CORRELATIONS AND FLUCTUATIONS '98

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1 Introduction

To begin my summary, let me first single out two contributions to the meeting which I consider as the real steps forward in our quest for understanding the complicated phenomena of multiple production, but which I am unable to summarize in short terms.

First, Hans Eggers showed an amazingly simple and elegant solution of the model of multiplicative cascade [1]. As we all know, the model played an important role in formulation of the subject of this meeting, i.e. studies of fluctuations in multiparticle production. It seems to me very likely that this new development will soon create further progress in our field.

The second contribution is that by Bo Andersson [2]. In the investigation of the structure of the QCD cascade at the end of the available phase-space, the Lund group arrived recently at the conclusion that emitted gluons are ordered not only in rapidity but also in azimuthal angle and – moreover – proposed the method to measure this effect. If this is indeed confirmed by experiment, this result would mean a significant step forward in understanding the QCD cascade which is the major problem in description of the multiparticle production processes.

2 Bose-Einstein interferrence

Coming to the bulk of the conference, it was clear to everybody that this year the discussion of the Bose-Einstein interference or, in other words, the Hanbury-Brown and Twiss effect was the dominating issue. Let me thus start by a brief reminder what is this all about[a].

The practical problem we face can be formulated as follows: given a calculation (or a model) which ignores identity of particles, how to "correct" it

[a] The *physics* of the HBT effect was recently extensively reviewed by G. Baym [3].

in order to take into account the effects of quantum interference (which is the consequence of the identity[b]). Let us thus suppose that we have an amplitude for production of N particles $M_N^{(0)}(q)$ $(q = q_1, ...q_N)$ calculated with the identity of particles being ignored. The rules of quantum mechanics tell us that, to take the identity of particles into account, we have to replace $M_N^{(0)}(q)$ by a new amplitude $M_N(q)$ which is a sum over all permutations of the momenta $(q_1, ...q_N)$

$$M_N^{(0)}(q) \to M_N(q) \equiv \sum_P M_N^{(0)}(q_P) \tag{1}$$

This would be the end of the story if particle production was described by a single matrix element. In general, however, we have to average over parameters which are not measured and therefore the correct description of the multiparticle final state is achieved in terms of the density matrix

$$\rho_N^{(0)}(q, q') = \sum_\omega M_N^{(0)}(q, \omega) M_N^{(0)*}(q', \omega), \tag{2}$$

rather than in terms of a single production amplitude. The sum in (2) runs over all quantum numbers ω which are not measured in a given situation. $\rho^{(0)}(q, q')$ gives all available information about the system in question. At this point it is useful to note that, when tranformed into (mathematically equivalent) Wigner representation

$$W_N(\bar{q}, x) = \int d(\Delta q) e^{ix\Delta q} \rho_N^{(0)}(\bar{q}, \Delta q) \tag{3}$$

$(\bar{q} = (q + q')/2; \quad \Delta q = q - q')$ it gives information about the distribution of momenta and positions of the particles (see, e.g.,[4] for a discussion of this point).

Using (1) one easily arrives at the formula for the *corrected* (i.e., with identity of particles taken into account) density matrix $\rho_N(q, q')$ and one finally obtains the observed multiparticle density

$$\Omega_N(q) = \frac{1}{N!} \sum_{P,P'} \rho^{(0)}(q_P, q_{P'}) \tag{4}$$

where the sum runs over all permulations P and P' of the momenta $(q_1, ...q_N)$. The factor $\frac{1}{N!}$ appears because the phase space for N identical particles is $N!$

[b]It should be understood that this problem is very common in quantum mechanical calculations, as illustrated, e.g., by eveluation of Feynman diagrams. I would like to thank J. Pišút and K. Zalewski for discussions of this question.

times smaller than the phase space for N non-identical particles. The formula (4) is in common use[c] and is the basis of our further discussion.

