Survival of Back-to-Back Correlations

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Introduction

- Late 90's → Back-to-Back Correlations (BBC) among boson-antiboson pairs were shown to exist if the masses of the particles were modified in a hot and dense medium [Asakawa, Csörgo" & Gyulassy, P.R.L. 83 (1999) 4013].
- 2001 → It was shown that similar BBC existed among fermion-antifermion pairs with medium modified masses [Panda, Csörgo", Hama, Krein & SSP, P. L. B512 (2001) 49].

- **Some properties:**
  - Similar formalism for both bosonic (bBBC) and fermionic (fBBC) Back-to-Back correlations
  - Similar (and unlimited) intensity of fBBC and bBBC
  - Expected to appear for $p_T \leq 1-2\ \mathrm{GeV}/c$

Formalism

- **Infinite medium**

$$H = H_0 - \frac{1}{2}\int d\vec{x}\, d\vec{y}\, \phi(\vec{x})\, \delta M^2(\vec{x}-\vec{y})\phi(\vec{y})$$ → In-medium Hamiltonian

$$H_0 = \frac{1}{2}\int d\vec{x}\,(\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2)$$ → Asymptotic (free) Hamiltonian, in the rest frame of matter

- Scalar field $\phi(x)$ $\leftrightarrow$ quasi-particles propagating with momentum-dependent medium-modified effective mass, m_* , related to the vacuum mass, m , by

♦ $$m_*^2\left(\left|\vec{k}\right|\right) = m^2 - \delta M^2\left(\left|\vec{k}\right|\right)$$

- Consequently: $\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2\left(\left|\vec{k}\right|\right)$

Ω_k → frequency of the in-medium mode with momentum $\vec{k}$

Some comments (bosonic BBC):

- $b_k\,(b^\dagger{}_k)$ → in-medium annihilation (creation) operator
- $a_k\,(a^\dagger{}_k)$ → annihilation (creation) operator of the asymptotic quanta with 4-momemtum $k^\mu = (\omega_k, \vec{k})$

  They are related by the Bogoliubov transformation:

$$\begin{cases} a^\dagger{}_k = c_k\, b^\dagger{}_k + s_{-k}\, b_{-k} \\ a_k = c_k^*\, b_k + s_{-k}^*\, b^\dagger_{-k} \end{cases} \quad ; \quad \boxed{c_k = \cosh[f_k]} \quad ; \quad \boxed{s_k = \sinh[f_k]}$$

- $\boxed{f_k = \frac{1}{2}\ln(\omega_k\,/\,\Omega_k)}$ → squeezing parameter (the Bogoliubov transformation is equivalent to a squeezing operation)
- a-quanta → observed; b-quanta → thermalized in medium

- Full two-particle correlation (e.g., for π^0's)

$$\langle a^\dagger_{k_1} a^\dagger_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^\dagger_{k_1} a_{k_1} \rangle \langle a^\dagger_{k_2} a_{k_2} \rangle + \langle a^\dagger_{k_1} a_{k_2} \rangle \langle a^\dagger_{k_2} a_{k_1} \rangle + \langle a^\dagger_{k_1} a^\dagger_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

NOTATION

$$N_1(\vec{k}_i) = \omega_{k_1} \frac{d^3N}{d^3k} = G_c(\vec{k}_i, \vec{k}_i) \equiv G_c(i,i) = \omega_{k_1} \langle a^\dagger_{k_i} a_{k_i} \rangle$$ → **Spectra**

$$G_c(\vec{k}_1, \vec{k}_2) \equiv G_c(1,2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a^\dagger_{k_1} a_{k_2} \rangle$$ → **Chaotic amplitude**

$$G_s(\vec{k}_1, \vec{k}_2) \equiv G_s(1,2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1} a_{k_2} \rangle$$ → **Squeezed amplitude**

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)}$$

HBT **BBC**

$$\therefore \begin{cases} G_c(1,2) = \sqrt{\omega_k\,\omega_{k_2}}\left[\left\langle\left(c^*_{k_1} b^\dagger_{k_1}\right)\left(c_{k_2} b_{k_2}\right)\right\rangle + \left\langle\left(s_{-k_1} b_{-k_1}\right)\left(s^*_{-k_2} b^\dagger_{-k_2}\right)\right\rangle\right] \\ \\ G_s(1,2) = \sqrt{\omega_k\,\omega_{k_2}}\left[\left\langle\left(s^*_{-k_1} b^\dagger_{-k_1}\right)\left(c_{k_2} b_{k_2}\right)\right\rangle + \left\langle\left(c_{k_1} b_{k_1}\right)\left(s^*_{-k_2} b^\dagger_{-k_2}\right)\right\rangle\right] \end{cases}$$

