

3D Relativistic Hydrodynamic Computations Using Lattice-QCD-Inspired Equations of State

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Quark Matter 2005, **Budapest**

Outline

1 The Basic Ingredients of Hydrodynamic Models

- Equations of Motion - SPheRIO Code
- Equations of State
- Initial Conditions
- Decoupling Procedure

2 Our Results

- Pseudorapidity Distribution
- Transverse-Momentum Distribution
- Elliptic-Flow Parameter v_2
- HBT Radii

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Equations of Motion

SPheRIO code

- Energy-Momentum Conservation:

$$\partial_\nu T^{\mu\nu} = 0, \quad \text{where} \quad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu};$$

- Baryon-Number Conservation:

$$\partial_\mu (n_B u^\mu) = 0;$$

- Other Quantum-Number Conservation (n_S, n_{ch}, \dots):

$$\partial_\mu (n_S u^\mu) = 0, \quad \dots$$

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\dots .

Equations of Motion

SPheRIO code

We solve these equations with

SPheRIO

(**S**moothed **P**article hydrodynamic **e**volution of **R**elativistic
heavy **I**on collisions) code,

[C.E. Aguiar, T. Kodama, T. Osada and Y. Hama, *J. Phys. G* **27** (2001) 75]

based on SPH (Smoothed-Particle Hydrodynamics) algorithm.

[L.B. Lucy, *Astrophys. J.* **82** (1977) 1013;

R.A. Gingold and J.J. Monaghan, *Mon. Not. R. Astro. Soc.* **181** (1977) 375]

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Equations of State

- EoS with 1st.-Order Phase Transition (**1OPT EoS**):

- QGP phase: MIT Bag Model
- Hadron phase: Resonance gas, with excluded-volume

[see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:

- Critical End Point in the Phase Diagram

[Sándor Katz, Previous talk in this Conference]

- Phenomenological Parametrization of Lattice QCD EoS:
We call this parametrization **CP EoS**.

- Comparisons of **CP EoS** with **1OPT EoS**.

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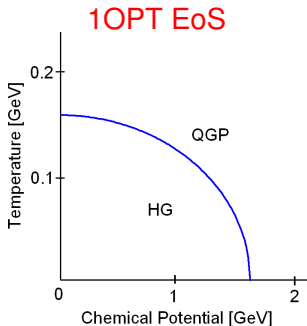
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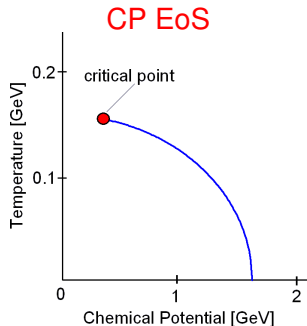
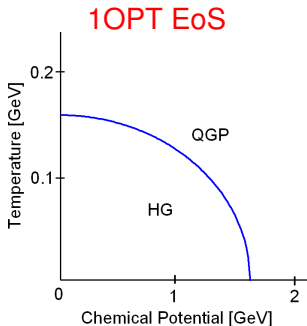
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Phse Diagram in (T, μ) Plane

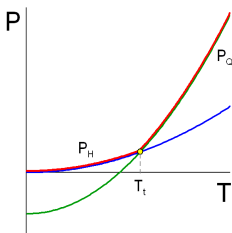


Phse Diagram in (T, μ) Plane



Parametrization of Lattice QCD EoS

In **1OPT EoS**:

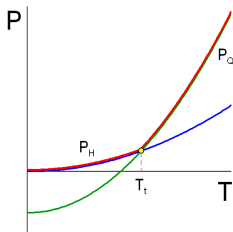


P is given by

$$(P - P_Q)(P - P_H) = 0.$$

Parametrization of Lattice QCD EoS

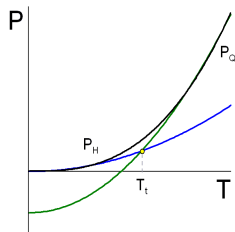
In **1OPT EoS**:



P is given by

$$(P - P_Q)(P - P_H) = 0.$$

In CP EoS:



given μ , we parametrize P as

$$(P - P_Q)(P - P_H) = \delta(\mu),$$

with $\delta(\mu) = \delta_0 \exp[-(\mu/\mu_c)^2]$.

Parametrization of Lattice QCD EoS

Then,

$$P = \lambda P_H + (1 - \lambda) P_Q + \frac{2\delta}{\sqrt{(P_Q - P_H)^2 + 4\delta}}$$

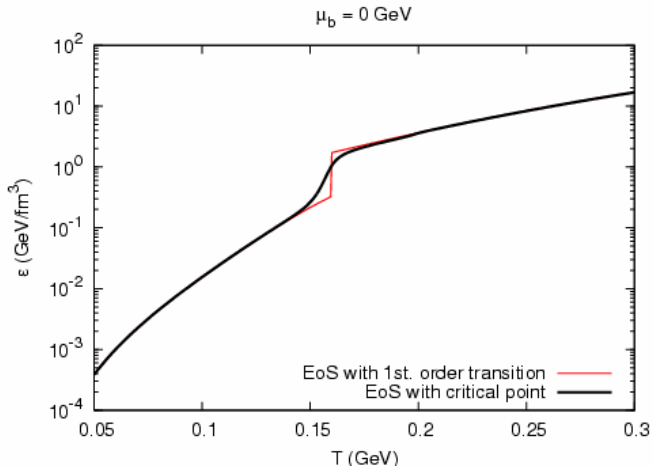
$$s = \lambda s_H + (1 - \lambda) s_Q$$

$$n_B = \lambda n_H + (1 - \lambda) n_Q - \frac{2(\mu/\mu_c^2)\delta}{\sqrt{(P_Q - P_H)^2 + 4\delta}}$$

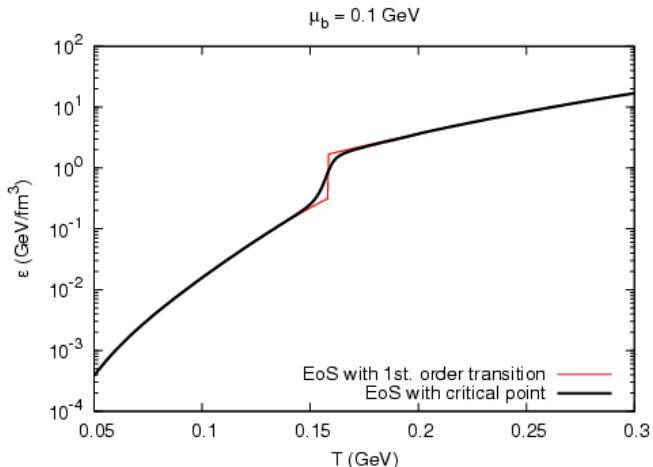
$$\epsilon = \lambda \epsilon_H + (1 - \lambda) \epsilon_Q - \frac{2[1 + (\mu/\mu_c)^2]\delta}{\sqrt{(P_Q - P_H)^2 + 4\delta}}$$

where $\lambda \equiv \left[1 - (P_Q - P_H) / \sqrt{(P_Q - P_H)^2 + 4\delta} \right] / 2$.