2.1 A theoretical laboratory: independent particle production

The case of independent particle production is an attractive theoretical laboratory which, although not expected to describe all details of the data, reveals - nevertheless - some general (and generic) features of the problem. This was first recognized by Pratt [5]. In terms of the density matrix, the independent production means that the density matrix factorizes into a product of single-particle density matrices

$$\rho_N^{(0)}(q,q') = \rho^{(0)}(q_1,q_1')\rho^{(0)}(q_2,q_2')....\rho^{(0)}(q_N,q_N') \tag{5}$$

and that the multiplicity distribution is the Poisson one

$$P^{(0)}(N) = e^{-\nu}\frac{\nu^N}{N!}. \tag{6}$$

Several contributions to this problem were presented at the meeting [6,7,8,9]. It turns out [6,7] that in the case of a Gaussian density matrix the problem can be solved analytically. The main results (valid also in the general case of an arbitrary density matrix [10]) can be listed as follows.

(a) All correlation functions $K_p(q_1,...,q_p)$ and the single particle distribution $\Omega(q)$ can be expressed in terms of one (hermitian) function $L(q,q') = L^*(q',q)$ of two momenta:

$$\Omega(q) = L(q,q); \quad K_2(q_1,q_2) = L(q_1,q_2)L(q_2,q_1)$$
$$K_3(q_1,q_2,q_3) = L(q_1,q_2)L(q_2,q_3)L(q_3,q_1) + L(q_1,q_3)L(q_3,q_2)L(q_2,q_1), \tag{7}$$

and analogous formulae for higher correlation functions.

(b) At very large phase-space density, particle distribution approaches a singular point representing the phenomenon of Bose-Einstein condensation: almost all particles populate the eigenstate of $\rho^{(0)}(q,q')$ corresponding to the *largest eigenvalue*. The resulting multiplicity distirbution is very broad (almost flat) so that, e.g., probability of an event with no single π^0 produced is non-negligible. Such a situation may perhaps be a possible explanation of the somewhat elusive "centauro" events [11], as suggested by Pratt [5][d].

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[c] Using the hermiticity property of the density matrix, the double sum in (4) can be reduced to a single sum. The factor $\frac{1}{N!}$ is then absent.

[d] This effect was also considered in connection with the possible production of the Disoriented Chiral Condensate [12,13]. The present argument adds another obstacle on the difficult road to observation of DCC, as discussed thoroughly at this meeting by the Bergen group [14].

(c) At high density, the parameters extracted from the observed spectra have little in common with the input parameters characterizing the source. In particular, for the Gaussian source in the BE condensation limit we have

$$R^2_{eff} = \frac{R^2}{2R\Delta} < R^2; \quad \Delta^2_{eff} = \frac{\Delta^2}{2R\Delta} < \Delta^2 \tag{8}$$

where $R^2 =< x^2 >$ and $\Delta^2 =< q^2 >$ are the average values of the position and momentum of the particles (uncertainty condition implies $R\Delta \geq 1/2$).

It should be not surprizing that the very restrictive condition of independent production, as expressed by (5,6), is not realized in nature. This was shown at the present meeting by Lörstad [15], who demonstrated that there are practically no genuine three-particle correlations[e] in S-Pb collisions at CERN SPS. Since the two-particle correlations are clearly visible, this observation cannot be reconciled with Eq. (7). It was also shown by Eggers et al. [17] that the UA1 data are in contradiction with (7), although in this case the 3-body correlations seem to be too large to satisfy (7). This striking difference between the behaviour of heavy ion and "elementary" collisions is certainly very interesting and deserves further attention.

We cannot thus consider the results obtained from (5,6) to be realistic description of the data. Nevertheless, the main conclusion about the possibility of Bose-Einstein condensation remains an interesting option which is worth serious consideration.

2.2 Monte Carlo simulations

In this situation, the practical method to study the effects of BE symmetrization on particle spectra is to implement it into the Monte Carlo codes. A "minimal" method of performing this task was suggested some time ago [4]. The idea is to take an existing code (which reproduces the distribution of particle momenta, i.e. the diagonal elements of the density matrix) and to modify only the off-diagonal elements of the multiparticle density matrix (2)[f]. Each event generated by the MC code is then given a weight which is calculated as the ratio of symmetrized distribution, Eq. (4), and the unsymmetrized one. In this way the modification of the original spectra is kept at the minimum.

A practical realization of this idea has been developopped by the Cracow group [18] and was presented by Fiałkowski at this meeting. They propose the

[e] The importance of the absence of 3-particle correlations in heavy ion collisions was emphasized already some time ago [16].