- After performing the thermal averages → $\langle\mathcal{O}\rangle = Tr(\hat{\rho}\mathcal{O})$
  - If the thermal b gas freezes out suddenly at some time, at temperature T, the observed *single-particle distribution* for a is written as

$$N_1(\vec{k}) = \frac{V}{(2\pi)^3}\omega_k\left[|c_k|^2 n_k + |s_{-k}|^2 (n_{-k}+1)\right] \equiv \frac{V}{(2\pi)^3}\omega_k\, n_1(\vec{k})$$

  - And the *squeezed correlation function* is given by

$$C_s(\vec{k},-\vec{k}) = 1 + \frac{\left|c_k\, s^*_k n_k + c_{-k} s^*_{-k}(n_{-k}+1)\right|^2}{n_1(\vec{k})\, n_1(-\vec{k})}$$

Finite size medium moving with collective velocity

- For a hydrodynamical ensemble → amplitudes can be written as [Makhlin & Sinyukov, N.P. A566 (1994) 598c]:

$$G_c(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x)\,K^\mu_{1,2}\,e^{iq_{1,2}.x}\left[\left|c_{1,2}\right|^2 n_{1,2} + \left|s_{-1,-2}\right|^2\left(n_{-1,-2}+1\right)\right]$$

$$G_s(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x)\,K^\mu_{1,2}\,e^{i2K_{1,2}.x}\left[s^*_{-1,2}\,c_{2,-1}\,n_{-1,2} + c_{1,-2},1\,s^*_{-2,1}\left(n_{1,-2}+1\right)\right]$$

- where $c_{i,j} = \cosh[f_{i,j}]$; $s_{i,j} = \sinh[f_{i,j}]$
- Squeezing coefficient:

$$f(i,j,x) = \tfrac{1}{2}\ln\left[\left(K^\mu_{i.j}\,u_\mu(x)\right)/\left(K^{*\nu}_{i.j}\,u_\nu(x)\right)\right] = \tfrac{1}{2}\ln\left[\frac{\omega_{k_i}(x)+\omega_{kj}(x)}{\Omega_{k_i}(x)+\Omega_{kj}(x)}\right]$$

  - » Two-particle momenta: $K^\mu_{i,j} = \frac{1}{2}(k_i + k_j)$; $q^\mu_{i,j} = (k_i - k_j)$
  - » u^μ → local flow vector at freeze-out

- $d^4\sigma^\mu(x) = d^3\Sigma^\mu(x,\tau_f)\,F(\tau_f)\,d\tau_f$ → product of the normal-oriented volume element depending parametrically on τ_f (freeze-out hypersurface parameter) and the inv. distrib. of τ_f, $F(\tau_f)$.

- σ^μ $\leftrightarrow$ hydrodynamical freeze-out surface

- Freeze-out possibilities:

<u>Instant freeze-out</u> → $\int dt\, E_{i,j} e^{-2iE_{i,j}.\tau}\,\delta(\tau - \tau_0)\, d\tau_f = E_{i,j} e^{-2iE_{i,j}.\tau_0}$

<u>Finite emission interval</u> → $\int dt\, E_{i,j}\, F(\tau_f)\, e^{-iE_{i,j}(\tau-\tau_0)}\, d\tau_f = \dfrac{E_{i,j}}{[1 + (E_{i,j}\Delta t)^2]}$

(corresponding to: $F(\tau) = \dfrac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau-\tau_0)/\Delta t}$)

- $n_{i,j}$ → Boltzmann limit of Bose-Einstein distribution:

$$n_{i,j}(x) \sim \exp\Big[-\Big(K^\mu_{i,j}u_\mu - \mu(x)\Big)/T(x)\Big]$$

Hydro solution → $\dfrac{\mu(x)}{T(x)} = \dfrac{\mu_0}{T(x)} - \dfrac{\vec{r}^{\,2}}{2R^2}$

Non-relativistic limit

- The flow 4-velocity is written as:

$$u^\mu = \gamma(1,\vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

- In the non-relativistic limit → $\gamma = (1-\vec{v}^2)^{-1/2} \approx 1 + \frac{1}{2}\vec{v}^2$
- and keeping all moment up to $\mathcal{O}(mv^2)$.
  - For large mass m and small mass shifts $(m_* - m)/m \ll m$ → flow effects on squeezing parameter:

$$\mathcal{O}\left[\left(\frac{Kin.En.}{m}\right)\left(\frac{\delta M^2}{m^2}\right)\right]$$