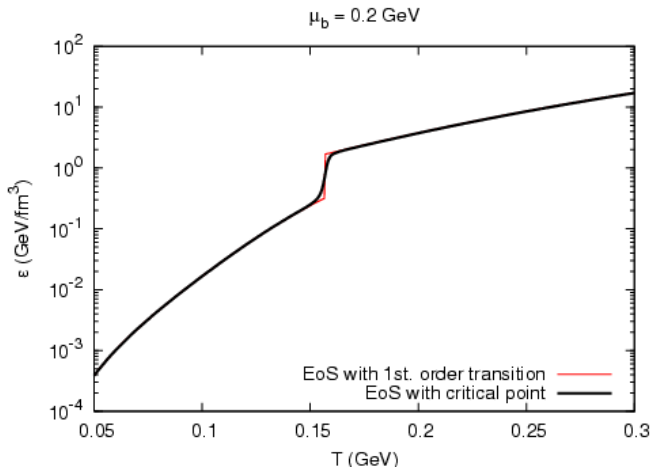
Equations of State: $\epsilon(T)$



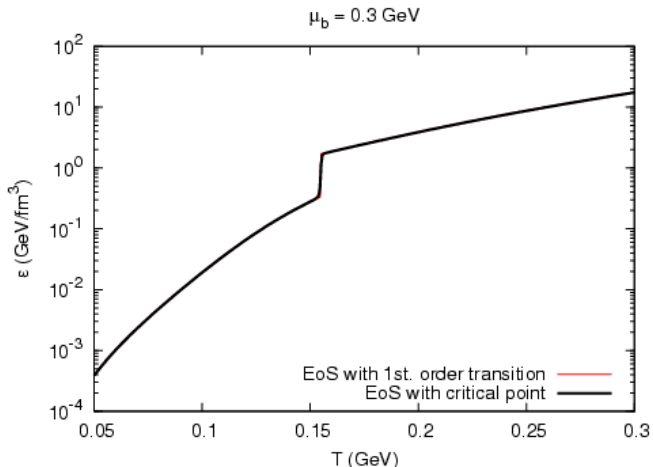
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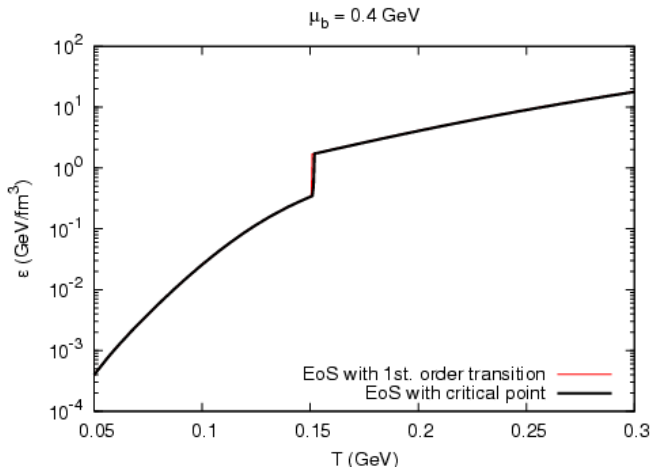
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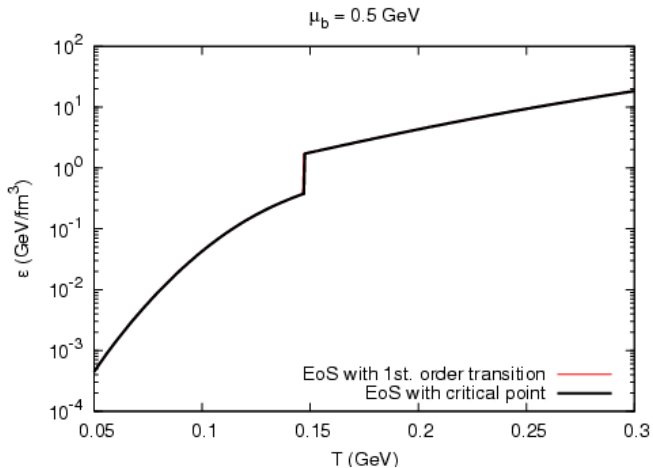
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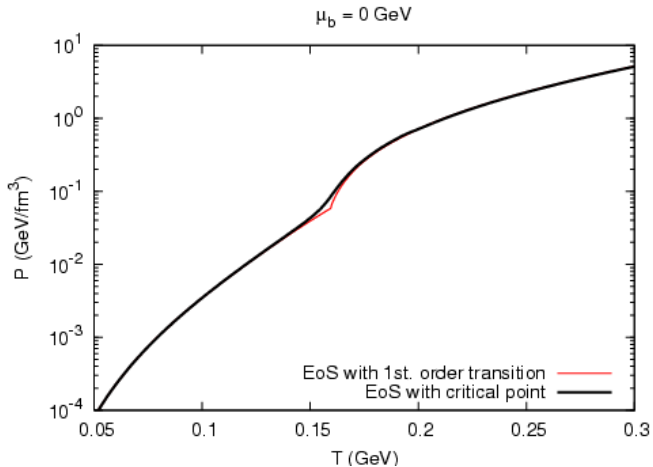
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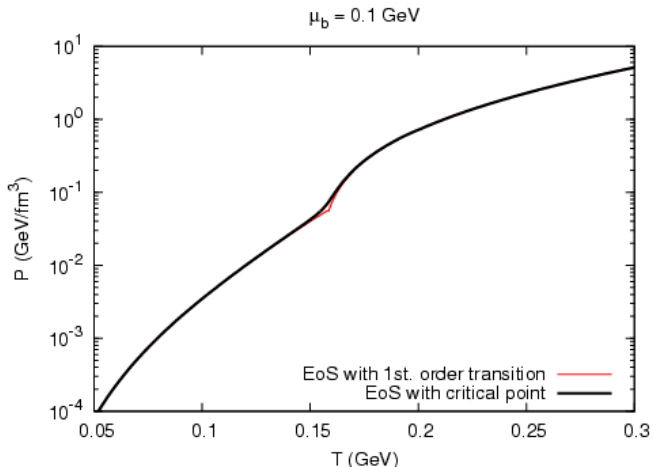
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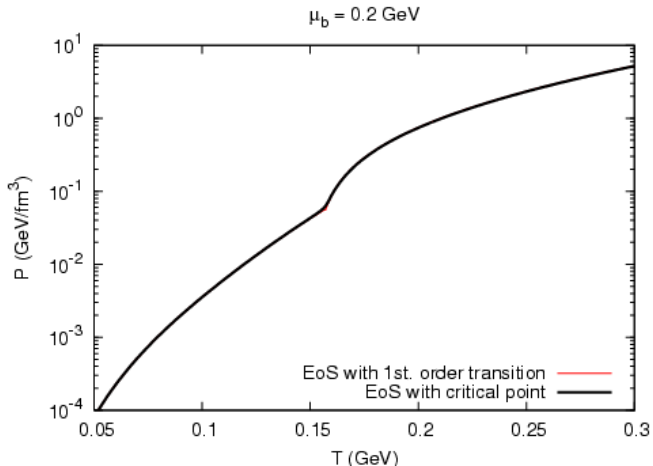
Equations of State: $p(T)$



