Strangeness and statistical models

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- Definition and observables
- Physical meaning, relationship to previous lecture
- Ensembles
- Strangeness: Why is it interesting?
- Strangeness: Experimental situation
- Interpretations: Kinetic or canonical
  (or both? or neither?)
- Interpretations: Equilibrium or non-equilibrium
  (or neither?)
- What next?

This talk spans a <u>broad</u> range of topics. I will <u>not</u> give you many details (that's what the references are for), but I'll try to motivate each idea and tell you what to expect if it applies to reality

Please ask questions if you think something is not clear/wrong/both!

Mine (and hopefully your) philosophy:

We are not economists, politicians or theologians! We are scientists!

- The best physicist is not the one with the correct theory, but the one with the best understanding of their theory, it's implications, and it's connections to reality

- One should always try to test each theory to the limit against the real world.
  Proving a model false does everyone a favour, including the people who invented it, since they can now devote their talents to something better.

- If there are different models explaining the same data, the priority is to experimentally distinguish them, to falsify some (or possibly all!)

Another word on philosophy...

Statistical models are theories, stemming from the common assumption that some or all dynamics factors out and observables are determined by phase space.

As theories, they are testable. Some have more parameters,and therefore they might be more difficoult to falsify, but all are falsifiable:

To what extent are yields,ratios and fluctuations describable through a given statistical model?

The statistical model[1]

$$N = \int \mathcal{M} \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

$\mathcal{M} \rightarrow constant$ (dynamics $\rightarrow$ phase space)

$$P_N = \frac{\Omega_N}{\sum_n \Omega_n} \qquad \Omega = \int \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

Observables:

$$< N > \quad , \quad \omega = \frac{< N^2 > - < N >^2}{< N >}, \quad \text{higher comulants}$$

calculable through partition function

[1] Relationship to previous lecture: Non-extensive effects higher order.

$$S_{12} = S_1 + S_2 + h(S_1, S_2) \qquad E_{12} = E_1 + E_2 + h(E_1, E_2)$$

Phenomenology described here first order, hence the $\sum_i$ and $\prod_i$

What's happening physically?

- Fermi:Many particles$\rightarrow$ phase space dominance
- Landau,...: strong interactions+ high density $\rightarrow$ Local thermalization
  Phase space dominance in each element of system

Results can be the same, but systems different.

- Is collective flow there?
- Is a charge absent at the start (e.g.: strangeness) statistically distributed?

Ensembles , or how to deal with conservation laws
$\lim_{V \to \infty}^{N/V=const} < N >$ same in $\forall$ ensembles. not ω

Micro-canonical : EbyE conservation

$$\delta_E \delta_Q = \delta\left(\sum_i E_i - E_T\right)\delta\left(\sum_i Q_i - Q_T\right) \quad \omega_E = \omega_Q = 0$$

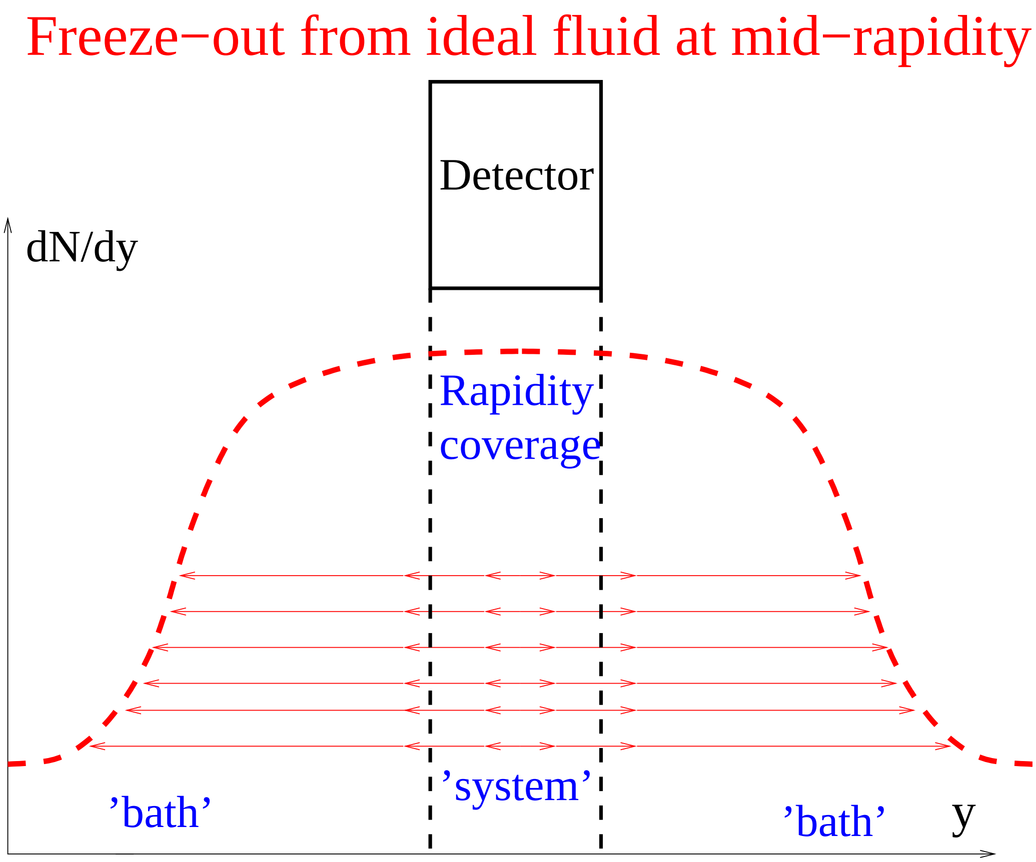
Canonical : Energy conserved on average
Appropriate for system in equilibrium with bath

$$\delta_E \to \delta\left(E_T - < E >\right) \qquad \omega_E \sim 1$$

Grand Canonical : Charge conserved on average

$$\delta_Q \to \delta\left(Q_T - < Q >\right) \qquad \omega_E \sim \omega_Q \sim 1$$

Appropriate for detector sampling part of a fluid



Boost invariance: Rapidity ⇔configuration space

- Mid-rapidity ⇔system
- Peripheral regions ⇔bath

⇒ Grand Canonical ensemble needs to be used!

Cleymans, Redlich, PRC 60, 054908 (1999):

$$\left[\frac{dN}{dy}\right]_{b.i.} \sim < N >_{4\pi} \quad \left[\frac{d(\Delta N)^2}{dy}\right]_{b.i.} \sim (\Delta N)^2_{4\pi}$$

- All details of flow and freeze-out integrate out

- Up to Normalization, $< N >, \omega$ calculable from Grand Canonical T, λ_i

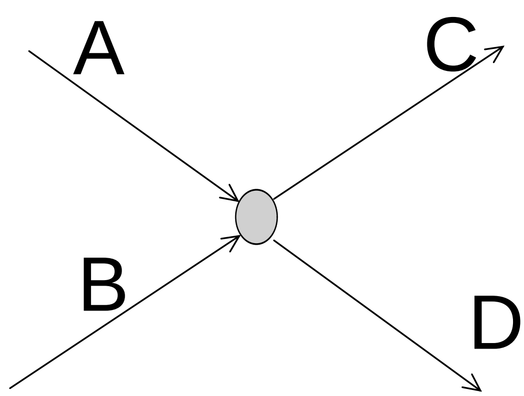
- Assumes fluid in equilibrium up to breakup $\rightarrow$ see next lecture for a critique of that.

GC calculation

$$N_i = \int \frac{d^3p \lambda e^{-\frac{\sqrt{p^2+m_i^2}}{T}}}{1 \pm \lambda e^{-\frac{\sqrt{p^2+m_i^2}}{T}}} \qquad (\Delta N_i)^2 = \int \frac{d^3p \lambda e^{-\frac{\sqrt{p^2+m_i^2}}{T}}}{\left(1 \pm \lambda e^{-\frac{\sqrt{p^2+m_i^2}}{T}}\right)^2}$$

$$T^{-1} = \frac{\partial S}{\partial E} \qquad \lambda_Q = e^{\mu_Q/T} \qquad \mu_Q = T\frac{\partial S}{\partial N_Q}$$

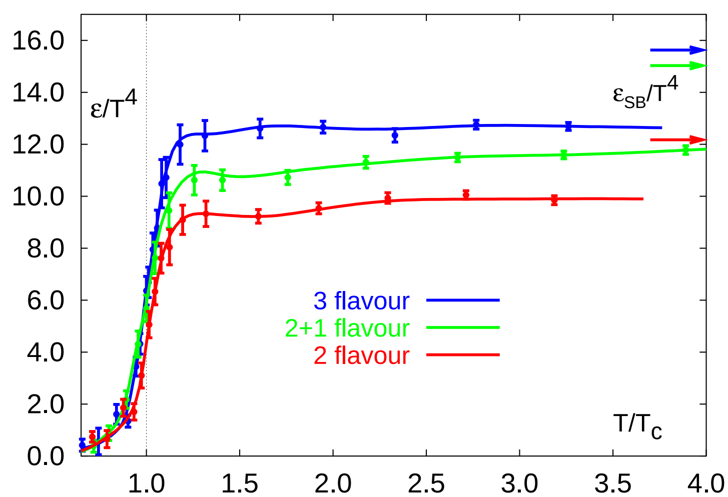
Chemical equilibrium ("detailed balance")



$$\lambda_A \lambda_B = \lambda_C \lambda_D \longrightarrow \lambda_A = \lambda_{\bar{A}}^{-1}$$

But should anything be in chemical equilibrium?

