Isotropization and QCD Plasma Instabilities

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based on the collaboration with
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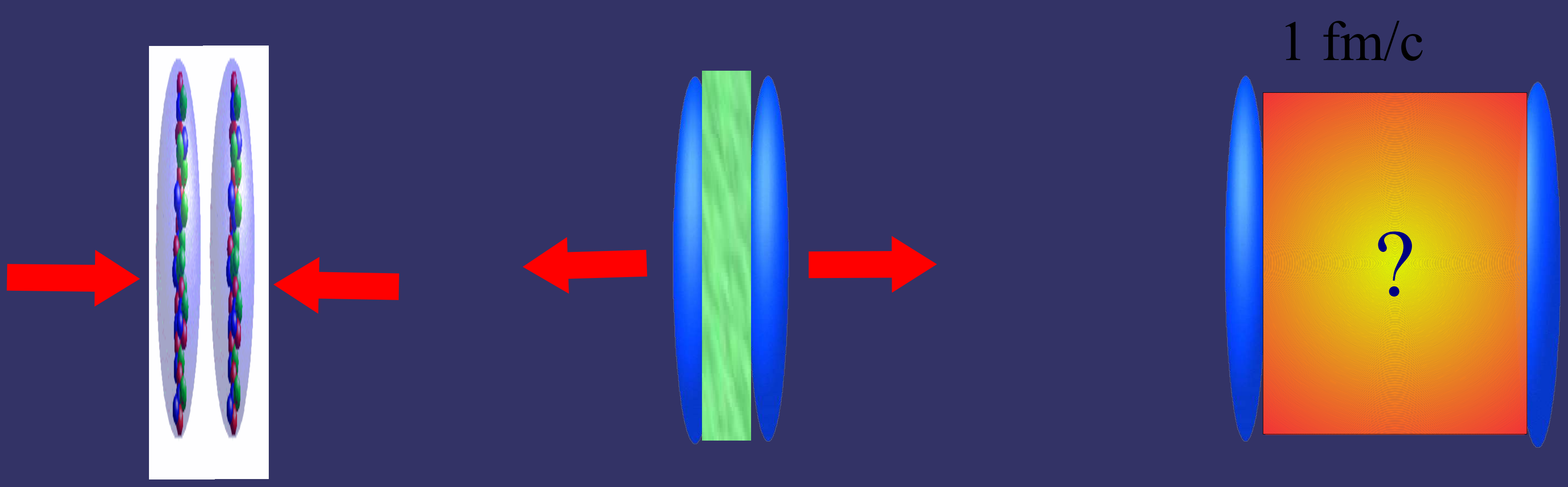
Long-term Goal

- Understanding the pre-equilibrium dynam-ics in high energy heavy ion collisions.
- Do we have thermal deconfined matter (QGP) at RHIC/LHC?
- If so, How? When? Which T?



Non-equilibrium dynamics within kinetic theory

Traditional plasma physics   L.M.Lifshitz and L.P.Pitaevskii, Physical kinetics

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_x f + g\left(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}\right)\cdot\nabla_p f = C[f]$$

$$\dot{\boldsymbol{E}} = \nabla\times\boldsymbol{B} - \boldsymbol{J} \qquad \boldsymbol{J} = g\int\frac{d^3p}{(2\pi)^3}\boldsymbol{v}\, f$$

$$\dot{\boldsymbol{B}} = -\nabla\times\boldsymbol{E}$$

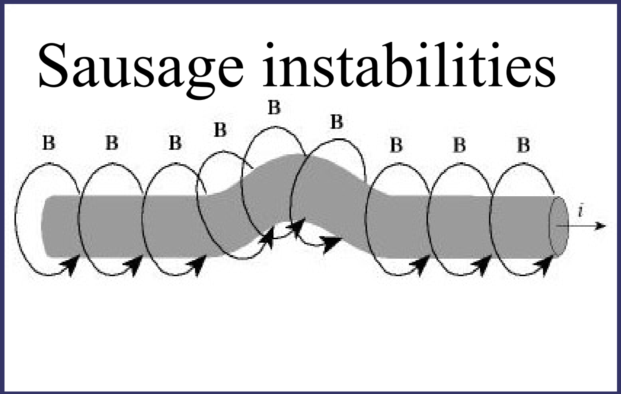
$f(\boldsymbol{x},\boldsymbol{p})$ One-particle distribution function

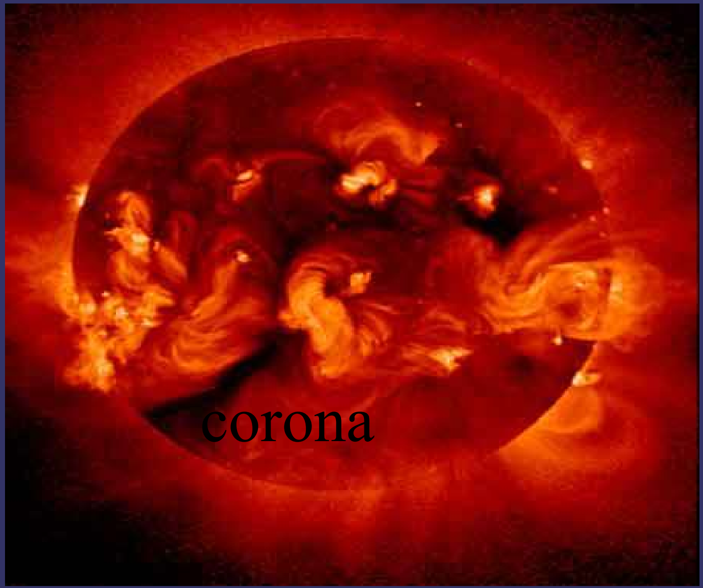
- Collision only: Boltzmann equation (PCM: Bass&Muller, Zhang&Molnar&Gyulassy, Xu&Greiner...)
- Mean field only: Vlasov equation. Collective effects.