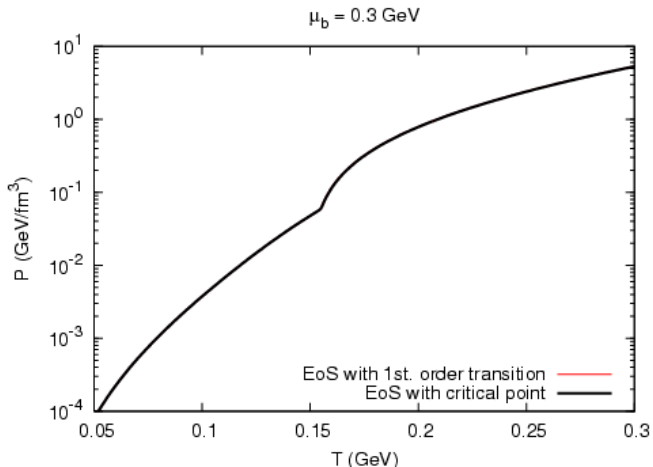
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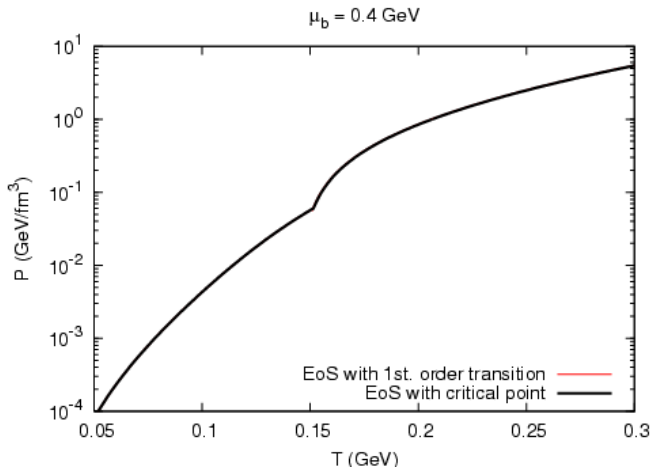
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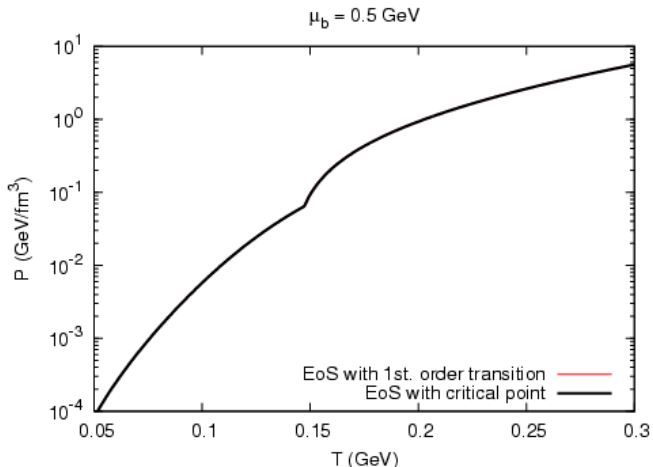
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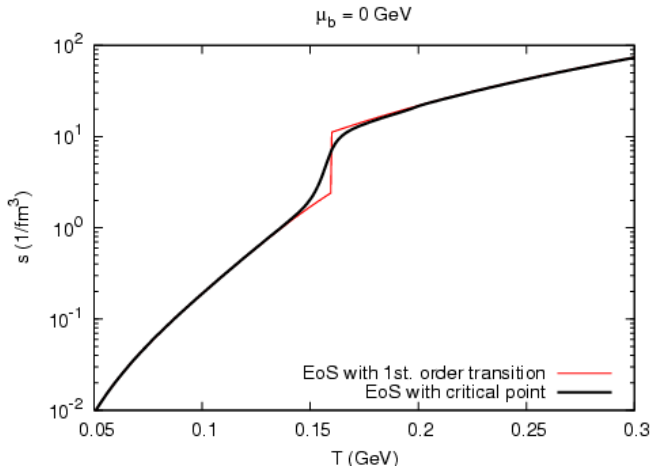
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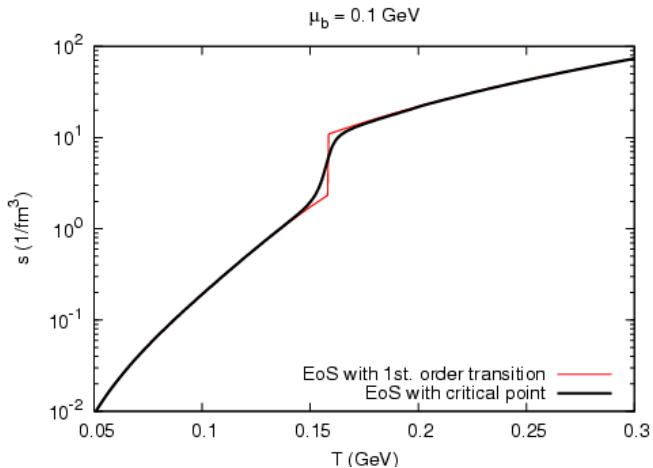
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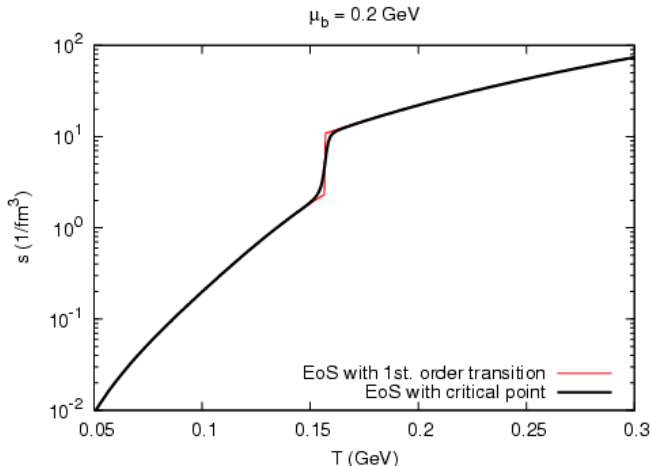
Equations of State: $s(T)$