System is expanding and cooling fast.
A sharp, possibly first order phase transition, two phases (HG) and (QGP) with different phase space structure



QGP produces lots of entropy which has to be conserved at hadronization. $\longrightarrow$ chemical non-equilibrium for some/all flavors?[2]

[2]NB:No one expects Charm to be kinetically equilibrated.

But some (Gorenstein, Becattini, Rafelski,...) think its statistically distributed (Ratios such as $\frac{\psi}{D^+ D^-}$, $\frac{D^*}{D}$, $\frac{\psi'}{\psi}$ to follow statistical model). So equilibration/detailed balance $\neq$ statistical production

QGP Dynamics behind statistical hadronization?

Consider a boiling equilibrated "pot" of QGP.

Hadronization conserves entropy		Statistical production
but	$\Rightarrow$	but
Quark content from equilibrium QGP		No detailed balance

NB: Neglect hadronization dynamics $\Rightarrow \Delta S_{hadronization}$ small

Statistical coalescence of pre-existing $Q, \overline{Q}$

$$P_N = \int \prod_i \rho_Q \rho_{\overline{Q}} \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q \quad N_i^{GC} = \int \frac{d^3 p \rho_Q^Q \rho_{\overline{Q}}^{\overline{Q}} e^{-\frac{\sqrt{p^2+m_i^2}}{T}}}{1 \pm \rho_Q^Q \rho_{\overline{Q}}^{\overline{Q}} e^{-\frac{\sqrt{p^2+m_i^2}}{T}}}$$

Parametrize non-equilibrium $\rho_Q, \rho_{\overline{Q}}$ by

$$\lambda_q \to \lambda_q \gamma_q \qquad \gamma_q = \gamma_{\overline{q}} (= 1)_{equilibrium}$$

Incomplete strangeness equilibration $\Rightarrow \gamma_s < 1.$

Fast transition,Expansion $\Rightarrow$ Possibly $\gamma_{q,s} > 1$

- $q\overline{q}$ mesons (eg ϕ if $\gamma_s \neq 1$ or π if $\gamma_q \neq 1$) aquire chemical potential
- If $\gamma_q > 1$ BE corrections important for π
  - γ_q increases: More π yields, More fluctuations!
  - T increases OR more resonances: More π yields, less fluctuations!
- Mixed states like $\pi^0, \rho^0, \phi, ...(\alpha \left| u\overline{u} \right\rangle + \beta \left| d\overline{d} \right\rangle + \gamma \left| s\overline{s} \right\rangle)$ enhanced if $\gamma \neq 1$ by mixing

Resonance decay

Hagedorn : Resonances are energy levels in interacting hadronic system

So strong interactions ⇔ "ideal gas" of all resonances

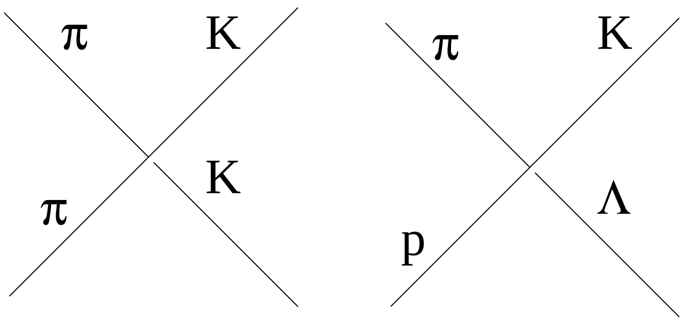
for spectra, resonance contribution non-trivial (kinematics and flow). For yields, simple sum .

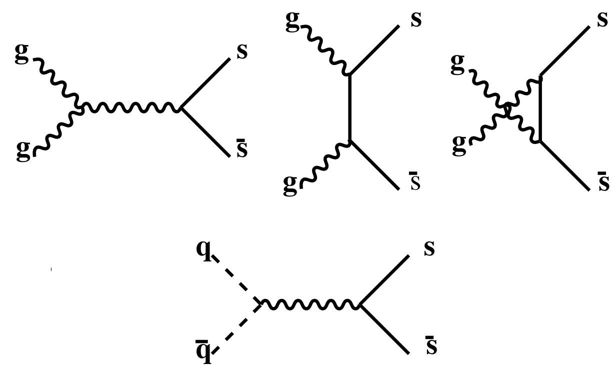
$$N_i = N_i^{direct} + \sum_j b_{j\to i} N_j$$

$$\Delta N_i^2 = \Delta N_i^2 + \sum_j \left[b_{j\to i} \left(1 - b_{j\to i}\right) N_j + b_{j\to i}^2 \Delta N_j^2) \right]$$

Strangeness: Why is it such a good probe?

Koch, Rafelski, Muller 1982, 1986: QGP kinetics more efficient at producing $s\bar{s}$ than HG kinetics



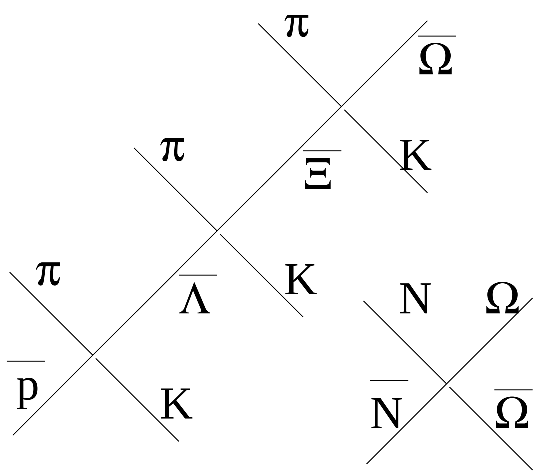


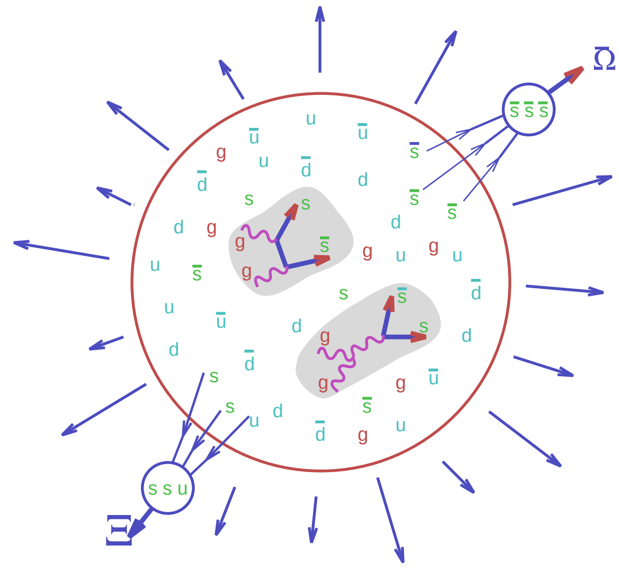
- Faster equilibration time

$$Q_{hadrons} \sim 500MeV \qquad Q_{QGP} = 2m_s \sim 200MeV$$

- More $s\bar{s}$ at equilibrium ($\gamma_s > 1$ in HG phase?)

$$\frac{m_{K,\Lambda,...}}{T} \ll \frac{m_s}{T}$$





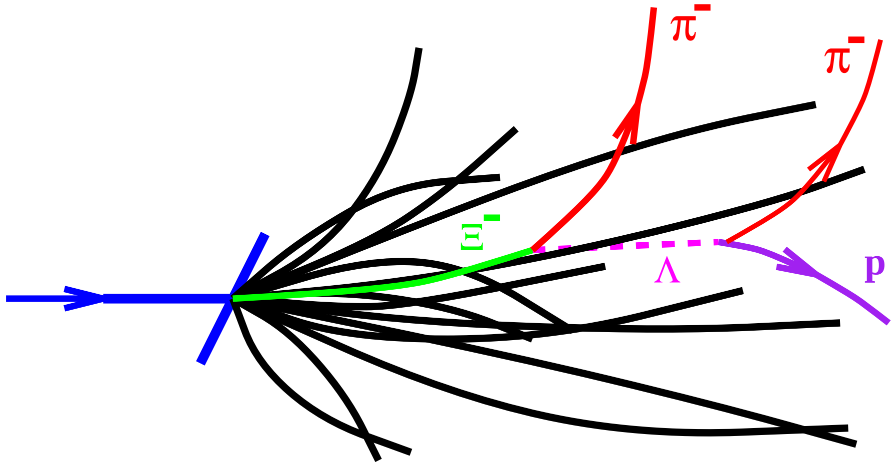
strange quark coalescence enhances multistrange ANTIbaryons with respect to hadronic production