[f] As seen from (3) this corresponds to introducing an – a priori arbitrary – distribution of particle emision points in configuration space.

unsymmetrized density matrix in the form

$$\rho_N^{(0)}(q,q') = P_N(\bar{q}) \prod_{i=1}^{N} w(q_i - q_i') \tag{9}$$

where $P_N(q)$ is the probability of a given configuration obtained in JETSET and w is a Gaussian. This prescription does not modify the diagonal elements of the unsymmetrized density matrix ($w(0) = 1$) and, moreover, does not introduce any new correlations between emission points of the produced particles (when transformed into Wigner representation, the product $\prod w(q_i - q_i')$ becomes the product $\prod w(x_i)$). Thus (9) can indeed be considered as a minimal modification of the existing code. The autors find that this prescription represents well the existing data on two-particle correlations and that they can recover the experimental multiplicity distribution by a simple rescaling with the formula $P(N) \rightarrow P(N)cV^N$, without the necessity of refitting the JETSET parameters.

The results presented at this meeting concerned the W production in e^+e^- collisions. The authors find that the expected mass shift is very small (less than 20 MeV). They also predict a shift of multiplicity observed in hadronic decay of one and two W's:

$$n(2W) - 2n(W) = 2.1 \pm 0.9. \tag{10}$$

This may be an overestimate because (as seen from (9)), in present version of the model the position of particle emission point is not correlated with its momentum, whereas this effect is likely to be present in reality.

A more fundamental approach has been pursued since some time by Andersson and Ringnér [19]. It is based on the famous paper by Andersson and Hoffman [20] and was presented here by Ringnér and by Todorova-Nova [21]. They write the "uncorrected" matrix element for the deacay of one Lund string in the form

$$M_N^{(0)}(q) \sim \exp\left[\left(\frac{b}{2} + \frac{i}{2\kappa}\right) A(q)\right] \prod_{i=1}^{N} e^{-\frac{\pi}{2\kappa} q_{\perp i}^2} \tag{11}$$

and then follow the procedure explained in introduction. Two particle correlations are well described and several interesting effects are predicted. Among them: (a) The longitudinal and transverse correlations are expected to be different because they are controlled by two different physical mechanisms; (b) Three particle correlations are predicted non-vanishing and were actually calculated; (c) WW production was studied and no significant mass shift is expected; (d) No multiplicity shift in the W decay is predicted.

This last conclusion is a consequence of the fact that, in case of more then one string present in the final state, no symmetrization between particles stemming from different strings is performed. This corresponds to the assumption that the strings are created at a very large distance from each other. One thus may expect that in a more realistic treatment some multiplicity shift should be present[g].

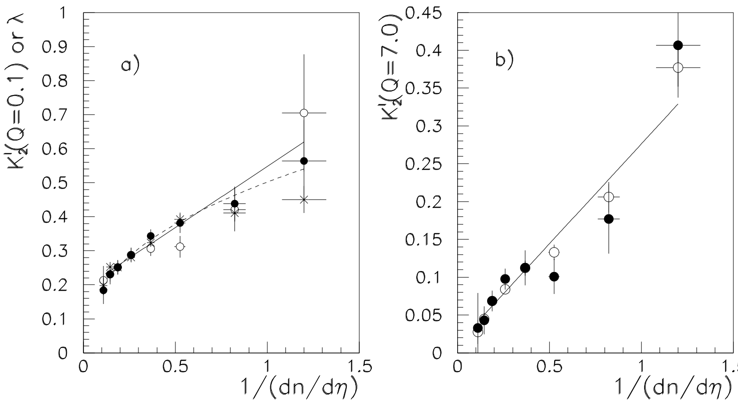

Figure 1: The second order cumulant plotted versus inverse of the rapidity density. Data from UA1 [23].

The problem of quantum interfererence between particles from different strings is certainly the important one and its solution may be crucial for the success of the Lund model in processes which are more complicated than e^+e^- annihilation. In this context interesting data of UA1 collaboration were presented by B. Buschbeck [23]. The authors studied the dependence of the correlations between like- and unlike- pairs as function of the particle density. The data are shown in Fig. 1. One sees linear dependence of the normalized cumulants on $\frac{1}{dN/dy}$. One observes, furthermore, that in the region $Q = 7$ GeV (where no HBT effect is expected) the cumulant vanishes in the limit of large density. On the other hand, in the region $Q = .1$ GeV (which is likely to be dominated by BE correlations) the cumulant tends to a finite value in this limit.