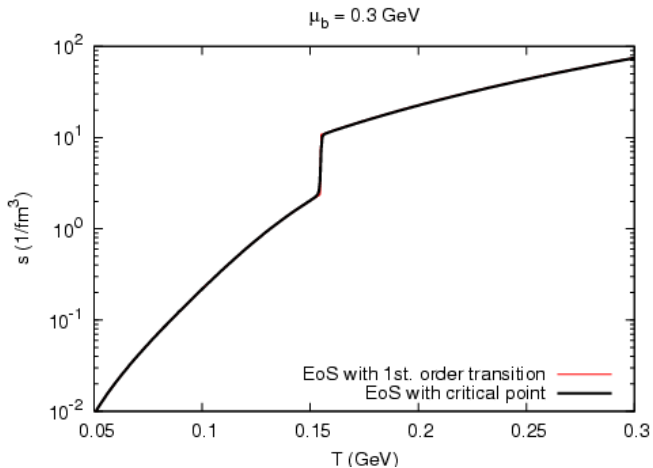
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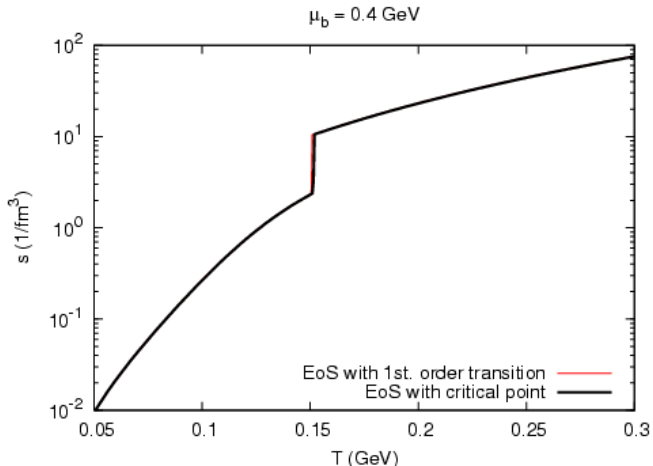
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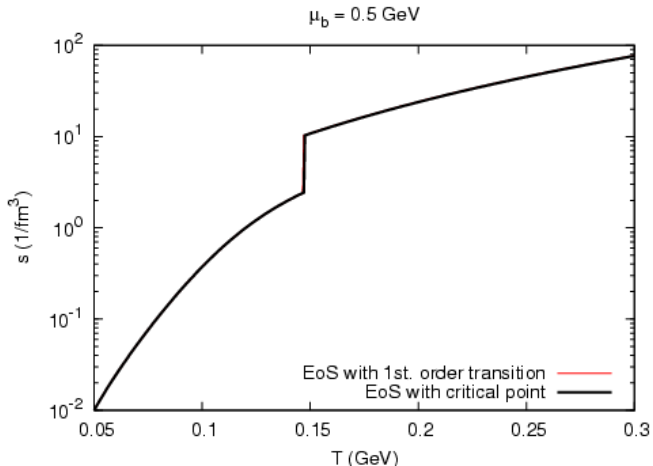
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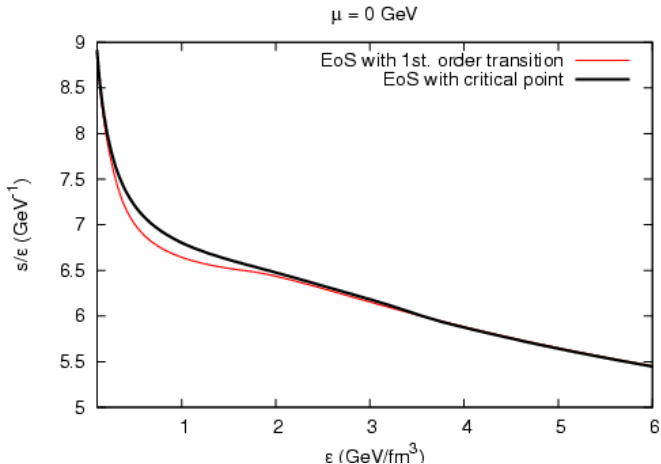
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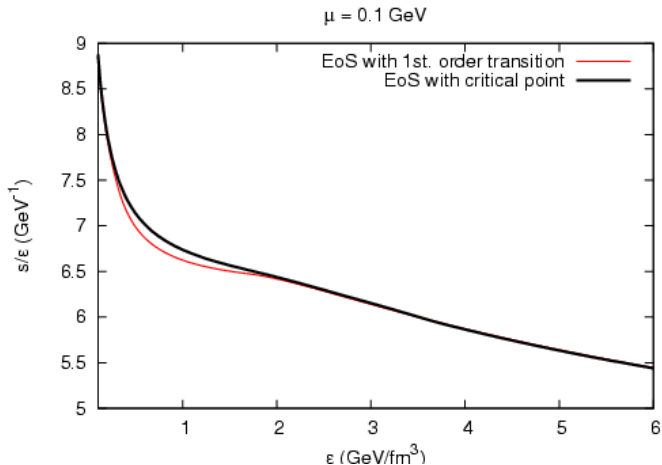
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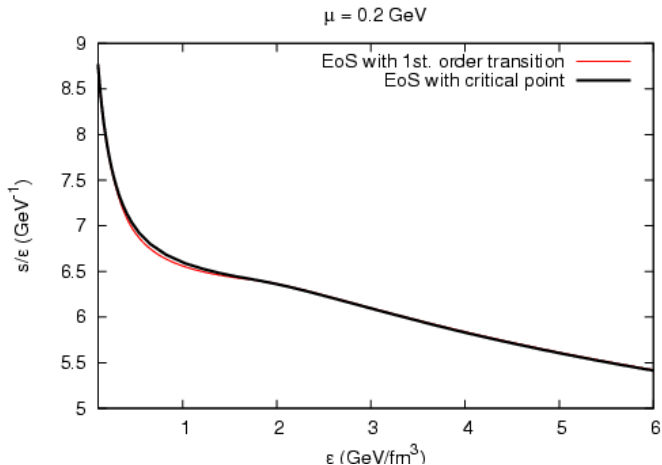
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



