

Parton cascade and coalescence

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Plenary talk at Quark Matter 2005

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- **parton coalescence - review**
 - motivation, formalism
 - main successes - baryon enhancement, elliptic flow scaling
- **parton cascade**
 - introduction
 - highlights: large opacities at RHIC, charm flow
 - **implications of space-time dynamics for coalescence**
- **summary and outlook**

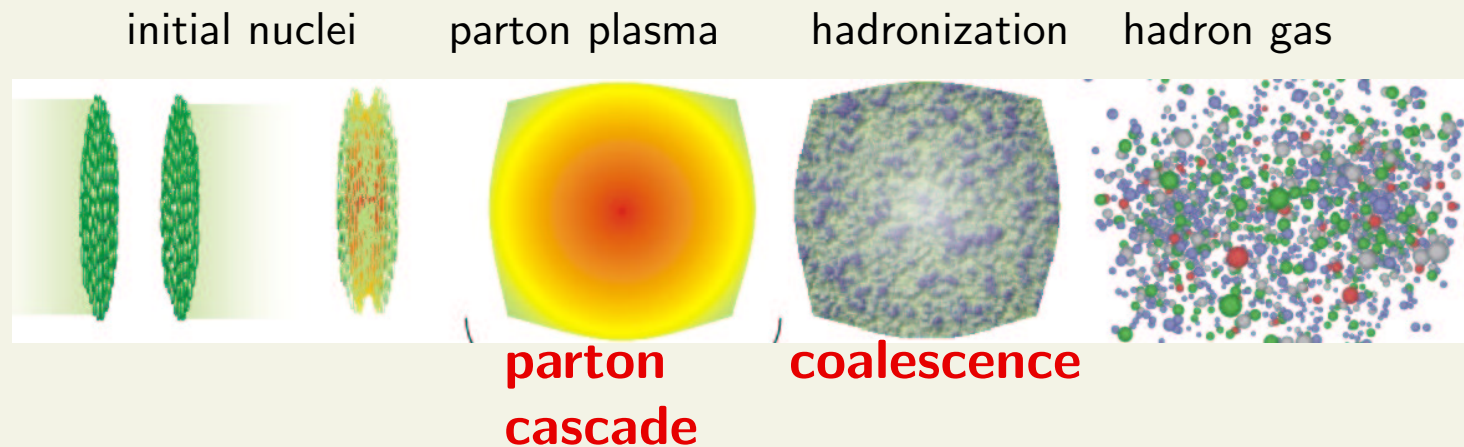
Heavy-ion double challenge

- **partonic condensed matter physics** Kajantie '96

many-body system $\gg \sum$ constituents

- **collision dynamics** (\sim plasma physics, nonlinear systems)

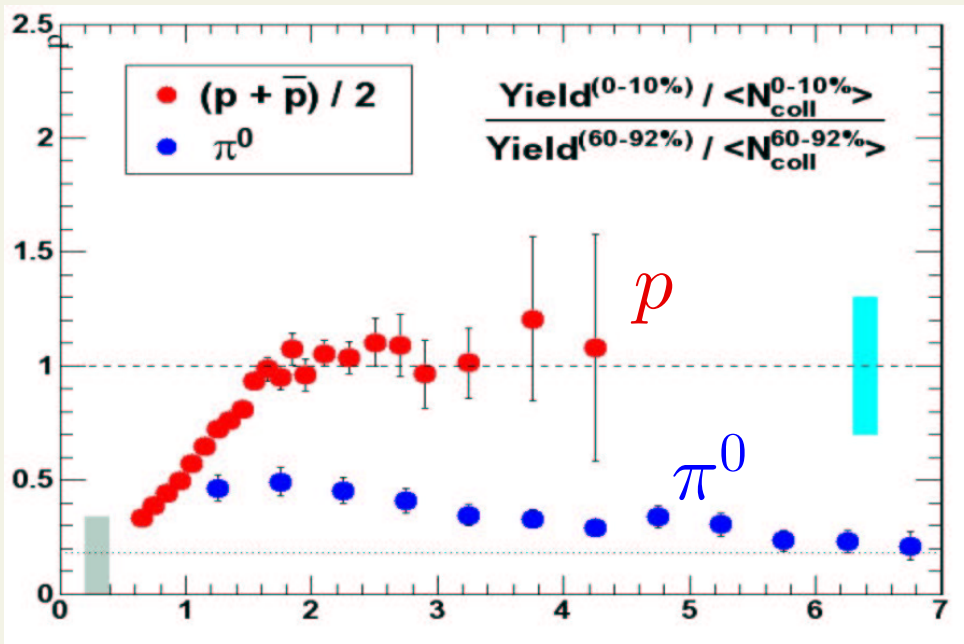
evolving system \gg system in a box



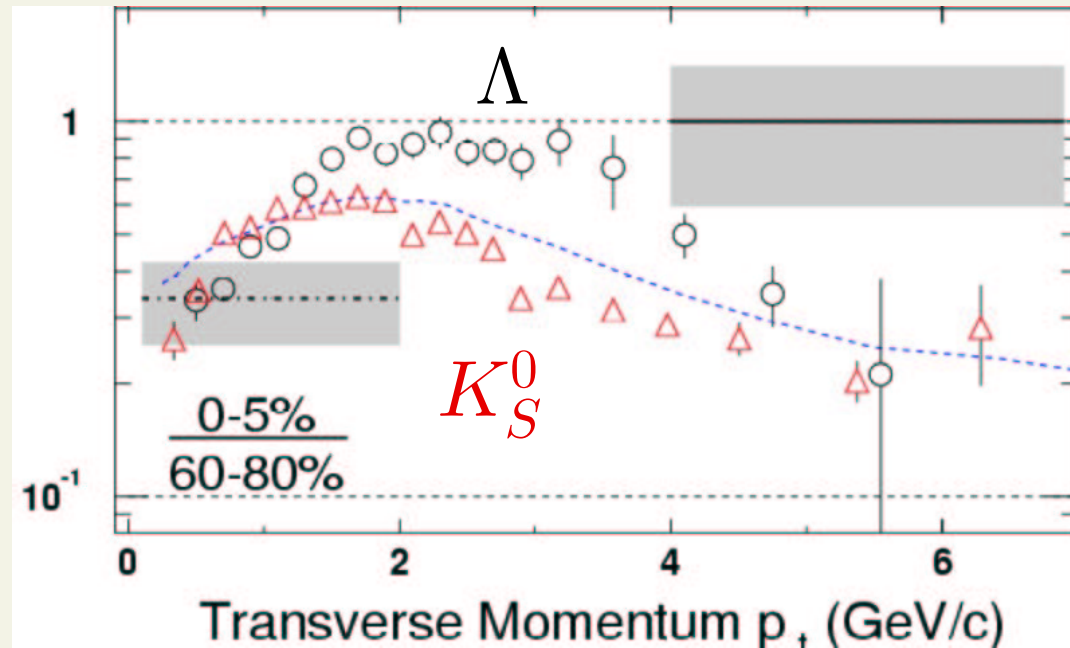
Baryon non-suppression @ RHIC

$$R_{cp} \equiv \frac{\text{central yield rel. to binary scaled } p + p}{\text{peripheral yield rel. to binary scaled } p + p}$$

d'Enterria [PHENIX] @ INT March '03:



Sorensen [STAR] @ SQM2003:

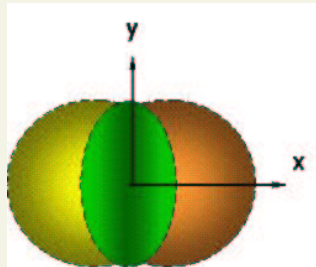


→ cannot be explained by jet quenching + factorized perturbative QCD

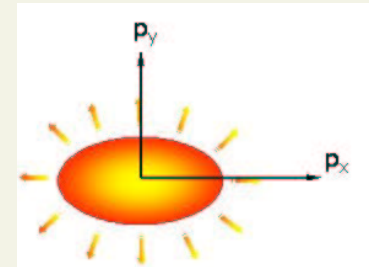
Baryon-meson elliptic flow splitting

spatial anisotropy \rightarrow final azimuthal momentum anisotropy

$$\epsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

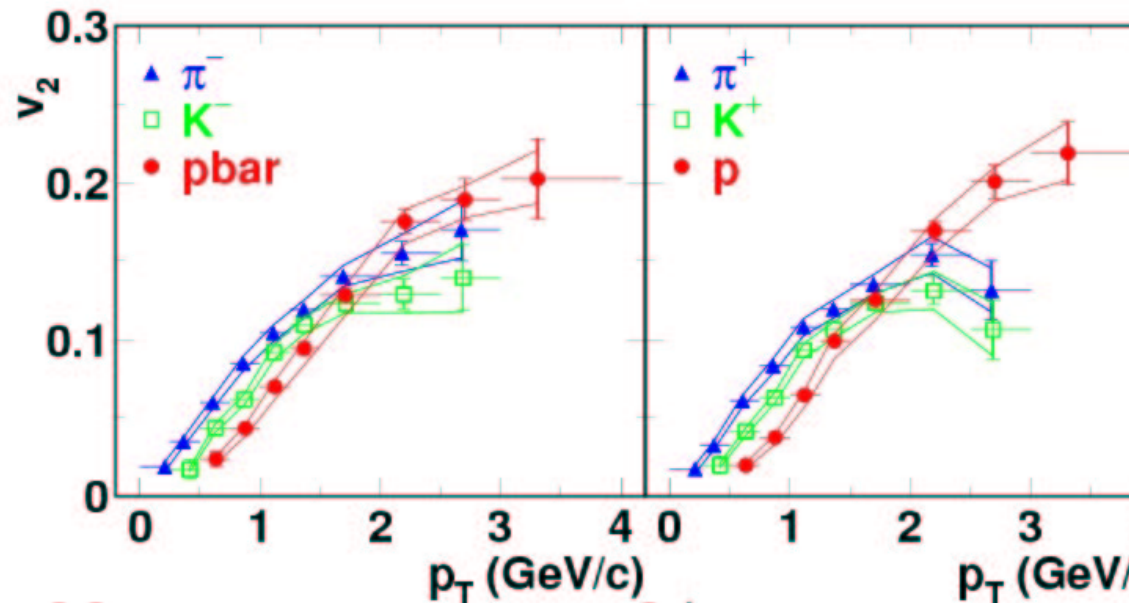
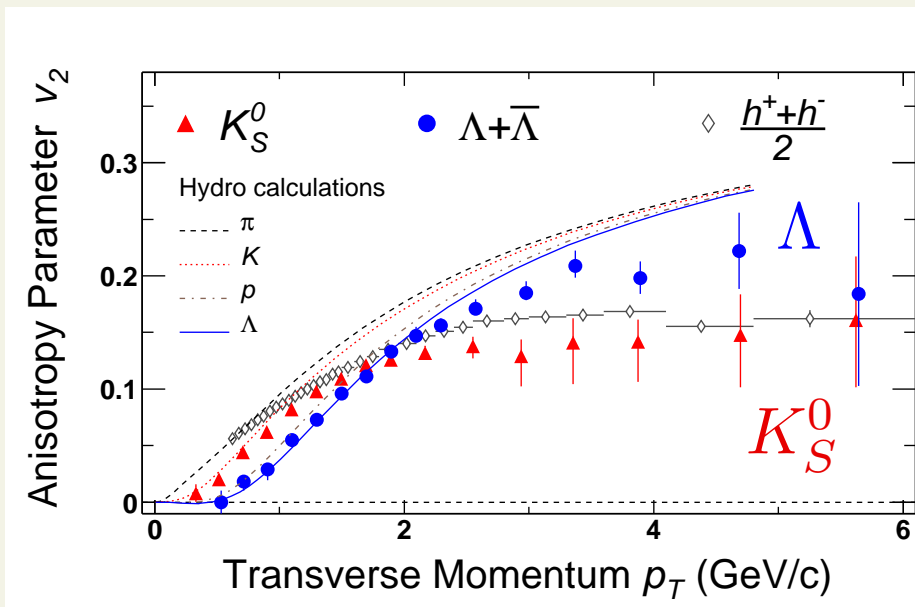


$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$



Sorensen [STAR] @ SQM2003:

Esumi [PHENIX] '03:



\rightarrow not hydrodynamic behavior either

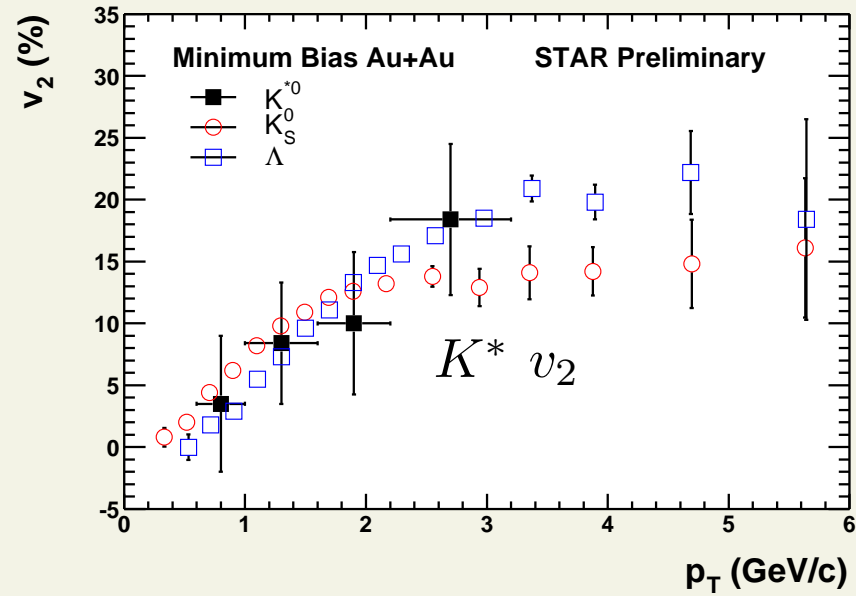
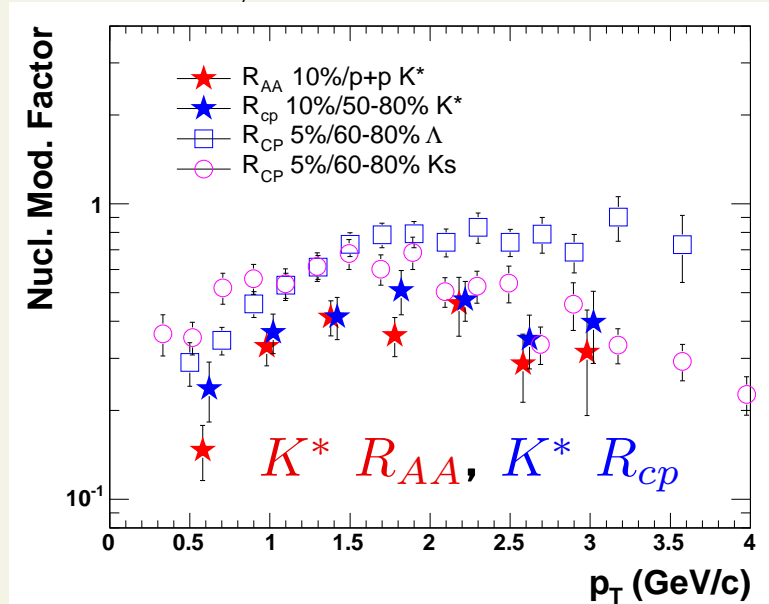
Something new is going on at intermediate pT

- possible origin
 - baryon/meson mass difference
 - quark content ($n_v = 2$ vs 3)
 - bosons vs fermions
 - ...

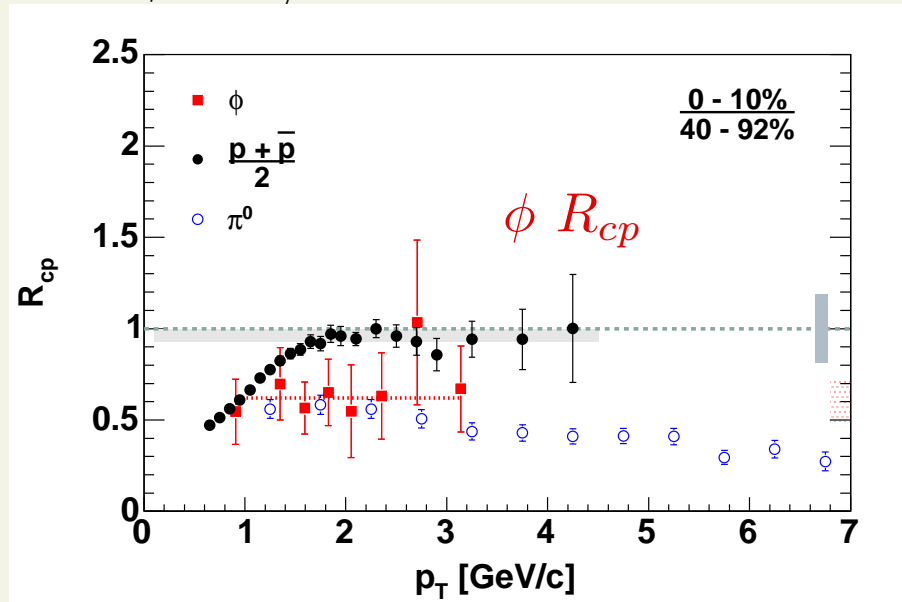
important test: $\phi(1020), K^*(892)$ vs $p(938), \Lambda(1116)$

- where in pT does the “real” perturbative QCD regime start?

STAR, nucl-ex/0403010:



PHENIX, nucl-ex/0410012:



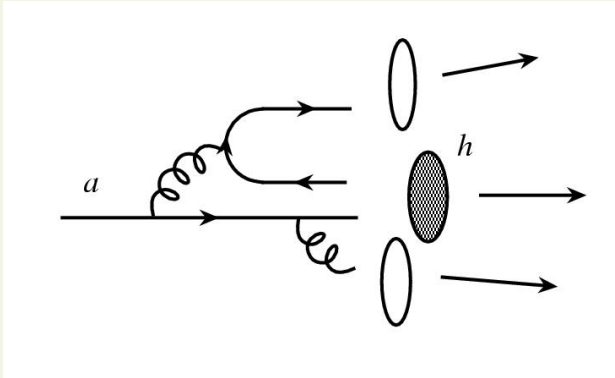
Rcp suggests it is not mass effect

QM2005: corroborated by v_2 data (ϕ)

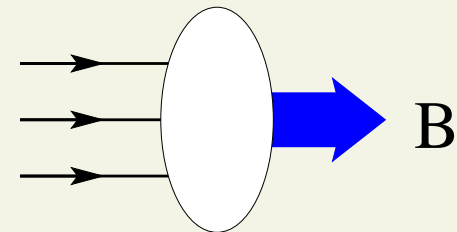
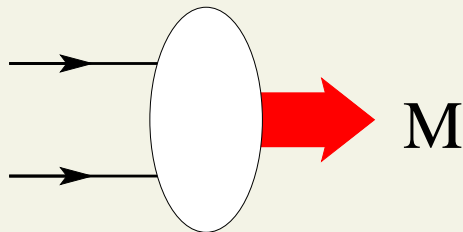
Parton coalescence

Hwa, Yang, Biró, Zimányi, Lévai, Csizmadia, Ko, Lin, Voloshin, DM, Greco, Fries, Müller, Nonaka, Bass, ...

In addition to jet fragmentation



other hadronization channels via parton coalescence/recombination



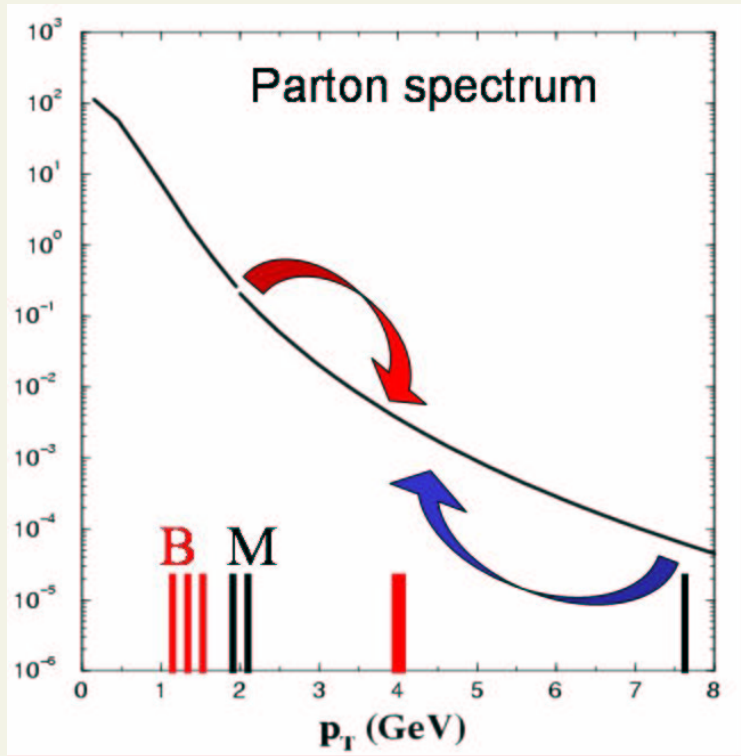
to lowest order $q\bar{q} \rightarrow M$, $qqq \rightarrow B$

coalescence becomes important when parton phasespace densities are high

References:

- hadron multiplicity:** Das & Hwa, PLB68 ('77)
Biró et al, PLB347 ('95) - ALCOR
Csizmadia & Lévai, JPG28 ('02) - MICOR
- baryon/meson ratio:** Hwa & Yang, PRC65 ('02)
Greco, Ko, Levai, PRL90 ('03); PRC68 ('03)
Fries, Müller, Nonaka, Bass, PRL90 ('03); PRC68 ('03)
Hwa, Yang, PRC67 ('03)
Fries, Müller, Nonaka, Bass, JPG30 ('04)
Hwa & Yang, PRC70 ('04)
- resonances:** Nonaka, Müller, Asakawa, Bass, Fries, PRC69 ('04),
Zimányi & Lévai, nucl-th/0404060
- elliptic flow:** Ko & Lin, PRL89 ('02)
Voloshin, NPA715 ('02)
DM & Voloshin, PRL91 ('03)
Nonaka, Fries, Bass, PLB583 ('04)
DM, JPG30 ('04)
Greco, Ko, PRC70 ('04)
DM, nucl-th/0403035
- charm hadron elliptic flow:** Lin & DM, PRC68 ('03)
Greco, Ko, Rapp, PLB595 ('04)
- hadron correlations:** Fries, Bass, Muller, PRL94 ('05)
Hwa, Tan, nucl-th/0503052
- spacetime dynamics:** DM, JPG30 ('04), nucl-th/0406066, nucl-th/0408044
Pratt & Pal, PRC71 ('05)

Coalescence vs fragmentation



- fragmentation requires energetic parton

$$p_{hadron} = z \cdot p_{parton}, \quad z < 1$$

$$N_{had} \sim N_{par}$$

M/B yields controlled by frag. fn. $D(z)$

- coalescence needs “near-by” partons

$$p_{hadron} = n \cdot p_{parton}, \quad n = 2, 3$$

$$N_{had} \sim (N_{par})^n$$

M/B yields depend on phasespace density

for exponential, coal wins:

$$N_{parton} \sim A e^{-p_T/T}$$

$$N^{coal} \sim A^2 e^{-p_T/T}$$

$$N^{frag} \sim A e^{-p_T/T \langle z \rangle}$$

at high- p_T , fragmentation wins:

$$N_{parton} \sim p_T^{-\alpha} \quad (\alpha \sim 5 - 6)$$

$$N^{coal} \sim p_T^{-2\alpha}$$

$$N^{frag} \sim p_T^{-(\alpha+2,3)}$$

AuAu at RHIC: coal dominates up to $p_T \sim 4 - 6$ GeV Greco et al, Fries et al PRL 90 ('03)

“Sudden” approximation

Greco, Ko, Levai et al; Voloshin, Lin, DM, et al; Fries, Bass, Mueller et al, ...

- originally for $n + p \rightarrow d$

Butler & Pearson, PR129 ('63); Schwarzschild & Zupancic, PR129 ('63); Sato & Yazaki, PLB98 ('81); Gyulassy, Frankel & Remler, NPA402 ('86); Dover et al PRC44 ('91); Nagle et al PRC53 ('96) Kahana et al PRC54 ('96); Scheibl & Heinz, PRC59 ('99); ...