[g]In both [18] and [19] the "interconnection effect" [22] (which has tendency to reduce the multiplicity) is neglected. The full phenomenological analysis of the data is therefore certainly more complicated.

To understand the meaning of these data, consider particle emission from a number N of *independent* sources. In this case the particle density is

$$\frac{dn}{dy} = N\frac{d\nu}{dy} \tag{12}$$

where $\frac{d\nu}{dy}$ is the particle density from one source. The normalized two-particle correlation function is

$$\left.\frac{\frac{d^2n}{dy\,dy'}}{\frac{dn}{dy}\frac{dn}{dy'}}\right|_{y=y'} - 1 = \frac{1}{\frac{dn}{dy}}\,K(y) \tag{13}$$

where $K(y)$ depends only on the particle distribution from one source.

Thus the emission from several independent sources implies that the normalized cumulant is *inversely proportional* to the particle density. As seen from Fig. 1, this is in good agreement with the data at large Q. At small Q, however, dominated by BE correlations, the normalized correlation function approaches a constant different from zero at large densities, in disagreement with (13). This implies correlations between the sources which may well originate from the quantum interference between them. The qualitatively different behaviour in the two regions supports this idea. On the other hand, the fact the that correlations between the like- and unlike charges behave similarly (see Fig. 1) casts a doubt on this interpretation. Further investigations along these lines are thus certainly needed.

Finally, let me comment on the contribution of Lönnblad [24]. He presented the method of describing the HBT effect by shifting the momenta of identical particles so that the observed two-particle correlations are reproduced. Personally, I do not believe that this is a correct way of treating the problem of BE interference[h]. I like thus only mention the contribution to this meeting by Smirnova [25], who showed that the method presented by Lönnblad does not reproduce the values of the parameters used as an input.

2.3 Probing the space-time structure

Much attention during the meeting was devoted to the information one may obtain from the data on quantum interference about the space-time structure of the multiparticle system created in the collision. Although such analyses can have at most a limited scope, as they only provide the information about

[h]It seems to be an attempt to treat the effects of quantum interference as a final state interaction.

the system at the freeze-out and, as emphasized by Weiner [26], require several additional assumptions – they provide nevertheless a unique opportunity to investigate this problem. Most of the *caveats* are thus usually postponed to the future (and better data) and the analysis is carried on.

The presented investigations were based on the hydrodynamic approach. The general framework was explained by Csörgő [27] who advocated a new Buda-Lund parametrization, as an improvement with respect to the standard YKP one [28]. There were four presentations of the experimental results.

Lörstad discussed the m_t dependence of the data of NA44 and LEP [15]. The radius of the system systematically decreases with increasing transverse mass of the particles. In case of heavy ion collisions this is usually interpreted as evidence for hydrodynamic flow. However, the same phenomenon is observed also in e^+e^- annihilation where the notion of hydrodynamic flow is perhaps not so easy to accept.

Seyboth presented data of NA49 experiment on Pb-Pb collisions at SPS [29]. He showed rather convincingly that (i) The longitudinal flow of particles is well consistent with the Bjorken in-out model [30] and (ii) Particle emission starts rather late and it lasts not very long: life time of the system is of the order of 8 fm, while the duration of pion emission is only of the order of 3 fm.

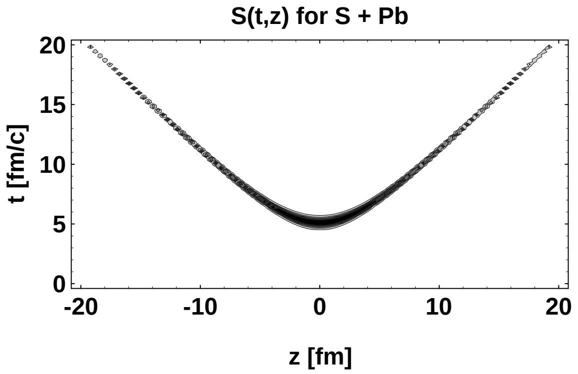

Figure 2: Space-time region of particle emission in S-Pb collisions [31].

These two features are also present in the NA44 data on S-Pb collisions, discussed by Ster [31]. This is seen in Fig. 2 where the reconstructed space-time distribution of the source of particles is shown. One clearly sees a characteristic Bjorken shape of the source. One also sees that particle emission in the central region starts only at about 4 fm and is practically finished ∼1.5 fm later.