$$\frac{3m_s}{T} >>>>> \frac{m_\Omega}{T}$$

$$Q_{N\overline{N}\to\Omega\overline{\Omega}} <<<< 3m_s$$

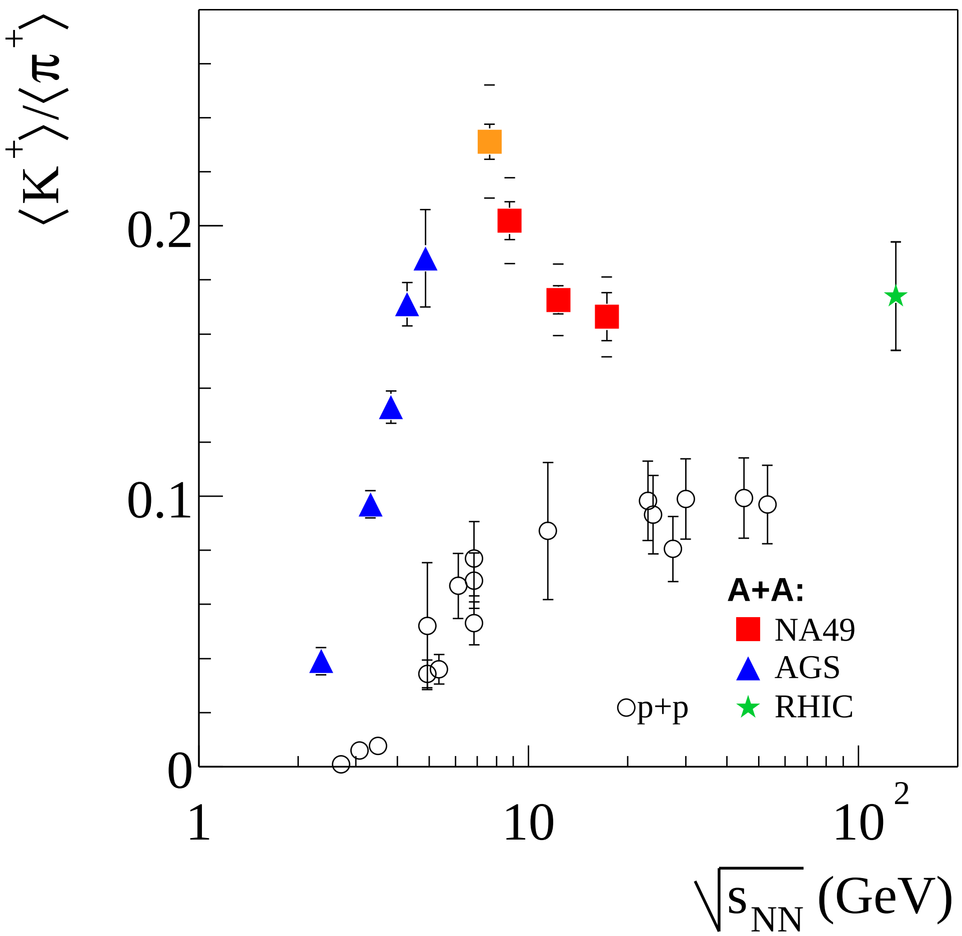
$$\tau_{p\pi\to\Lambda\pi\to\Xi\pi\to\Omega} <<<< \tau_s^{QGP}$$

finally...



Hyperon decays self-analyzing ("easy" to measure)
Yields of K, Λ, Ξ, Ω can hopefully be measured by both wide and narrow acceptance detectors with systematic errors under control.

Experiment I: The "horn"

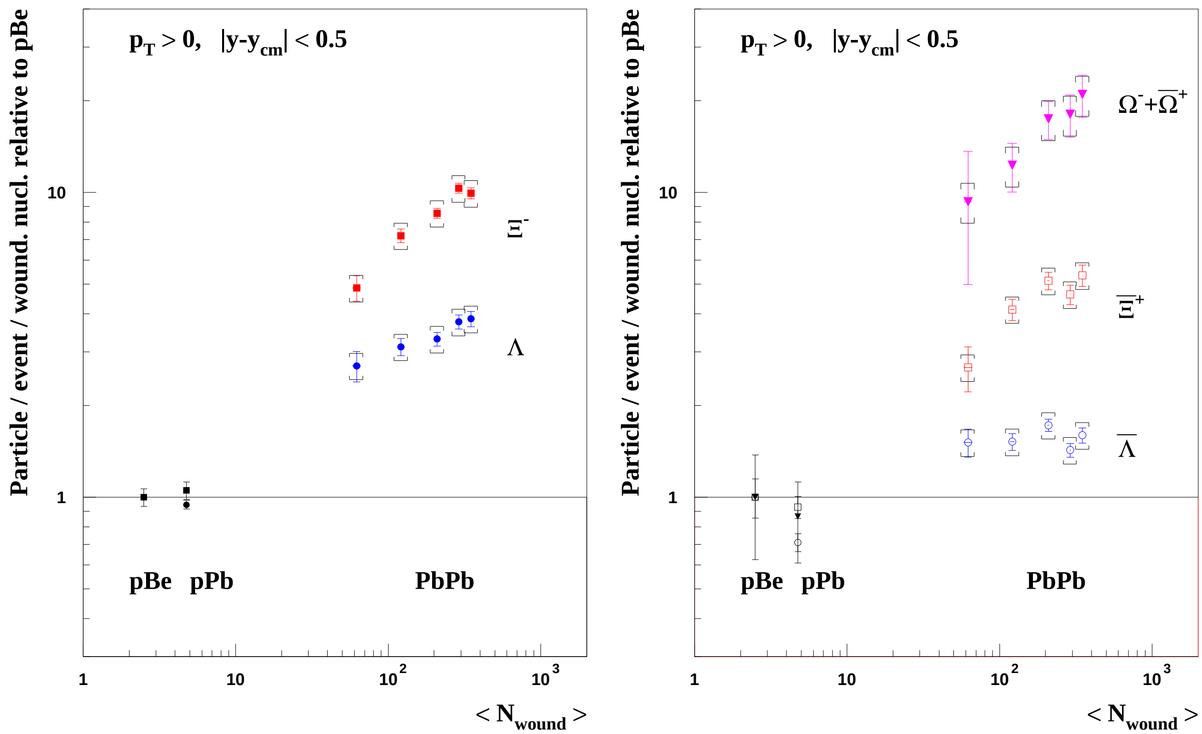


A discontinuity is observed @ $\sqrt{s}/A \sim 8GeV$

when plotting $\frac{s}{\pi}\left(\frac{K^-}{\pi^-}, \frac{\Lambda}{\pi}, ...\right)$ ratios with $\sqrt{s}$.

Nothing similar in p-p collisions

Experiment II: Enhancement, defined as



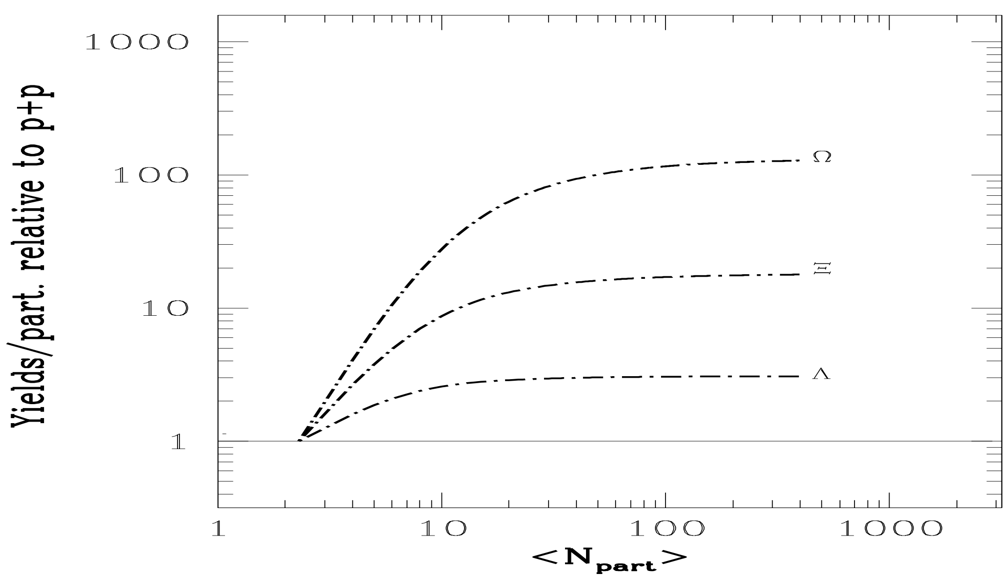
$$\frac{N^{AA}/N^{AA}_{part}}{N^{pp}/N^{pp}_{part}}$$

is definitely there, as much as ~ 20 for $\overline{\Omega}$. But the interpretation of this has been subject to controversy

QGP enhancement or Canonical suppression

$$\lim_{V\to\infty} \frac{< N >_{CE}}{< N >_{GCE}} = 1$$

but away from thermodynamic limit→ additional suppression, nonlinear in volume[3] due to comparatively fewer degrees of freedom that conserve the charge (here, strangeness) exactly. (Hamieh,Tounsi,Becattini,Keranen,...)



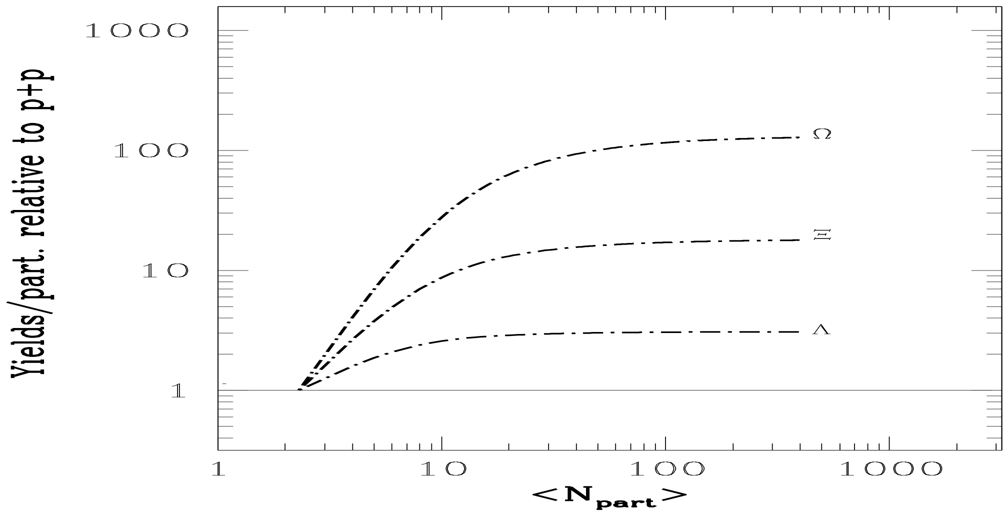
[3] NB:This is different from non-extensivity. Non-linearity arises from constrains (δ) in phase space, not microscopically

- Could strangeness enhancement be caused by the fact that p-p is far from the thermodynamic limit, while A-A is close to it? Is p-p particle production also governed by equilibrium statistics?