● basic equations:

$$“N_M(t) = N_{pairs} \text{Tr}[|\phi_M\rangle\langle\phi_M| \hat{\rho}_{q,\bar{q}}(t)]”$$

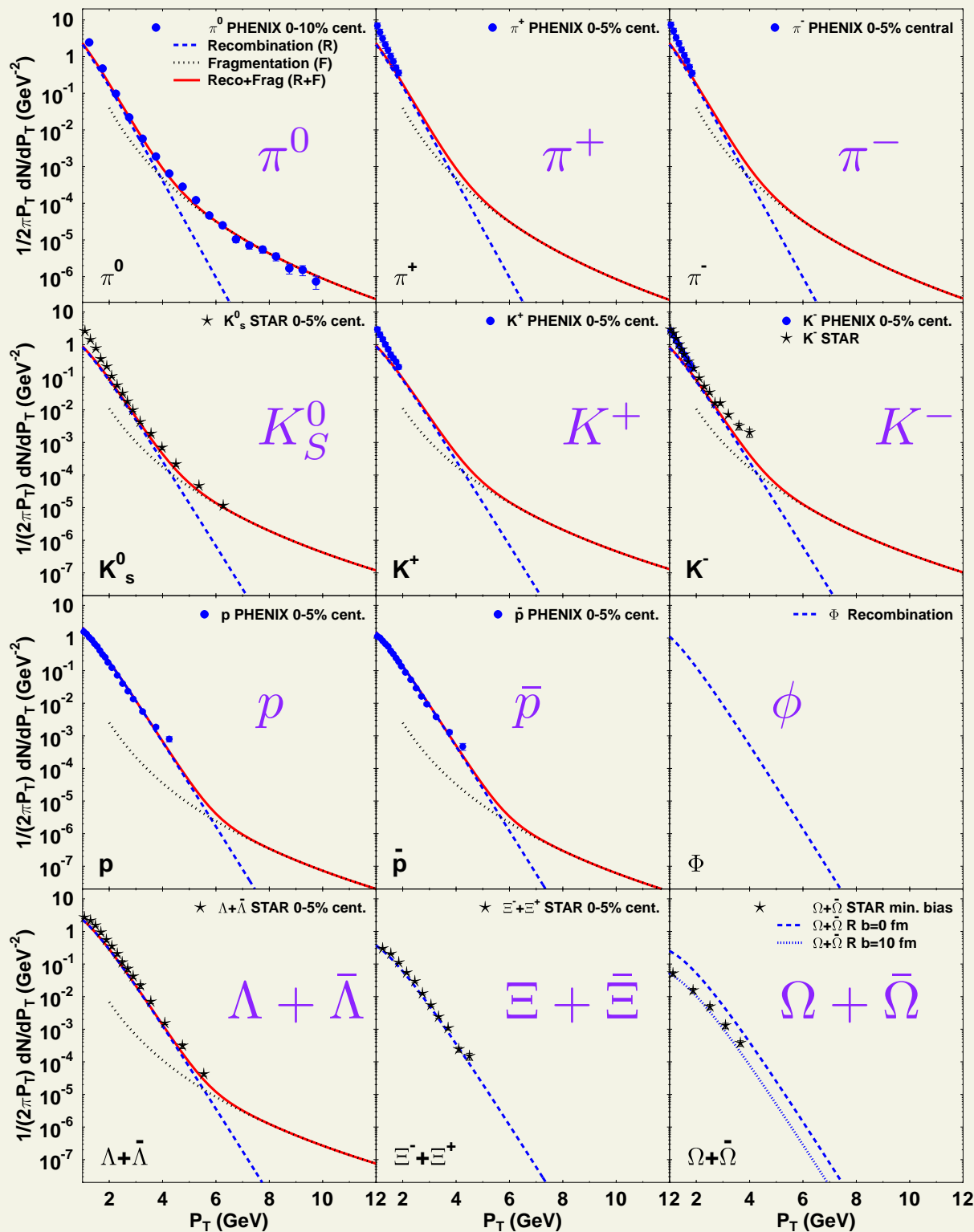
$$\frac{dN_M(\vec{p})}{d^3p} = g_M \int \left(\prod_{i=1,2} d^3x_i d^3p_i \right) W_M(x_1 - x_2, \vec{p}_1 - \vec{p}_2) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

$$\frac{dN_B(\vec{p})}{d^3p} = g_B \int \left(\prod_{i=1,2,3} d^3x_i d^3p_i \right) W_B(x_{12}, x_{13}, \vec{p}_{12}, \vec{p}_{13}) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) f_\gamma(\vec{p}_3, x_3) \delta^3(\vec{p} - \sum \vec{p}_i)$$

hadron yield space-time hadron wave-fn. quark distributions

- assumes:**
- weakly-bound hadrons
 - sudden hadronization (3D hypersurface, constant “time”)
 - rare process (not valid at very low pT → unitarity problem)
 - no 2-body or 3-body correlations (straightforward to introduce Fries et al PRL94 '04)

→ 8 distributions ($f_u, f_d, f_s, f_c + \text{anti}$) give spectra for all hadron species



Spectra at RHIC are fairly well reproduced with 2-component system: Bass et al PRC68 ('03)

- i) $q-\bar{q}$ "plasma" at $T \approx 175$ MeV with strong radial flow $v_m \approx 0.55c$
- ii) quenched jets

not a perfect agreement (factor ~ 2 deviations)

but captures the trends well

perhaps fits could be improved?

Elliptic flow scaling

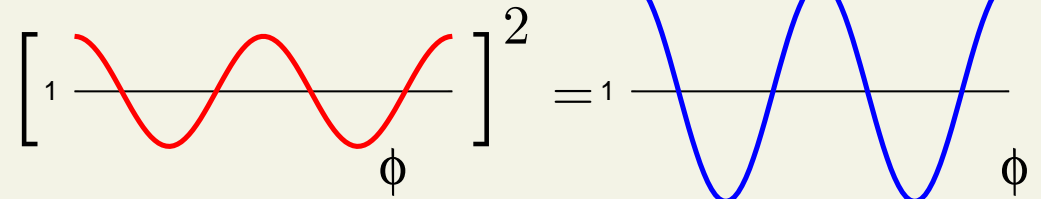
Ko, Lin, Voloshin, DM, Greco, Levai, Mueller, Fries, Bass, Nonaka, Asakawa ...

coalescence of comoving quarks: $q\bar{q} \Rightarrow \longrightarrow M$ $3q \Rightarrow \longrightarrow B$

DM & Voloshin, PRL91 ('03)

$$\frac{dN_M(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/2)}{d\phi} \right]^2$$

$$\frac{dN_B(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/3)}{d\phi} \right]^3$$

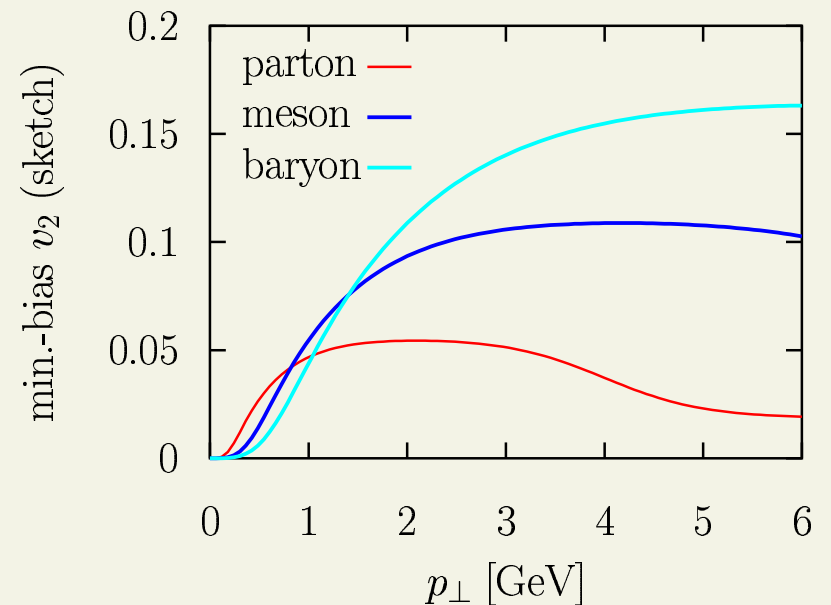


squared/cubed probability \rightarrow amplified v_2

$$v_2^{hadron}(p_\perp) \approx n \times v_2^{quark}(p_\perp/n)$$

$3 \times$ for baryons } 50% larger v_2
 $2 \times$ for mesons } for baryons

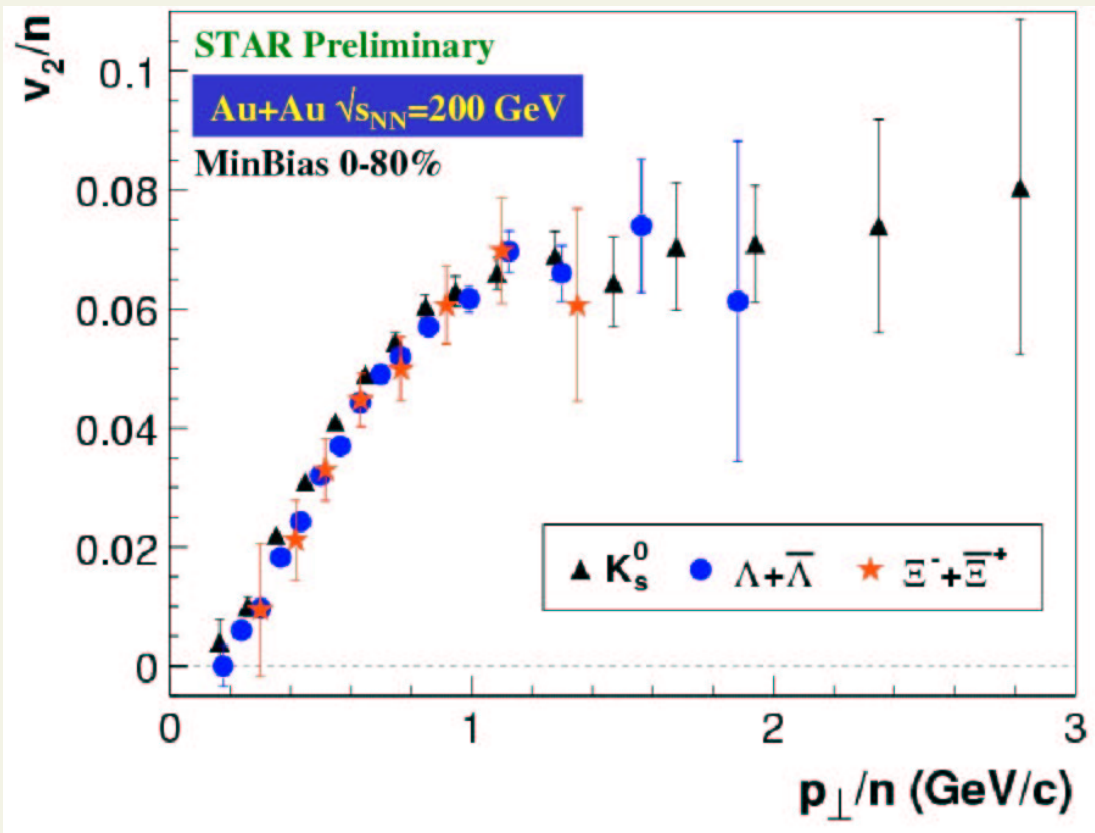
$\rightarrow 5 \times$ for pentaquark, $6 \times$ for deuteron



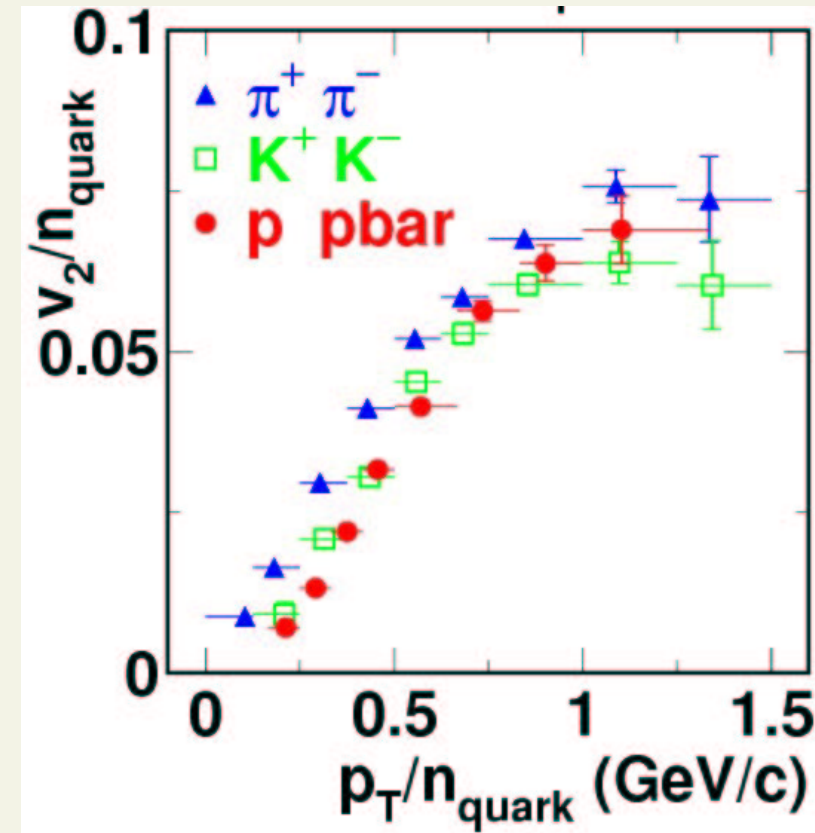
simple but naive DM '04: ignores space-time, other hadronization channels

