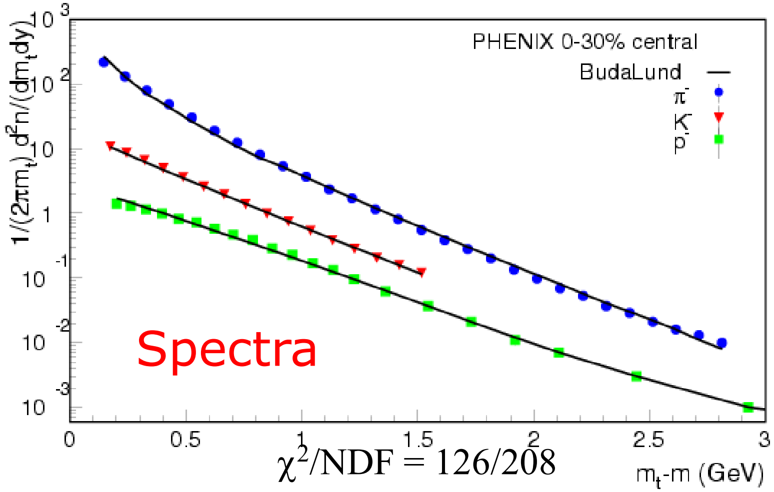
"In general we look for a new law by the following process.
First we guess it.
Then we compare the consequences of the guess to see
what would be implied if this law that we guessed is right.
Then we compare the result of the computation to nature,
with experiment or experience, compare it directly with observation,
to see if it works.
If it disagrees with experiment it is wrong.

In that simple statement is the key to science.
It does not make any difference how beautiful your guess is.
It does not make any difference how smart you are,
who made the guess, or what his name is —
if it disagrees with experiment it is wrong.

/R.P. Feynman/"

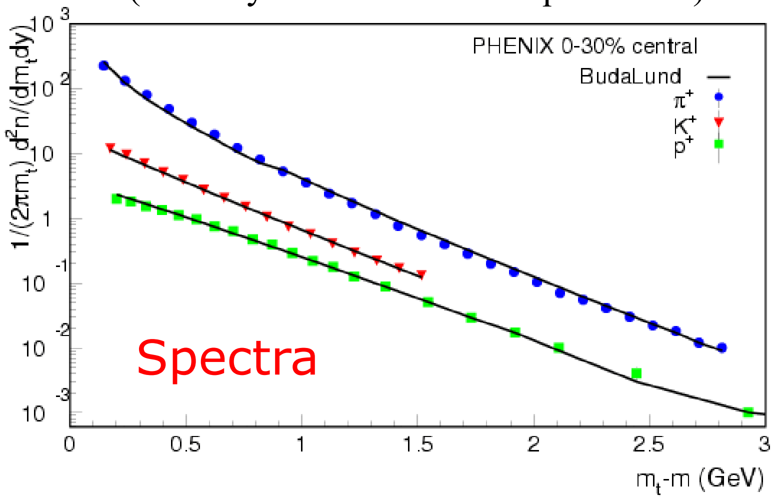
Buda-Lund hydro and Au+Au@RHIC

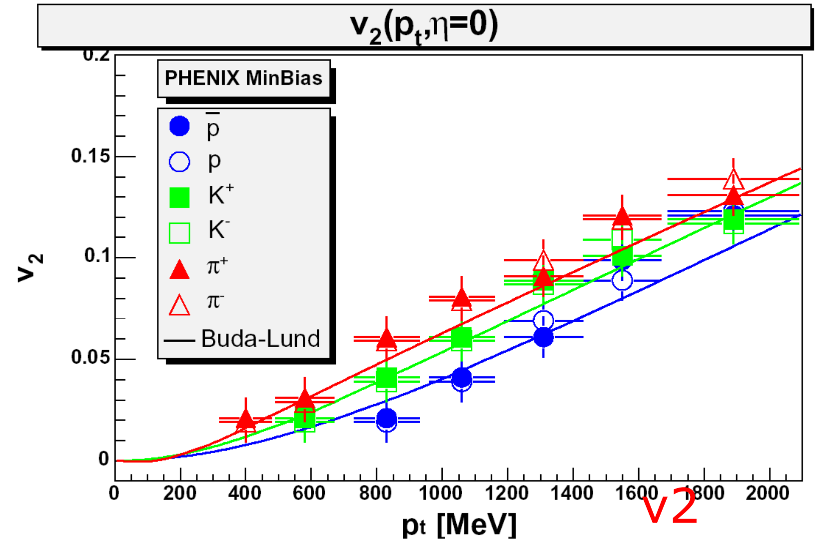
BudaLund v1.5 hydro fits to 200 AGeV Au+Au

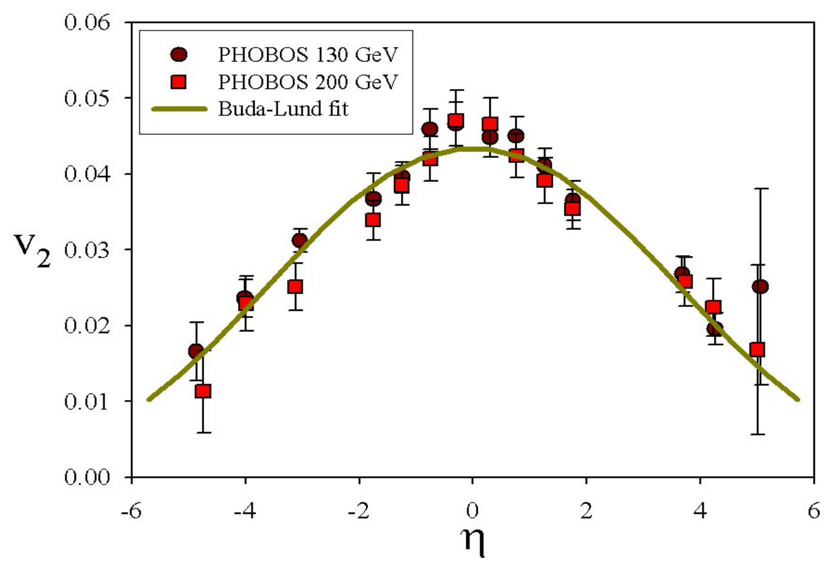


$\chi^2/NDF = 126/208$

(stat + syst errors added in quadrature)





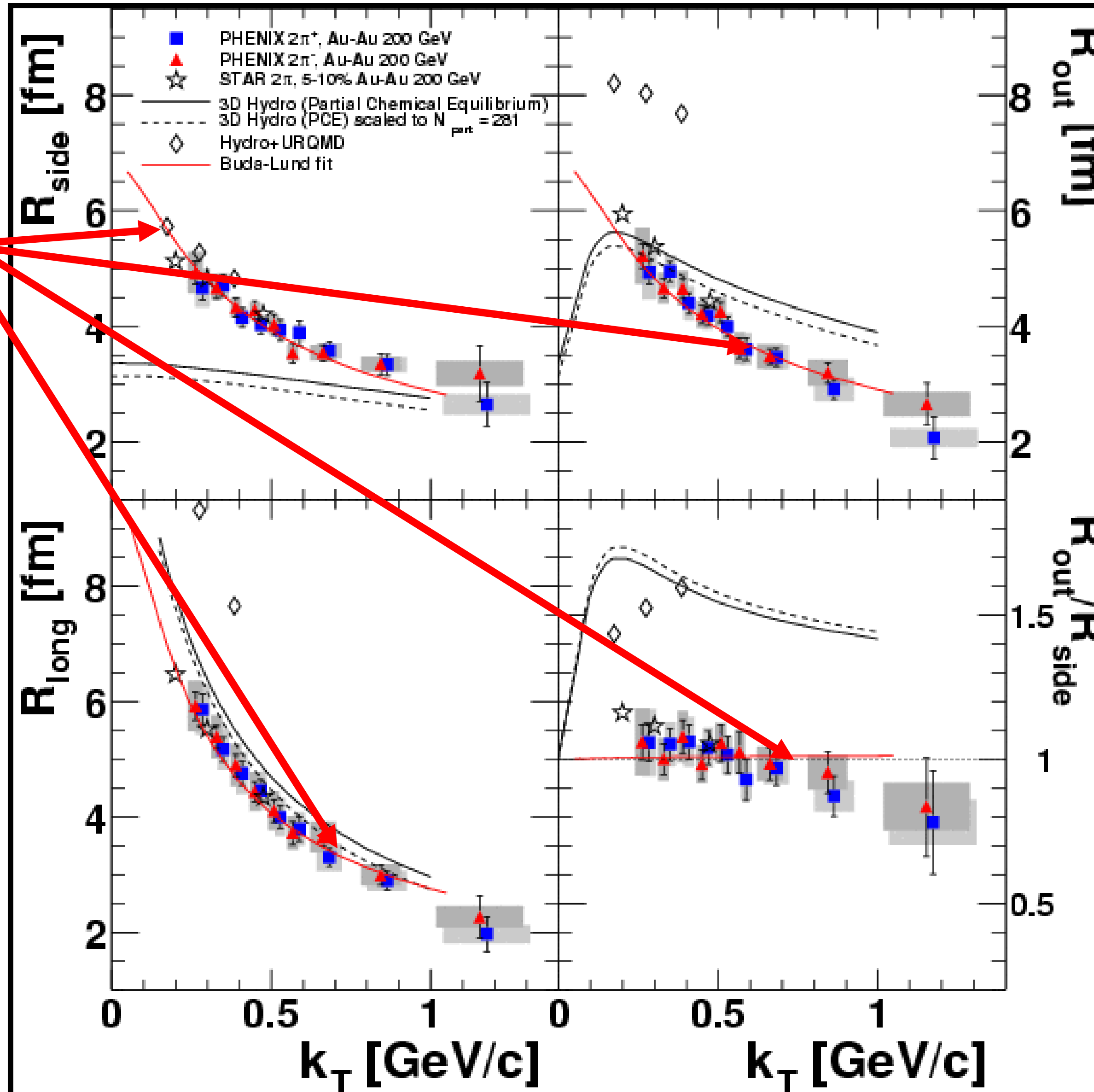


nucl-th/0311102, nucl-th/0207016, nucl-th/0403074

Femptoscopy signal of supercooled QGP

Buda-Lund hydro fit indicates
- scaling of HBT radii
- sudden hadronization
- a hint for supercooled QGP.