- Or could we be seeing 2 different production mechanisms, one (p-p) based on hadronic physics, the other one on QGP?
  (Hadronic transport models such as uRQMD can explain, without equilibrium p-p strangeness production but not A-A, e.g. NA57, Eur. Phys. J. C11 1999 79-88)

You decide! See the philosophy stuff

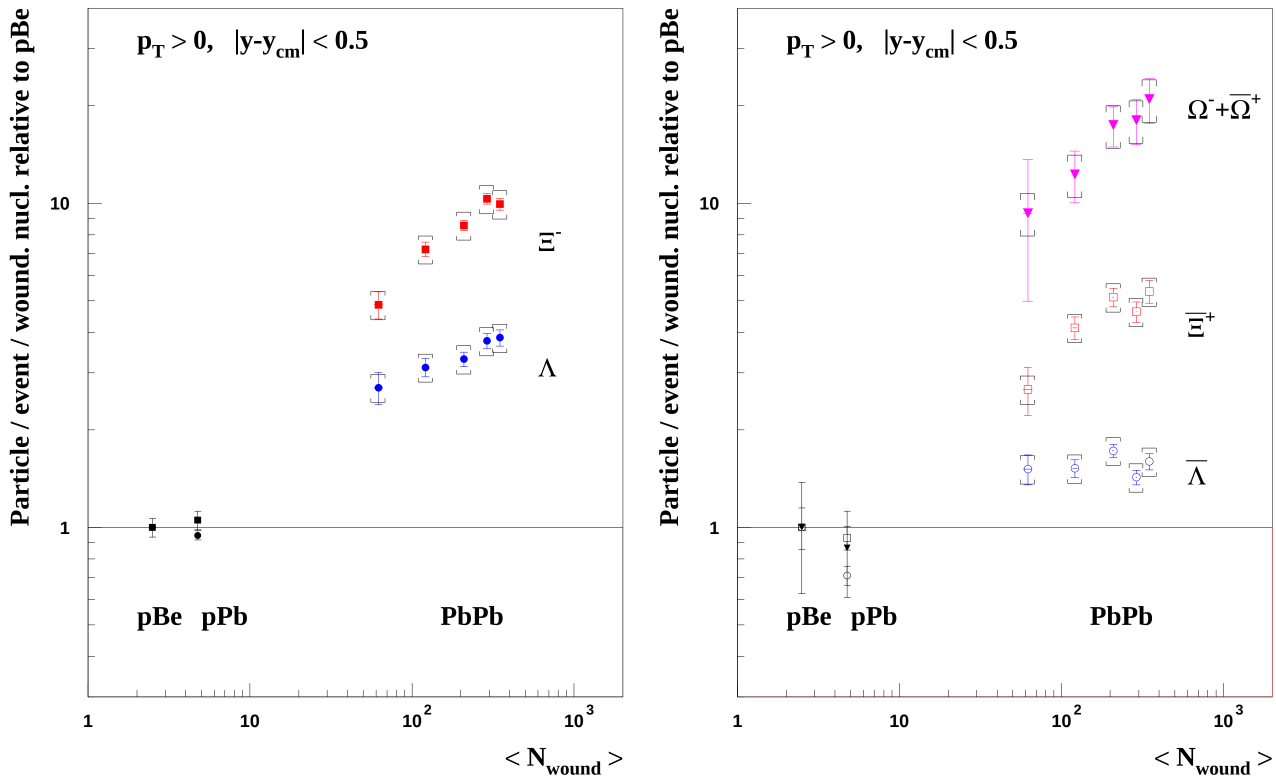
Canonical suppression and centrality



- Canonical suppression ⇒ enhancement rises at small V, then saturates at "thermodynamic limit"

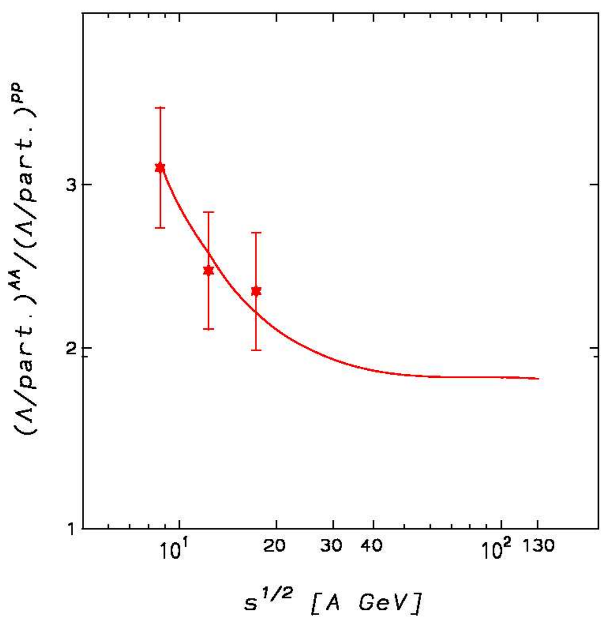
- QGP model ⇒ enhancement starts at critical system size (when QGP forms). Then it probably always goes up with N_{part}, as bigger volume → bigger efficient strangeness producer

NA57, 158 GeV/A p-Pb,P-Be,Pb,Pb



Keep an eye on new enhancement measurements in various systems/centralities!!!

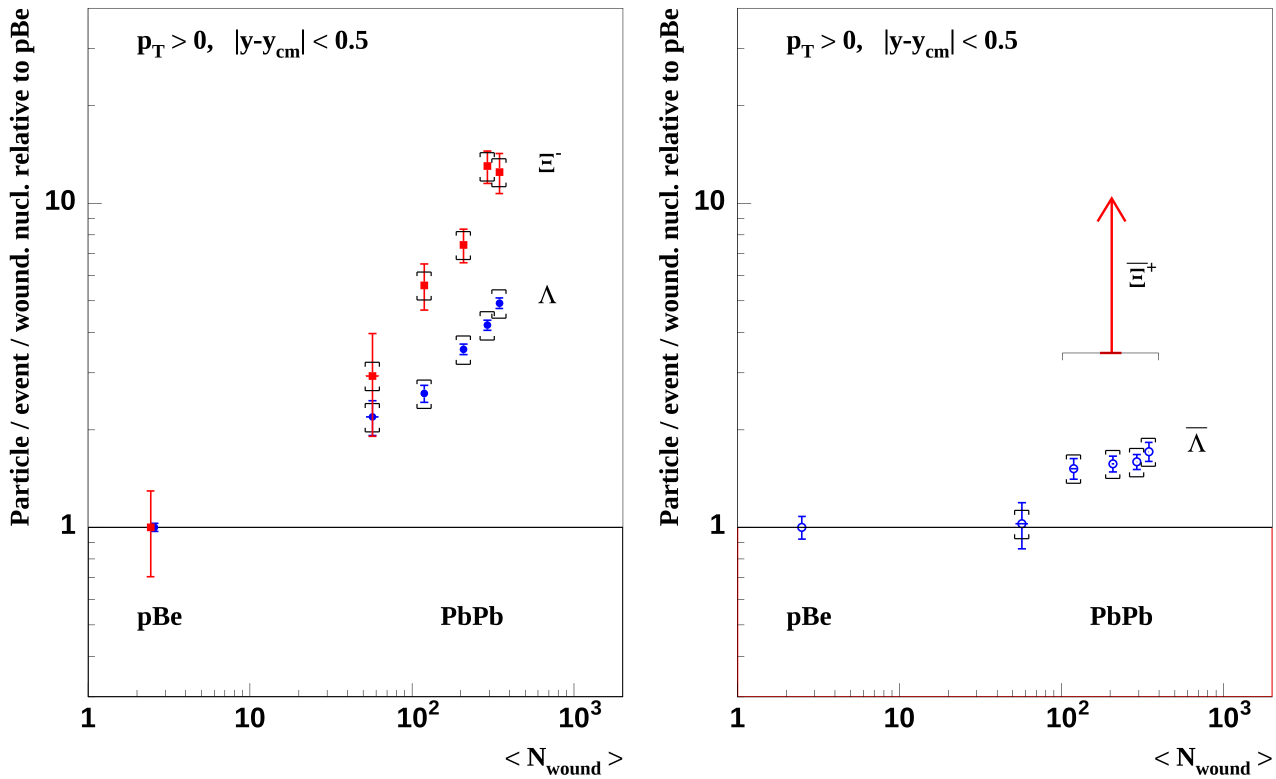
Canonical suppression and energy



- Canonical suppression $\Rightarrow$ enhancement <u>increases</u> as $\sqrt{s}$ goes <u>down</u>: Less E/V $\longrightarrow$ less heavier strange particles $\longrightarrow$ Further from thermodynamic limit

- QGP model $\Rightarrow$ enhancement <u>goes up</u> with $\sqrt{s}$, same reasons as before