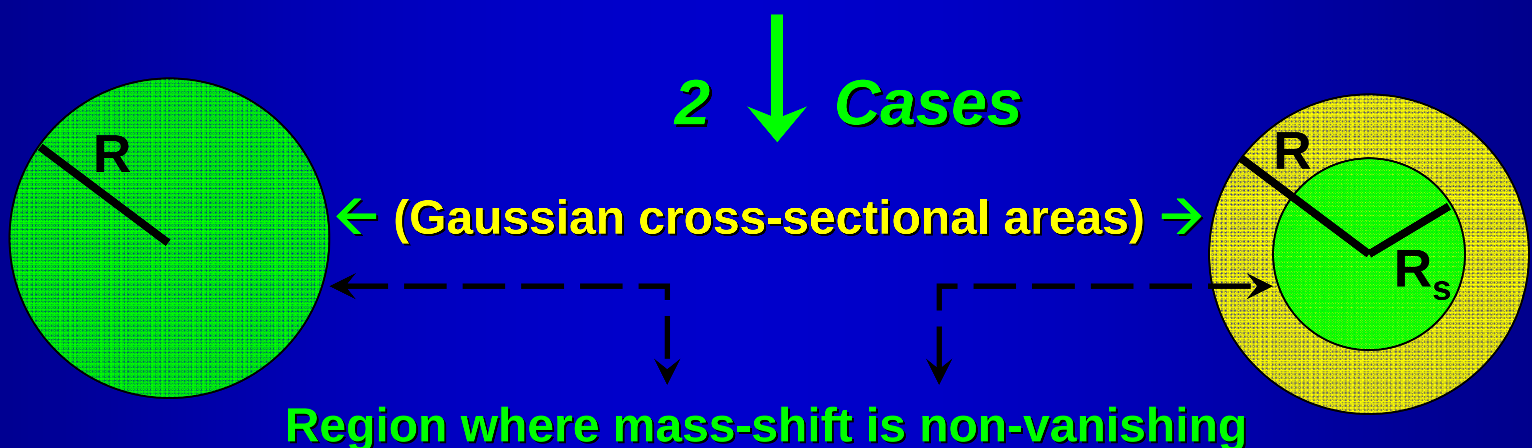
→ flow effects on $f_{i,j}$ are negligible.

  - So, $c_{i,j}$ and $s_{i,j}$ become flow independent, but could be r-dependent if m_* is (e.g., through $T(x)$, as in hydro)
  - Hypotheses: mass-shift is independent on position within a fireball, which is assumed to have a sharp surface, i.e., $\delta M=0$, and density also vanishes outside this volume.

- Integration is over the region where the mass-shift is non-vanishing, which is NOT infinitely large
- In Heavy Ion Collisions: $V \sim (R \simeq 5-10\, fm)^3$

- What is the volume V ?
  - » The coefficient of the vacuum term in the integrands → $s_{i,i}=0$ outside mass-shift region
  - » Terms proportional to $n_{i,j}$ are finite (hydro solution)
  - » → Integration could be extended to infinity:

- $\Rightarrow$ Profile for V → easiest: Gaussian $\approx \exp[-\vec{r}^2/(2R^2)]$