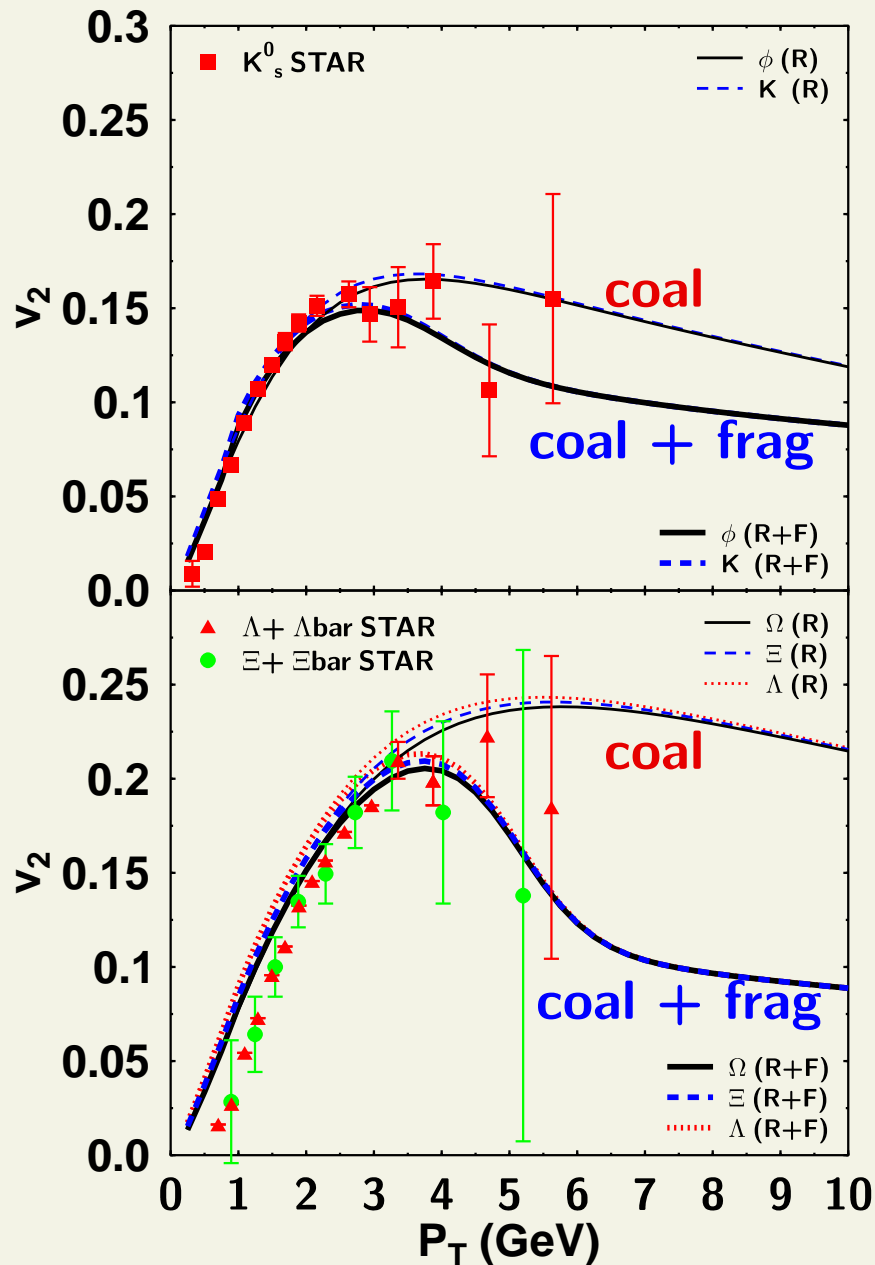
quark number scaling **OBSERVED**: v_2/n_v vs p_T/n_v universal

Castillo [STAR] '03



Esumi [PHENIX] '03





at very high p_T , baryon-meson flow separation should of course disappear

pure fragmentation regime

$\rightarrow v_2^M = v_2^B$, scaling breaks down

e.g., Nonaka et al, PLB583 ('04)

\Rightarrow should be verified experimentally

Picture seems to hang together

- **coalescence describes enhanced baryon yields** Fries et al, Ko et al

- **meson/baryon elliptic flow difference also explained**

[● robust against resonance decays, hadron wave fn details Greco et al, Fries et al]

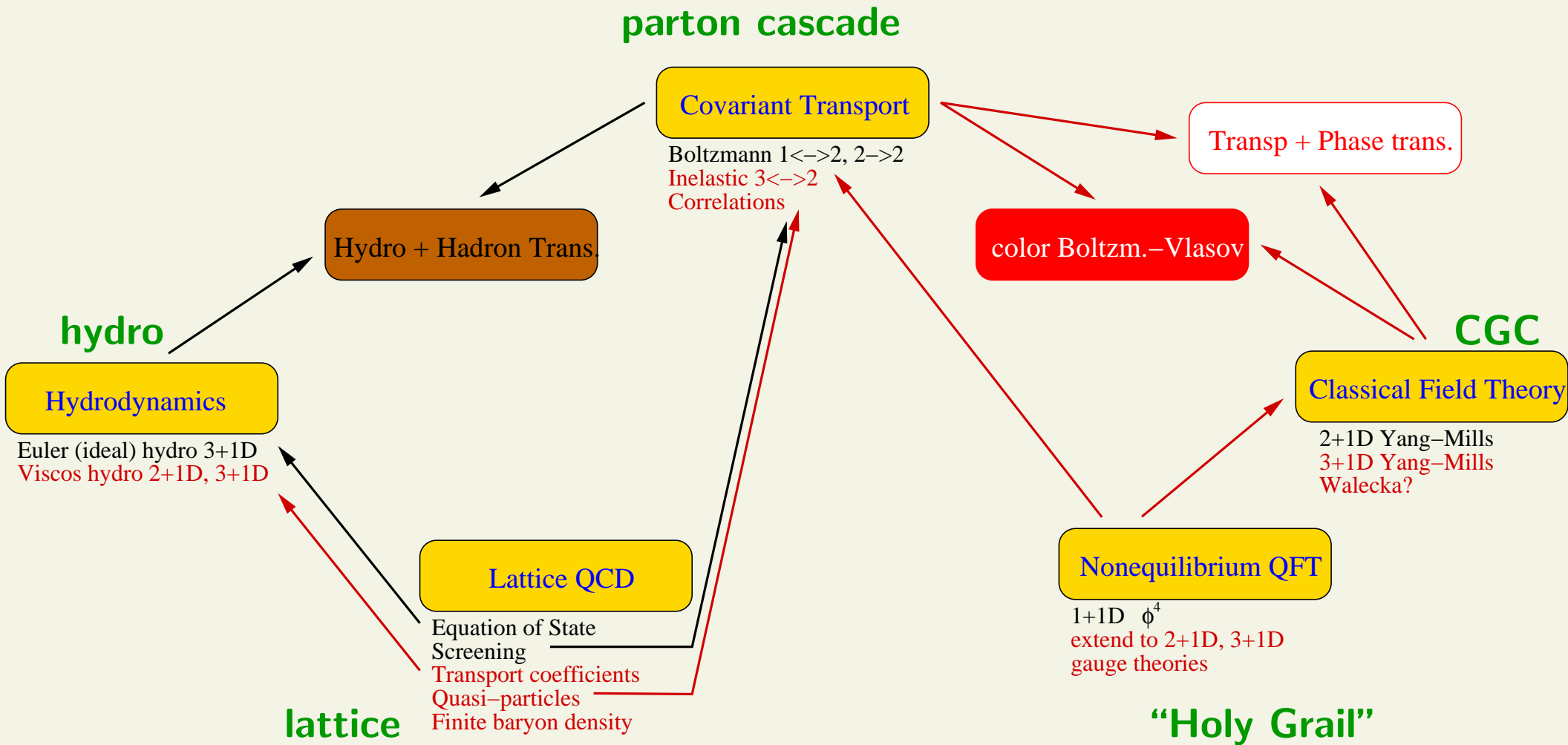
[● correlations at intermediate p_T Fries et al, Hwa et al] → Rudy Hwa (yesterday)

[● (charge) fluctuations Bass, Jeon, et al] → Steffen Bass (Today 5.20pm)

(!) provided quark phasespace distribution is a fit parameter

are parametrizations consistent with a dynamical approach?

Dynamical frameworks



Quark-gluon kinetic theory

Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, ...

Incoherent, particle limit of underlying quantum theory (QCD). Nonequilibrium approach.

Boltzmann transport eqn: $f_i(\vec{x}, \vec{p}, t)$ - quark/gluon phase space distributions

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = \overbrace{S_i(\vec{x}, \vec{p}, t)}^{\text{source } 2 \rightarrow 2 \text{ (ZPC, GCP, ...)}} + \overbrace{C_i^{el.}[f](\vec{x}, \vec{p}, t)}^{2 \leftrightarrow 3 \text{ (MPC, Xu-Greiner)}} + \overbrace{C_i^{inel.}[f](\vec{x}, \vec{p}, t)} + \dots$$

- on-shell dynamics ($p^2 = m^2 \geq 0$)

→ see this afternoon Bin Zhang (ZPC) and Zhe Xu (X-G)

other variants: e.g., VNI(b), off-shell partons Kinder-Geiger, Shrivastava, Mueller, Bass, ...

* OSCAR code repository @ <http://nt3.phys.columbia.edu/OSCAR> *

An example: MPC

Processes: Debye-screened elastic $2 \rightarrow 2 + gg \leftrightarrow q\bar{q}, q\bar{q} \rightarrow q'\bar{q}' + ggg \leftrightarrow gg$

Equation for $f^i(x, \vec{p})$: $i = \{g, d, \bar{d}, u, \bar{u}, \dots\}$

$$\begin{aligned}
 p_1^\mu \partial_\mu \tilde{f}^i(x, \vec{p}_1) &= \frac{\pi^4}{2} \sum_{jkl} \int_2 \int_3 \int_4 \left(\tilde{f}_3^k \tilde{f}_4^l - \tilde{f}_1^i \tilde{f}_2^j \right) \left| \bar{\mathcal{M}}_{12 \rightarrow 34}^{i+j \rightarrow k+l} \right|^2 \delta^4(12 - 34) \quad \swarrow 2 \rightarrow 2 \\
 &+ \frac{\pi^4}{12} \int_2 \int_3 \int_4 \int_5 \left(\frac{\tilde{f}_3^i \tilde{f}_4^i \tilde{f}_5^i}{g_i} - \tilde{f}_1^i \tilde{f}_2^i \right) \left| \bar{\mathcal{M}}_{12 \rightarrow 345}^{i+i \rightarrow i+i+i} \right|^2 \delta^4(12 - 345) \quad \swarrow 2 \leftrightarrow 3 \\
 &+ \frac{\pi^4}{8} \int_2 \int_3 \int_4 \int_5 \left(\tilde{f}_4^i \tilde{f}_5^i - \frac{\tilde{f}_1^i \tilde{f}_2^i \tilde{f}_3^i}{g_i} \right) \left| \bar{\mathcal{M}}_{45 \rightarrow 123}^{i+i \rightarrow i+i+i} \right|^2 \delta^4(123 - 45) \quad \swarrow 3 \leftrightarrow 2 \\
 &+ \tilde{S}^i(x, \vec{p}_1) \quad \leftarrow \text{initial conditions}
 \end{aligned}$$

with shorthands:

$$\tilde{f}_i^q \equiv (2\pi)^3 f_q(x, \vec{p}_i), \quad \int_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1 + p_2 - p_3 - p_4) \equiv \delta^4(12 - 34)$$

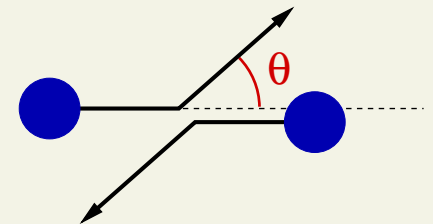
Relevant parameters

mean free path: characterizes local conditions

$$\lambda(x) \equiv \frac{1}{\text{cross section} \times \text{density}(x)} \quad \begin{cases} \lambda = 0 & \text{– ideal hydrodynamics} \\ \lambda = \infty & \text{– free streaming} \end{cases}$$