NA57, 40 GeV/A P-Be,Pb,Pb

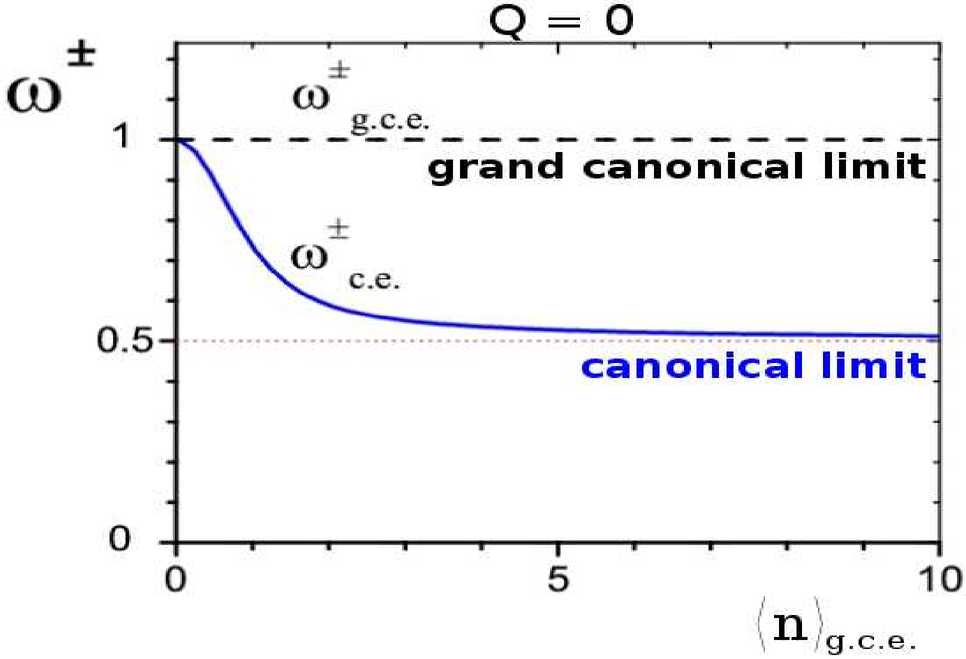


Keep an eye on new enhancement measurements at low energy!!!

Canonical suppression and fluctuations

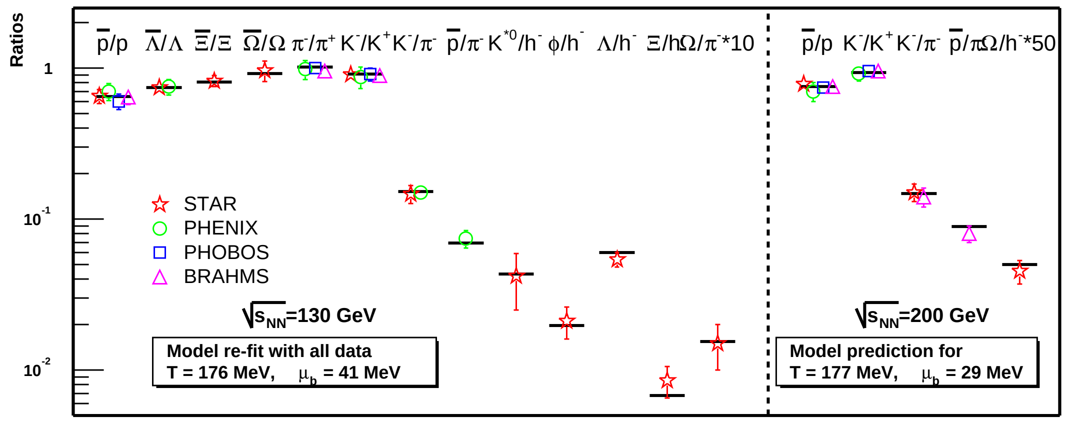
Begun, Gazdzicki, Gorenstein,Zozulya: Fluctuations are not the same in the ensembles

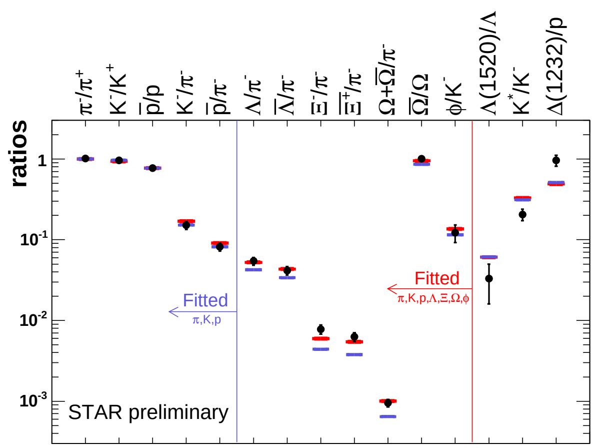
$$\lim_{V\to\infty} \omega_{CE} \neq \lim_{V\to\infty} \omega_{GCE}$$



Keep an eye on strangeness fluctuations wrt centrality/system size

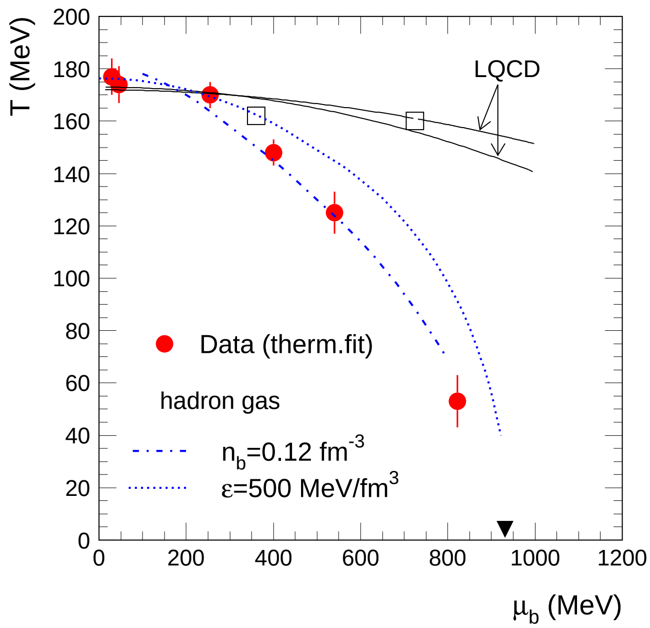
Equilibrium statistical mechanics(Braun-Muntziger, Stachel, Magestro, Florkowski, Broniowski, Redlich, Barannikova, Kaneta, Xu,Becattini,...)





Fit T,μ from particle ratios. Resonances don't fit, but authors don't expect them to (re-equilibration between thermal and chemical freeze-out).

but why are resonances <u>enhanced?!</u> More later!



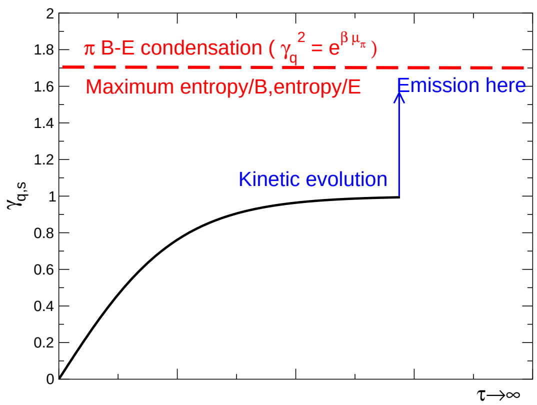
- $\sqrt{s}$ dependance expected $\frac{dT}{ds} > 0, \frac{d\mu_B}{ds} < 0$
  Freeze-out criterion, $\frac{E}{V} \sim 1GeV$? (Redlich,...)

- No temperature dependence with centrality

- "Horn" (not really) explained by Canonical effects?

- $\gamma_s^{fitted} \sim 1$, strangeness equilibration (Kaneta,Xu)

Alternatively... Supercooling+oversaturation
(chemical Non-equilibrium)

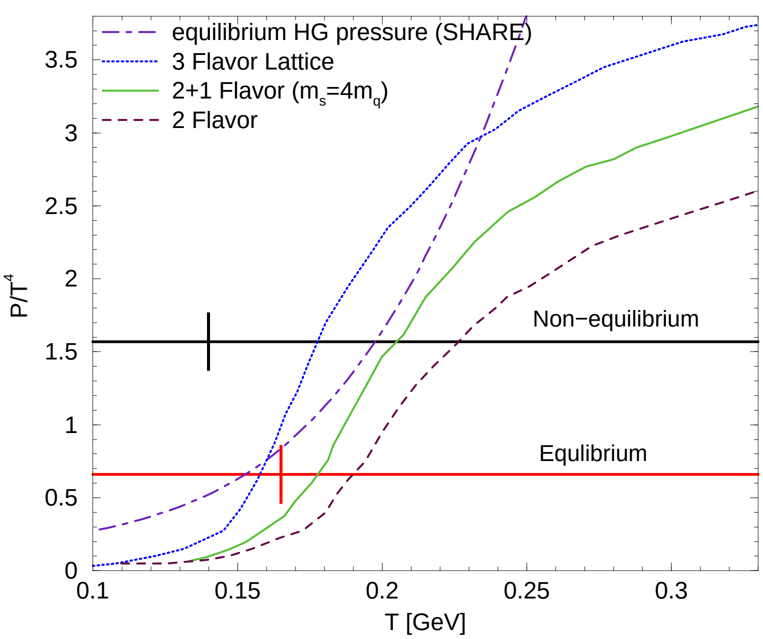


<u>inaccessible</u> by kinetic evolution
(if put "in a box", this system would heat)
<u>but</u> accessible in a fast phase transition
from a high entropy phase $\gamma_q > 1, \gamma_s/\gamma_q > 1$.