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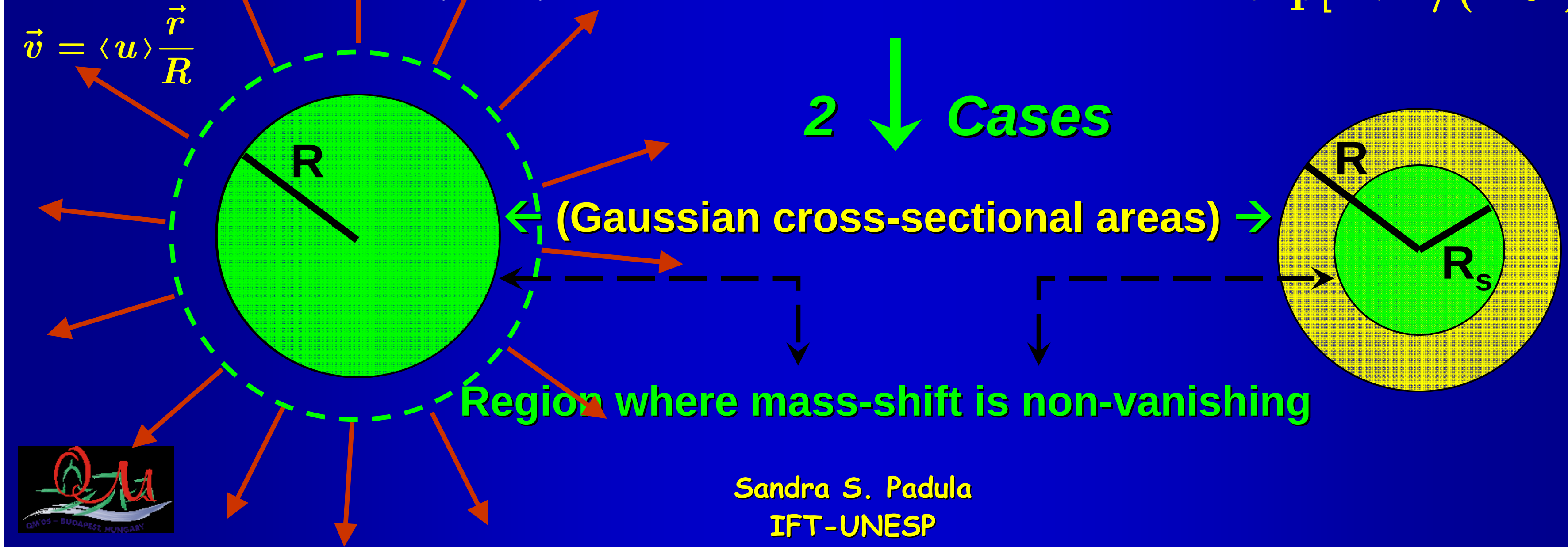
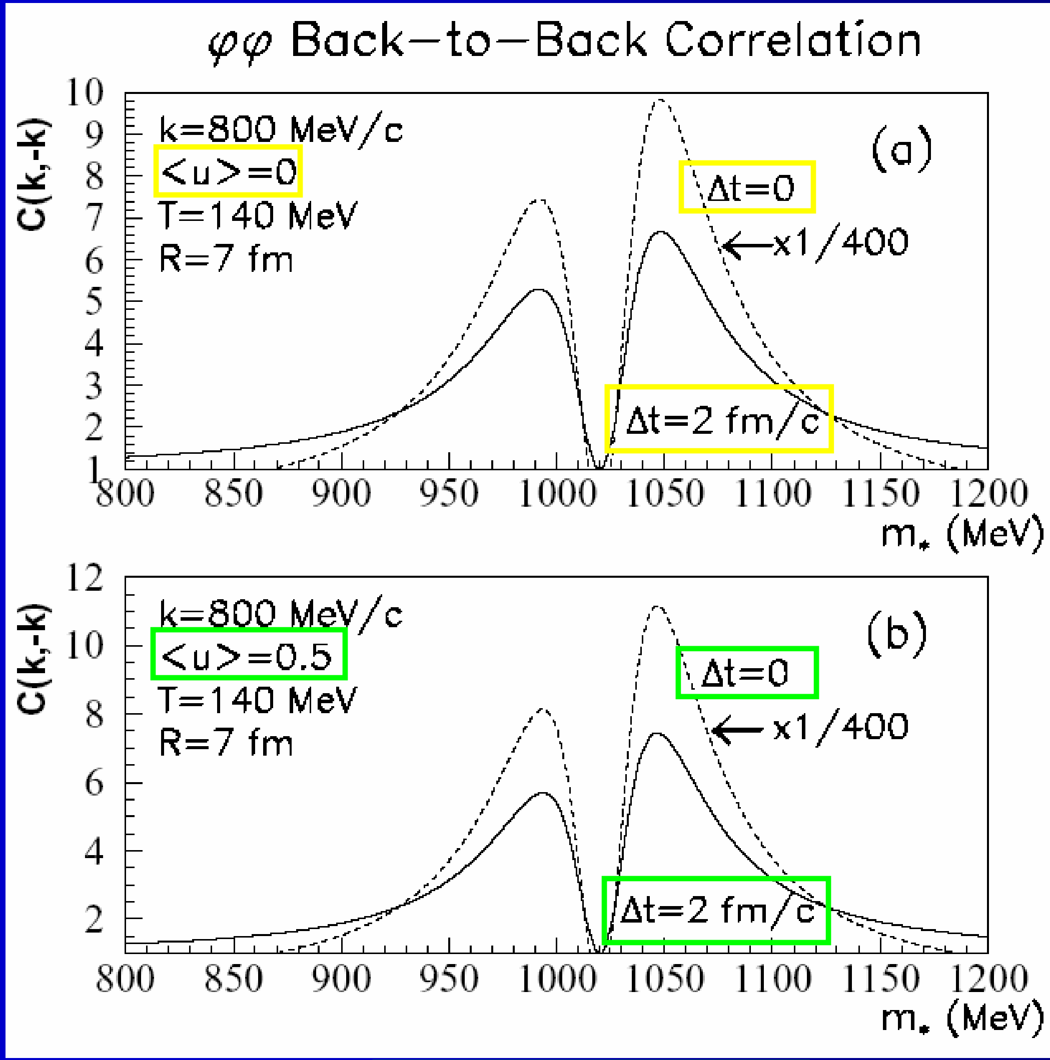