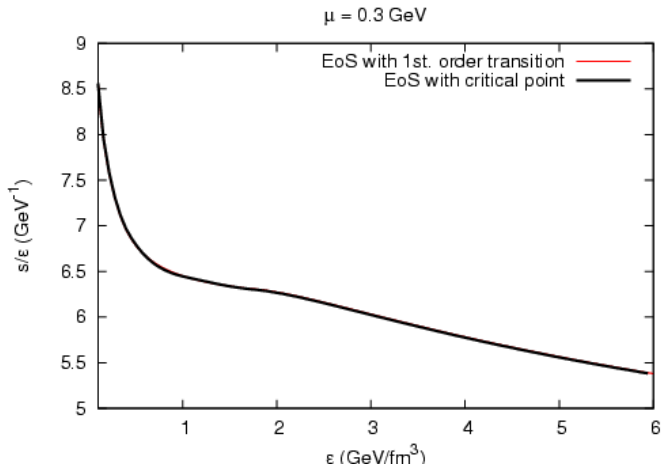
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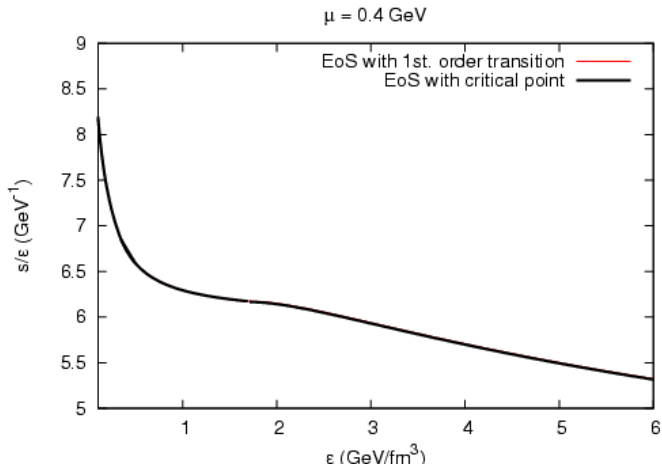
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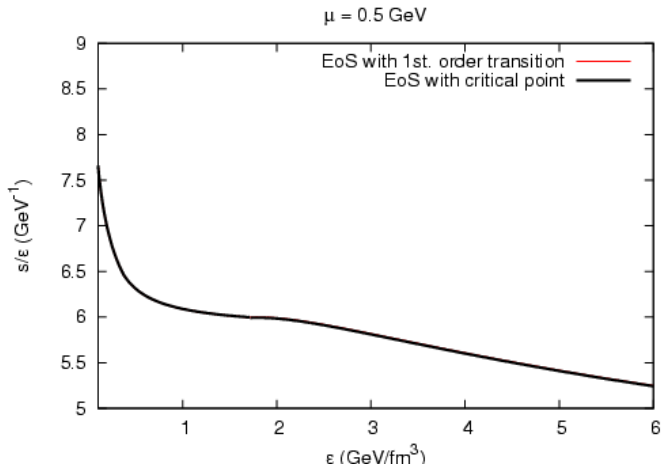
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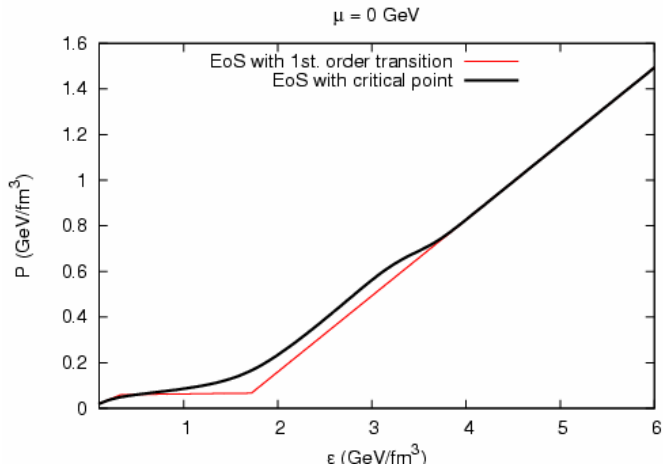
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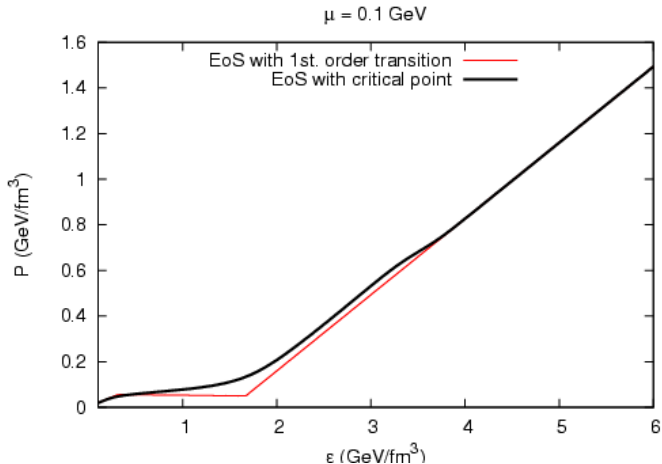
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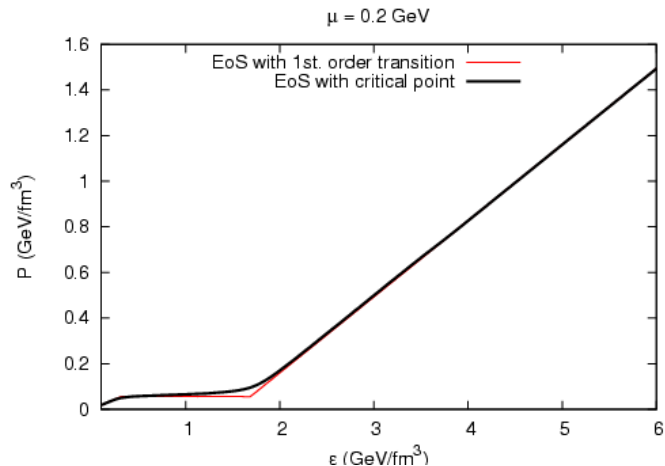
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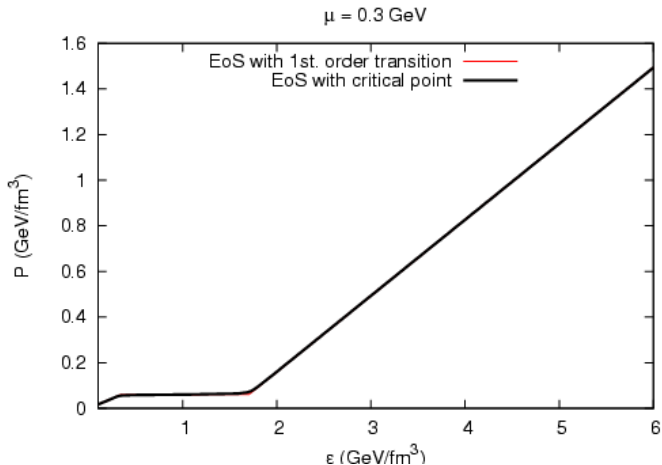
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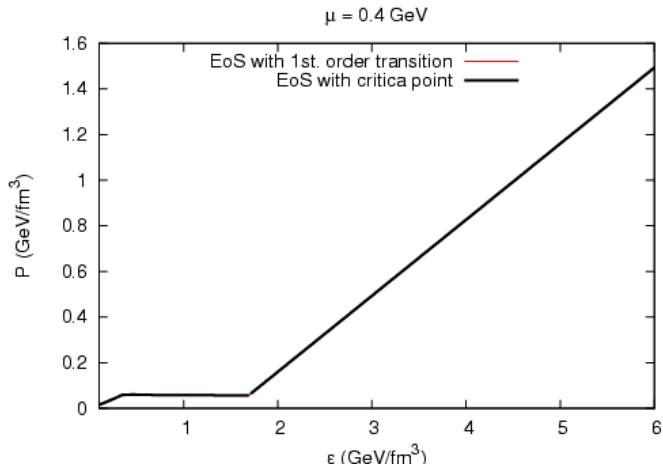
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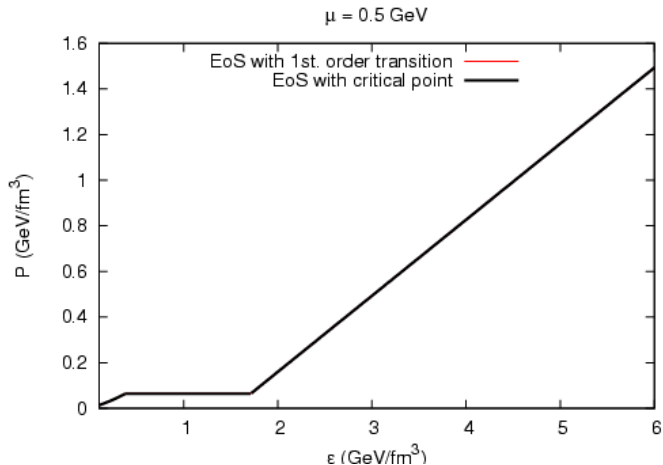
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Some Preliminary Conclusions