Plasma instabilities

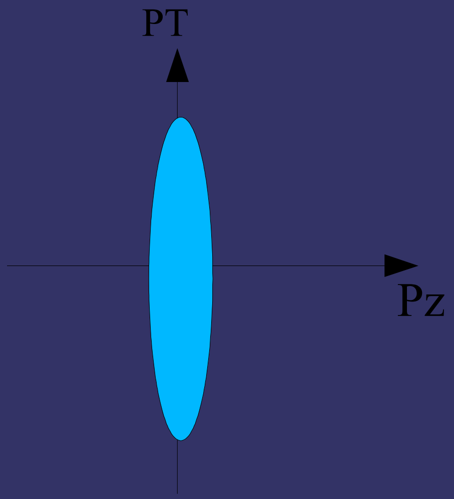
Exponential growth of wave solutions: $A \sim e^{\omega t}$

- **Macroscopic instabilities:**
  coordinate space nonuniformity





- **Microscopic instabilities:**
  momentum space anisotropy

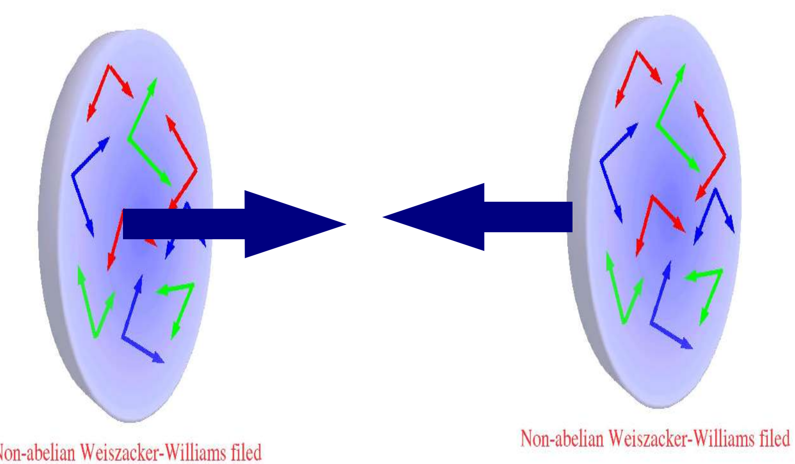


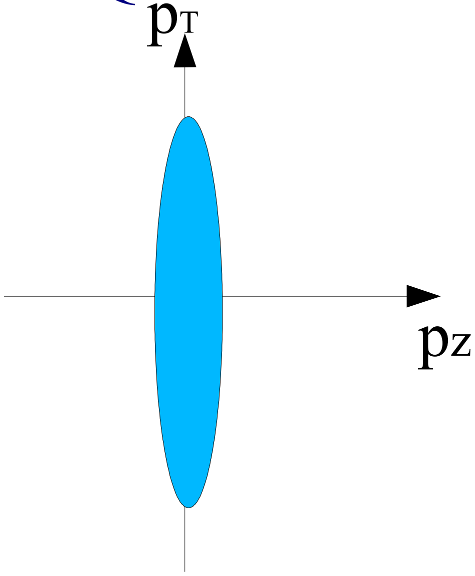
A Picture in the limit $\alpha_s \ll 1$

"Bottom-up", Phys. Lett. B502 (2001)51

- Gluons are produced at $\tau \sim 1/Qs$,
  $f \sim 1/\alpha_s$ typical momentum $p \sim Qs$

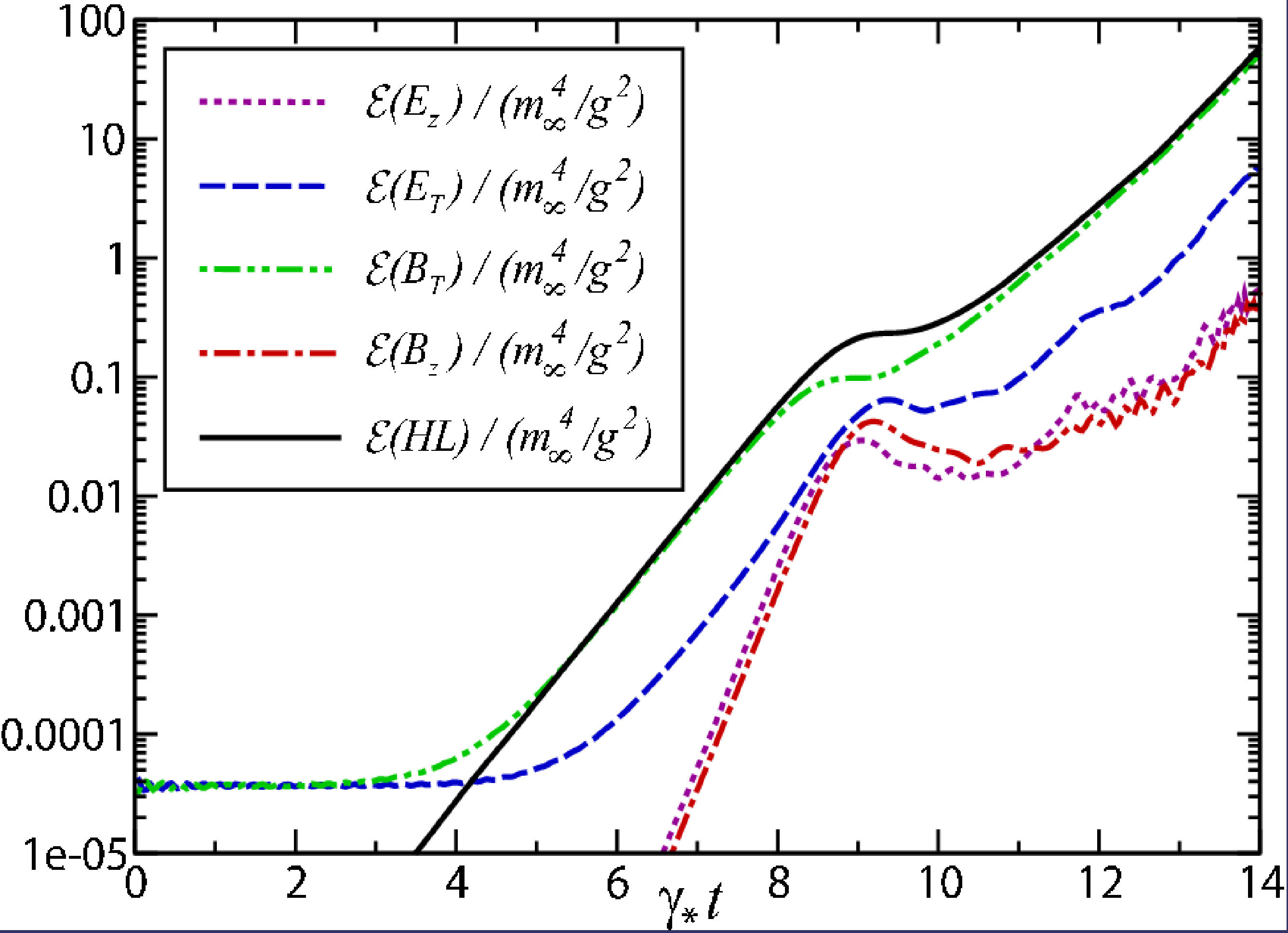
- Due to a longitudinal expansion, typical longitudinal momentum of gluons becomes
  $p_z \sim 1/\tau$, $p_x \sim p_y \sim Q_s$ at $\tau >> 1/Qs$





1+1D hard loop field simulations

A. Rebhan, P. Romatschke and M. Strickland, hep-ph/0412016.



Linear Vlasov $f \simeq f_0 + \delta f$

First confirmation of instabilities in SU(2) gauge theory predicted by

Mrowczynski,
Arnold, Lenaghan, Moore
Romatschke, Strickland

Initial Conditions:

- random fields
- squeezed particle momentum distr.