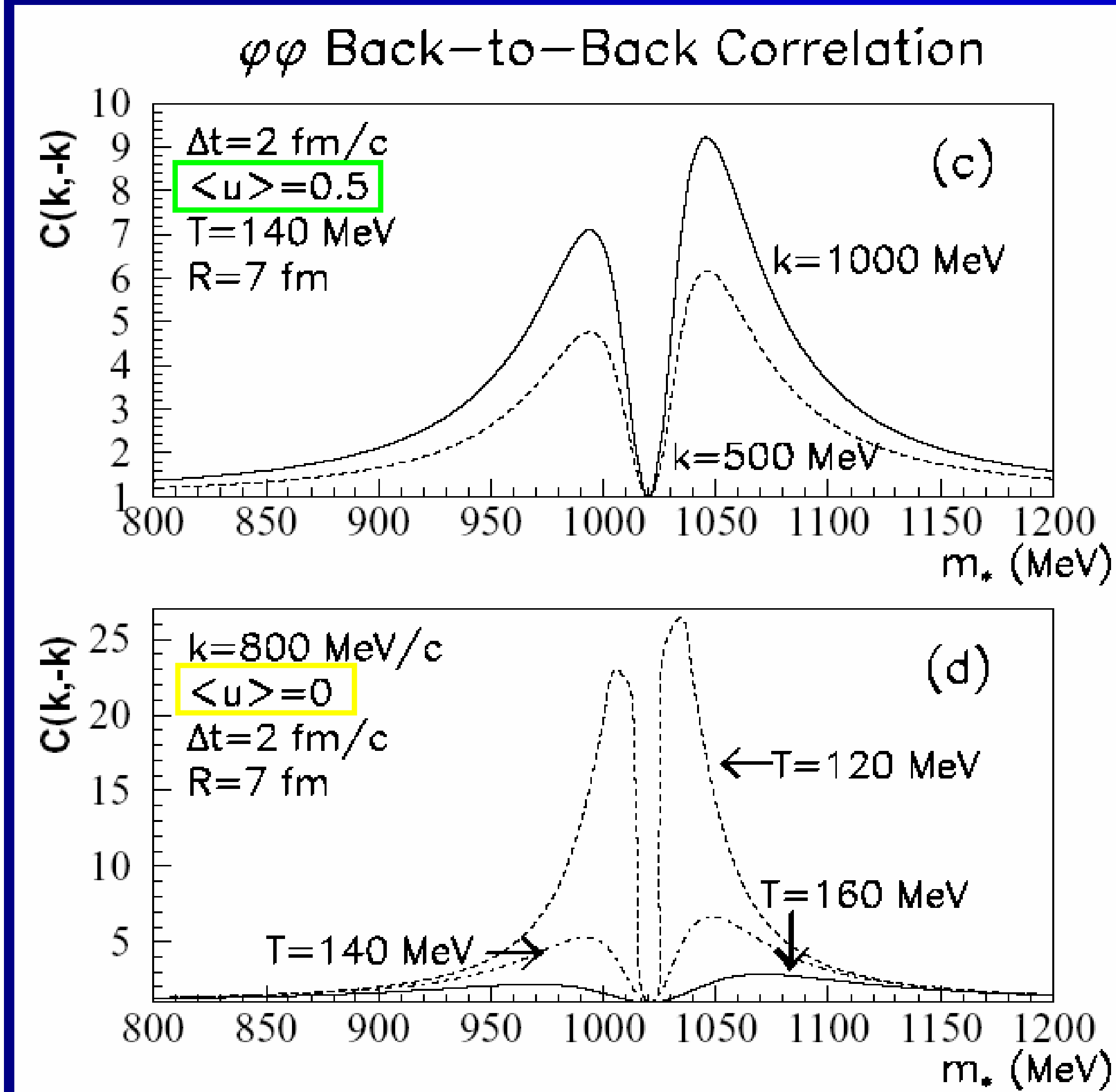
Squeezing region of radius R

- **Flow, finite size & time**
  - Finite emission times → have the effect of significantly suppress BBC signal intensity but it still is strong (already known)
  - Finite size systems → BBC signal $\propto$ size of mass-shifting region
  - For weak flow coupling → signal strength sensitive to momenta; BBC with $<u>=0.5$: comparable, weaker or even mildly enhanced than for $<u>=0$ for different k



$m_\phi = 1020 MeV$

Rôle of momenta and temperatures



- **Momenta ($<u>=0.5$):**
  - Increasing modulus of the back-to-back momenta of the pair enhances the effect