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the multiplicity is larger for CP EoS, as compared with the one for 1OPT EoS.

- The acceleration is larger in the transition (crossover) region for CP EoS.

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Initial Conditions (IC)

- In usual hydrodynamic approach, one assumes some highly symmetric and smooth IC.
- However, our systems are small, so large fluctuations are expected in real collisions.
- Many simulators, based on microscopic models, e.g.
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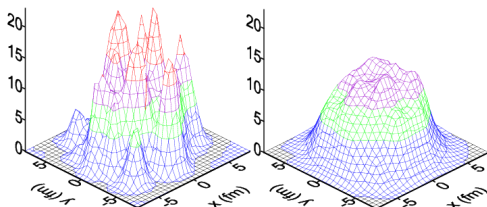
Initial Conditions:

Event-by-Event Fluctuating vs. Smooth Averaged

Energy density for central Au+Au collisions at 130 GeV, given by NeXus simulator,* at mid-rapidity

One random event

Average over 30 events



* [H.J. Drescher, F.M. Liu, S. Ostrapchenko, T. Pierog and K. Werner, *Phys. Rev. C* **65** (2002) 054902.]

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Decoupling Procedure

- Often one assumes decoupling on a sharply defined hypersurface. We call this Sudden Freeze Out (FO).
- However, our systems are small, so particles may escape from a layer with thickness comparable with the systems sizes.
- We introduce, at each space-time point x^μ , a certain momentum-dependent escaping probability

$$\mathcal{P}(x, k) = \exp \left[- \int_{\tau}^{\infty} \rho(x') \sigma v d\tau' \right].$$

This is the Continuous Emission Model (CE).*

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Decoupling Procedure

In order to make the computation practicable,

- we take \mathcal{P} on the average, *i.e.*, $\mathcal{P}(x, k) \rightarrow \langle \mathcal{P}(x, k) \rangle \equiv \mathcal{P}(x)$
and
- we approximate linearly the density $\rho(x') = \alpha s(x')$.