A qualitatively similar behaviour is found in πp collisions of NA22 experiment, as presented by Hakobyan [32]. The picture shown in Fig. 3 looks qualitatively rather similar to that in Fig. 2. Note, however, an important *quantitative* difference: In hadron-hadron reaction the particle emission in the central region starts almost immediately after collision and lasts about 1.5 fm.

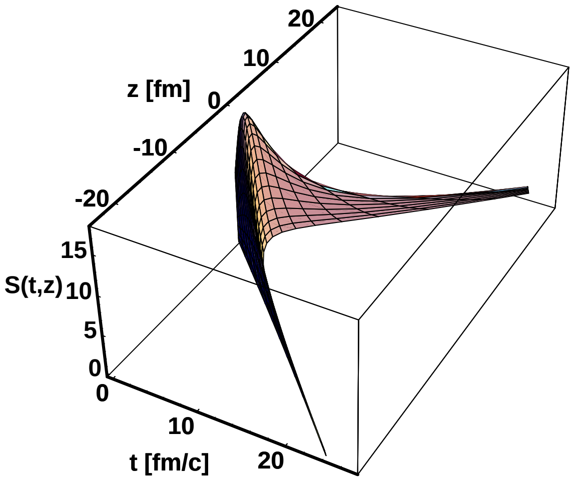

Figure 3: Space-time region of particle emission in πp collisions [32].

We have also seen from a contribution of Schlei [33] that the BE correlations may serve as a tool for analysis (and improvement) of the equation of state of the strongly interacting matter. The reason is that the volume occupied by the system at freeze-out depends on the equation of state and thus information on this volume provided by BE correlations restricts severely the possible equations of state. For esentially the same reason information from BE correlations helps to estimate the particle density in phase-space, as was pointed out by Pratt [34].

This completes the list of contributions discussing the data on BE interference. Several other results about particle correlations related to the space-time structure were also shown.

Lednicky [35] presented an interesting idea that correlations between the *non-identical* particles can provide information on the time sequence of their production. Indeed, consider two particles moving in the same direction and suppose that they are subject to a final state interaction (for instance Coulomb interaction). If the faster one is emitted before the slower one, the effect of the interaction shall be smaller (because they move apart from each other), otherwise it will be stronger (because the faster particle will catch the other one). The feasibility of such measurements was discussed and the prospects seem to be promising.

Kuvshinov[36] discussed production of instantons in deep inelastic collisions. The most striking effect seems to be a very narrow multiplicity distribution in the instanton decay, which may serve as a good signal of such a phenomenon.

Particle-antiparticle correlations were discussed by Andreev [37] and by Csörgő [38]. They considered a passage of a particle through a region of the false vacuum (DCC) if such a region was produced in a collision. Since the particle in the false vacuum cannot be on its mass shell, an adjustment of the wave function is necessary when it leaves the DCC region. This manifests itself as additional particle production. Since the quantum numbers of the produced system must be those of vacuum, one concludes that a particle-antiparticle pair with opposite momenta must show up. This would be certainly a very attractive signal for the DCC. The estimates of Csörgő et al. [38] are that the effect is expected to be fairly strong (although precise estimates are not possible at the moment) and thus deserves attention of experimenters.

Several papers on possible evolution of DCC were presented by the Bergen group[14]. Unfortunately I have neither space nor competence to comment on them.

3 Multiplicity distributions

Multiplicity distributions were discussed in several contributions.

Giovannini and Ugoccioni[39] presented estimates of the KNO scaling violation expected because of the onset (and eventual dominance) of hard scattering (mini-jets and jets) at high energies. Hegyi[40] discussed a generalized negative binomial distribution along the lines proposed some time ago by Carruthers: Poisson transforming the generalized gamma distribution.

Płoszajczak [41] discussed scaling laws in the systems which undergo the 2nd order phase transition. The numerical analysis leads to the conclusion that such systems obey the scaling law of the form

$$<n>^{\delta} P(n) = \Phi(z_\delta) \tag{14}$$

with

$$z_\delta = \frac{n- <n>}{<n>^{\delta}} \tag{15}$$

where δ is a number between 1 and $\frac{1}{2}$. It turns out, furthermore, that the value of δ is determined by the nature of the variable n. If the system is at the phase transition and n is the correct order parameter, then $\delta = 1$. If the system is at phase transition but n is not the correct order parameter, $\frac{1}{2} < \delta < 1$. What is most unexpected, however, even if the system is *not* at phase transition one still has the scaling law (14) with $\delta = \frac{1}{2}$. This intriguing property certainly requires (and deserves!) further investigation.