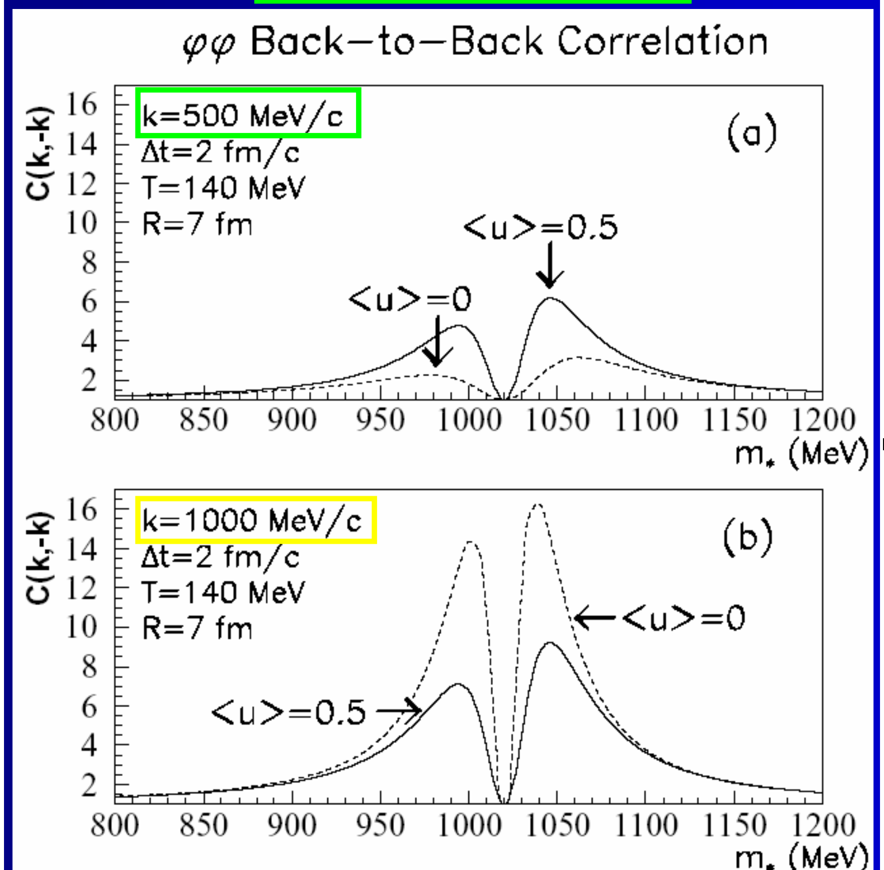
$$m_\phi = 1020 MeV$$

- **Temperature ($<u>=0$):**
  - BBC signal enhanced for lower emission temperatures, for finite sized systems emitting during finite time interval