Purpose of our Study

solving full Vlasov equation for gluons,
i.e. dynamics of fields and particles.

- non-linear effects: saturation of instabilities.
- study isotropization of the system (particles and field) by interactions between particles and field.
- weak and strong initial field.

Particle in cell simulation for colored particles

$$\frac{d\,\boldsymbol{x}_i}{dt}=\boldsymbol{v}_i,\quad \frac{d\,\boldsymbol{p}_i}{dt}=gQ_i^a(\boldsymbol{E}_i^a+v_i\times\boldsymbol{B}_i^a),\quad \frac{d\,\boldsymbol{Q}_i}{dt}=igv_i^\mu[A_\mu,Q_i].$$

$$D_\mu F^{\mu\nu}=J^\nu=g\sum Q\,v^\mu\,\delta(x-x_i(t))$$

Non-Abelian version of the charge conservation algorithm

C. R. Hu and B. Muller, Phys. Lett. B409 (1997)377

G. D. Moore, C.R. Hu and B. Muller, Phys. Rev. D58 (1998)045001.

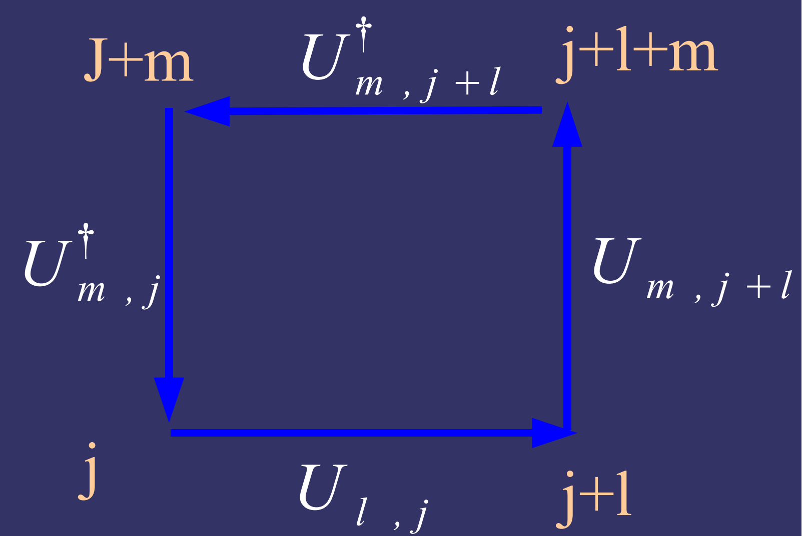
Yang-Mills Hamiltonian on the lattice

Numerical implementation of real time simulation in classical YM is well established. Kogut-Susskind Hamiltonian:

$$H_{YM} = \frac{1}{2}\sum_{l} E_l^2 + \frac{\tau}{2}\sum_{\square}\left(N_C - \Re\, Tr\, U_{\square}\right)$$

Link $\quad U_{j,i} = \exp[-iga\, A_j(i)]$

Plaquette $\quad U_{\square} \equiv U_{l,j}\, U_{m,j+l}\, U^{\dagger}_{l,j+m}\, U^{\dagger}_{m,j}$



Equations of motion for dynamical variable v

$$\frac{dv}{d\tau} = \{H_L, v\}$$

Initial conditions for simulations

- Fields depend only on x-direction (1D), but particle motion is in 3D.
- Initial field configuration: random white noise.

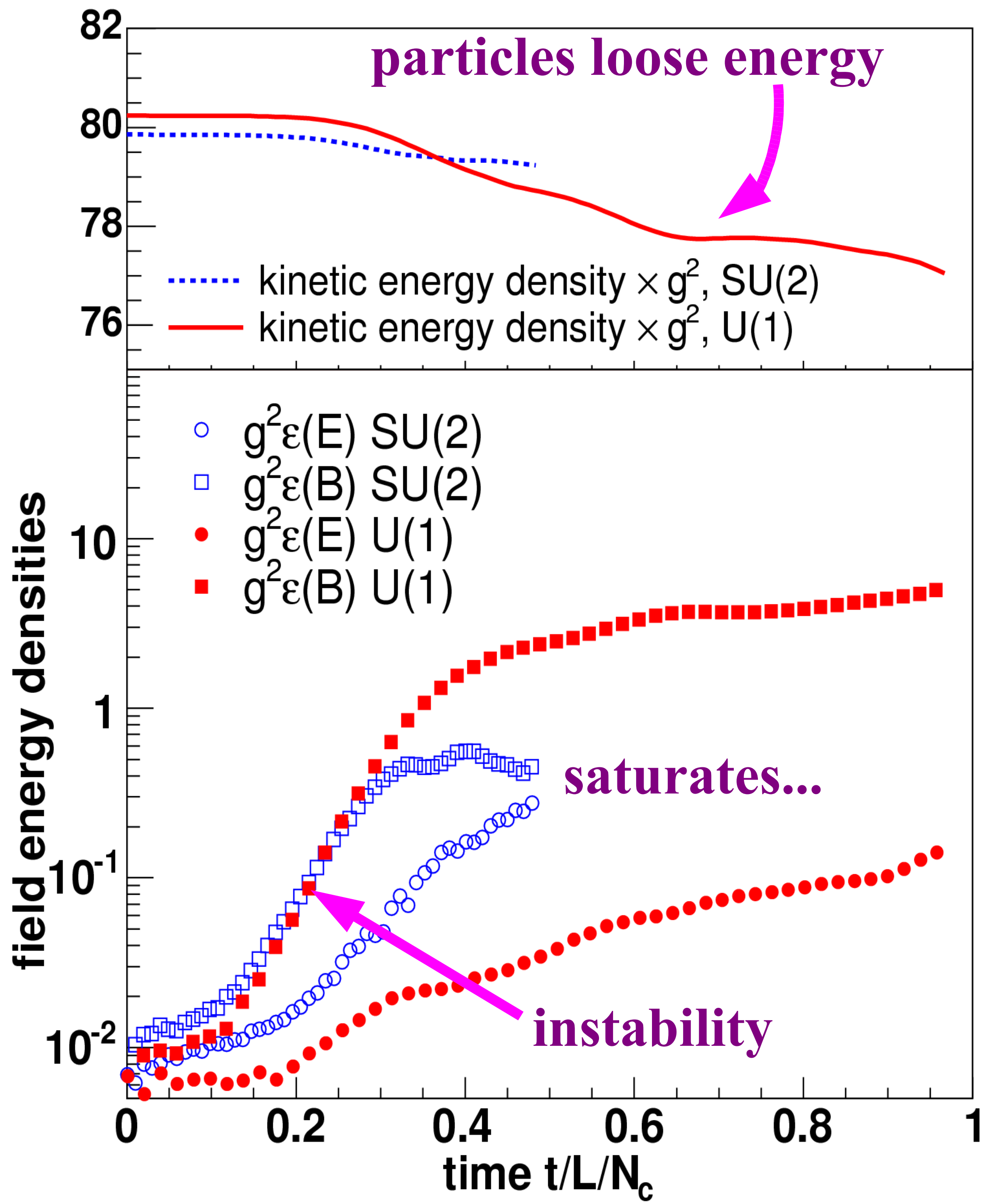
1) Weak field: i.e. large separation between soft and hard modes.
   $P_{hard} = 10$ GeV