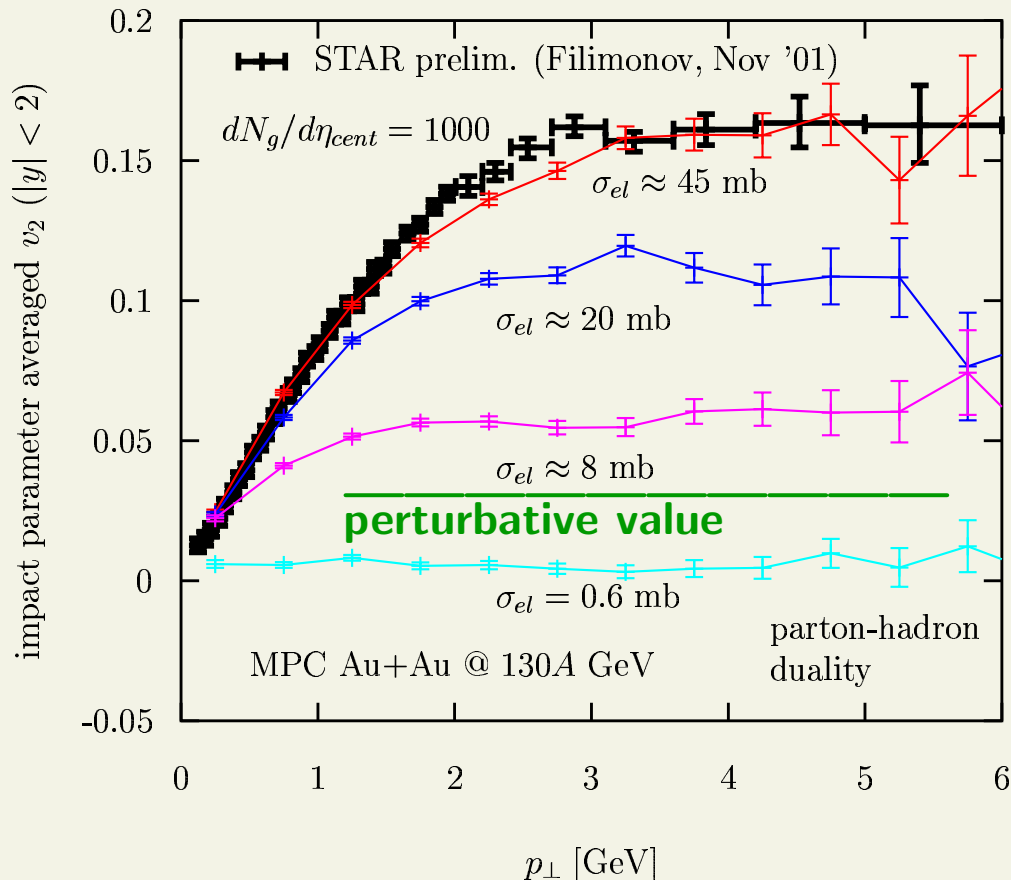
transport opacity: time-integrated, spatially averaged [DM & Gyulassy NPA 697 ('02)]

$$\chi \equiv \langle n_{coll} \rangle \langle \sin^2 \theta_{CM} \rangle \sim \# \text{ of collisions per parton} \times \text{mom. transfer efficiency}$$



Strong interactions at RHIC

DM & Gyulassy, NPA 697 ('02): $v_2(p_T, \chi)$



Au+Au @ 130 GeV, $b = 8$ fm

nonlinear opacity dependence

$$v_2^{max} \sim \chi^{0.61} \sim (\sigma \times dN/dy)^{0.61}$$

need $15\times$ perturbative opacities - $\sigma_{el} \times dN_g/d\eta \approx 45 \text{ mb} \times 1000$

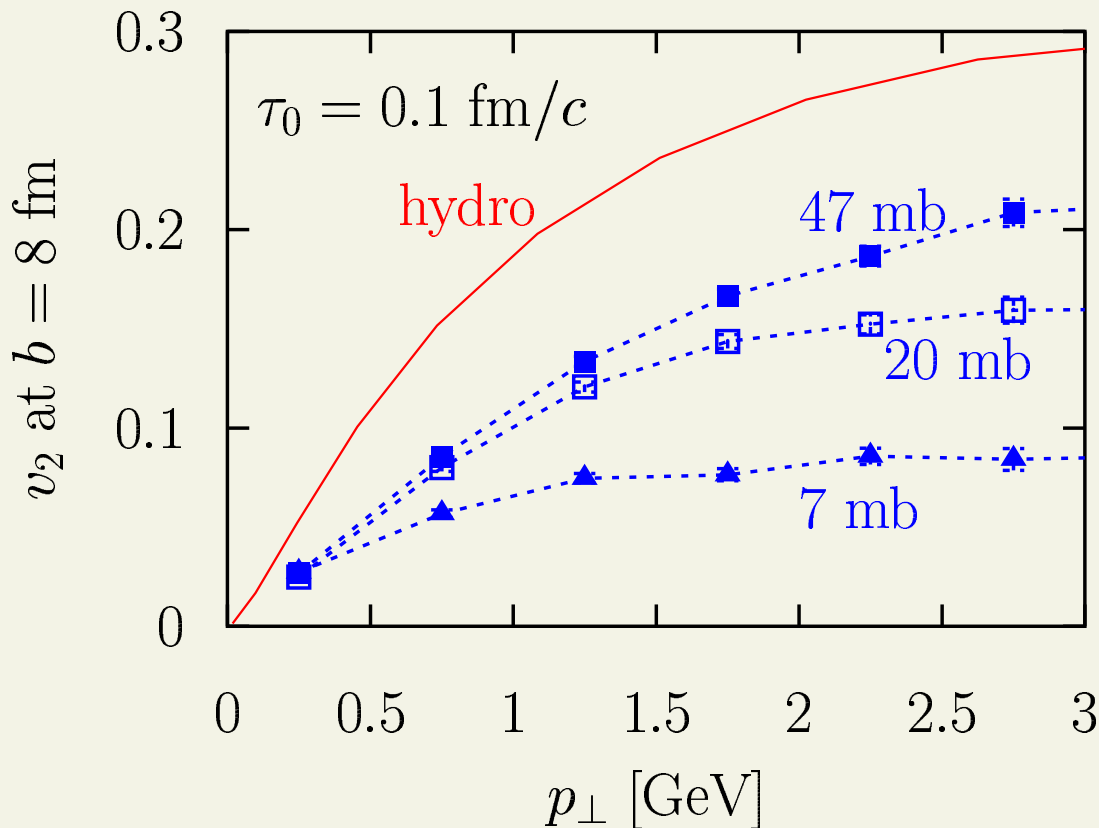
(saturated gluon $\frac{dN^{cent}}{d\eta} = 1000$, $T_{eff} \approx 0.7$ GeV, $\tau_0 = 0.1$ fm, 1 parton \rightarrow 1 π hadronization)

\Rightarrow **strongly-interacting quark-gluon plasma (sQGP)** \rightarrow see Shuryak's talk

Still not ideal fluid(!)

Even $\sigma_{gg \rightarrow gg} \sim 50 \text{ mb}$ is insufficient for ideal hydro (perturbative QGP: $\sim 3 \text{ mb}$)

DM & Huovinen, PRL94 ('05):



- **dissipation** reduces v_2 by 30 – 50%

- in addition, it **slows cooling**

was already seen in 1+1D transport

Gyulassy, Pang & Zhang ('97)

also there in 1+1D **viscos** hydro

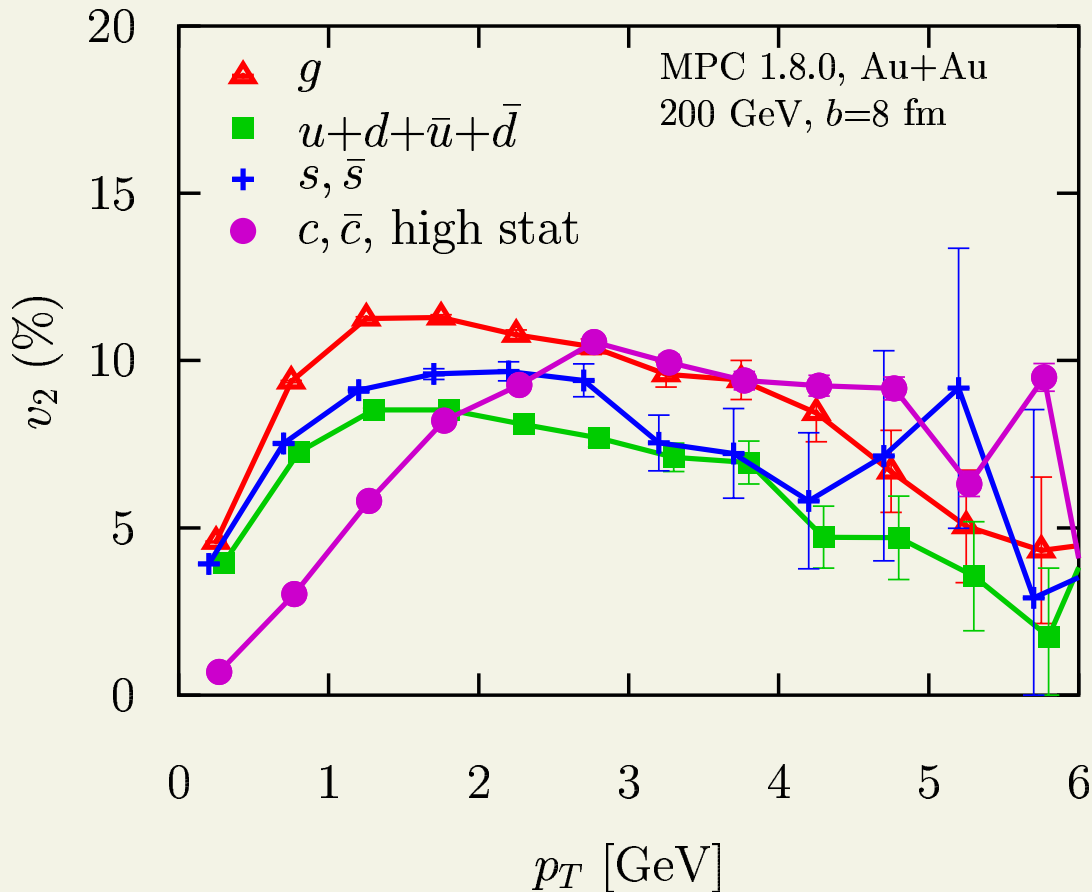
Danielewicz & Gyulassy ('85)

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon+p}{\tau} = \frac{(\zeta + \frac{4}{3}\eta)}{\tau^2}$$

viscosity very small - but gradients very large

Should see charm flow(!)

DM, JPG 31 ('04): **parton** $v_2(p_T)$

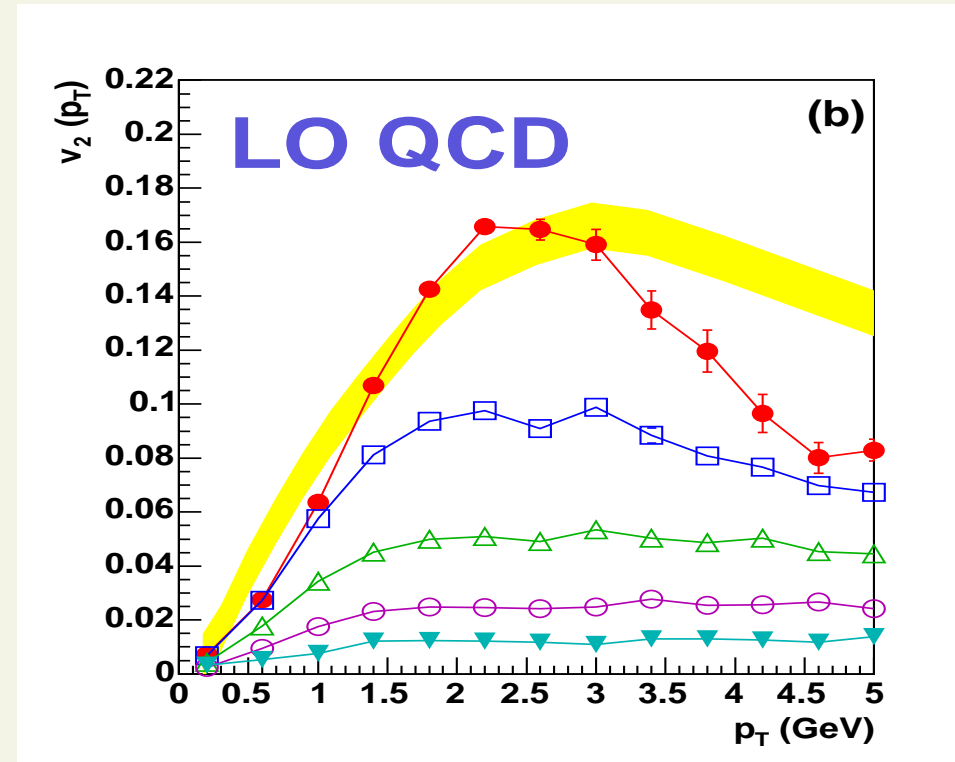
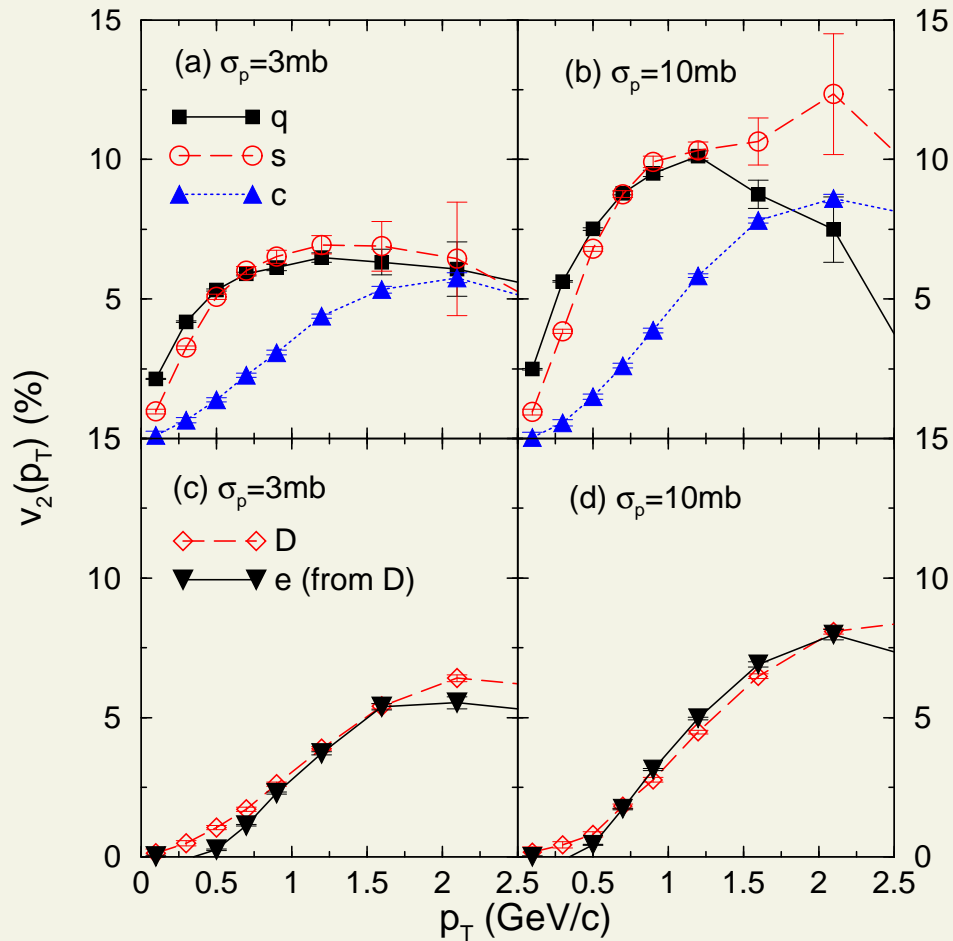