Hadrons with $T>T_c$ escape-
a hint also for cross-over transition



Phases of QCD Matter, EoS

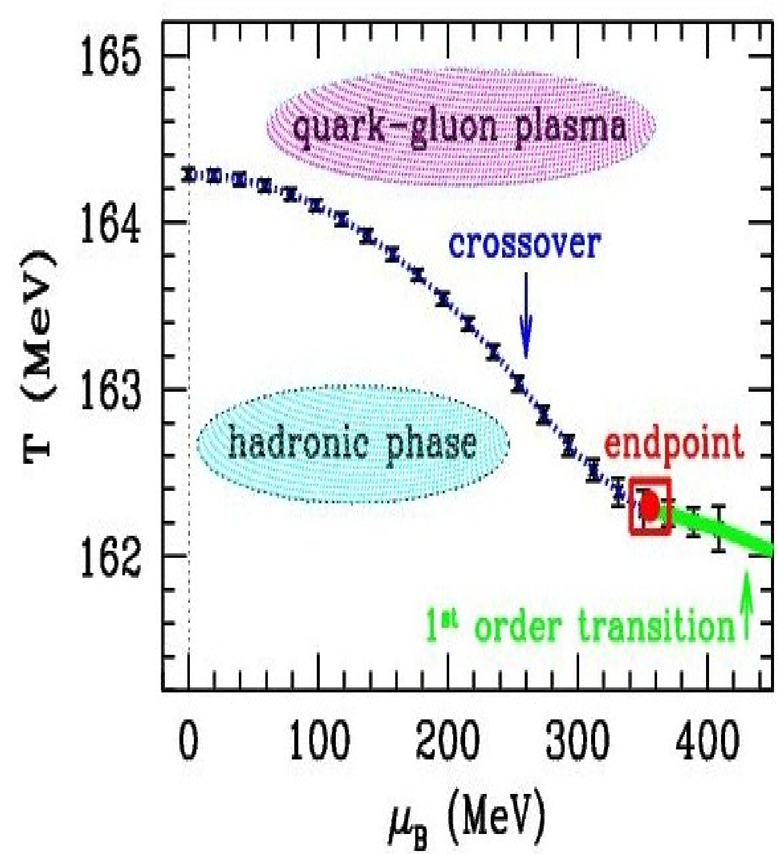
Quark Gluon Plasma

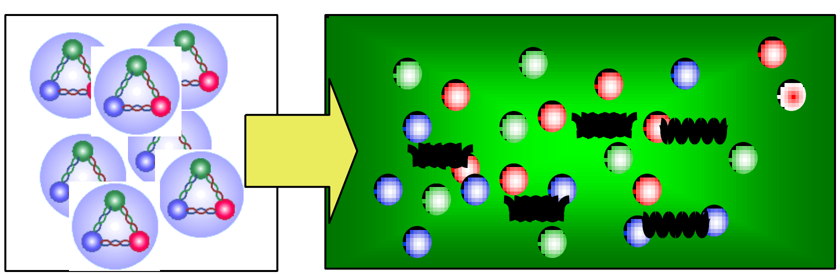
"Ionize" nucleons with heat

"Compress" them with density

New state(s?) of matter

Z. Fodor and S.D. Katz:
T_c = 164 ± 2 -> 189 ± 8 MeV, QM'05(?)
even at finite baryon density,
Cross over like transition.
(hep-lat/0106002, hep-lat/0402006)





For most recent results

S. D. Katz, http://qm2005.kfki.hu/

lattice QCD -> Equations of State

Input for hydrodynamics

Nonrelativistic hydrodynamics

- **Equations of nonrelativistic hydro:**

$$\begin{aligned} \partial_t n + \nabla(n\mathbf{v}) &= 0 \\ \partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} &= -(\nabla p)/(mn) \\ \partial_t \epsilon + \nabla(\epsilon\mathbf{v}) &= -p\nabla\mathbf{v} \end{aligned}$$

- **Not closed, EoS needed:**

$$\begin{aligned} \epsilon &= \kappa p \\ p &= nT \end{aligned}$$

- **Perfect fluid: no viscosity and heat conductivity**
- **We use the following scaling variable:**

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

- **X, Y and Z are characteristic scales, depend on (proper-) time**

Exact nonrelativistic solutions

- **A general group of scale-invariant solutions (hep-ph/0111139):**

$$
\begin{aligned}
n(t,\mathbf{r}) &= n_0 \frac{V_0}{V}\nu(s) \\
\mathbf{v}(t,\mathbf{r}) &= \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z\right) \\
T(t,\mathbf{r}) &= T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \mathcal{T}(s) \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0}\int_0^s \frac{du}{\mathcal{T}(u)}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{r}' &= (r_x \frac{X_0}{X}, r_y \frac{Y_0}{Y}, r_z \frac{Z_0}{Z}) \\
n\,(t,\mathbf{r}) &= n(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ}\right) \\
v_x\,(t,\mathbf{r}) &= v_x(t_0,\mathbf{r}') \frac{\dot X}{\dot X_0}, \ldots \\
T\,(t,\mathbf{r}) &= T(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ}\right)^{1/\kappa}
\end{aligned}
$$

- **This is a PARAMETRIC but exact solution, if the scales fulfill:**

T. Csörgő, Acta Phys. Polonica B37 (2006) 1001

□Tale of 1001 nights”

$$
X\ddot X = Y\ddot Y = Z\ddot Z = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa}
$$

- **Temperature scaling function is arbitrary,**
  e.g. Constant temperature $\Rightarrow$ Gaussian density

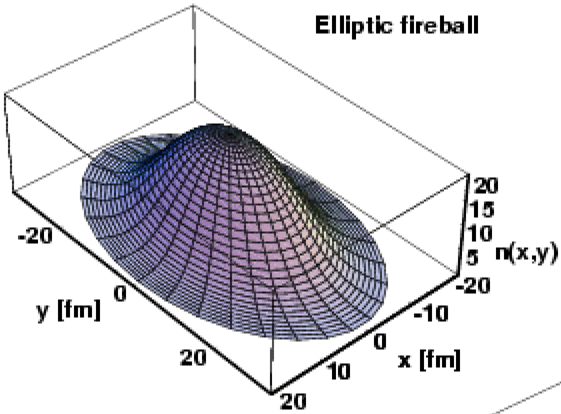
Buda-Lund profiles:

$$
\begin{aligned}
\mathcal{T}(s) &= \frac{1}{1+bs} \\
\nu(s) &= (1+bs)\exp\left[-\frac{T_i}{2T_0}(s+bs^2/2)\right]
\end{aligned}
$$

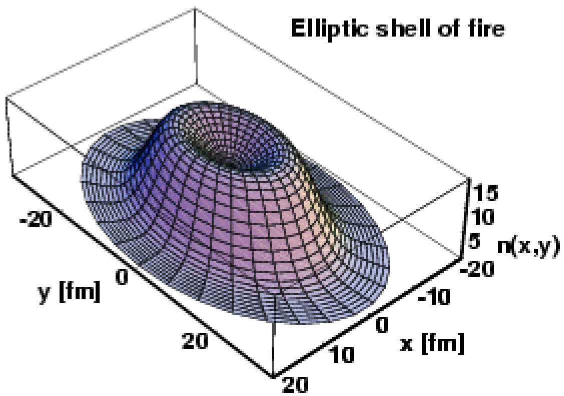
Zimányi-Bondorf-Garpman profiles:

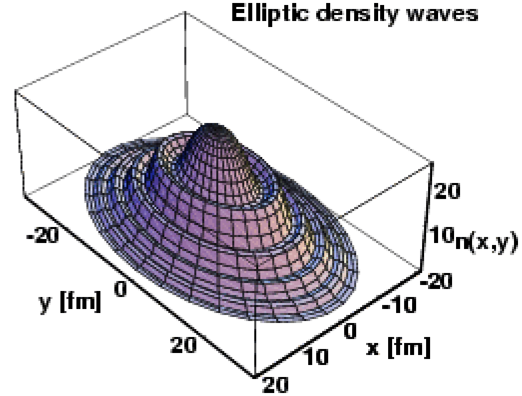
$$
\begin{aligned}
\mathcal{T}(s) &= (1-s)\,\Theta(1-s) \\
\nu(s) &= (1-s)^{\alpha}\,\Theta(1-s)
\end{aligned}
$$

Some new solutions of hydro



- Non-relativistic as well as relativistic generalizations
- 1d, 3d axial, 3d ellipsoidal
- arbitrary temperature profile functions
- Excellent for illustration
- Good tool for students
- Asymptotic solutions vs. collisionless Boltzmann gas





Time evolution of the scales follows a classical motion!

Scale parameters determine observables - info on history LOST!

Friedmann eq. of heavy ion physics

- **Scale invariant solutions of fireball hydro, hep-ph/0111139:**

$$X\ddot{X} = Y\ddot{Y} = Z\ddot{Z} = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{1/\kappa}$$

- **From global energy conservation -> "Friedmann equation"**

$$\frac{\partial}{\partial t}\int d^3r(\varepsilon + \frac{nmv^2}{2}) = 0$$

$$\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 + 3\frac{T_0}{m}\left(\frac{V_0}{V}\right)^{2/3} = A = const.$$

$$R^2(t) = X^2(t) + Y^2(t) + Z^2(t) \; = \; A(t-t_0)^2 + B(t-t_0) + C,$$

$$H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{3}{2}T_0\left(\frac{X_0Y_0Z_0}{XYZ}\right)^{2/3} \qquad (P_x, P_y, P_z) = m(\dot{X}, \dot{Y}, \dot{Z})$$

Hamiltonian motion in heavy ion physics

- **Direction dependent Hubble flow**

$$\boxed{H = \frac{1}{2m}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{3}{2}T_0\left(\frac{X_0 Y_0 Z_0}{XYZ}\right)^{2/3}}$$

$$\begin{aligned} v_x(t,\mathbf{r}) &= \frac{\dot X(t)}{X(t)} r_x, \\ v_y(t,\mathbf{r}) &= \frac{\dot Y(t)}{Y(t)} r_y, \\ v_z(t,\mathbf{r}) &= \frac{\dot Z(t)}{Z(t)} r_z. \end{aligned}$$