4 Perturbative and non-perturbative QCD

Application of perturbative QCD calculations to multiparticle spectra was a subject of a hot discussion. It is now rather well established that the average multiplicity and single particle spectra are well described by perturbative QCD supplemented with the principle of parton-hadron duality [42]. At this meeting Lupia [43] presented a calculation of the cumulants of the multiplicity distribution and showed that they also agree with the data. Thus the principle of parton-hadron duality is now extended even for integrated correlation functions. This statement was challenged by Mangeol [44] who analyzed the data on cumulants in jets and found that the results do not obey the predictions of perturbative QCD in the region where one expects them to be the best, i.e. in high energy jets. The problem requires certainly further discussion but it is clear that the Lupia calculation marks an important step towards understanding the meaning of perturbative QCD predictions and of parton-hadron duality.

The real challenge to the idea of parton-hadron duality is to explain the data on diffrential correlaton functions. Indeed, it is hard to understand how the momenta of the produced hadrons can follow so closely the momenta of the created partons that the correlations between them are not washed out[i]. Therefore a non-trivial extension of the principle of parton-hadron duality must be formulated in order to give quantitative meaning to perturbalive calculations of multiple production. This point of view was substantiated by Kittel [45] who showed that the predictions of perturbative QCD formulated some time ago [46], are badly violated by the L3 data. On the other hand, the same data are well described by the JETSET code. The conclusion is that the hadronization part is probably not correctly taken into account by the simple (naive?) parton-hadron duality. This conclusion was challenged by Ochs [47] who showed in his talk that the previously published calculations included several simplifying assumptions (the most important among them seems the neglect of energy-

[i]This problem does not arize, of course, if one considers only the integrated correlation functions.

momentum consevation) and thus it is not clear which part of the result is actually responsible for the failure. Ochs presented several improvements and indicated the kinematical regions where perturbative QCD effects have a better chance to be seen. In my opinion, further work on these lines is necessary, however, to establish a reliable, quantitative link between partons and hadrons and to determine its range of application.

Another philosophy was presented by Hwa [48] who considered a fundamentally non-perturbative approach to the problem of multiparticle production. The model is an implementation of an old idea of Feynman [49] in which the partons in the final state are just those present already in the initial state but rearranged during the collision. Hwa supplements this idea with a specific prescription for the transition from partons to hadrons: the neutralization of colour happens by random walk in colour space. According to the author, the main advantage of this mechanism is that it can provide a natural explanation for the fractal character of multiparticle spectra [50] which is now firmly established in e^+e^- and hadron-hadron data (see, e.g. [51]). Further work is needed, however, to confront the details of the model with experiment.

Another interesting contribution was presented by Chekanov [52] who discussed the forward-backward correlations in deep inelastic ep scattering. He showed that the perturbative QCD calculations predict negative correlation between multiplicities of the current and target jet (in the Breit system). The data appear to follow this prediction (within fairly large errors). This seems to be an important step in more precise definition of the parton-hadron duality, indicating that it works not only for fully integrated quantities but also for those integrated over a large enough regions of phase-space.

5 "Traditional" intermittency

"Traditional" intermittency analysis of the data was presented by Sarkisyan [53] representing OPAL experiment. He showed that these data cannot be fully explained by the MC codes (JETSET and HERWIG were used) at very small phase-space intervals. The data in three-dimensional bins show a rather clean power-law behaviour and it will certainly be very interesting to see the results of the fit determining the intermittency parameters. It shall be also interesting to see if inclusion of BE correlations into the MC codes can bring the theory to agree with the data.

Let me also mention two theoretical contributions by Blazek [54] and Yang [55] who proposed new ways to analyse the multiparticle data in small phase-space intervals. As I have no space to describe their proposals in detail, I refer the reader directly to their written versions published in this volume.