$\sim 6 \times$ perturbative opacities

in opaque plasma, charm v_2 reaches light parton v_2 above $p_T > 2-3$ GeV

similar results from: **parton cascade ZPC** \rightarrow Bin Zhang (Today 2.40pm)
Fokker-Planck \rightarrow Derek Teaney (Today 4.40pm)

decay electron $D \rightarrow K^{(*)} e \nu$ data seem to support this \rightarrow Laue (this aftern.)



Parton cascade and coalescence

need approach consistent with coalescence formulas:

- **evolve parton system until freezeout** (consistent with weak binding assumption)

- **coalesce partons that “fit” into hadron Wigner function (in their CM)**

use box functions $W_M(\Delta x, \Delta p) \sim \Theta(x_m - |\vec{\Delta}x|)\Theta(p_m - |\vec{\Delta}p|)$ [$x_m \cdot p_m = const$]

if unequal times, first propagate earlier particle(s) to latest of times

Gyulassy, Frankel, Rehmler '83

- **fragment partons without coalescence partner**

use MPC for transport (2 → 2 mode), JETSET 7.4.10 for jets

initconds: $p_T > 2$ GeV - perturbative minijets

$p_T < 2$ GeV - saturated partons

($\tau_0 = 0.1\text{fm}/c$, $dN/dy^{cent} = 2000$)

Coalescence and space-time

for smooth distributions over length and mom scales of hadron wave fns:

$$\rightarrow W_M \sim \delta^3(\Delta x)\delta^3(\Delta p), \quad W_B \sim \delta^3(\Delta x_{12})\delta^3(\Delta x_{13})\delta^3(\Delta p_{12})\delta^3(\Delta p_{13})$$

$$\frac{dN_M(\vec{p})}{d^3p} \approx g_M (2\pi)^3 \int d^3x f_\alpha(\vec{p}/2, \mathbf{x}) f_\beta(\vec{p}/2, \mathbf{x})$$

$$\frac{dN_B(\vec{p})}{d^3p} \approx g_B (2\pi)^6 \int d^3x f_\alpha(\vec{p}/3, \mathbf{x}) f_\beta(\vec{p}/3, \mathbf{x}) f_\gamma(\vec{p}/3, \mathbf{x})$$

from “ v_2 amplification”, it follows DM nucl-th/0408044:

$$v_2^{Meson}(p_T) \approx \frac{2 \langle n_q^2(\mathbf{x}, p_T/2) v_{2,q}(\mathbf{x}, p_T/2) \rangle_x}{\langle n_q^2(\mathbf{x}, p_T/2) \rangle_x} \neq 2 v_{2,q}(p_T/2)$$

$$v_2^{Baryon}(p_T) \approx \frac{3 \langle n_q^3(\mathbf{x}, p_T/3) v_{2,q}(\mathbf{x}, p_T/3) \rangle_x}{\langle n_q^3(\mathbf{x}, p_T/3) \rangle_x} \neq 3 v_{2,q}(p_T/3)$$

flow scaling results only if spatial dependence cancels out and $|v_{2,q}(x)| \ll 1$:

- **spatially uniform** $n(x, p_T) = n(p_T)$ DM & Voloshin assumption
- **or local v_2 is spatially uniform** $v_2(x, p_T) = v_2(p_T)$ Fries et al assumption

uniform density $n(x, p_T)$



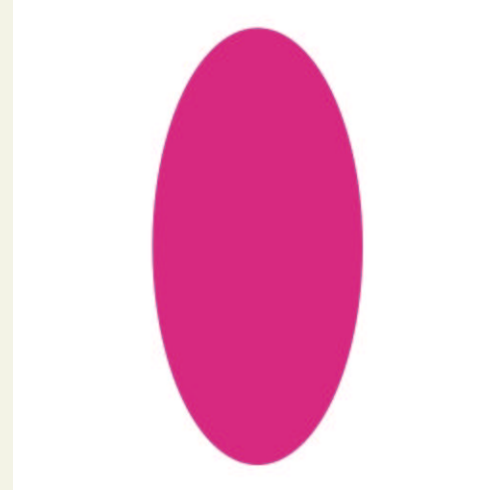
with nonuniform local $v_2(x, p_T)$



OR: nonuniform density $n(x, p_T)$



with uniform local $v_2(x, p_T)$

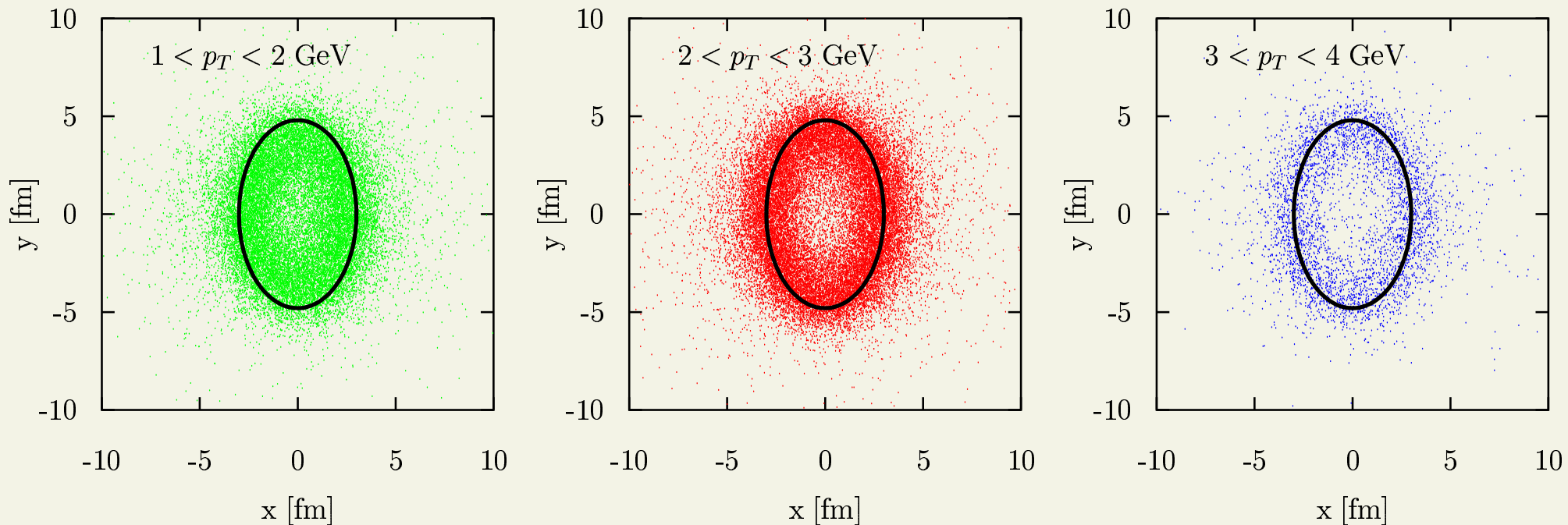


in the spatial region where the vast majority of particles are emitted from

Strong density variations

transverse position dN/d^2x_T dist. Au+Au @ 200GeV, $b = 8$ fm, $\sim 6\times$ pQCD opacity:

DM '04



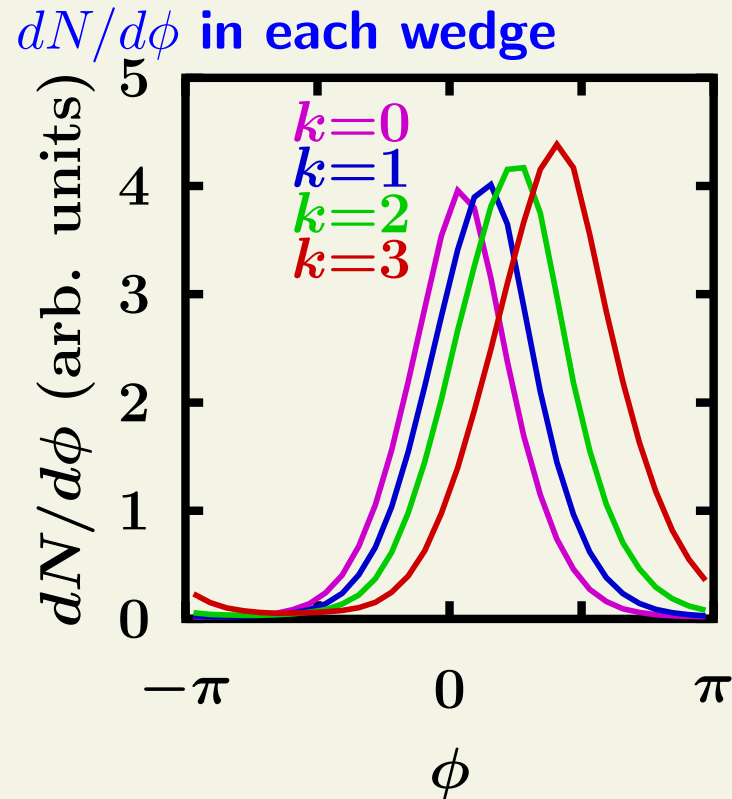
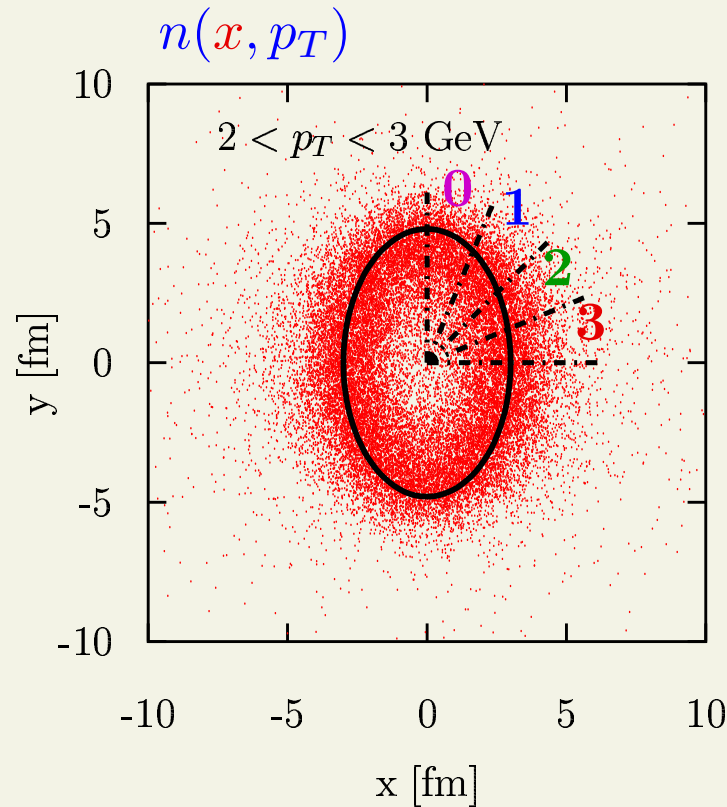
diffuse, nonuniform distributions, typical for transport

e.g., **UrQMD** Bleicher, Soff, Dumitru et al **or AMPT** Ko, Lin et al

even hydro freezeout is nonuniform (despite sharp hypersurface) Kolb, Heinz et al