$$(P_x, P_y, P_z) = m(\dot X, \dot Y, \dot Z)$$

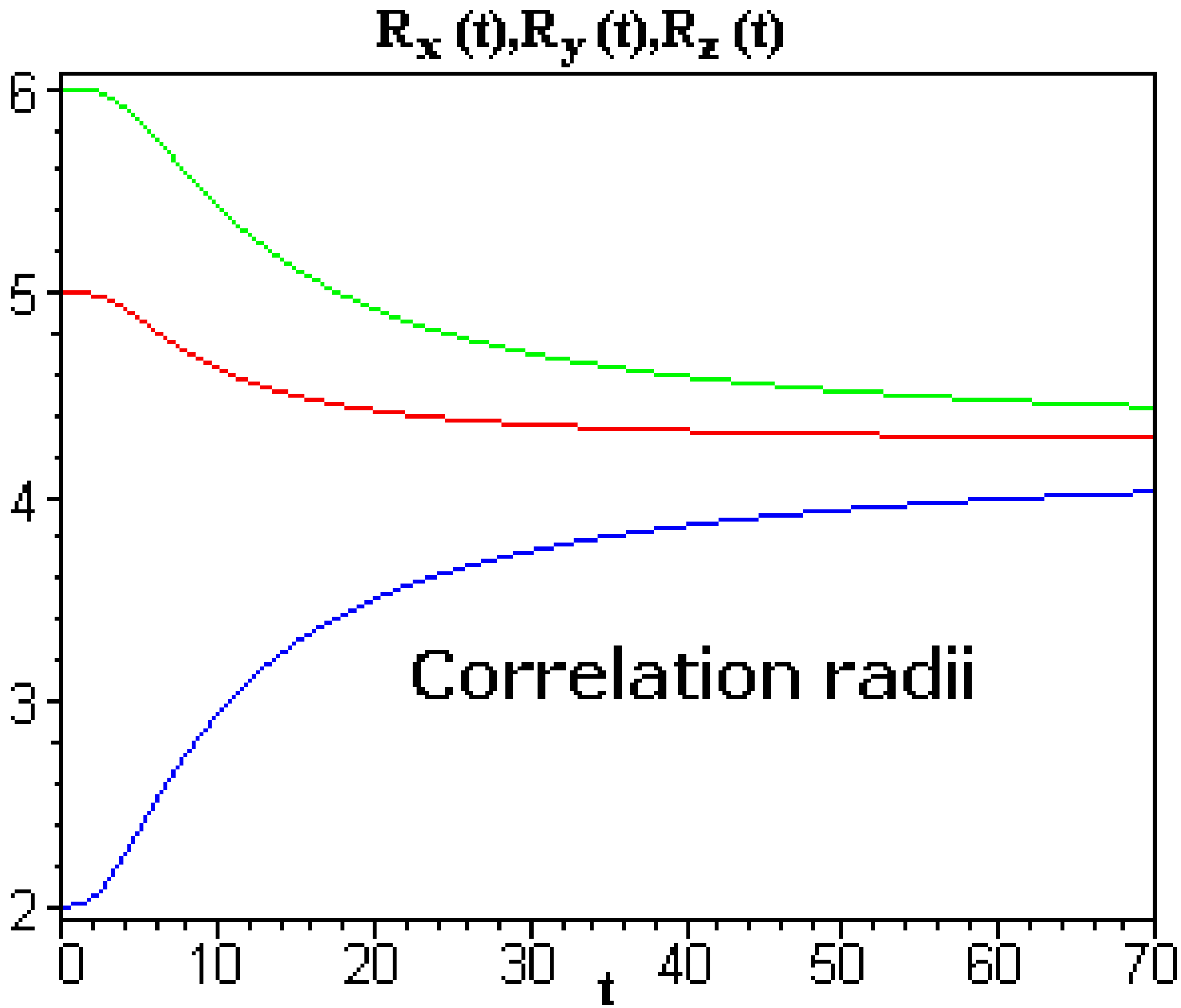
$$T(t) = T_0\left(\frac{V_0}{V(t)}\right)^{2/3},$$

$$n(t,\mathbf{r}) = n_0 \frac{V_0}{V(t)} \exp\left(-\frac{r_x^2}{2X(t)^2} - \frac{r_y^2}{2Y(t)^2} - \frac{r_z^2}{2Z(t)^2}\right),$$

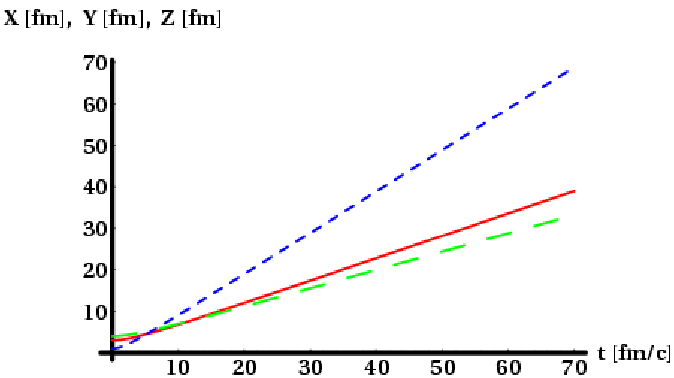
- **Late t -> v = H r, where H = 1/t. Spherical symmetry: R=X=Y=Z**
- **2/3 in general: c_s^2, if T_0 < 0, and c_s^2 = 1/3 -> Friedmann**

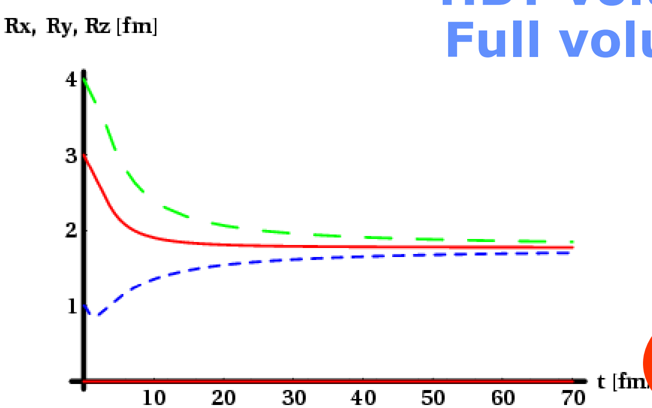
Examples of exact hydro results

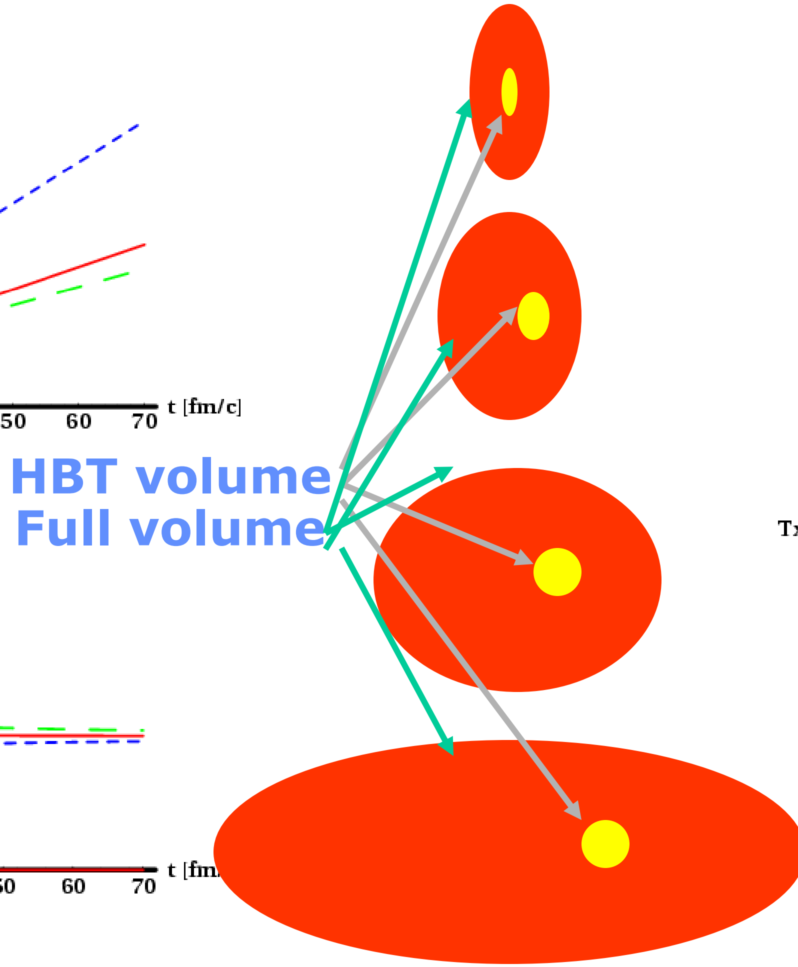
- **Propagate the hydro solution in time numerically:**