Thus,

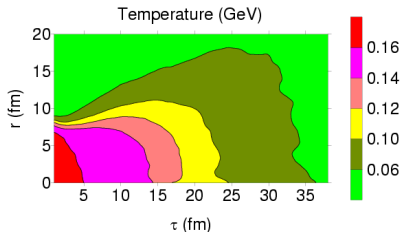
$$\mathcal{P}(x, k) \rightarrow \mathcal{P}(x) = \exp\left(-\kappa \frac{s^2}{|ds/d\tau|}\right),$$

where $\kappa = 0.5 \alpha \langle \sigma v \rangle$ is estimated to be 0.3, corresponding to $\langle \sigma v \rangle \approx 2 \text{ fm}^2$.

Sudden Freezeout vs. Continuous Emission

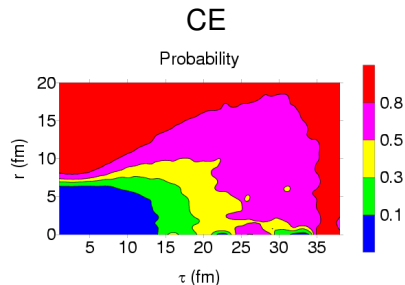
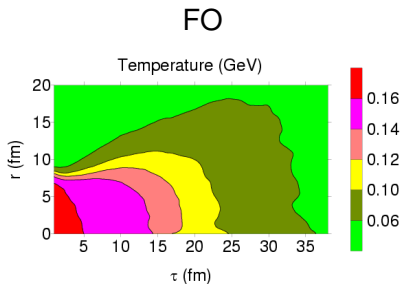
Central Au+Au collisions at 130 A GeV

FO



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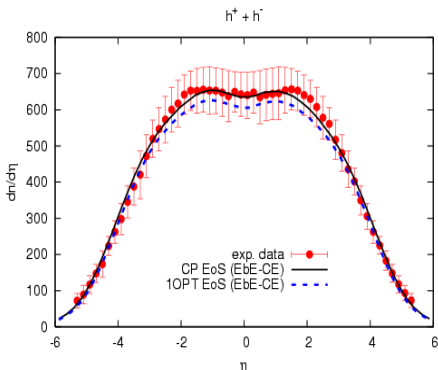
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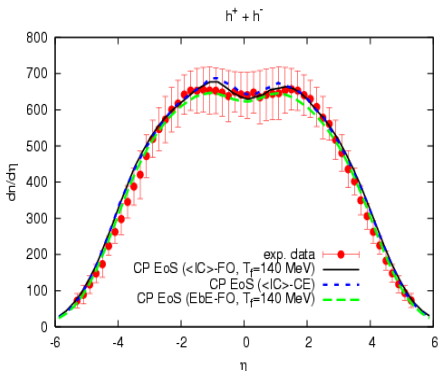
η distribution



Data: PHOBOS Collaboration, B.B.Back *et al.*,
Phys. Rev. **C65** (2002) 054902. (Central)

- The equations of state with critical end point (CP EoS) **increase the multiplicity** in the mid-rapidity region, as compared with those with first-order phase transition (**1OPT EoS**).

η distribution



Data: PHOBOS Collaboration, B.B.Back *et al.*,
Phys.Rev.C65(2002) 054902. (Central)

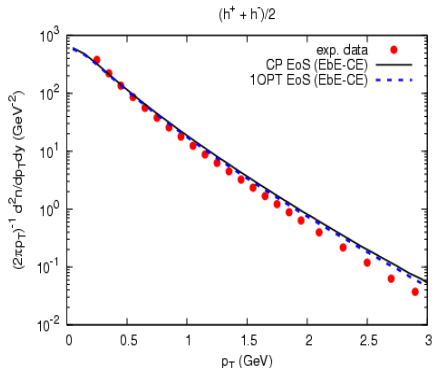
* [Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- The use of the averaged initial conditions ($\langle IC \rangle$) gives **larger multiplicity** as compared to event-by-event (EbE) ones.*
- As far as η distribution is concerned, the difference between the continuous emission (CE) and the sudden freezeout (FO) is small.

Outline

- 1 The Basic Ingredients of Hydrodynamic Models
 - Equations of Motion - SPheRIO Code
 - Equations of State
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 - Decoupling Procedure
- 2 Our Results
 - Pseudorapidity Distribution
 - **Transverse-Momentum Distribution**
 - Elliptic-Flow Parameter v_2
 - HBT Radii

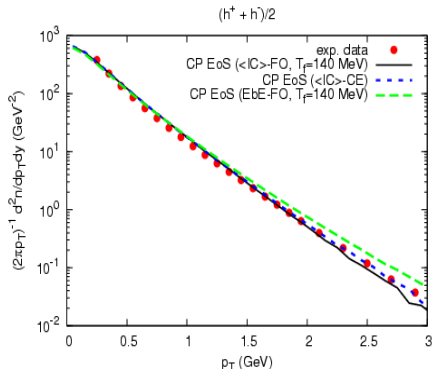
p_T distribution



Data: PHOBOS Collaboration, B.B.Back *et al.*,
Phys. Rev. Lett. **93** (2004) 052303.
 (Central collisions)

- The p_T distribution given by CP EoS is slightly flatter, as compared with the one predicted by 1OPT EoS. However, the difference is small.

p_T distribution



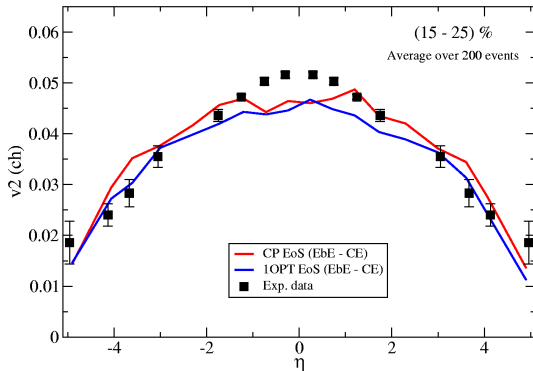
Data: PHOBOS Collaboration, B.B.Back *et al.*,
Phys. Rev. Lett. **93** (2004) 052303.
 (Central collisions)

- The averaged initial conditions (<IC>) give **steeper distribution** as compared to the event-by-event (**EbE**) ones. In the latter, the expansion is **more violent**.
- The difference between the continuous emission (**CE**) and the sudden freezeout (FO) is small. However, the interpretations are different.