2) Strong field: small separation between soft and hard modes.
   $P_{hard} = 1$ GeV

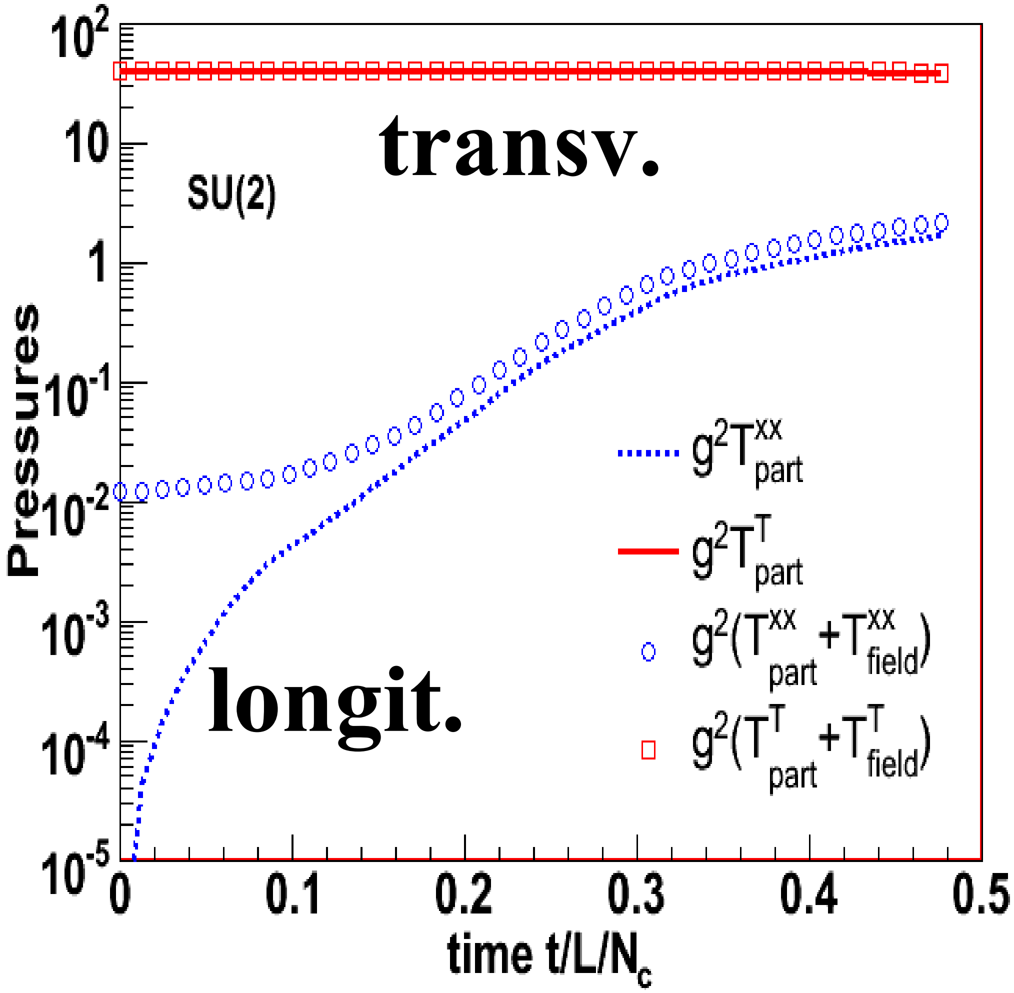
$$f(\boldsymbol{p}) \sim \delta(p_x)\, \mathrm{e}^{-\sqrt{p_x^2+p_y^2}/p_{\;har}}$$