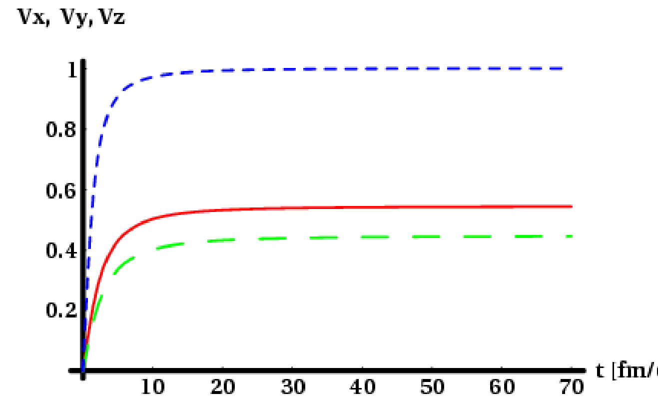


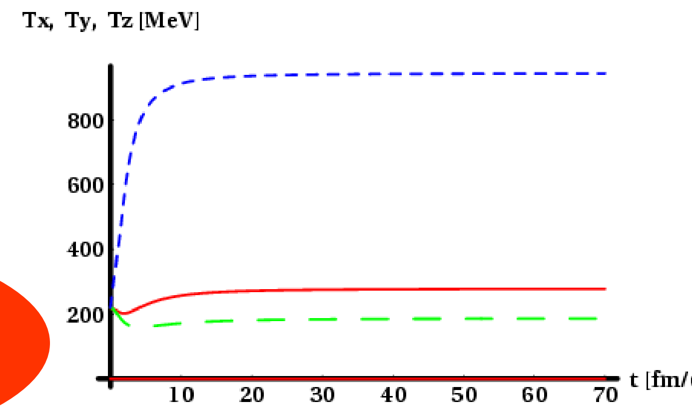
The RHIC horizont problem





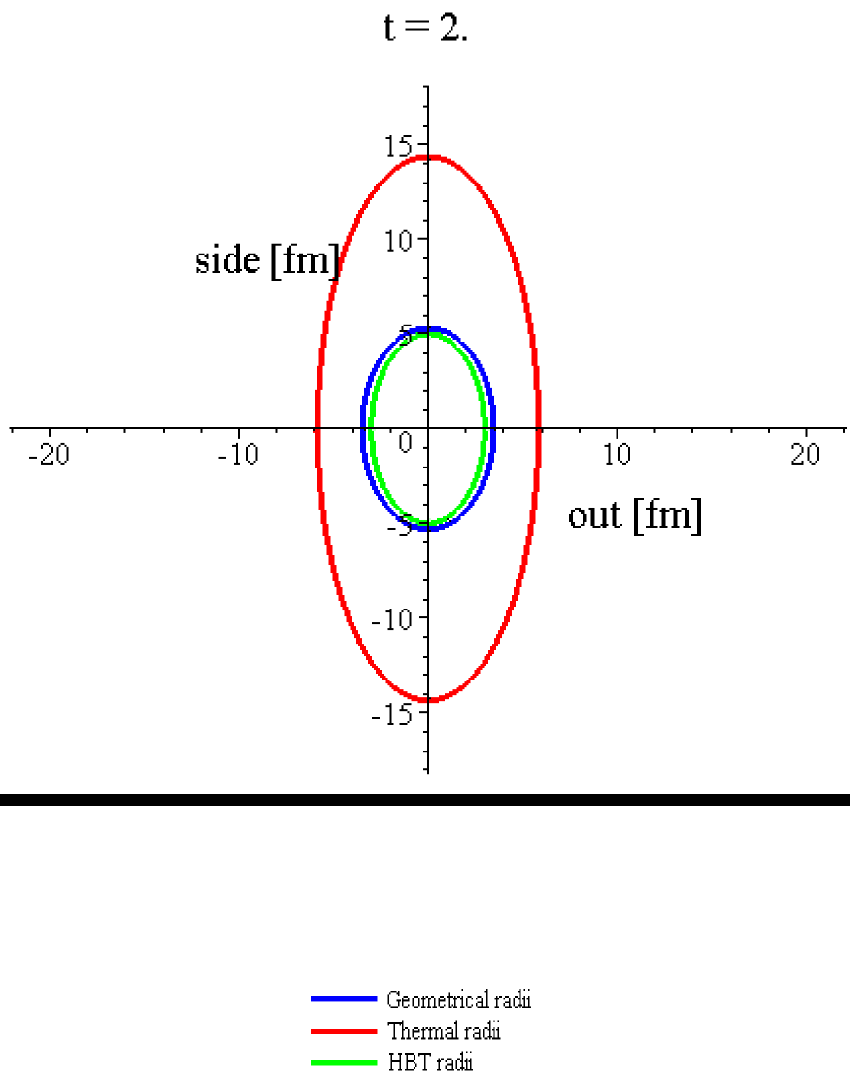






Geometrical sizes increase, velocities tend to a constant. Slope parameters and radii freeze out and Rx, Ry, Rz -> const! General property, analytic result!

Geometrical & thermal & HBT radii



3d analytic hydro: exact time evolution (!!)

geometrical size (fugacity ~ const)
Thermal sizes (velocity ~ const)
HBT sizes (phase-space density ~ const)

HBT dominated by the smaller of the geometrical and thermal scales

nucl-th/9408022, hep-ph/9409327
hep-ph/9509213, hep-ph/9503494

HBT radii approach a const(t) (!!!)
HBT volume -> spherical
HBT radii -> thermal, constant lengths!!

hep-ph/0108067, nucl-th/0206051
<-- Thanks to Máté Csanád for animation

Relativistic Perfect Fluids

- **Rel. hydrodynamics of perfect fluids is defined by:**

$$\partial_\mu \left(n u^\mu\right) = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \left(\varepsilon + p\right) u^\mu u^\nu - p g^{\mu\nu}$$

- **A recent family of exact solutions: (nucl-th/0306004):**

$$u^\mu = \frac{x^\mu}{\tau}$$
$$n(t,\mathbf{r}) = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \mathcal{V}(s)$$
$$p(t,\mathbf{r}) = p_0 \left(\frac{\tau_0}{\tau}\right)^{3+3/\kappa}$$
$$T(t,\mathbf{r}) = T_0 \left(\frac{\tau_0}{\tau}\right)^{3/\kappa} \frac{1}{\mathcal{V}(s)}$$

- **Overcomes two shortcomings of Bjorken's solution:**
  - **Rapidity distribution**
  - **Transverse flow**

$$u^\mu \partial_\mu u_\nu = 0$$

- **Hubble flow $\Rightarrow$ lack of acceleration.**
- **Accelerating exact rel. solutions: in preparation /Marci/**

Hubble from numerical rel. hydro

Assume net barion-free,
approx. boost invariant case
Rel. Euler equation
Entropy conservation
4 independent eqs, 5 variables

$$u^\mu \partial_\mu \left(T\, u^\nu\right) = \partial^\nu T,$$

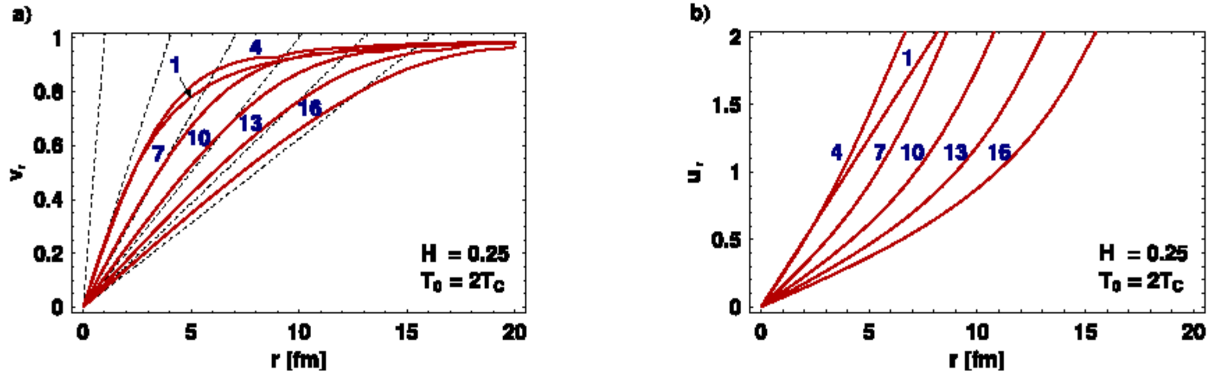
$$\partial_\mu \left(\sigma u^\mu\right) = 0,$$

$$d\varepsilon = T d\sigma, \qquad dP = \sigma dT, \qquad w = \varepsilon + P = T\sigma,$$

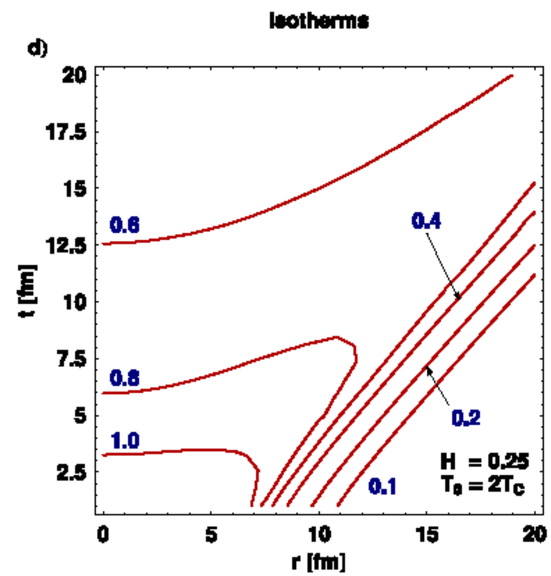
Closed by thermodynamical relationships.
key quantity:
temperature dependent speed of sound
can be taken from lattice QCD

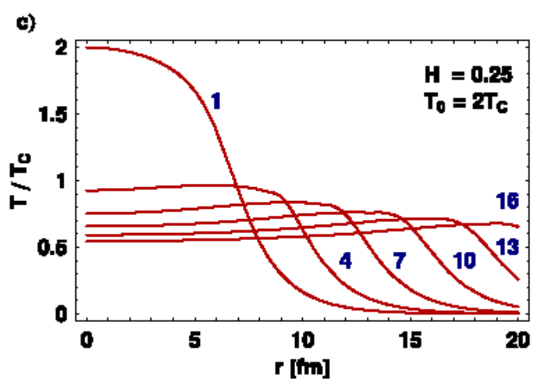
$$c_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{\sigma}{T}\frac{\partial T}{\partial \sigma}$$

Some num. rel. hydro solutions



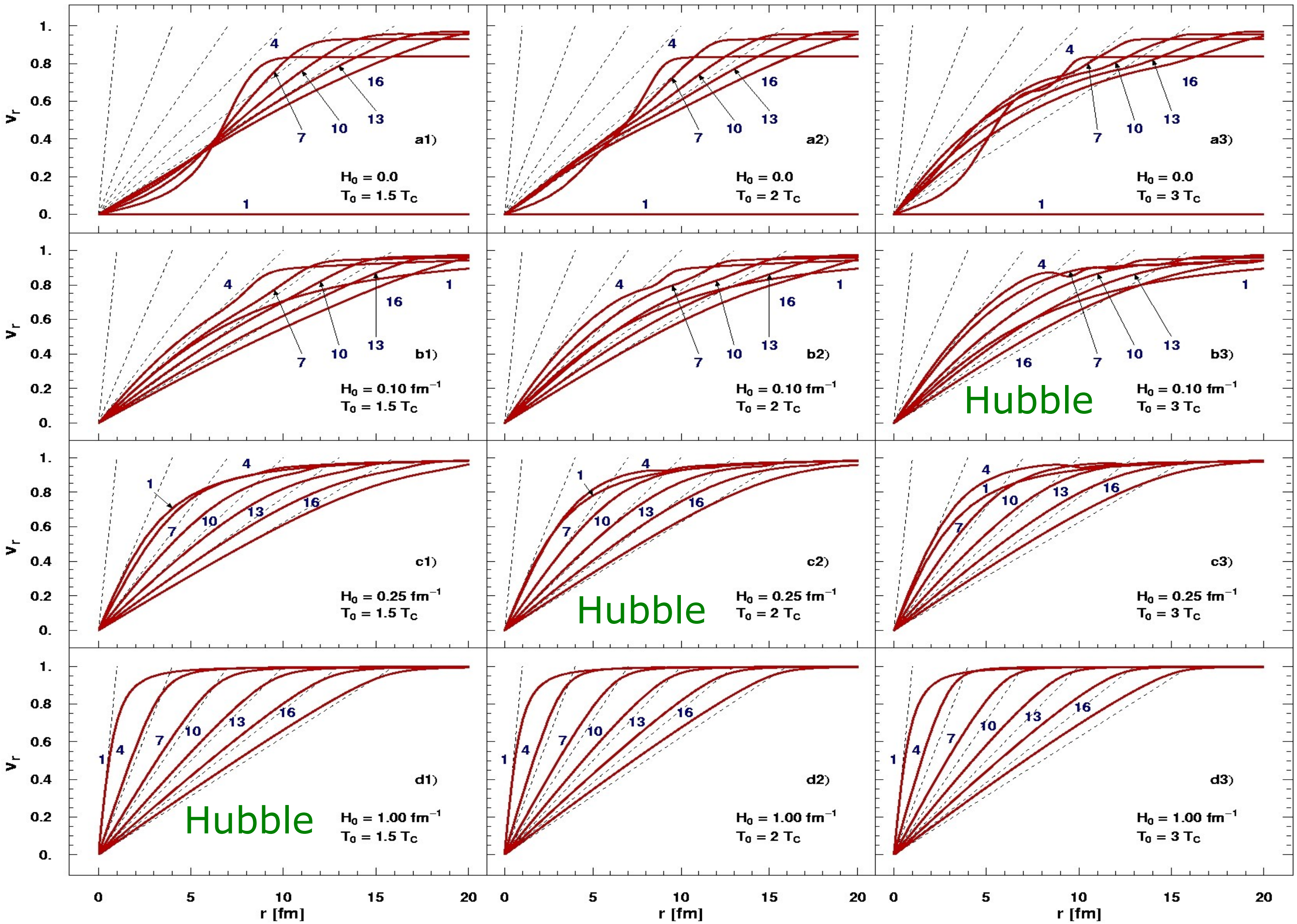
M. Chojnacki, W. Florkowski, T. Cs,
nucl-th/0410036
lattice QCD EOS (μ_B=0)
T_0(r) ~ initial entropy (Glauber)
H_0 ~ initial Hubble flow