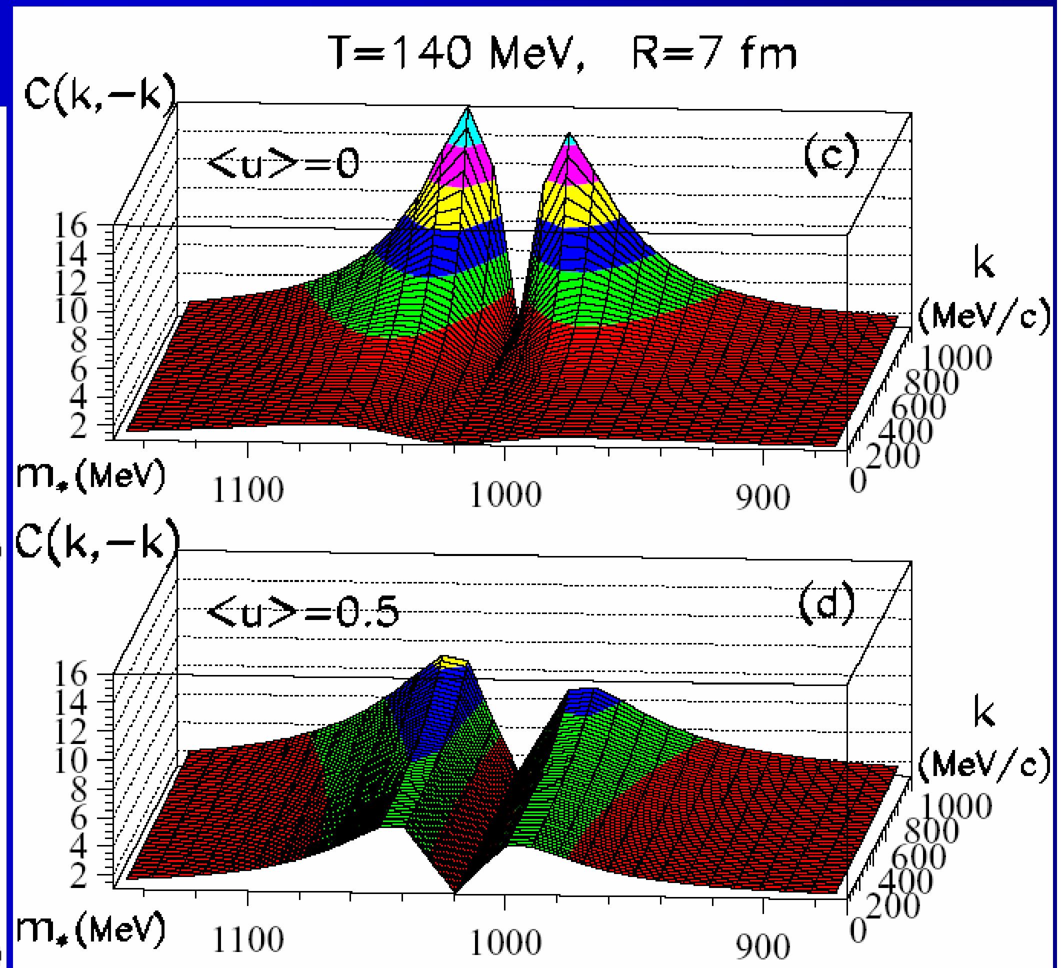
Squeezing region of radius R

- $T, R, \Delta t$ fixed for two values of k and $<u>$

$m_\phi = 1020 MeV$

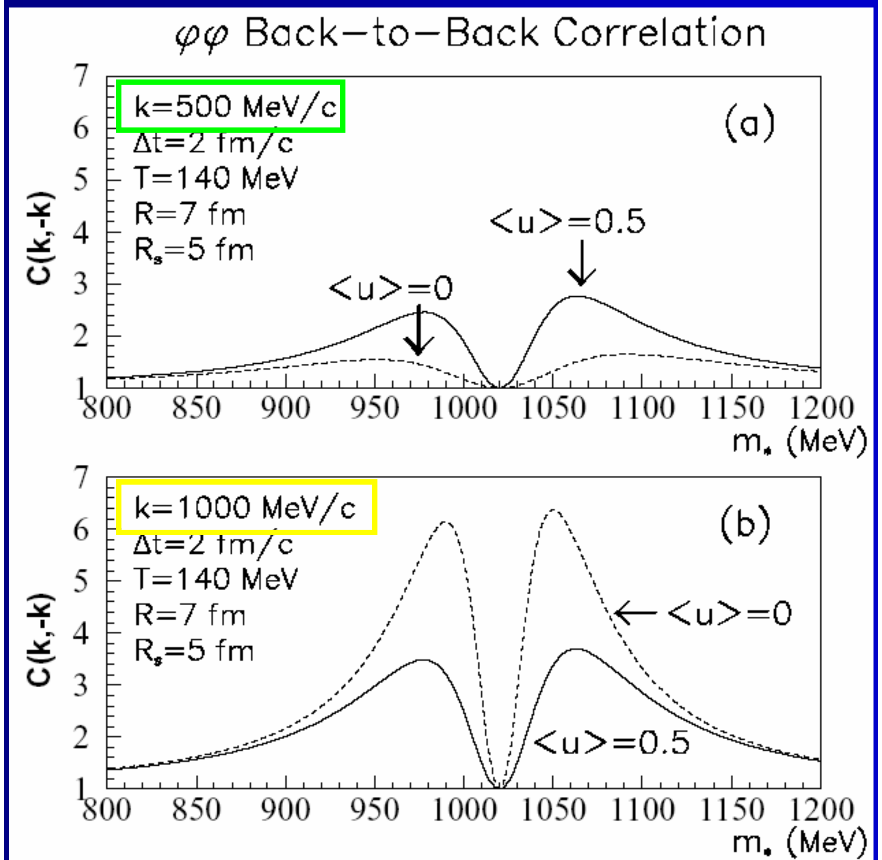


- Varying k as well:



Squeezing radius R_s, system radius R

– $T, R, \Delta t$ fixed for two values of k and $\langle u \rangle$



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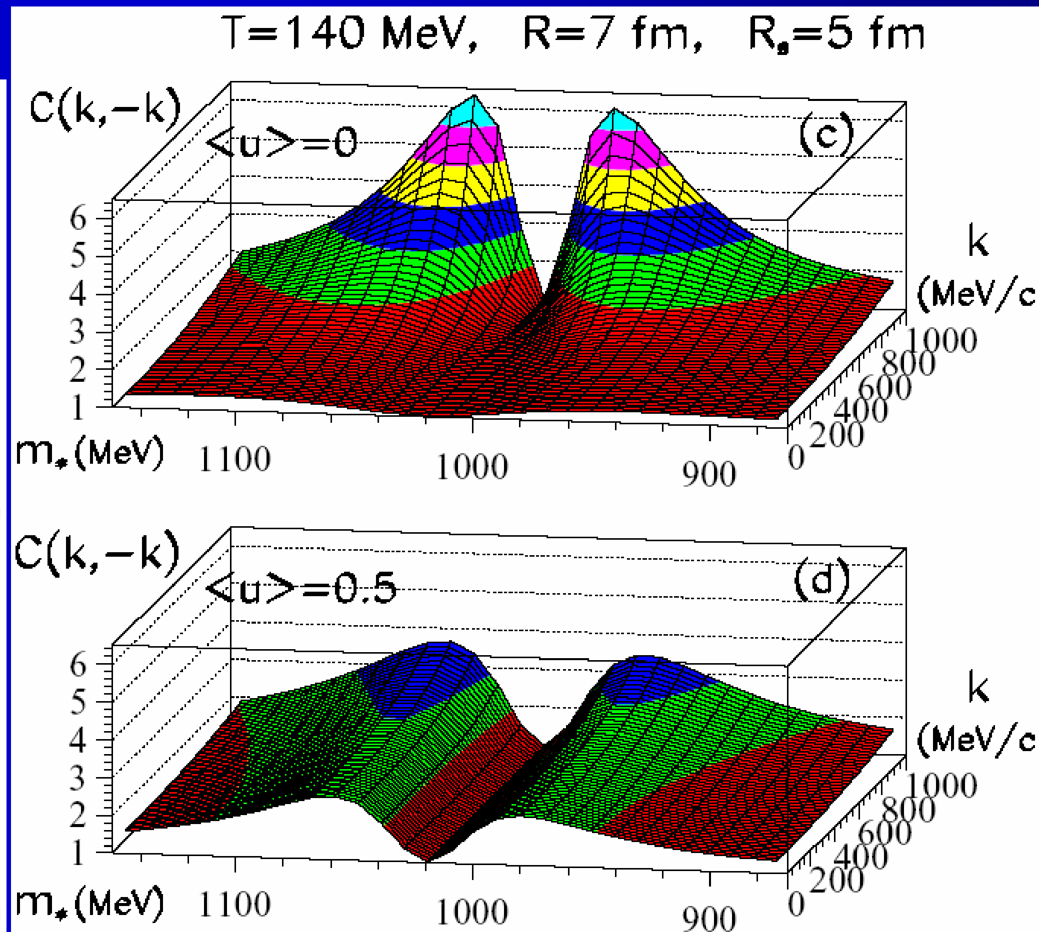