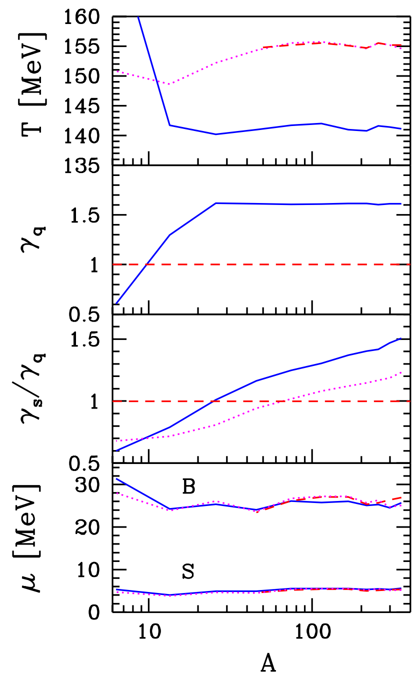
J. Rafelski, J Letessier, PRL 85:4695-4698,2000:
Explosive hadronization from supercooled QGP

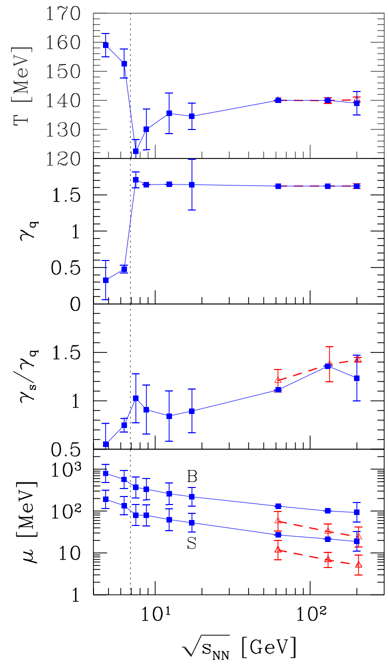
$$P_{vacuum} = P_{QGP} \qquad S_{HG} = S_{QGP} \qquad V \sim \frac{2}{3}V_{equilibrium}$$

$$T = 140MeV, \qquad \gamma_q \sim 0.9e^{m_\pi/2T} \sim 1.6$$

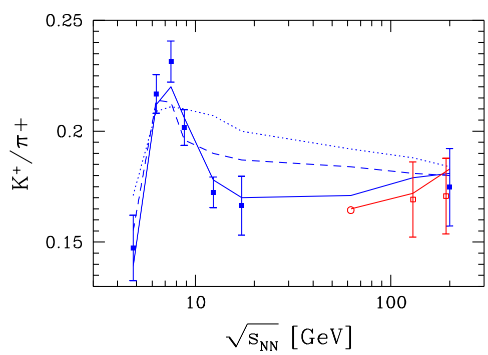


$\gamma_q > 1, T \to \sim 140 MeV$ @ critical $\sqrt{s}, N_{part}$



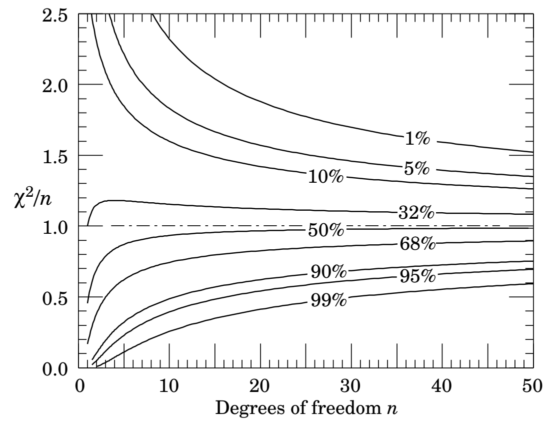


“Horn” explained by this critical point



Smoking gun 4 deconfinement?? Perhaps...

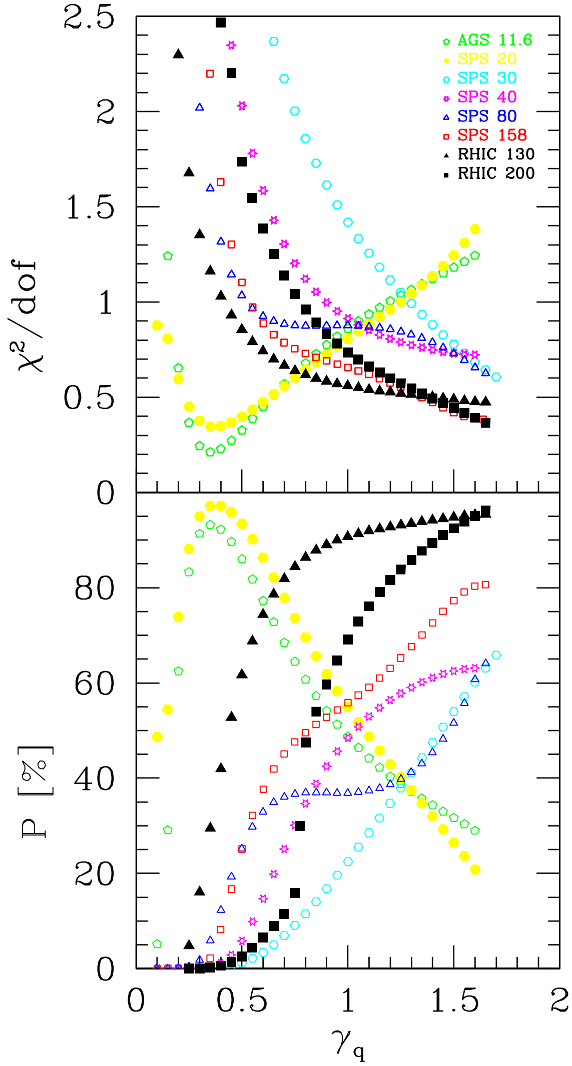
Equilibrium and non-equilibrium models have a different number of parameters.
Comparison standard: Statistical significance



- Statistical significance, the probability of getting χ^2 with n DoF given that "your model is true", is a quantitative measure of your fit's goodness

- With few DoF, "nice" looking graphs can have a very small statistical significance.

- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won't get a good statistical significance.



Maximum for SPS and RHIC is at $\gamma_q > 1$, but equilibrium not ruled out!

Need further data capable of determining γ_q.

Resonances and fluctuations

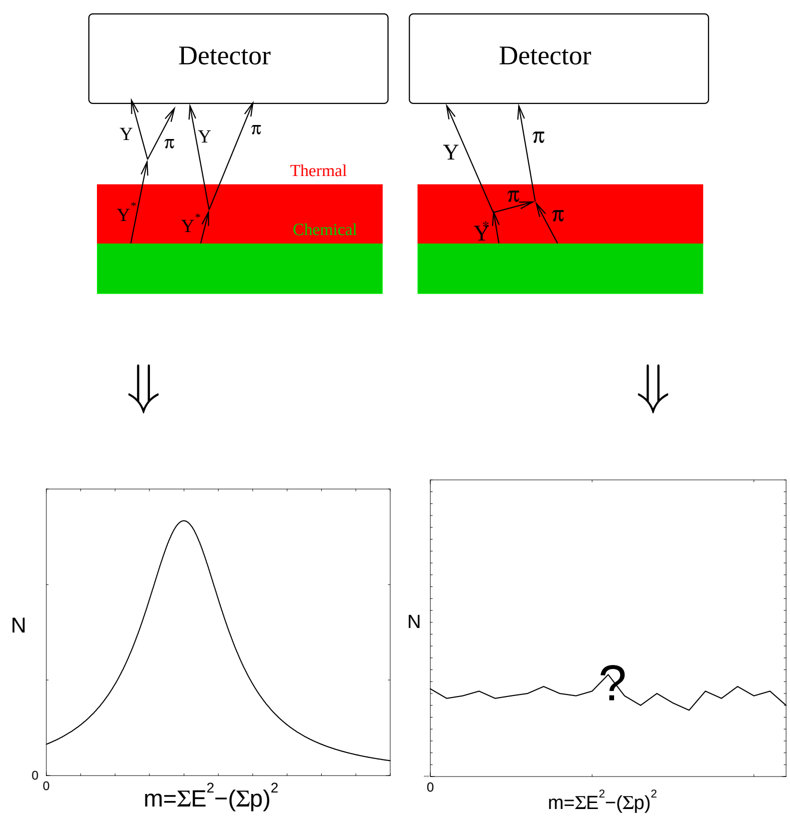
Resonances: a more careful look

Same quark composition as light particles

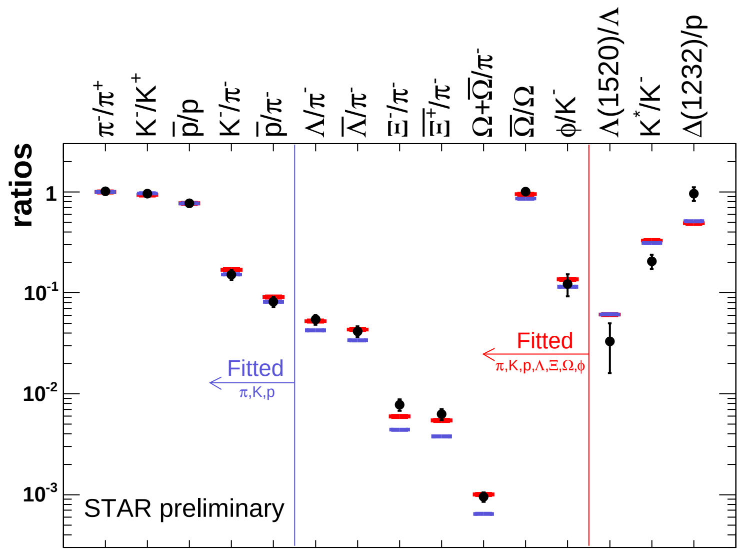
K^*/K, Σ^*/Λ,$\Lambda(1520)/\Lambda$,... All (equilibrium or not) chemical factors cancel out $\longrightarrow$ sensitive to T

but

Short-lived resonances are detected by Invariant mass reconstruction. Post-hadronization scattering of decay products makes them unobservable.



Problems with resonances in equilibrium models...