Support the quick development of the Hubble flow
and the Blast-wave, Buda-Lund and Cracow etc models

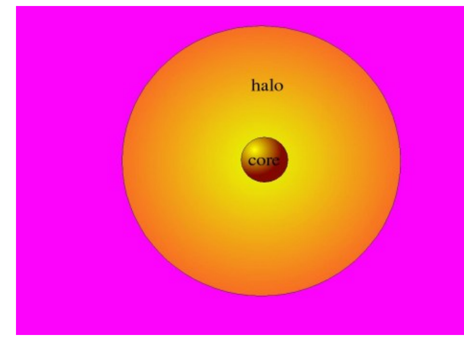
Effects of pre-equilibrium flow



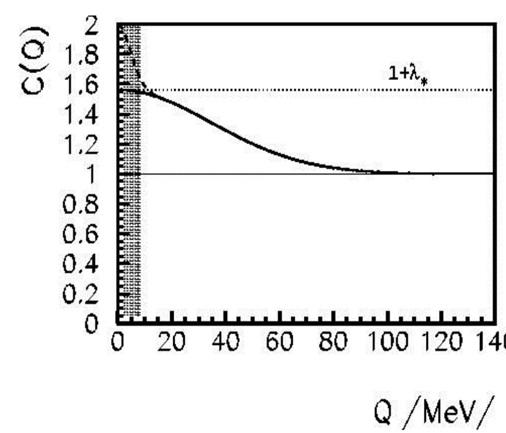
Initial temperature gradient and initial flow have to be co-varied to get Hubble in a sufficiently short time. $H_0 > 0$

Principles for Buda-Lund hydro model

- **Analytic expressions for all the observables**
- **3d expansion, local thermal equilibrium, symmetry**
- **Goes back to known hydro solutions in: nonrel, Bjorken, and Hubble limits - but smoothly extrapolates in between**
- **Separation of the Core and the Halo**
  - **Core: hydrodynamic evolution**
  - **Halo: decay products of long-lived resonances**



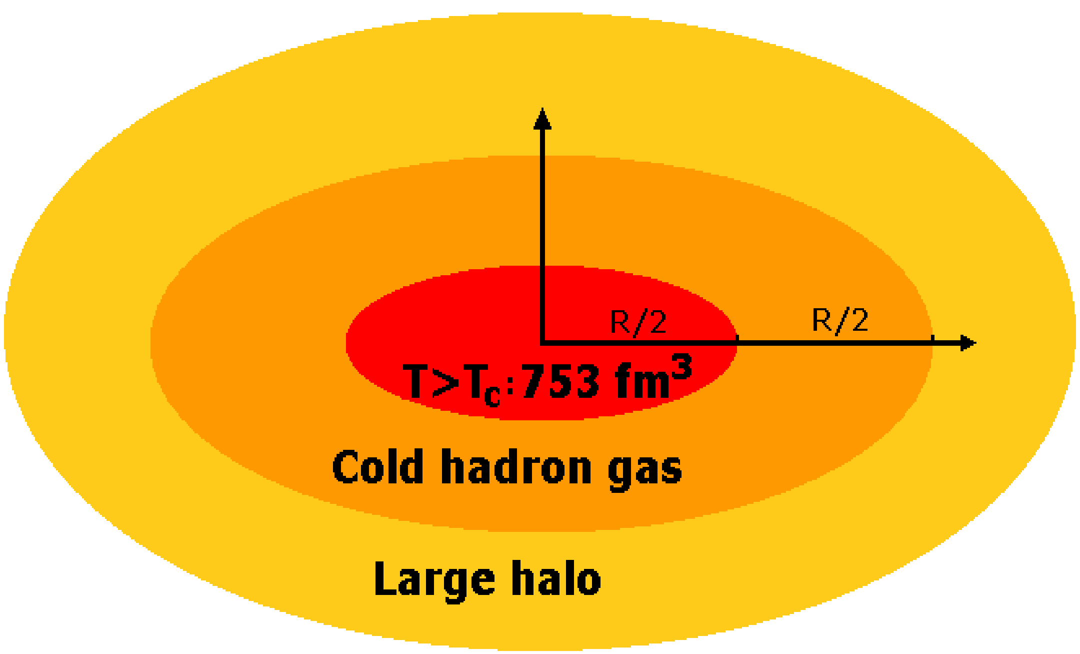
- **Missing link: accelerating simple solutions of rel. hydro**
  - **Yu. Karpenko, M. Nagy**

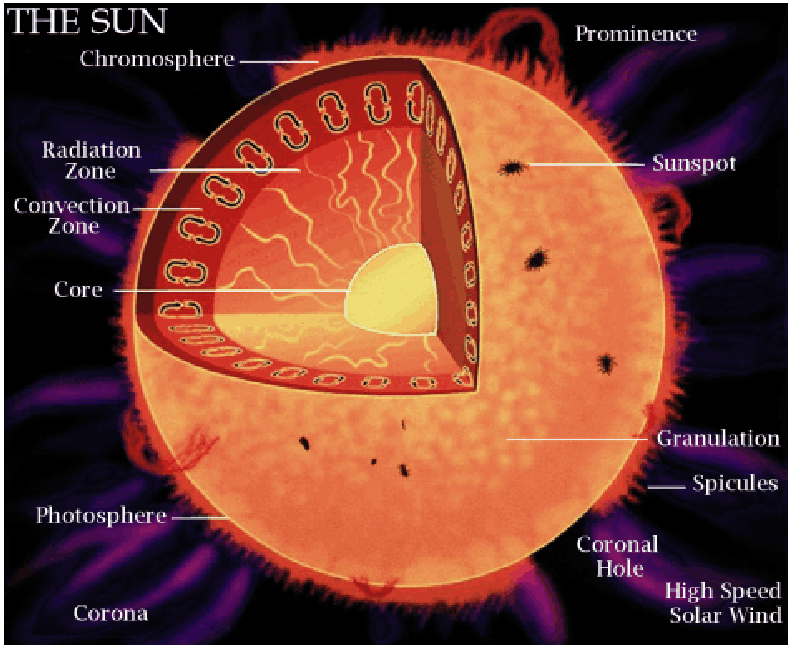


A useful analogy

Fireball at RHIC ⇔ our Sun

- Core ⇔ Sun
- Halo ⇔ Solar wind
- $T_{0,RHIC}$ ~ 210 MeV ⇔ $T_{0,SUN}$ ~ 16 million K
- $T_{surface,RHIC}$ ~ 100 MeV ⇔ $T_{surface,SUN}$ ~6000 K





Buda-Lund hydro model

The general form of the emission function:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

Calculation of observables with core-halo correction:

$$\mathrm{N}_1(p) = \frac{1}{\sqrt{\lambda_*}} \int d^4x S_c(p,x)$$

$$C(Q,p) = 1 + \left|\frac{\tilde{S}(Q,p)}{\tilde{S}(0,p)}\right|^2 = 1 + \lambda_* \left|\frac{\tilde{S}_c(Q,p)}{\tilde{S}_c(0,p)}\right|^2$$

Assuming special shapes for the flux, temperature, chemical potential and flow:

Buda-Lund hydro model

Invariant single particle spectrum:

$$N_1 = \frac{d^2n}{2\pi m_t dm_t dy} = \frac{g}{(2\pi)^3}\overline{E}\,\overline{V}\,\overline{C}\,\frac{1}{exp\left(\frac{p^\mu u_\mu(x_s)-\mu(x_s)}{T(x_s)}\right)+s_q}$$

Invariant Buda-Lund correlation function:
oscillating, non-Gaussian prefactor!

$$C_2(k_1,k_2) = 1+\lambda_*\Omega(Q_{||})\exp\left(-Q_{||}^2R_{||}^2 - Q_{=}^2R_{=}^2 - Q_{\perp}^2R_{\perp}^2\right)$$

Non-invariant Bertsch-Pratt parameterization,
in a Gaussian approximation:

$$C_2(k_1,k_2) = 1+\lambda_*\exp\left(-Q_o^2R_o^2 - Q_s^2R_s^2 - Q_l^2R_l^2 - 2Q_{os}^2R_oR_s\right)$$

Non-Gaussian BL form $\longrightarrow$ Gaussian BP *approximation*:

$$R_{||,\Omega}^2 = R_{||}^2\left(1+\frac{\overline{\Delta\eta}^2}{\overline{\eta}}\right)$$

The generalized Buda-Lund model

- The original model was for axial symmetry only, central coll.

- In the most general hydrodynamical form:

  'Inspired by' nonrelativistic 3d hydrodynamical solutions:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

- Have to assume special shapes:
  - Generalized Cooper-Frye prefactor:

$$p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x \qquad H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - Four-velocity distribution:

$$u^\mu = (\gamma, \sinh\eta_x, \sinh\eta_y, \sinh\eta_z)$$

  - Temperature:

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s} s\right)\left(1 + \frac{T_0 - T_e}{T_e} \frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - Fugacity:

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

Some analytic results

- **Distribution widths**

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2}+\frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2}+\frac{\dot{X}_i^2}{X_i^2}\right)$$

$$a^2 = \frac{T_0 - T_s}{T_s} = \left\langle \frac{\Delta T}{T} \right\rangle_r$$

- **Slopes, effective temperatures**

$$T_{eff} = \frac{1}{2}\left(\frac{1}{T_{*,x}}+\frac{1}{T_{*,y}}\right) \qquad T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