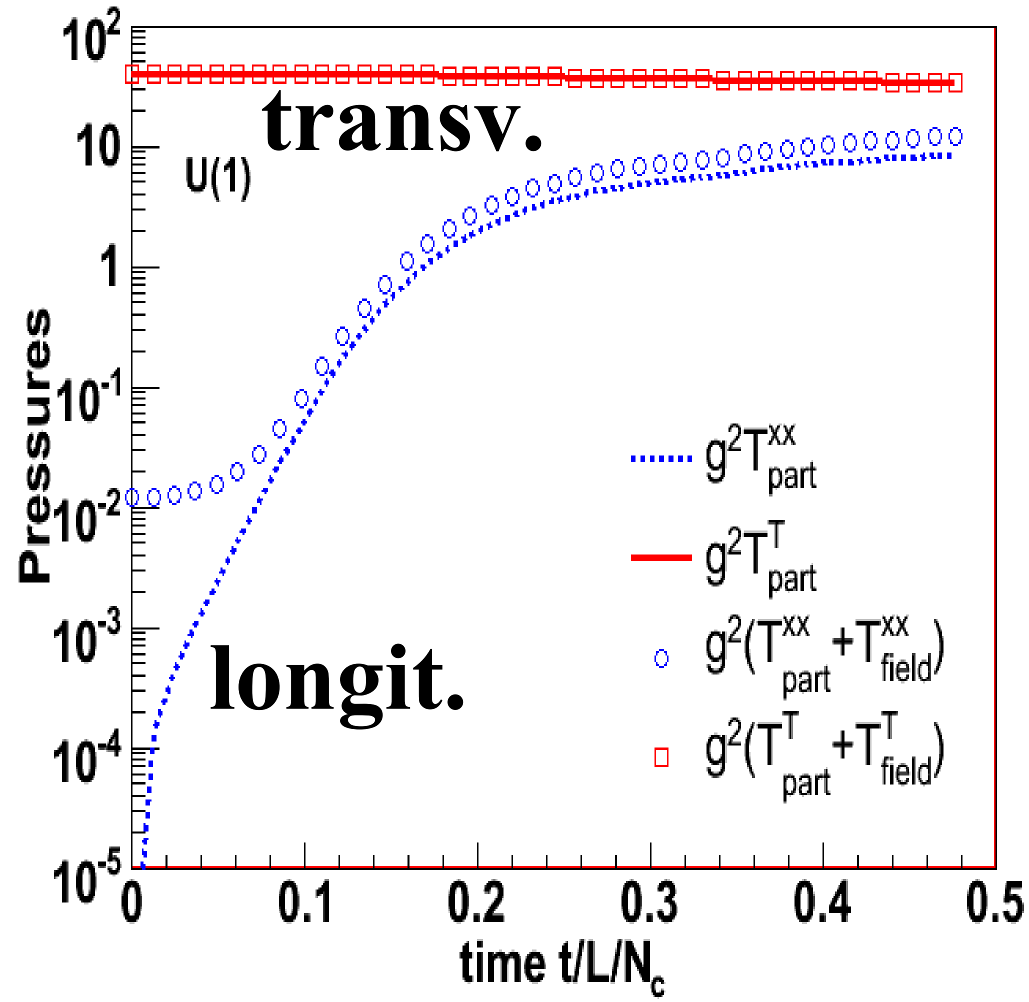
Weak initial fields

- Confirms existence of instabilities.
- Particles ARE affected
- Saturation well before fields reach particle scale.