Large, nonuniform local v_2

DM, nucl-th/0408044



nonuniform v_2 : out-of-plane regions have $v_2(\mathbf{x}) < 0$, in-plane $v_2(\mathbf{x}) > 0$
consistent with hydro picture (boosted sources)

narrow Gaussian peaks - $dN/d\phi \sim \exp[-(\phi - \phi_0)^2 / 2\sigma^2] \Rightarrow v_2 \approx \cos(2\phi_0) e^{-2\sigma^2} \sim 1$

\Rightarrow **power-law scaling:** $v_2^{had}(p_T, \mathbf{x}) \simeq v_{2,q}(p_T/n_v, \mathbf{x})^{1/n_v} \neq n_v v_{2,q}(p_T/n_v, \mathbf{x})$ [$\phi_0=0$]

Quark number scaling is nontrivial DM '04

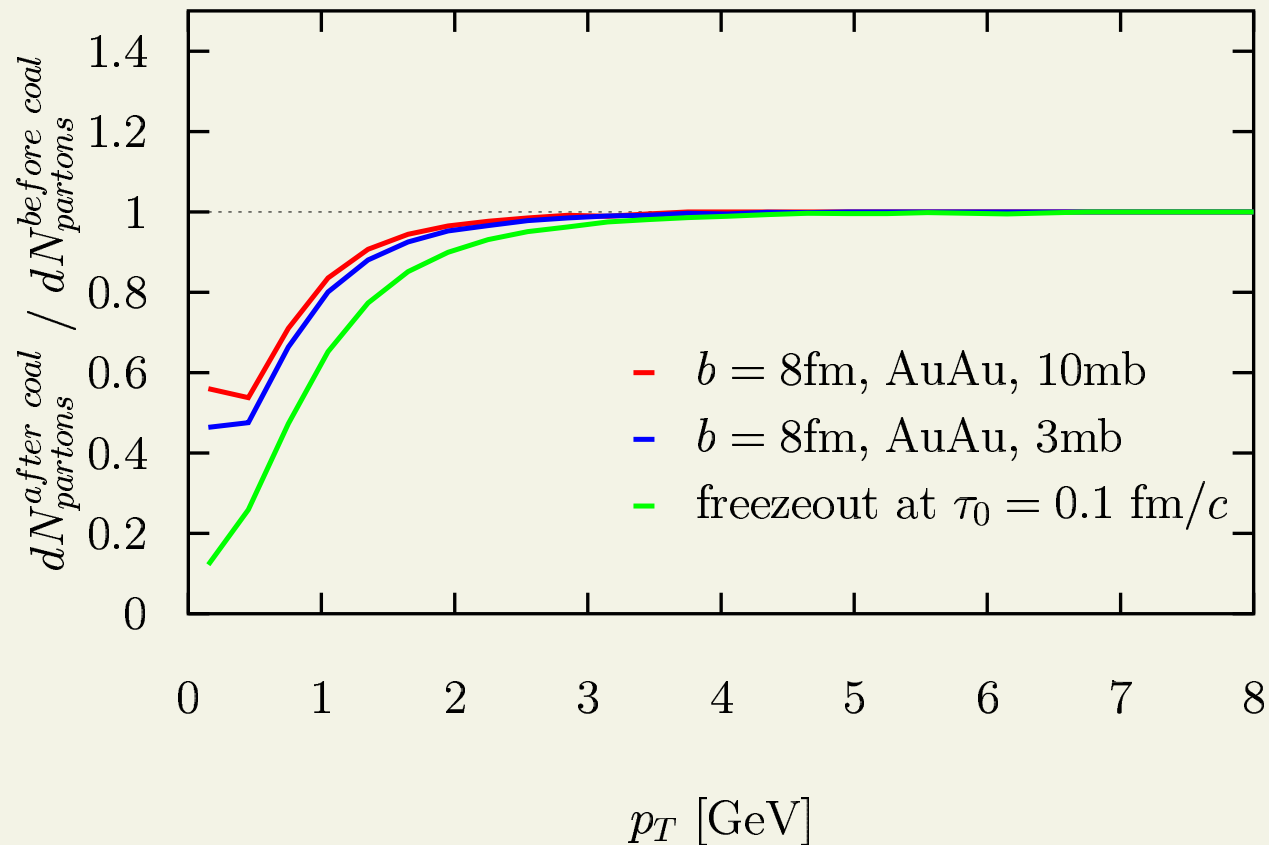
- realistic freezeout densities are nonuniform
- realistic flow anisotropies are large (locally) and nonuniform

several parameterizations have been studied Pratt & Pal PRC71 ('05)

- some scale (approximately), some do not

in addition, fragmentation contributions still missing

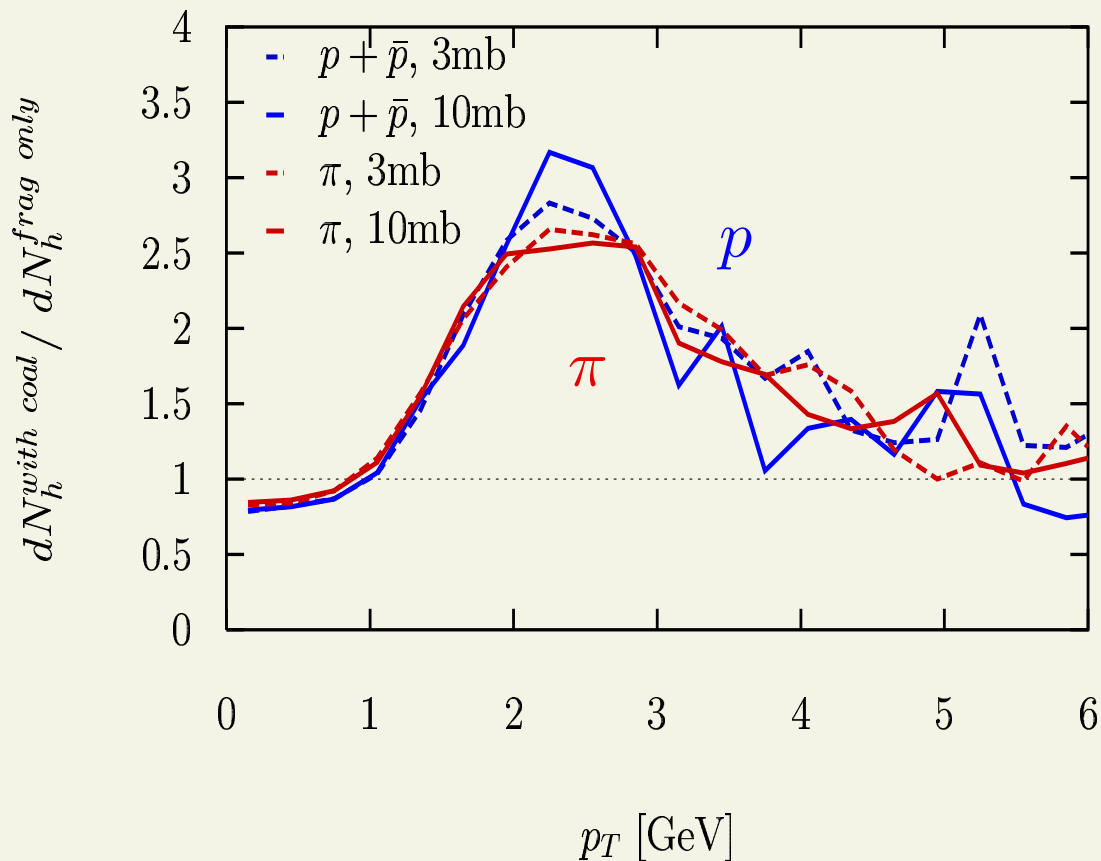
fraction of partons that fragment vs parton p_T DM, JPG31 ('04): **MPC + coal**



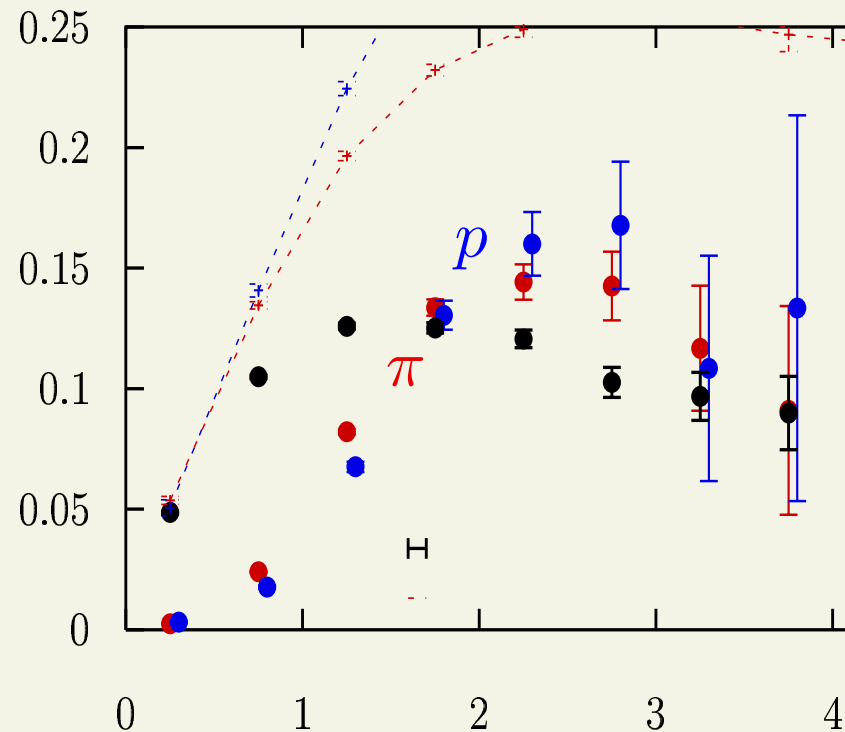
large fragmentation contributions, even for most optimistic earliest freezeout

\Rightarrow **spoils flow scaling and B/M enhancement**

hadron enhancement via coalescence



elliptic flow



2 – 3× yield enhancement for $1.5 < p_T < 4$ GeV but p/π almost unchanged

little flow amplification, baryon-meson splitting mostly gone

Summary

- **hadronization via parton coalescence is a promising explanation for several puzzles in the intermediate $2 < pT < 6$ GeV region:**
 - large baryon/meson ratios
 - quark number scaling of elliptic flow

can be explained with suitable choice of parton phase space distributions at hadronization.

⇒ we seem to be on the right track

- **however, the results rely on parametrizations that are inconsistent with dynamical models (parton cascade, hydro):**
 - space-time inhomogeneities, large local momentum anisotropies
 - fragmentation contributions

spoil the basic features and success of the simple models.

⇒ **the jury is still out...**

Outlook

- several improvements, studies possible:

- **nonzero binding energies** (“earlier” coalescence → higher coal yield)
- **nonlocal coalescence** in phase space
global correlations may exist that help maximize the number of coalesced hadrons
- **radiative processes** in transport ($3 \leftrightarrow 2$)
- ...