Due to

- Lower T $\leftrightarrow$ chemical non-equilibrium ($\gamma_q > 1$)
- Rescattering between T_{chem} and $T_{thermal}$?
- Both? Neither?

I'm not sure, but...

Issues to consider:

- Any re-interaction can usually only suppress resonances
  - A few → rescattering>regeneration → suppression
  - A lot → re-equilibration at lower T → suppression

  But some resonances appear ehanced!!!

- In general, rescattering will depend on Γ (dimensional analysis+optical theorem)

$$N_i\left(\frac{m_i}{T},\lambda\right) \rightarrow F\left[N_i\left(\frac{m_i}{T_{chem}},\lambda_{chem}\right),\Gamma_i\tau^{resc}\right]$$

2 resonances of different $m,\Gamma \Leftrightarrow T_{chem},\tau_{resc}$

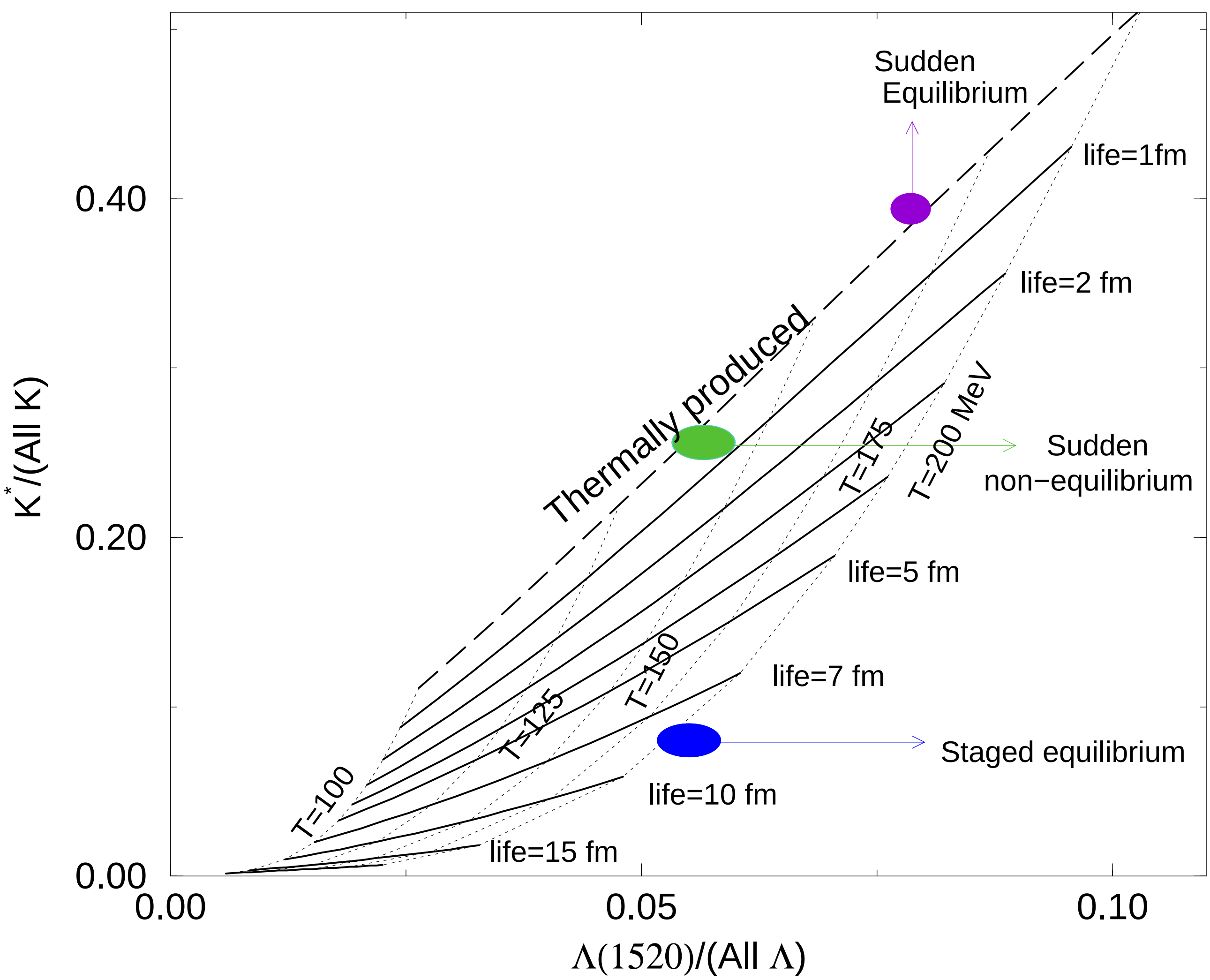
Rescattering model, GT and Rafelski, PLB, 509 239

$$\frac{dN^*}{dt} = -\Gamma N^*$$

$$\frac{d(N\pi)}{dt} = \Gamma N^* + (N\pi) < \sigma\gamma v > \frac{N_0}{V_0}\left(\frac{R_0}{R_0 + vt}\right)^3$$

- Observable $(N\pi)$ pairs created through decay and destroyed through rescattering

- Density $\frac{N_0}{V_0}$ fixed by statistical hadronization, R_0 by particle multiplicity, flow from spectral fits



But people doubt this since we neglected regeneration

Fluctuations (my current work)

Look at my poster (and talk @ ISMD) 4 details, but...

Statistical models should be able to describe both yields and fluctuations. This is because of

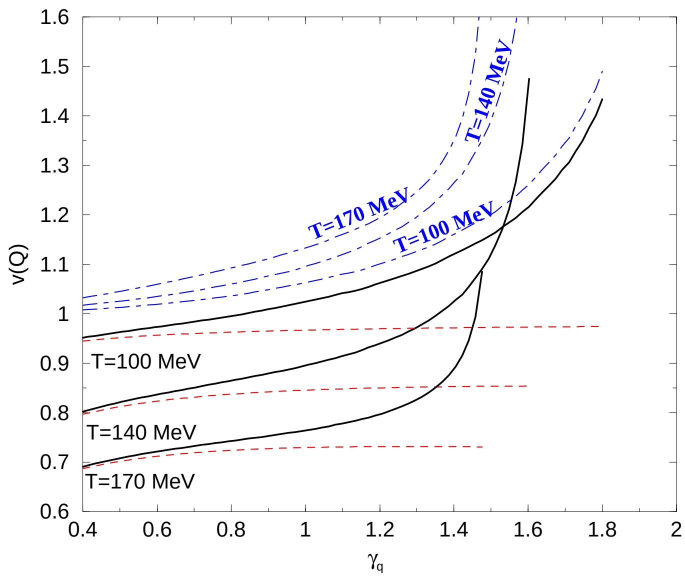
Fundamental physics Statistical mechanics predicts both. If it applies, both should be described.

Phenomenological power Simultaneus measurement of both yields and fluctuations is a powerful indicator of

- Ensemble choice
- Freeze-out temperature
- System volume
- Non-equilibrium
- Resonance rescattering/modification

Yet there is no quantitative study simultaneusly addressing fluctuations & yields... til now!

Diagnostics with fluctuations



T increase $\Rightarrow$ π Fluctuations decrease because of enhanced resonance production (Even when resonances undetectable!)

over-saturation ($\gamma_q > 1$) $\Rightarrow \pi$ Fluctuations increase because of BE corrections

ρ, σ **rescattering, regeneration, modification...** change $\pi^+\pi^-$ correlations $\Rightarrow$ Fluctuations of π^+/π^- ratio! Undetectable or rescattered resonances also work!

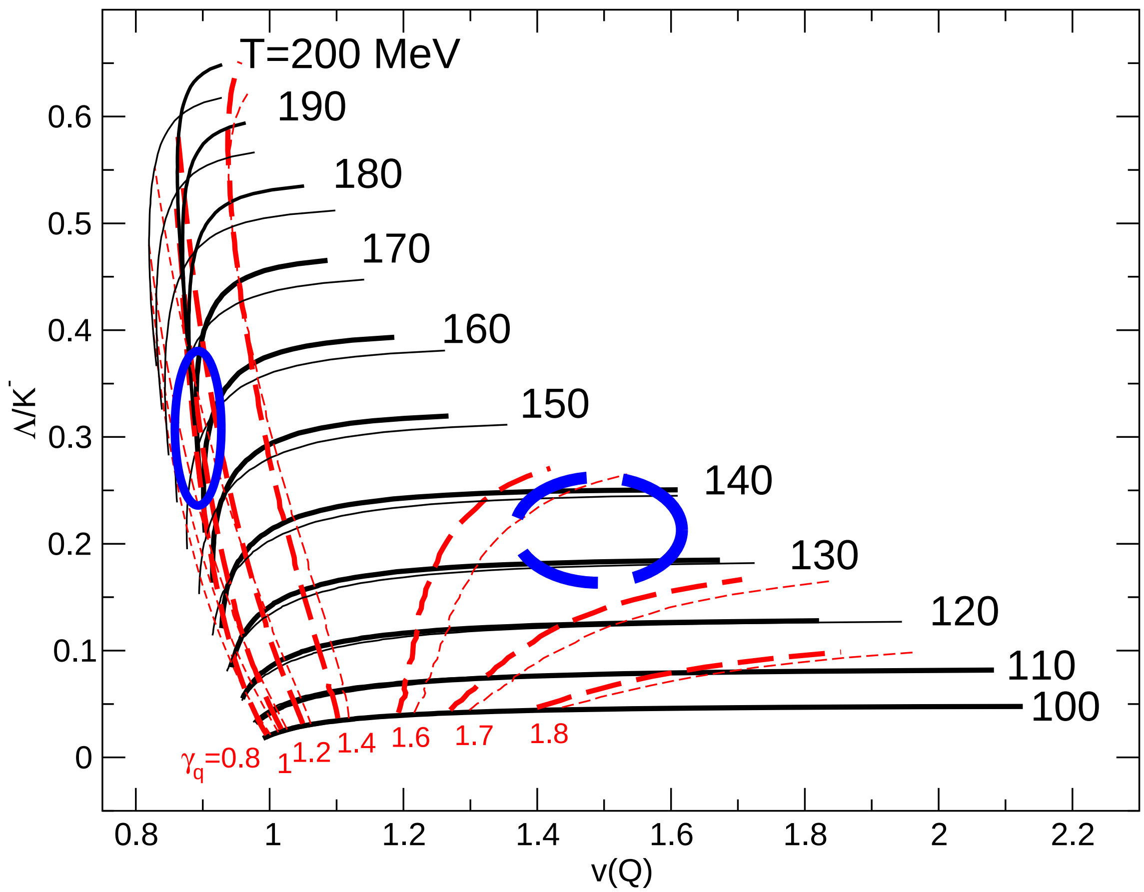
Finesse with Fluctuations I: Volume

Fluctuations are an extremely powerful tool capable,when analysed quantitatively together with yields, of falsifying every model I talked about. However, they are also very prone to systematic errors and misunderstanding.