(Partial) Isotropization



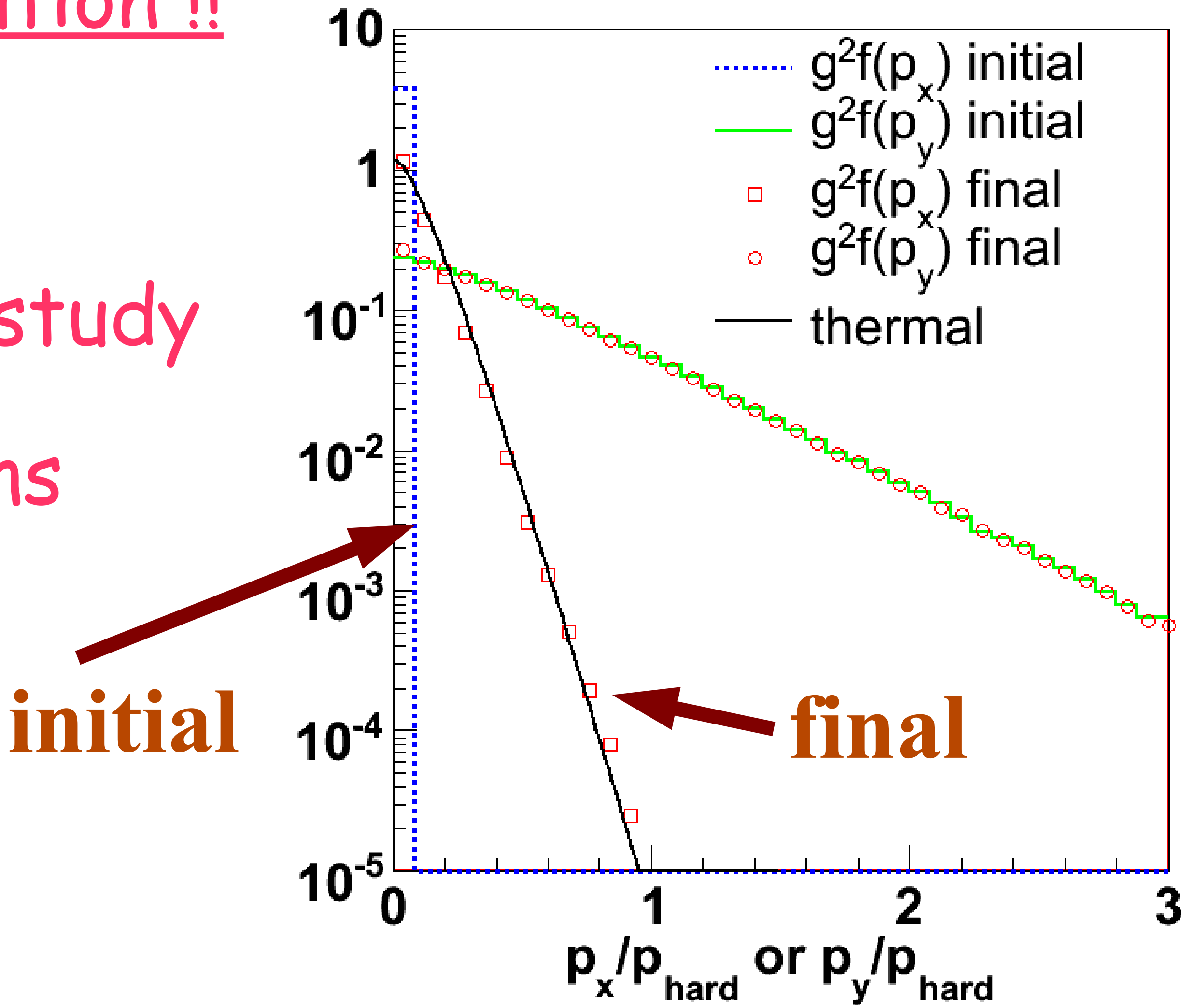


exponential growth of long. pressure

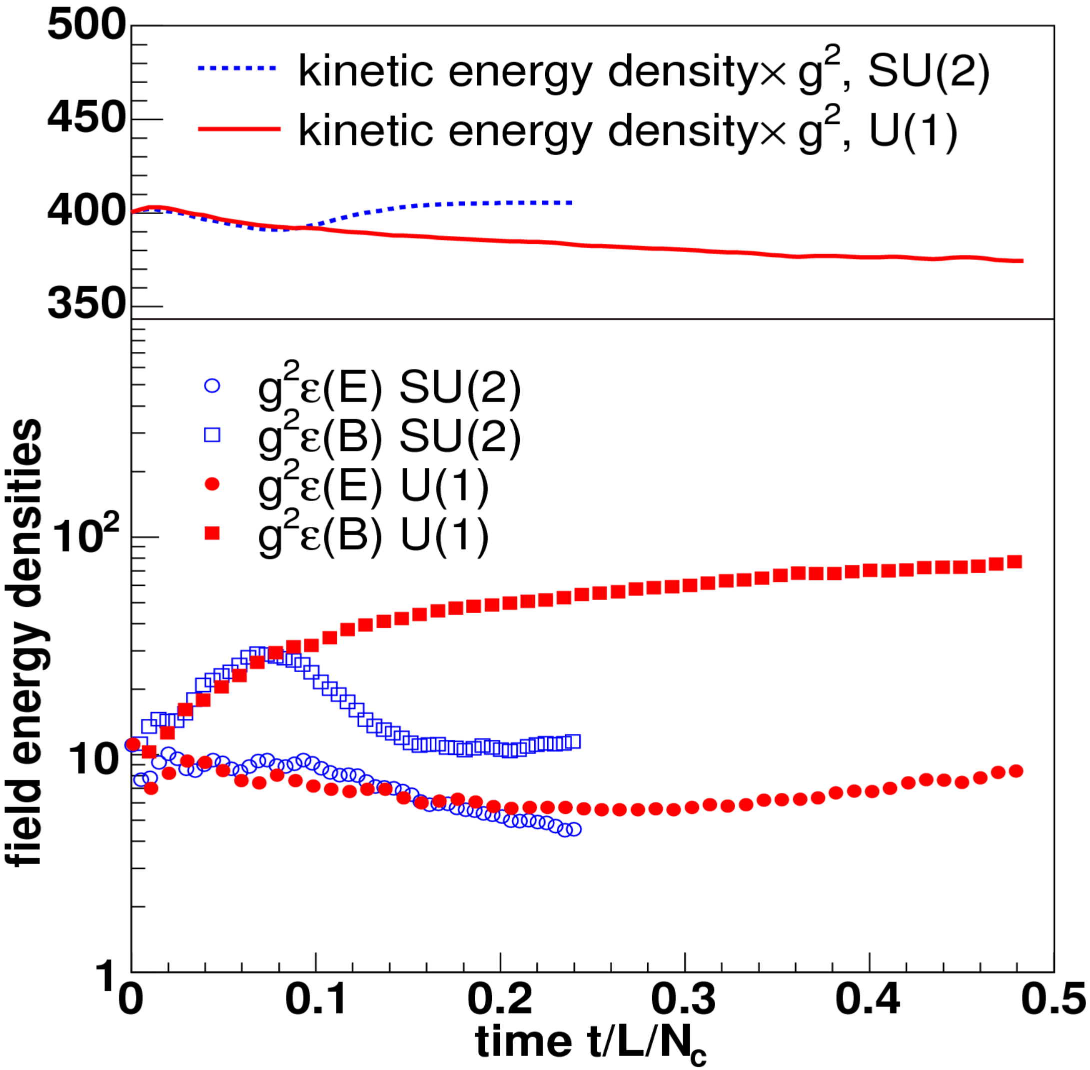
$$field\ part: T_{0x}=T_{transverse}=0, \quad T_{xx}=T_{00}$$

Momentum distributions of the hard particles

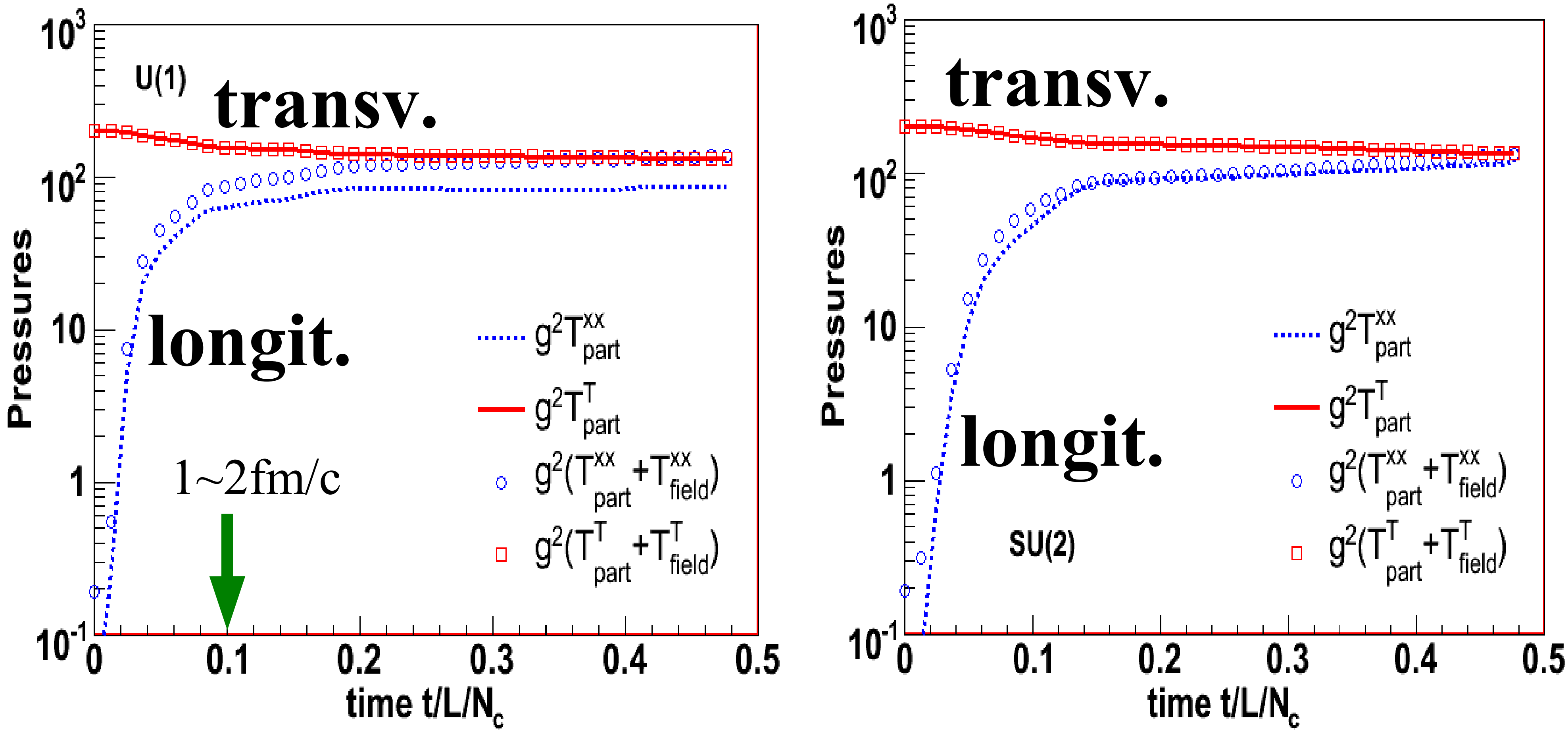
- Exponential distribution !!
- Longitudinal Cooler
- Full 3d required to study transverse directions