$m_\phi = 1020 MeV$

Sensitive to size of squeezing region

Comments and Conclusions

- **Main motivation: revive the discussion on BBC search**
- For this → estimated strength of BBC signal for:
  - Finite size systems emitting during finite time intervals
  - Expanding system with radial flow & non-relativistic limit
  - Illustration → Simplified situation of weak flow dependence of the BBC pair (close to central region) → $\phi\phi$ correlation
- For single (R) and two volumes (R & R_s) cases:
  - BBC vs. m_* and BBC vs. m_* vs. k
    - Pronounced maxima around $m_* \approx m$
    - Similar behavior with and without flow
    - BBC signal:
      - » increases for increasing size of mass-shifting region
      - » sensitive to spread in emission interval
      - » sizeable effect for particular cases studied here

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Next

- Introduce model-based mass-shift
- Perform more realistic calculations with flow
- Search for kinematical regions optimizing BBC signal
- Experimental feedback wanted for exp. acceptance
- Estimate shape and width of the BBC around the direction $k_2 = -k_1 = k$
- …

EXTRAS

BBC function

- Momenta of the pair

$$k_2 = -k_1 = k$$

- Back-to-Back correlation function

$$C_s(k_1,k_2) = 1 + \left\{ |c_0||s_0| \left[R^3 + 2\left(\frac{R^2}{(1+\frac{m^2\langle u\rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} \right) \right] \right\}^2 \times$$

$$\left\{ |s_0|^2 R^3 + \left(|c_0|^2 + |s_0|^2\right) \left(\frac{R^2}{(1+\frac{m^2\langle u\rangle^2}{m_* T})} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} - \frac{R^2(m\langle u\rangle k/m_*)}{(1+\frac{m^2\langle u\rangle^2}{m_* T})RT} \right) \right\}^{-2}$$

Correspondences

- **Bosonic BBC**

$$c_k = \cosh[f_k] \;;\; s_k = \sinh[f_k]$$

$$\begin{cases} a^{\dagger}{}_k = c_k\, b^{\dagger}{}_k + s_{-k}\, b_{-k} \\ a_k = c_k\, b_k + s^{*}_{-k}\, b^{\dagger}{}_{-k_1} \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \frac{1}{2}\log\left(\frac{\omega_k}{\Omega_k}\right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

- **Fermionic BBC**

$$c_k = \cos[f_k] \;;\; s_k = \sin[f_k]$$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}^{\dagger}_{\lambda',-k} \end{pmatrix} = \begin{pmatrix} c_k & \frac{f_k}{|f_k|} s_k A \\ -\frac{f_k^*}{|f_k|} s_k^* A^{\dagger} & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}^{\dagger}_{\lambda',-k} \end{pmatrix}$$

$$A = [\chi^{\dagger}_{\lambda}(\sigma.\hat{k})\tilde{\chi}_{\lambda'}] \;;\; A^{\dagger} = [\tilde{\chi}^{\dagger}_{\lambda'}(\sigma.\hat{k})^{\dagger}\chi_{\lambda}]$$

$$\begin{cases} \chi_{\lambda'} \rightarrow \text{ is a Pauli spinor} \\ \tilde{\chi}_{\lambda'} = -i\sigma^2\chi_{\lambda'} \;;\; \hat{k} = \vec{k}/|\vec{k}| \\ \tan(2f_k) = -\frac{|k|\Delta M(k)}{\omega_k^2 - \Delta M(k)M} \\ m_*(k) = m - \Delta M(k) \\ \omega_k^2 = m^2 + \vec{k}^2 \;;\; \Omega_k^2 = m_*^2 + \vec{k}^2 \end{cases}$$