6 Fluctuations at phase transition and event-by-event analysis

The last topic discussed at the meeting concerned fluctuations occuring at the phase transition. Antoniou presented results of the Athens group [56] who took as the starting point the instanton model of the QCD vacuum and looked at its behaviour close to the phase transition. They found rather large rapidity fluctuations of the fractal type but only in those events which happen to satisfy the phase-transition criteria. I find the problem important and thus I really look forward to see the results of three-dimensional calculations promised by the speaker.

Another important issue was stressed by Hwa [57]. He pointed out the essential difference between the determination of the fractal parameters in case of dynamical systems and in case of systems of many particles. In the dynamical system one can generate the *time sequence* and thus estimate how fast the different trajectories diverge. In case of multiparticle systems we do not have a time sequence and thus we have to rely on *patterns*. The question in this case is: how different are the patterns of different events. Hwa proposed to *measure* the pattern of an event by the factorial moment associated with it. One can then ask the question how this measure fluctuates from event to event. Studying moments of this distribution provides a measure of event-to-event fluctuation[j]. When they are considered as function of bin size, it is possible to define appropriate fractal dimensions which conveniently summarize the information. For the details the reader is referred to the original paper [57]. I personally feel that this is an important conceptual step in our thinking about the problem, although I am not fully convinced that the proposed measure cannot be improved.

It thus clearly emerged from the work reported above that it is very essential to be able to study the possible fractal behaviour in event-by-event analysis. The feasibility of this program was investigated in the paper presented by Ziaja [59]. It was shown that, in the framework of the α-model, it is possible to improve considerably the accuracy of the determination of the intermittency parameters, as compared with the original proposal [60]. In the discussion, it was pointed out by Eggers that the proposed corrections must be tested on other models before they can be considered reliable.

Let me thus end by showing the only example of the event-by-event analysis presented at the meeting [29]: The distribution of the HBT radii obtained from individual events by the NA49 collaboration is seen in Fig. 4. Although statistics is still limited (and the authors themselves do not attach too much

[j] To study the moments of the factorial moments was suggested, in a somewhat different context, already some time ago [58].

meaning to the details of the plot) one clearly sees that the distribution is not symmetric, with a long tail at large radii. I personally think that this is a hint of an interesting phenomenon but one should obviously wait for more data before one starts any theoretical speculations.

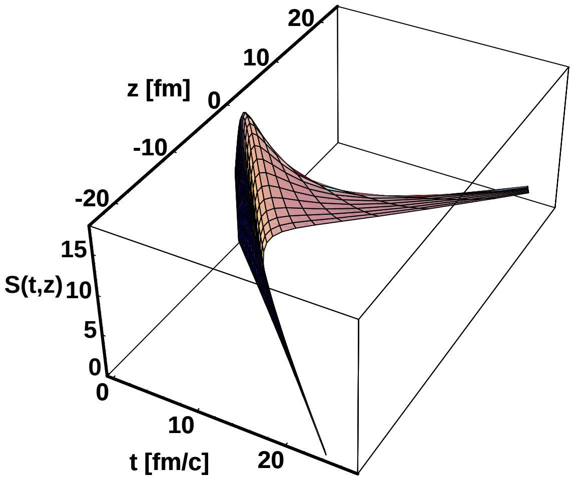

Figure 4: Distribution of HBT radii from event-by-event analysis of Pb-Pb collisions [29].

7 Conclusions

A tentative summary of this summary can be formulated as follows.

(i) Studies of HBT correlations using the hydrodynamical model became an effective tool for determining the space-time structure of particle emission.

(ii) Fast progress is being made in Monte Carlo implementation of the quantum interference, but controversies remain.

(iii) The problem of pion condensate is well understood and was even analytically solved for Gaussian distributions.

(iv) An important step was achieved in theory of branching processes.

(v) Perturbative QCD works for global quantities but still fails to describe local fluctuations.

(vi) Intermittency in e^+e^- annihilation is confirmed by high-statistics data from OPAL.

(vii) Fluctuations at phase transition are being intensively studied.

(viii) Clear need for event-by-event investigations emerges.

(ix) As expected, a new surprize from Bo arrived, and in time.

Let me close with apologies to all speakers whose work I have not been able to report here either for lack of space or (more often) for my inability to summarize shortly their results.

Acknowledgements

I would like to thank T. Csörgő for the kind hospitality at Mátraháza. This work was supported in part by the KBN Grant No 2 P03B 086 14.

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