- other dynamical effects may also play a role at relatively high p_T :

- **plasma “push”**: DM nucl-th/0503051

in opaque plasma, **initially soft partons** can get accelerated to surprisingly high p_T in multiple collisions

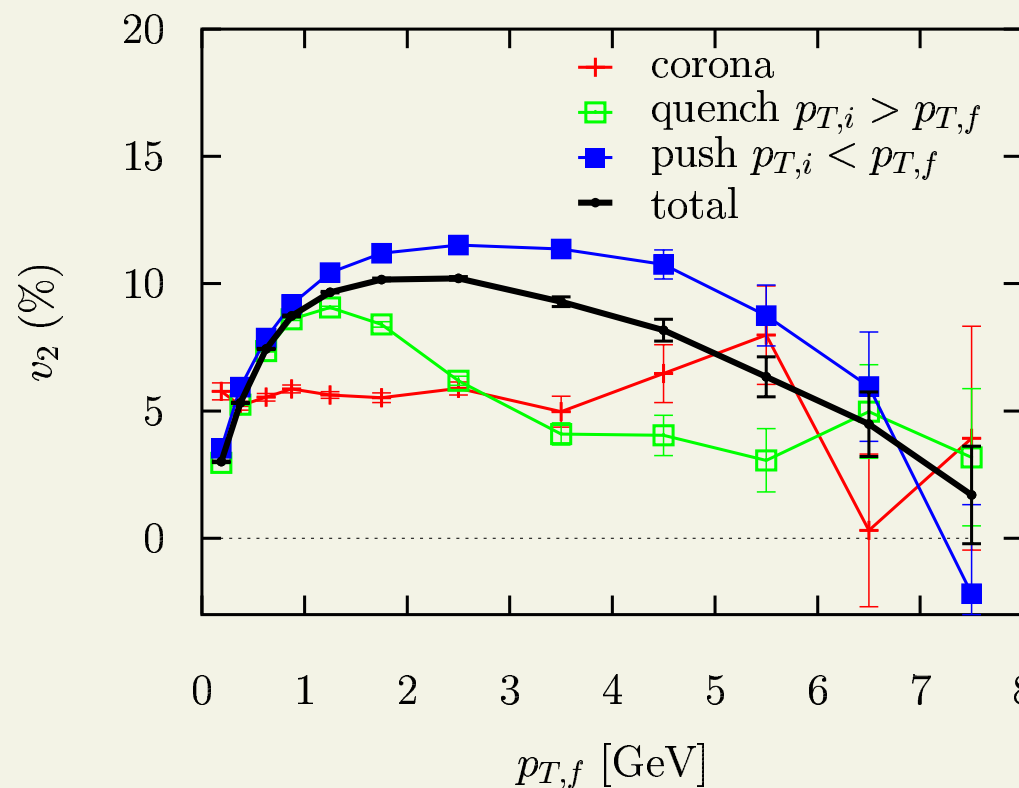
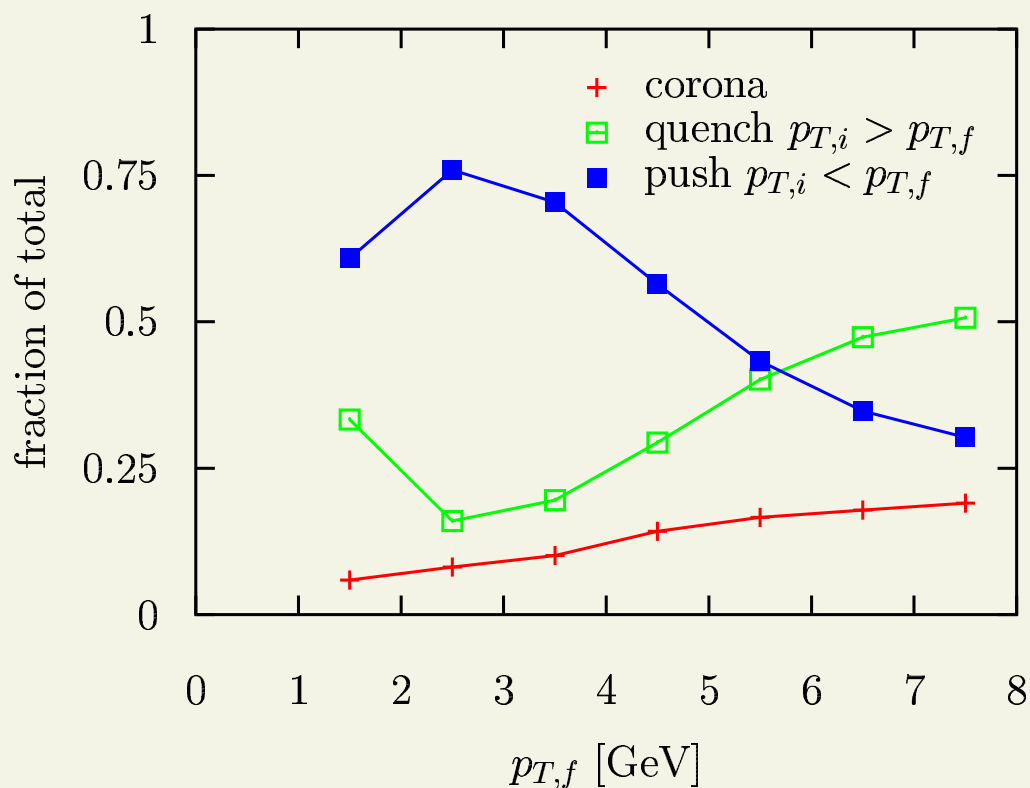
besides jet-quenching, two other components:

- **corona** → **partons that escape without interactions**
- **“push”** → **partons that interact and gain energy**

corona, quench, push fractions vs p_T

elliptic flow contributions vs p_T

DM, nucl-th/0503051: $\sim 6 \times$ pQCD opacities



significant $\sim 20\%$ “push” component even at $p_T \sim 8$ GeV

v_2 is higher and decreases slower at high p_T relative to pure quench case

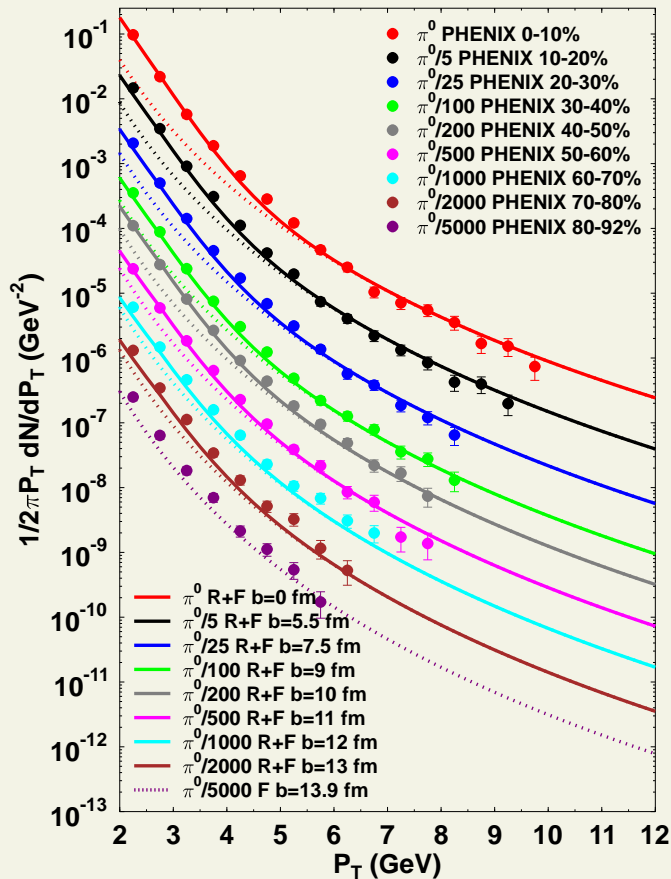
Backup slides

Spectra and baryon enhancement

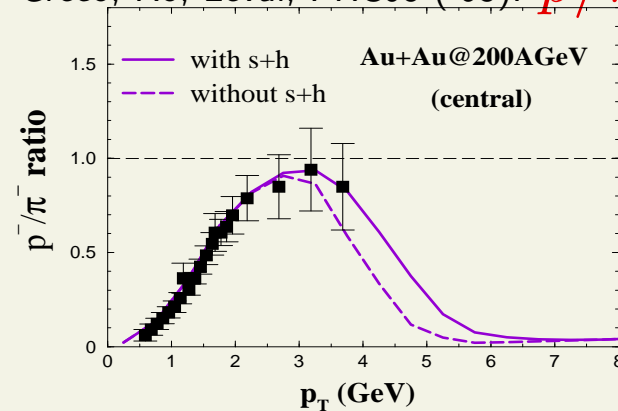
Spectra at RHIC are fairly well reproduced w/ 2-component system

i) $q-\bar{q}$ "plasma" at $T \approx 175$ MeV w/ strong radial flow $v_m \approx 0.55c$ + ii) quenched jets

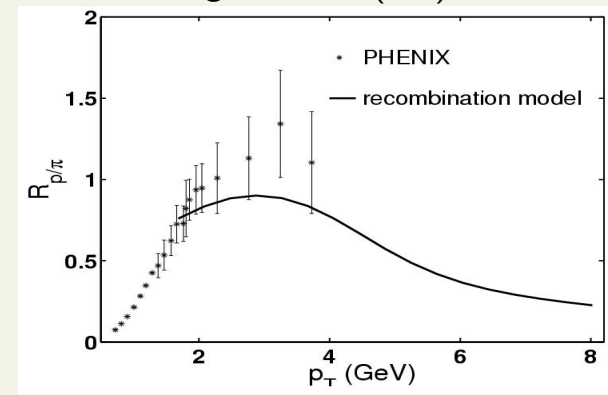
Fries et al, PRC68 ('03): π^0



Greco, Ko, Levai, PRC68 ('03): p/π ratio



Hwa & Yang, PRC71 ('04):

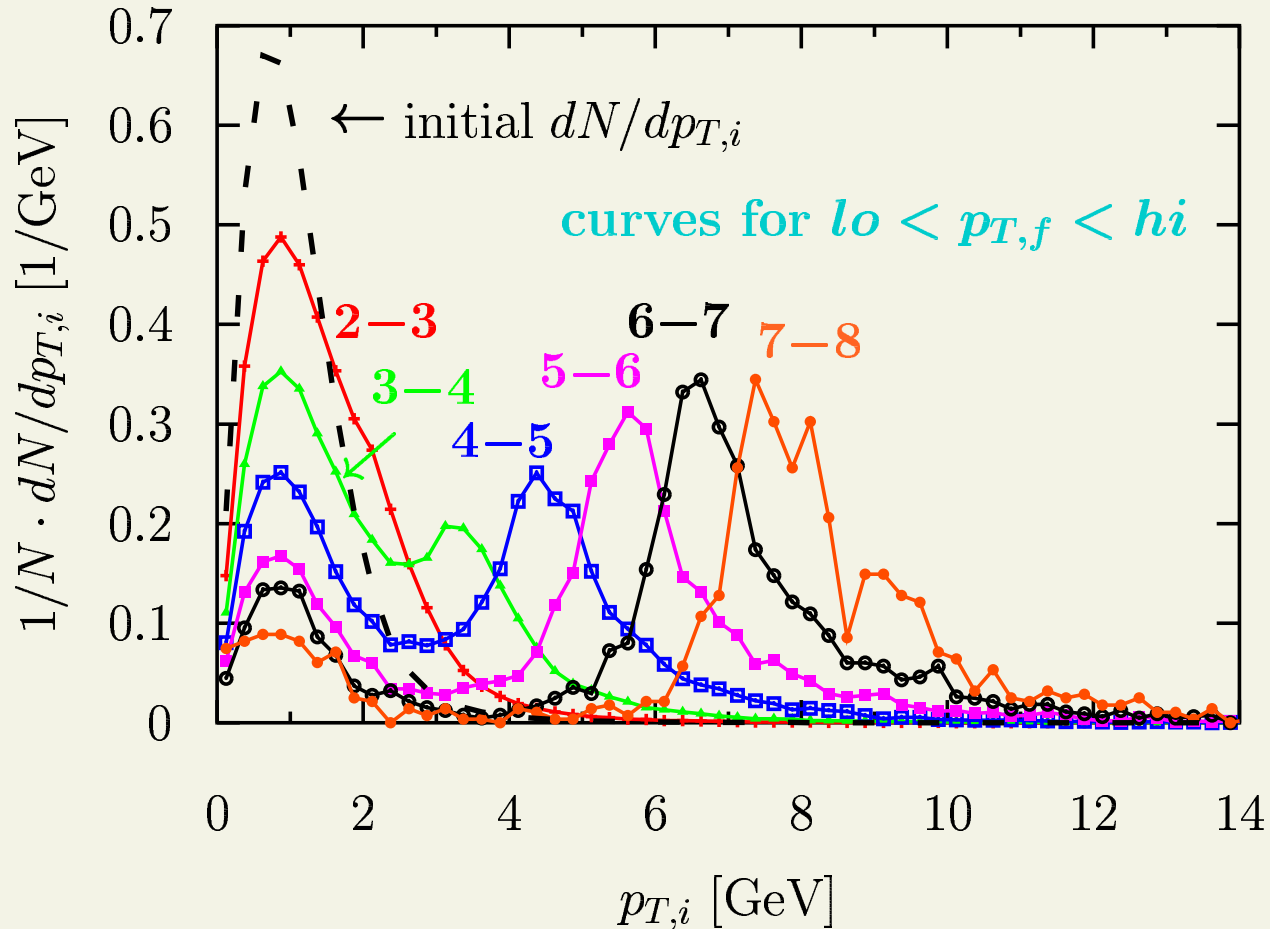


coal. extends thermal tails into the intermediate p_T regime \rightarrow cures p/π

distribution of initial momenta for fixed final momentum bins, $|y_{fin}| < 1$

(only quench + “push” plotted, normalized)

DM, nucl-th/0503051: $\sim 6 \times$ pQCD opacities



“lucky” $p_{T,i} \sim 1$ GeV soft partons can end up at $p_T \sim 7 - 8$ GeV