- **Flow coefficients**

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$

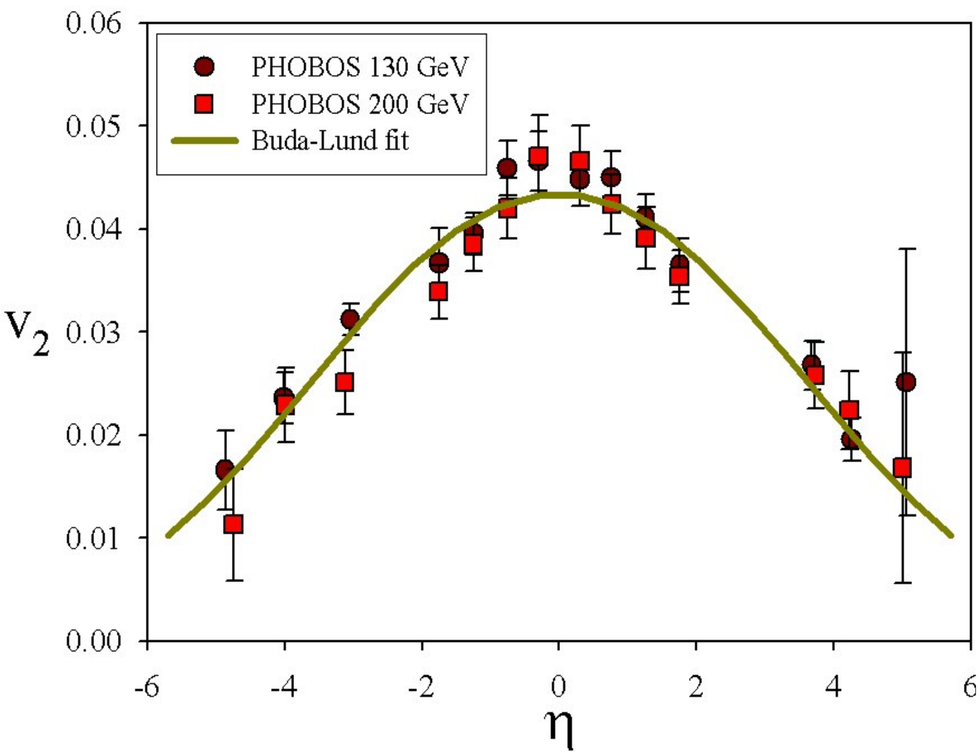
$$v_{2n+1} = 0$$

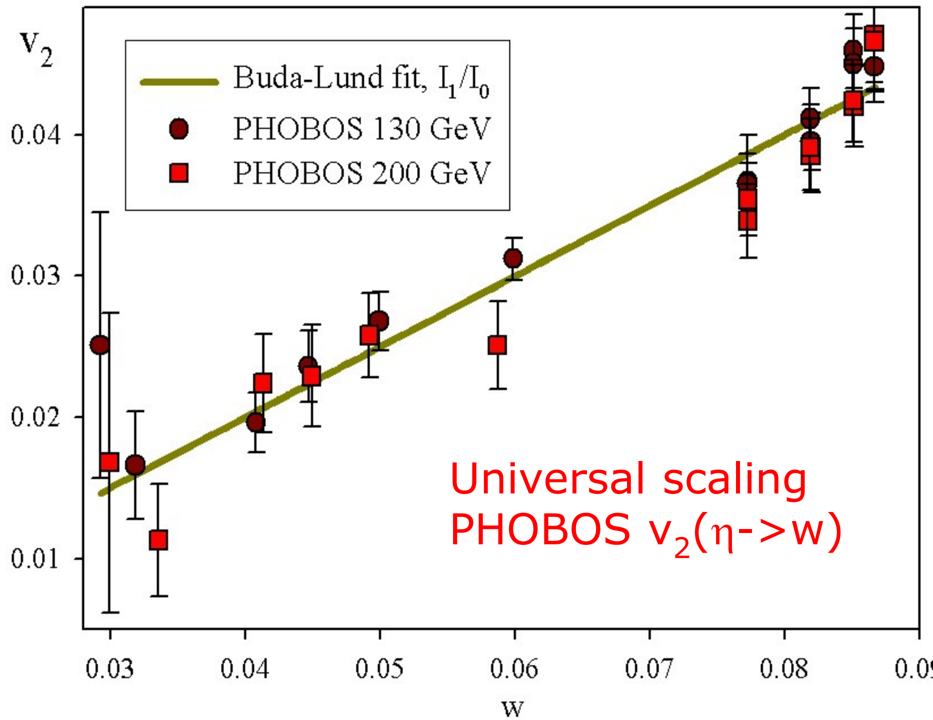
w ~ Energy x (slope difference)

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}} - \frac{1}{T_{*,x}}\right)$$

$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

Confirmation





see nucl-th/0310040 and nucl-th/0403074,
R. Lacey@QM2005/ISMD 2005
A. Ster @ QM2005.

Slope parameters

Buda-Lund rel. hydro formula:

$$T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

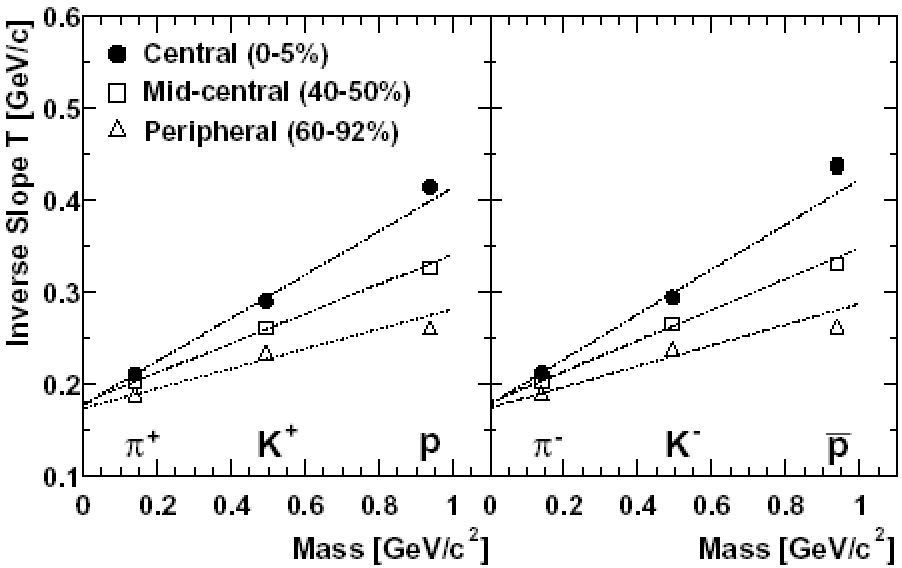
Experimental test: PHENIX, STAR

- **Exact non-rel. hydro solution:**

same, but m_t -> m, a-> 0

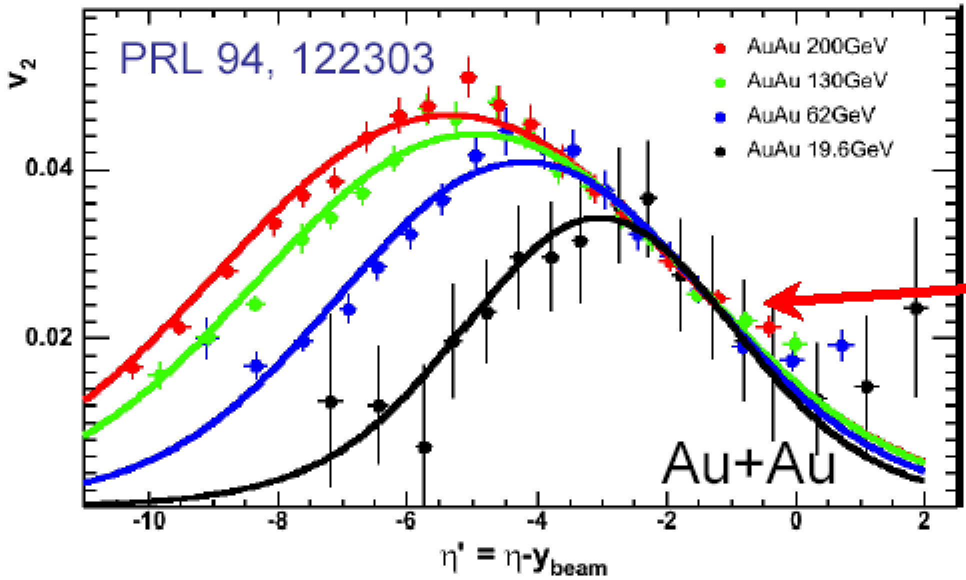
$$T'_x = T_f + m\dot{X}_f^2 \,,$$

$$T'_y = T_f + m\dot{Y}_f^2 \,,$$



Elliptic flow, limits

Extended longitudinal scaling: v_2



A surprising scaling!

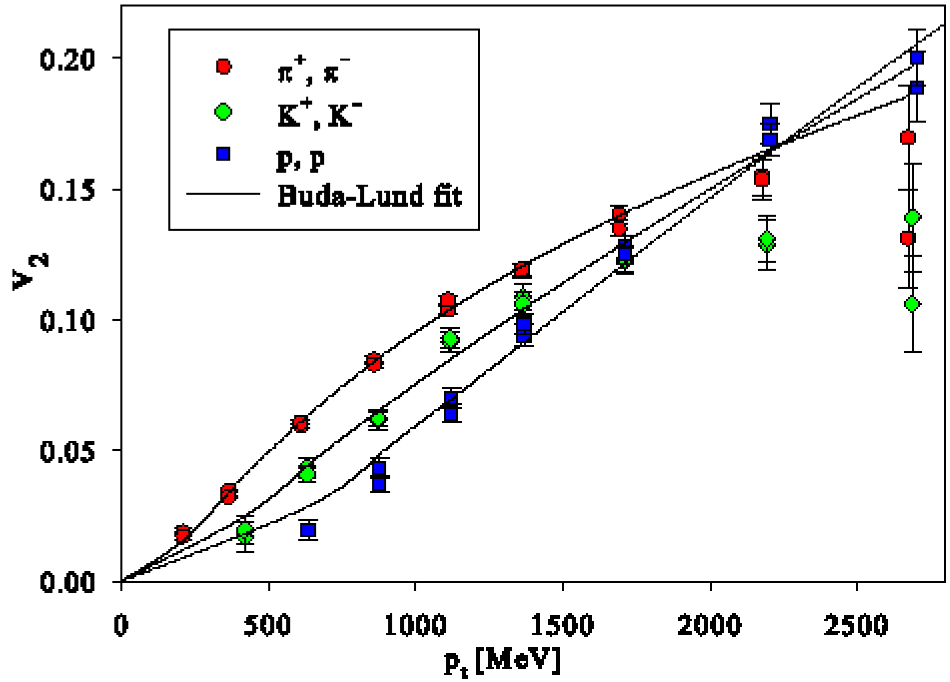
Not an initial state effect

nucl-th/0505019

Scaling reproduced by the Buda-Lund parametrization of the emitting source.