2) Strong initial fields



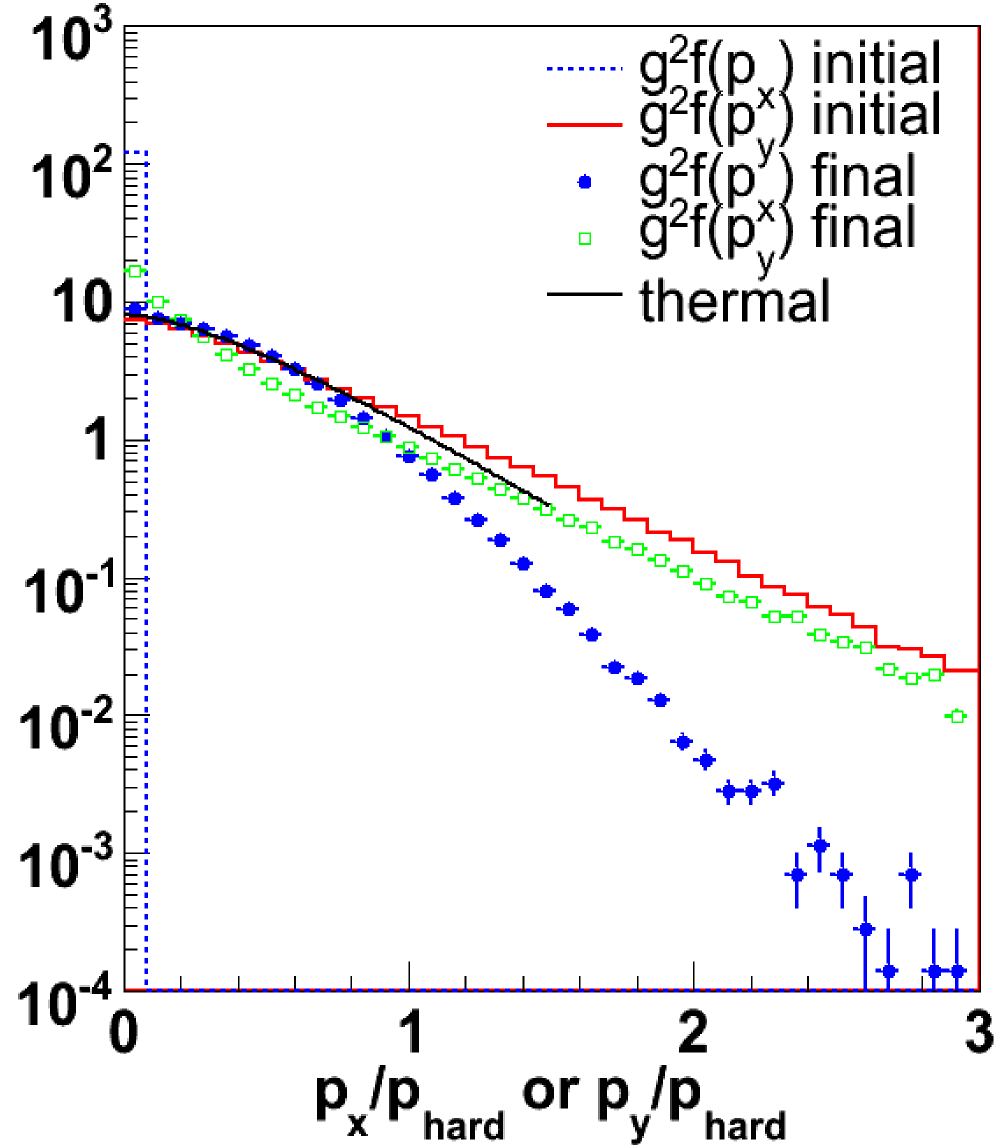
- No real instabilities seen...
- Soft fields irrelevant?



- Rapid isotropization by deflection in strong field.
- Shuts off instability.
- Even so, time scale at least as small as scattering.
- Soft fields need to be taken into account !

Momentum distributions of the hard particles

- Nearly thermal up to ~ p_{hard}
- ONE Temp !
- Stay tuned for long. Bj expansion, 3d, collisions, ...