Outline

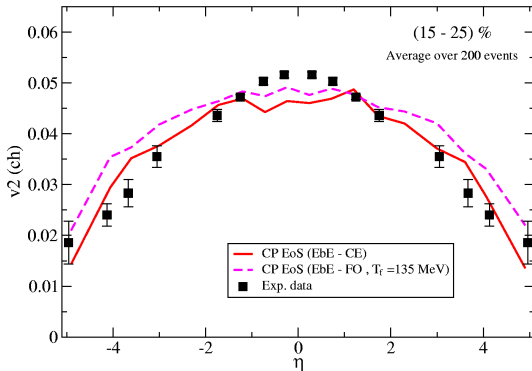
- 1 The Basic Ingredients of Hydrodynamic Models
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$v_2(\eta)$



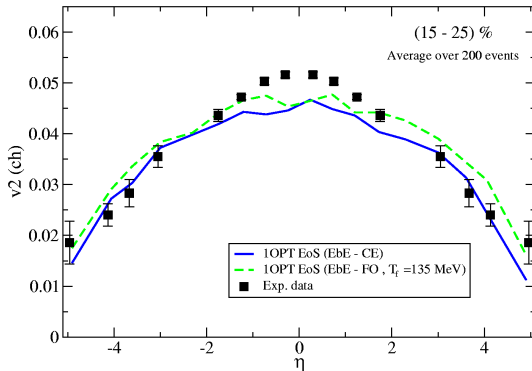
Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012
See more details in the [Poster 43: F. Grassi]

$v_2(\eta)$



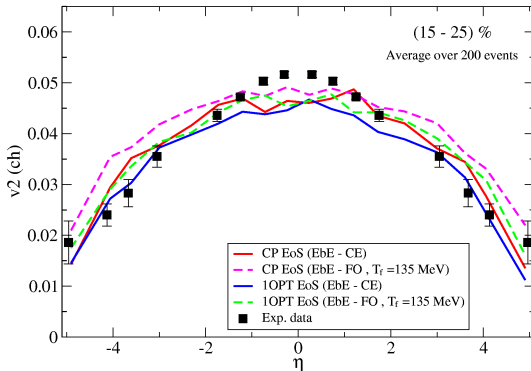
Data: [PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012](#)
See more details in the [[Poster 43: F. Grassi](#)]

$v_2(\eta)$



Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012
See more details in the [Poster 43: F. Grassi]

$v_2(\eta)$



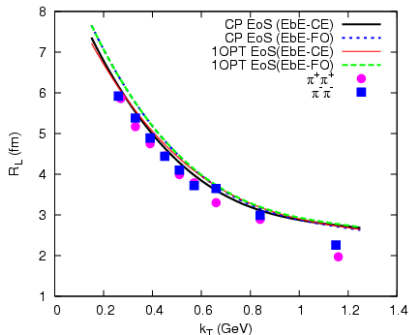
Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012
See more details in the [Poster 43: F. Grassi]

Outline

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HBT Radii

R_L (Event by Event)



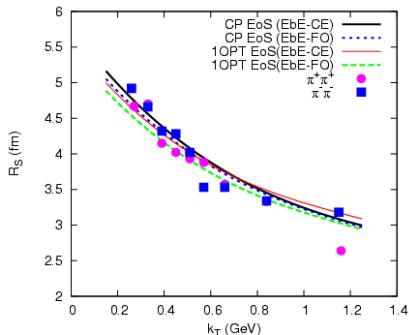
Data: PHENIX Collaboration, S.S. Adler *et al.*,
Phys. Rev. Lett. **93** (2004) 152302.

- Practically, there is no difference between the results of CP EoS and **1OPT EoS**.
- The data are well reproduced.
- The difference between Sudden Freezeout (FO) and the Continuous Emission (CE) is small.

See also the [Poster 116: O. Socolowski Jr.]

HBT Radii

R_s (Event by Event)



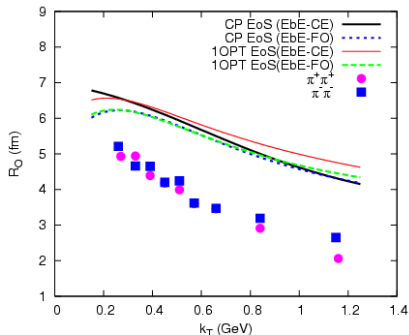
Data: PHENIX Collaboration, S.S. Adler *et al.*,
Phys. Rev. Lett. **93** (2004) 152302.

- k_T dependence predicted by CP EoS is steeper than the one given by 1OPT EoS.
- The data are more or well less reproduced, but the agreement is the best for CP EoS and with CE.

See also the [Poster 116: O. Socolowski Jr.]

HBT Radii

R_o (Event by Event)



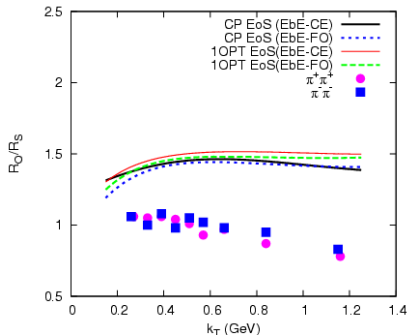
Data: PHENIX Collaboration, S.S. Adler *et al.*,
Phys. Rev. Lett. **93** (2004) 152302.

- k_T dependence predicted by CP EoS is steeper than the one given by 1OPT EoS.
- Quantitatively, the data are poorly reproduced.
- But, k_T dependence is best reproduced by CP EoS and with CE.

See also the [Poster 116: O. Socolowski Jr.]

HBT Radii

R_o/R_s (Event by Event)



Data: PHENIX Collaboration, S.S. Adler *et al.*,
Phys. Rev. Lett. **93** (2004) 152302.

See also the [Poster 116: O. Socolowski Jr.]

Summary

- The **multiplicity becomes larger** for **CP EoS** in the mid-rapidity.
- The p_T distribution becomes **flatter**. However, the difference is small.
- **Larger v_2** . Continuous Emission makes the η distribution narrower.
- HBT radii **slightly closer** to data.
- Outlook
 - The effect of the emission on the interacting component has not been taken into account. Probably it makes R_o smaller.