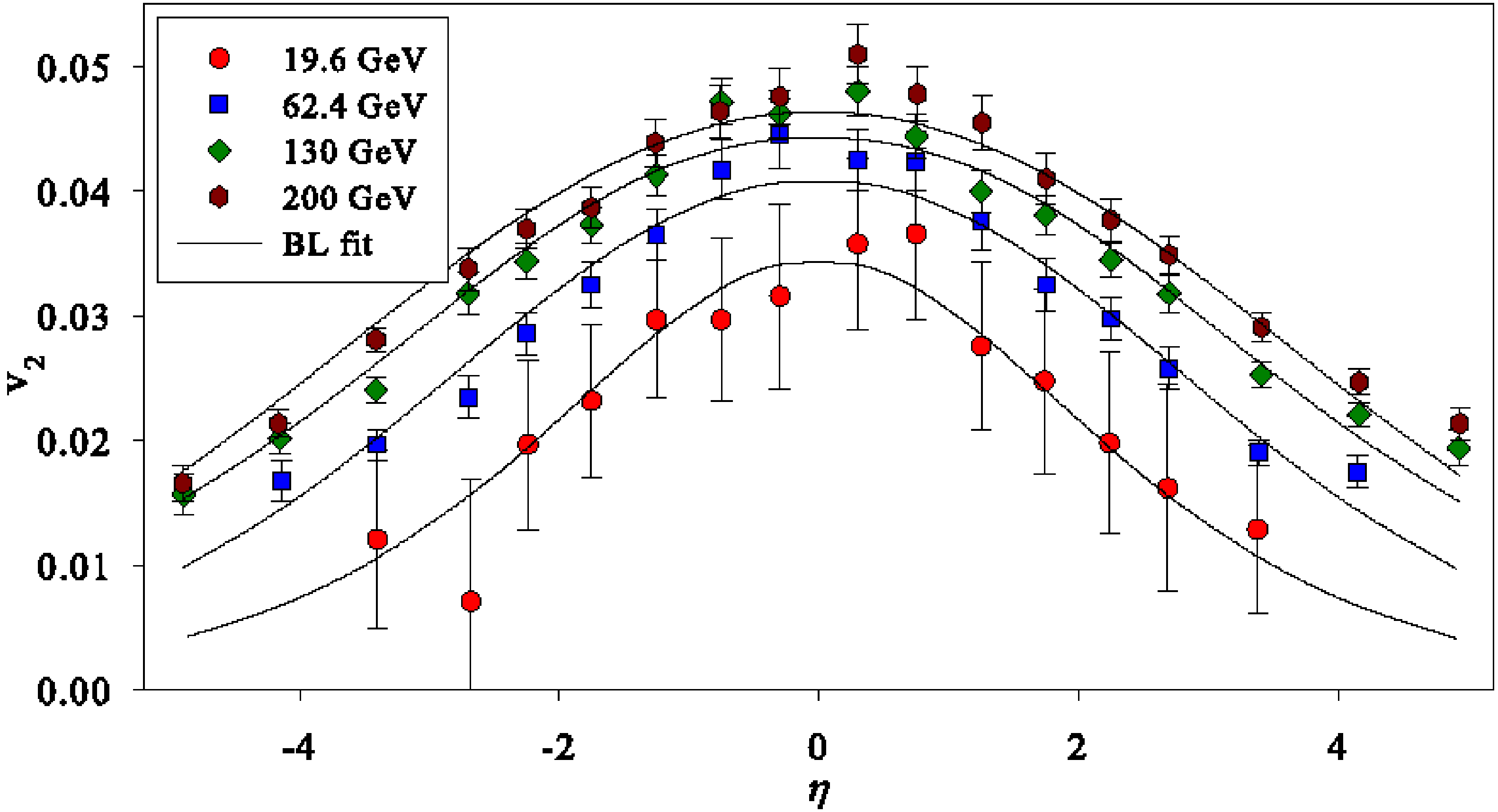
G. Veres, PHOBOS data, proc QM2005

Elliptic flow, PHENIX data



M. Csanád et al. nucl-th/0512078

Elliptic flow, PHOBOS data



M. Csanád et al. nucl-th/0512078

Elliptic flows

Buda-Lund rel. hydro formula nucl-th/1003040 (2003!):

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}} - \frac{1}{T_{*,x}}\right)$$

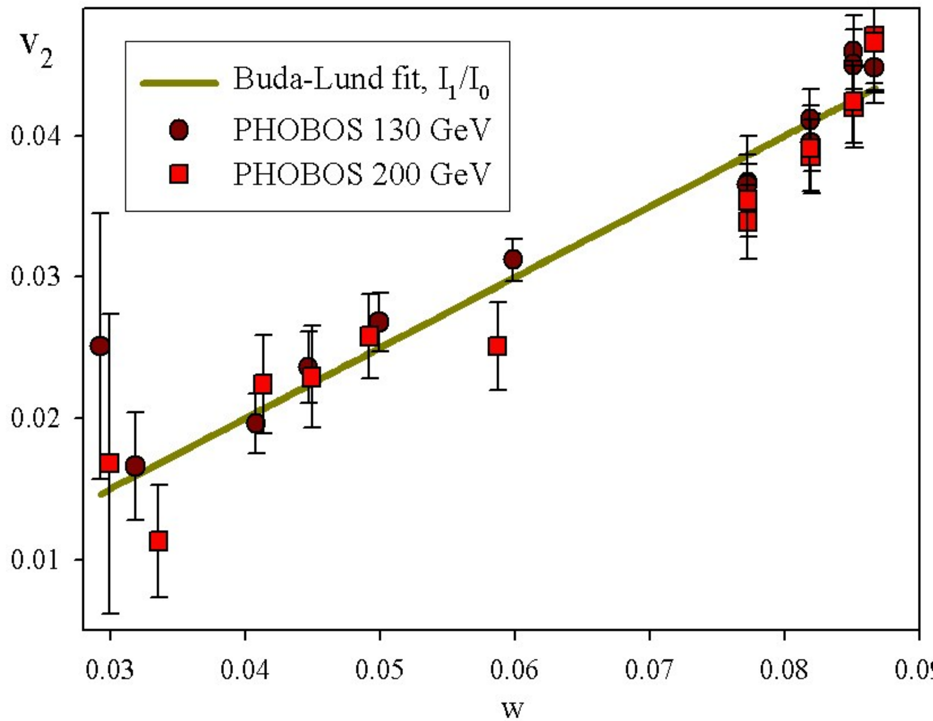
$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

Experimental test (on PHOBOS data, PRL 2005!)

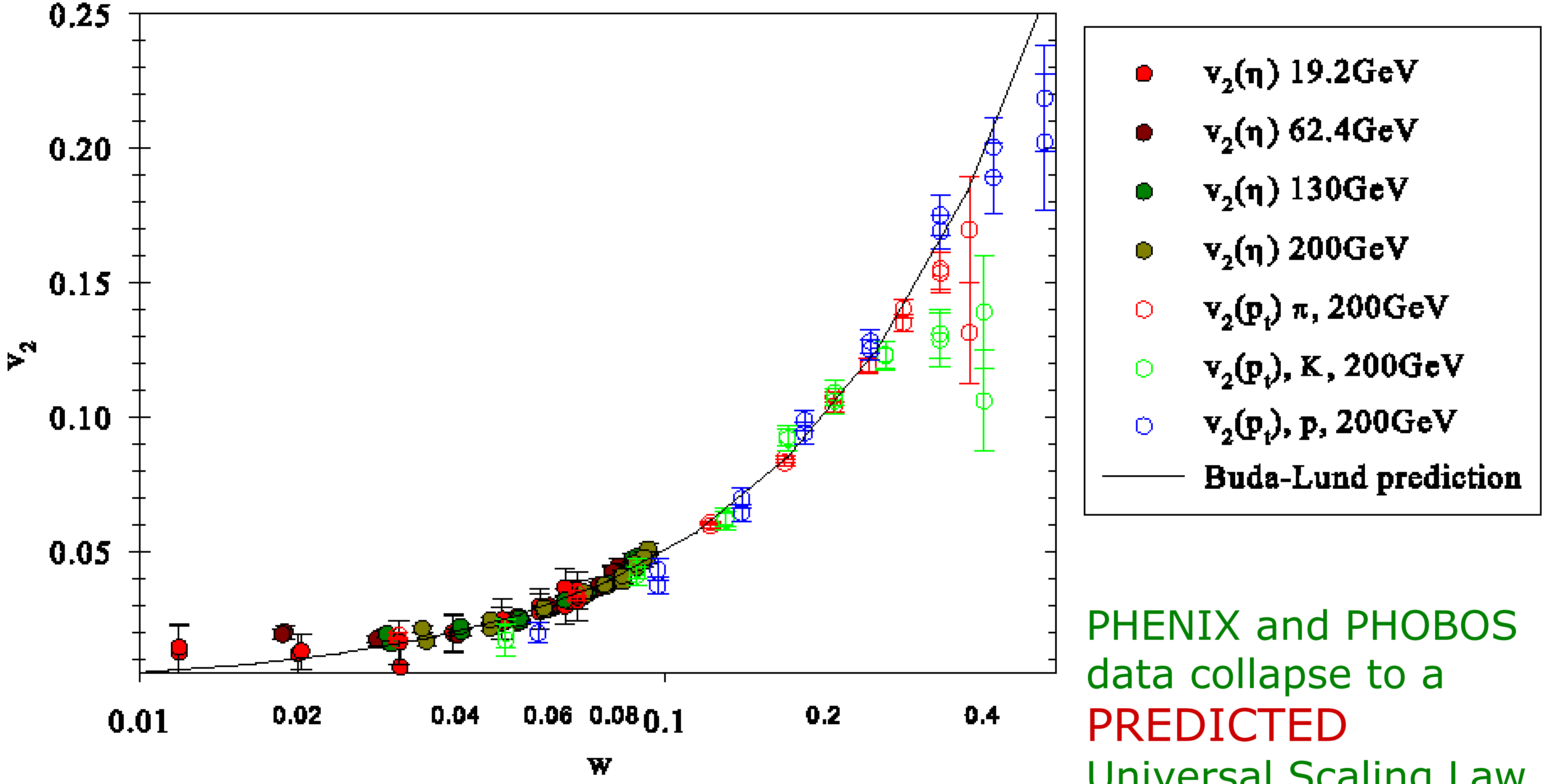
Exact non-relativistic result:

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

same, but $\underline{m_t}$ -> m



Universal v_2 scaling predicted in 2003



PHENIX and PHOBOS data collapse to a PREDICTED Universal Scaling Law of perfect fluid hydro

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

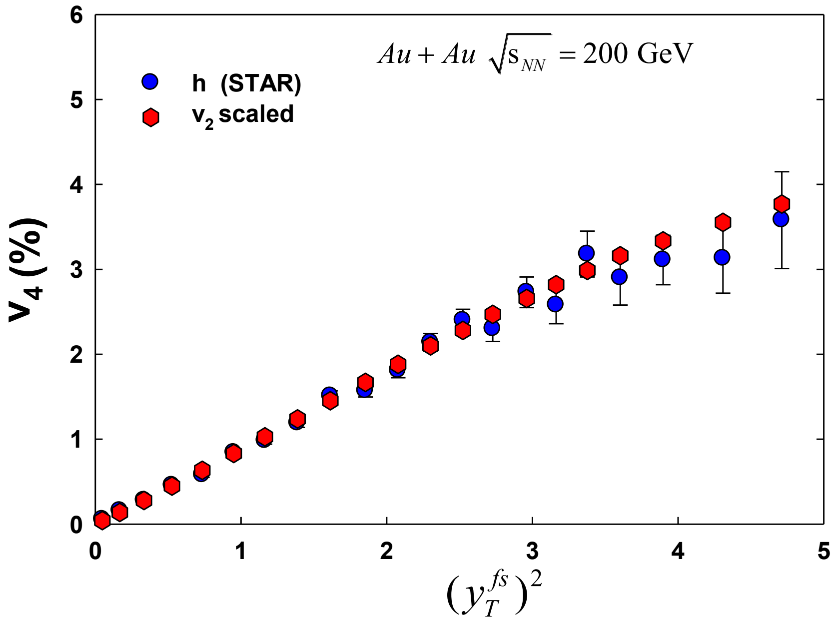
Higher flow coefficients

Buda-Lund rel. hydro formula:

$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$
$$v_{2n+1} = 0$$

Exact non-relativistic result:

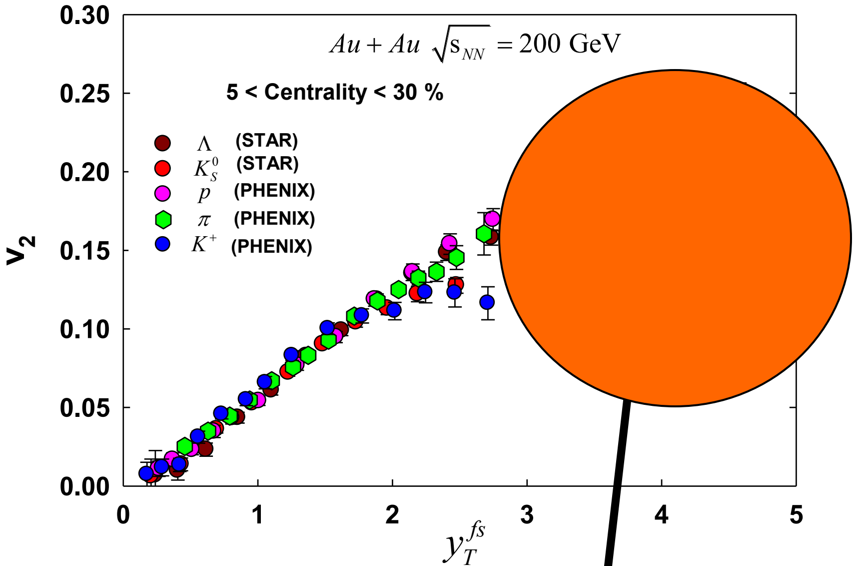
$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$
$$v_{2n+1} = 0$$

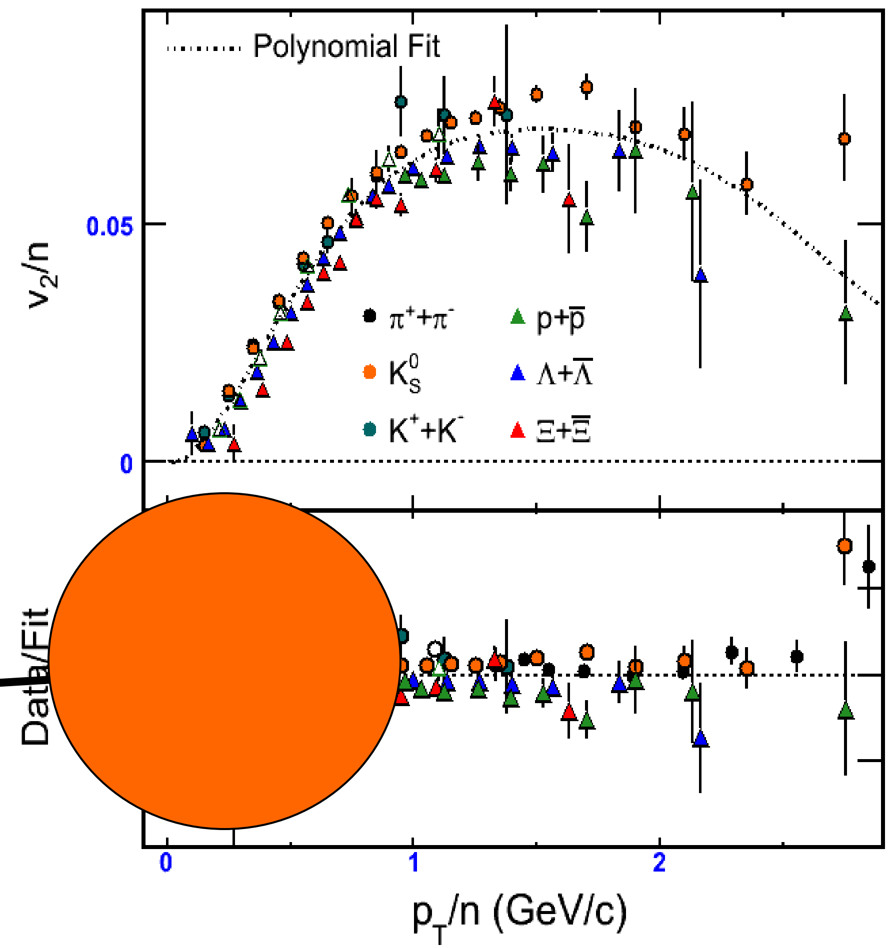


$$v_4 = \frac{v_2^2}{2} + k \times y_T^4$$

R. Lacey, Proc. QM 2005

Scaling and scaling violations





Universal hydro
scaling breaks
where quark number scaling
sets in, pt ~ 1-2 GeV
Fluid of QUARKS!!

R. Lacey and M. Oldenburg
Proc. QM 2005

Scaling laws from hydro

Exact non-rel. and Buda-Lund rel.

Single particle spectra
Slope
Rapidity width
Elliptic flow
Higher harmonics
HBT radius parameters
asHBT

Au+Au data at RHIC satisfy the scaling laws that were predicted by the Buda-Lund hydro model.

v2(y,pt, ...) is mapped already to a universal scaling function

-> compelling evidence for a perfect fluid at RHIC

scaling breaks between 1-2 GeV, where quark number scaling sets in.

Summary

Universal scaling of v_2 is observed

Au+Au data at RHIC satisfy the
UNIVERSAL scaling laws
predicted in 2003 by the Buda-Lund hydro model,
based on exact solutions of
PERFECF FLUID hydrodynamics

quantitative evidence for a perfect fluid in Au+Au at RHIC

scaling breaks in pt at ~ 1.5 GeV,
in rapidity at ~ ymax - 0.5
Search for establishing the domain of applicability started.
Further tests with STAR and BRAHMS data.

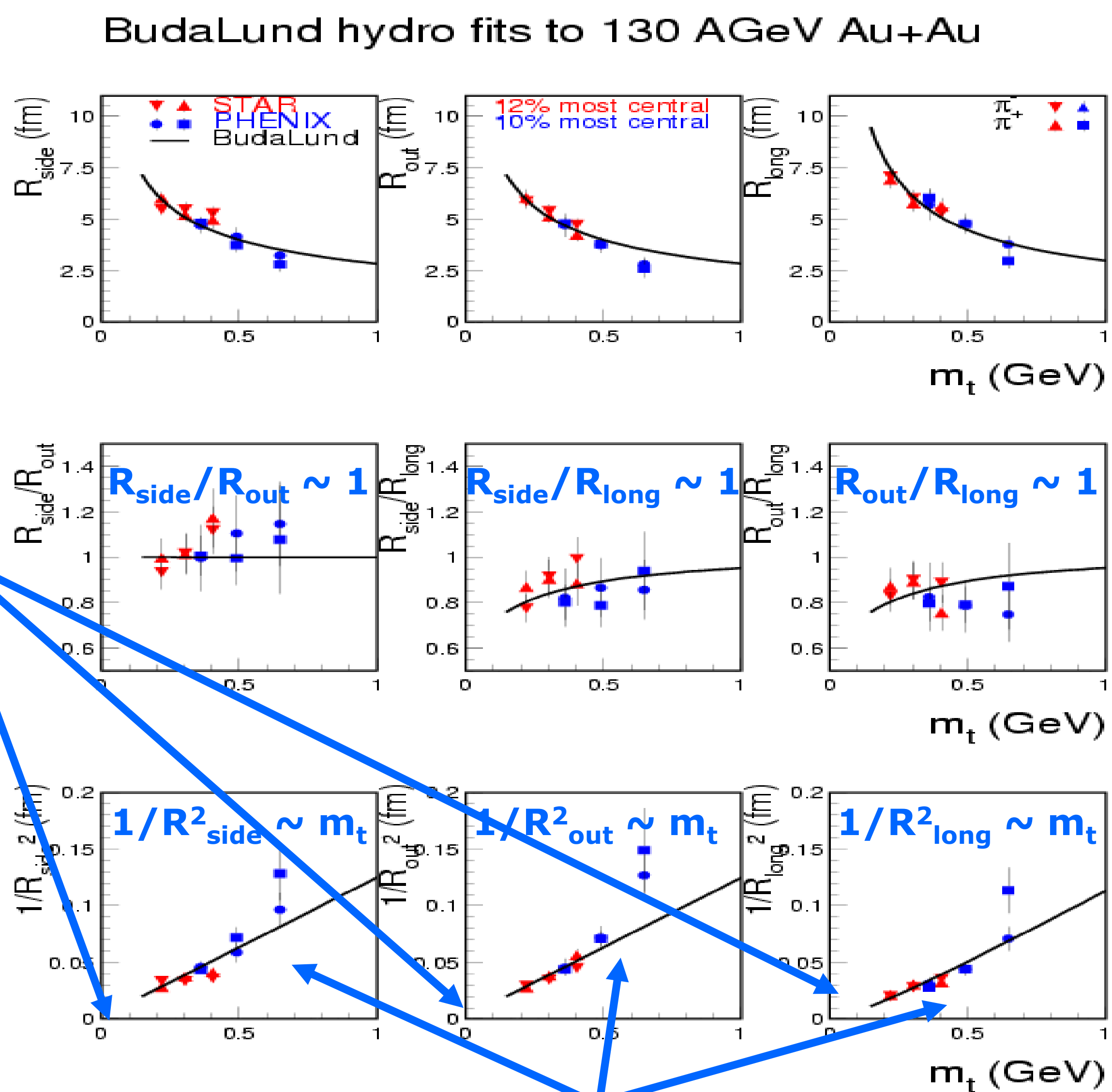
Similar scaling of Bose-Einstein/HBT RADII

$1/R^2_{eff} = 1/R^2_{geom} + 1/R^2_{therm}$
and $1/R^2_{therm} \sim m_t$

intercept is nearly 0,
indicating $1/R_G^2 \sim 0$,

thus $\mu(x)/T(x)$ = const!

reason for success of
thermal models @ RHIC!



same slopes ~ fully developed, **3d Hubble flow**