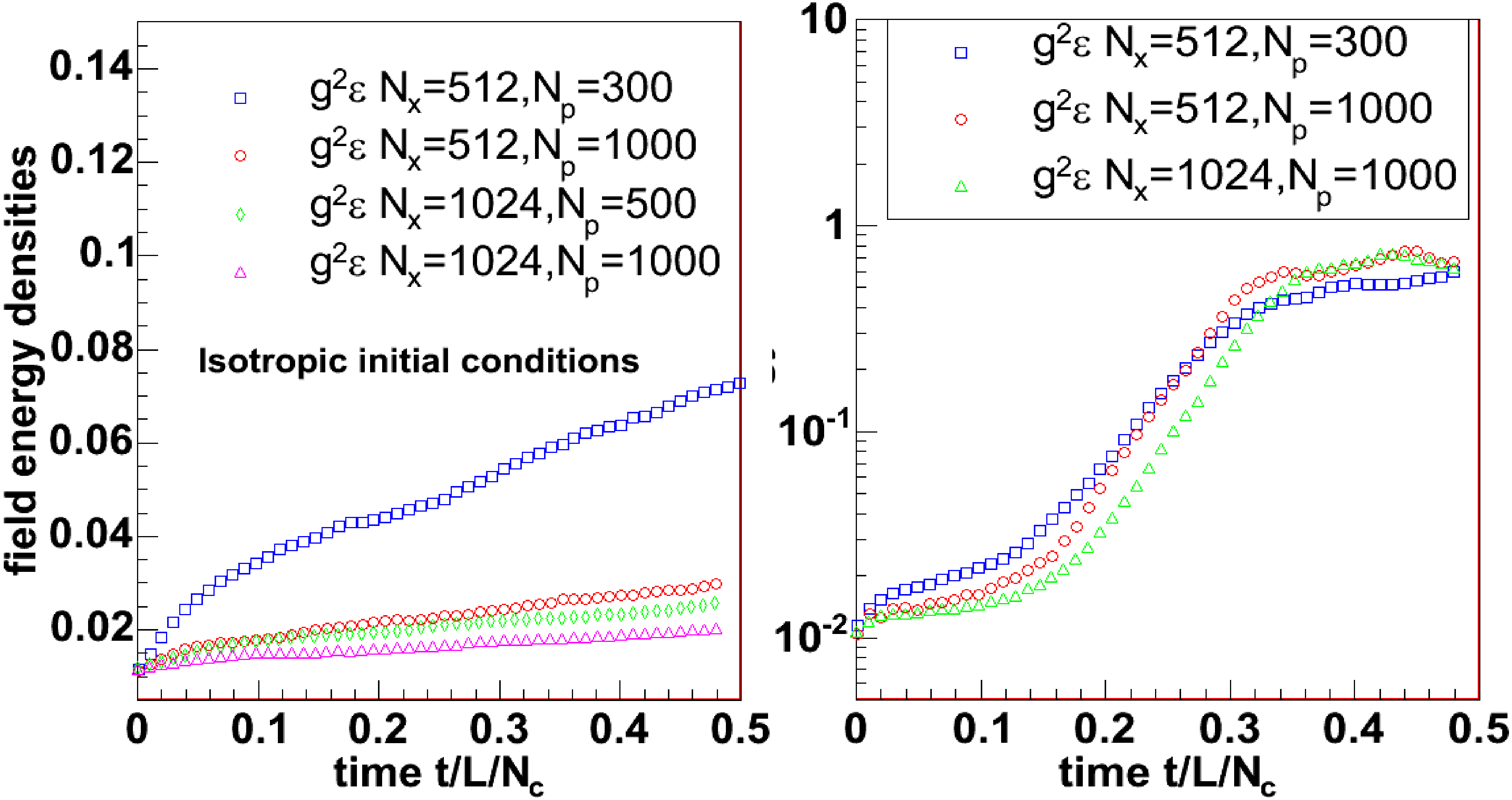


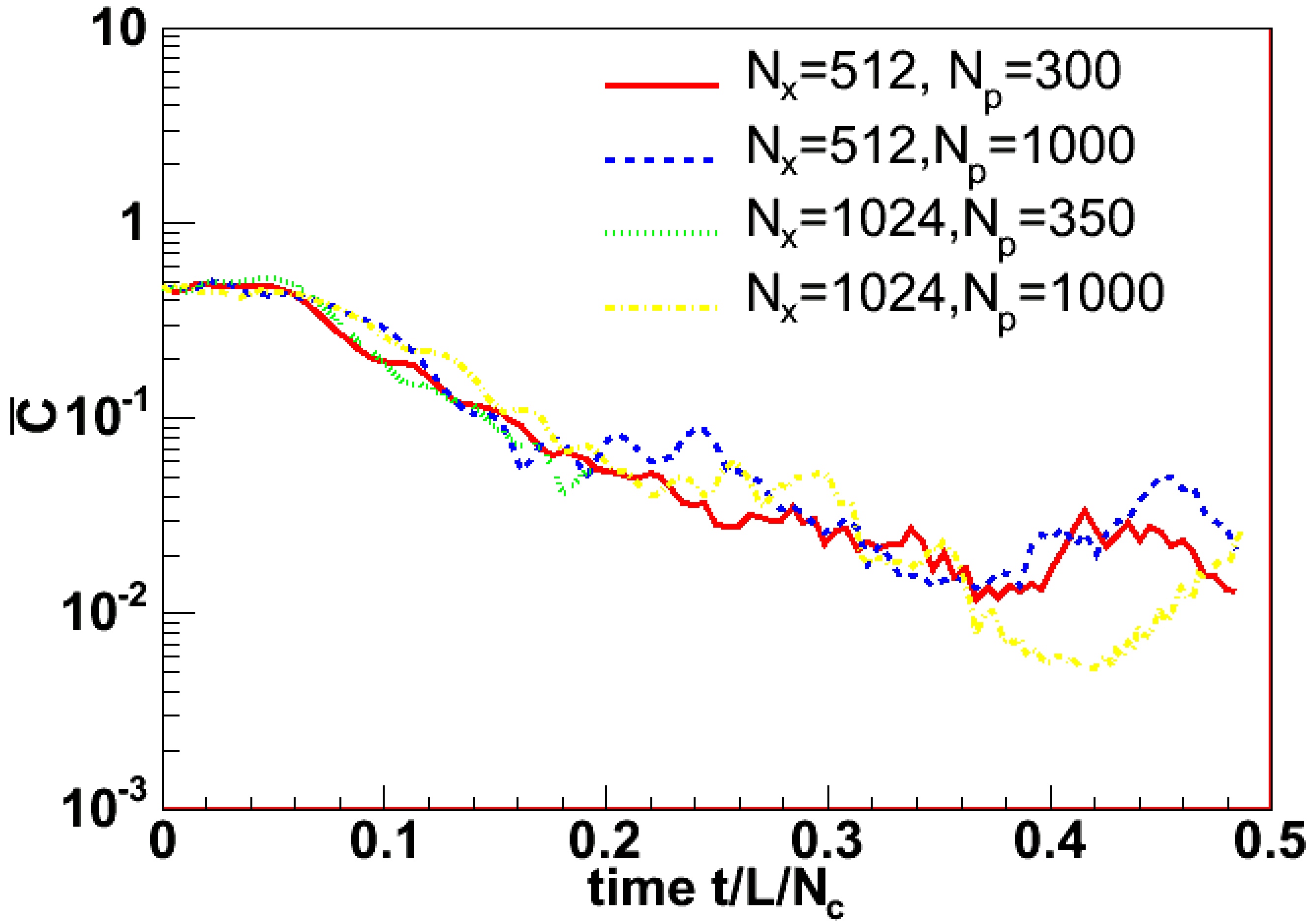
Summary and Outlook

- Particle-field (Vlasov-YM) simulations in QCD.
- So far only 1+1 D.
- Collective effects due to color field play important role for the pre-equilibrium dynamics.

- Include expansion of the system.
- Full 3D simulation
- Collision term

lattice size and test particle dependence





10
1
10^{-1}
10^{-2}
10^{-3}
$\bar{C}$
0
0.1
0.2
0.3
0.4
0.5
time t/L/N_c
N_x=512, N_p=300
N_x=512,N_p=1000
N_x=1024,N_p=350
N_x=1024,N_p=1000