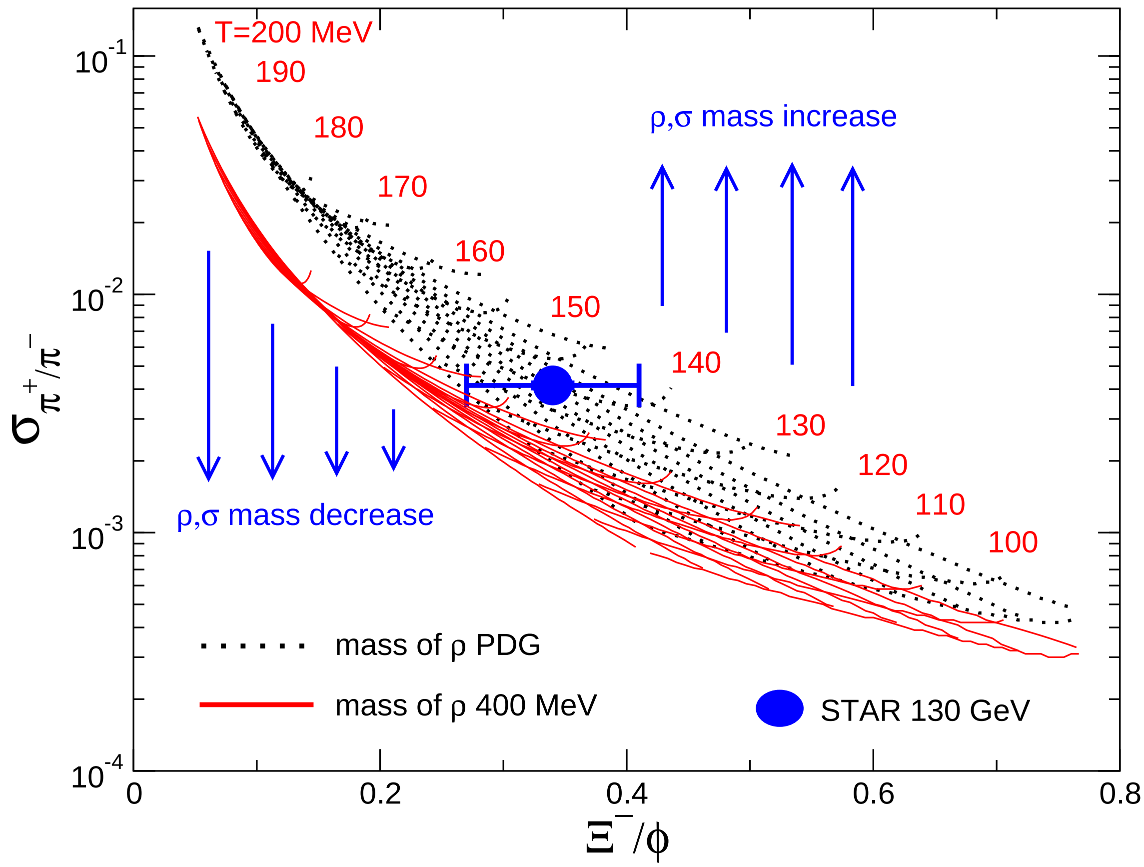
Volume fluctuations are not well understood, and show up in all $< N^2 > - < N >^2$. Avoid them choosing observables such as

- $(\Delta Q)^2$. $\frac{<Q>}{V}$small, so is $\Delta V \frac{<Q>}{V}$ (Jeon, Koch)
- Fluctuations of ratios(Jeon, Koch)
  Volume fluctuations irrelevant to 1st order
- For most other data-points can fit ΔV, $(\Delta N)^2 = V(\Delta\rho)^2 + [\Delta V < N >]^2$

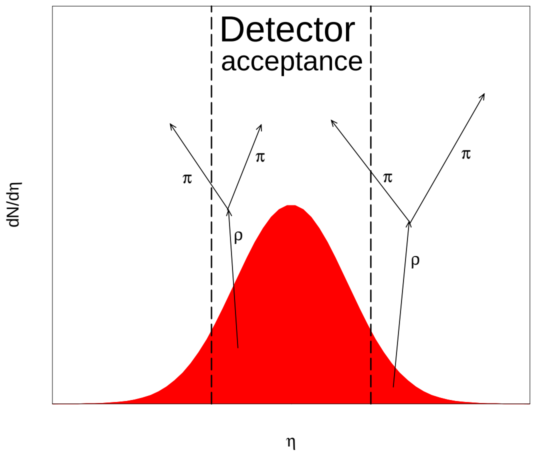
$v(Q) = \frac{<Q^2> - <Q>}{N_{ch}}$ vs Λ/K^-



σ_{π^+/π^-} (π^+/π^- fluctuation) vs Ξ^-/ϕ



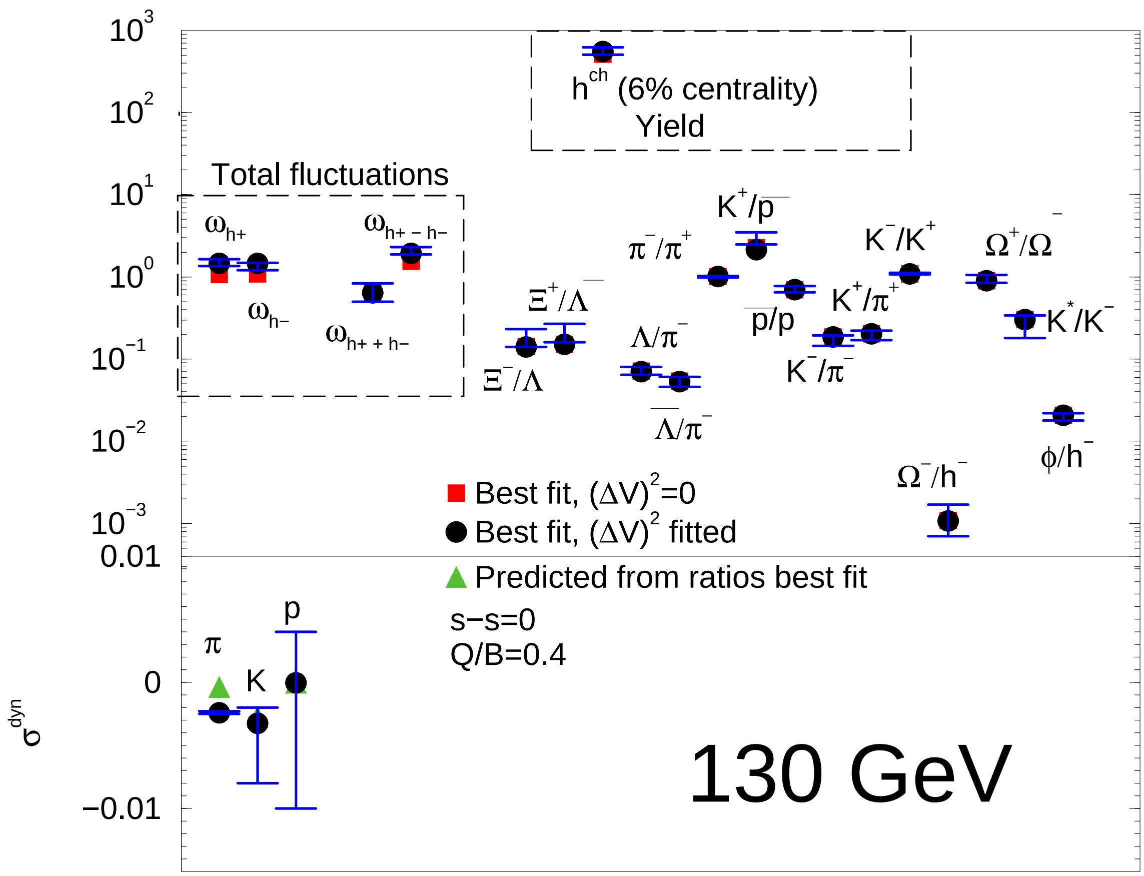
Finesse with fluctuations II:Detector response



use dynamical fluctuations $\sigma_{dyn} = \sigma - \sigma_{stat}$ Where $\sigma_{stat}$is obtained from event mixing (fake events with tracks from different events, same centrality) Poisson probability, no resonance correlations, so

$$\sigma_{stat} \sim < N >^{-1} + \sigma_{\mathrm{detector\ \ acceptance}}$$

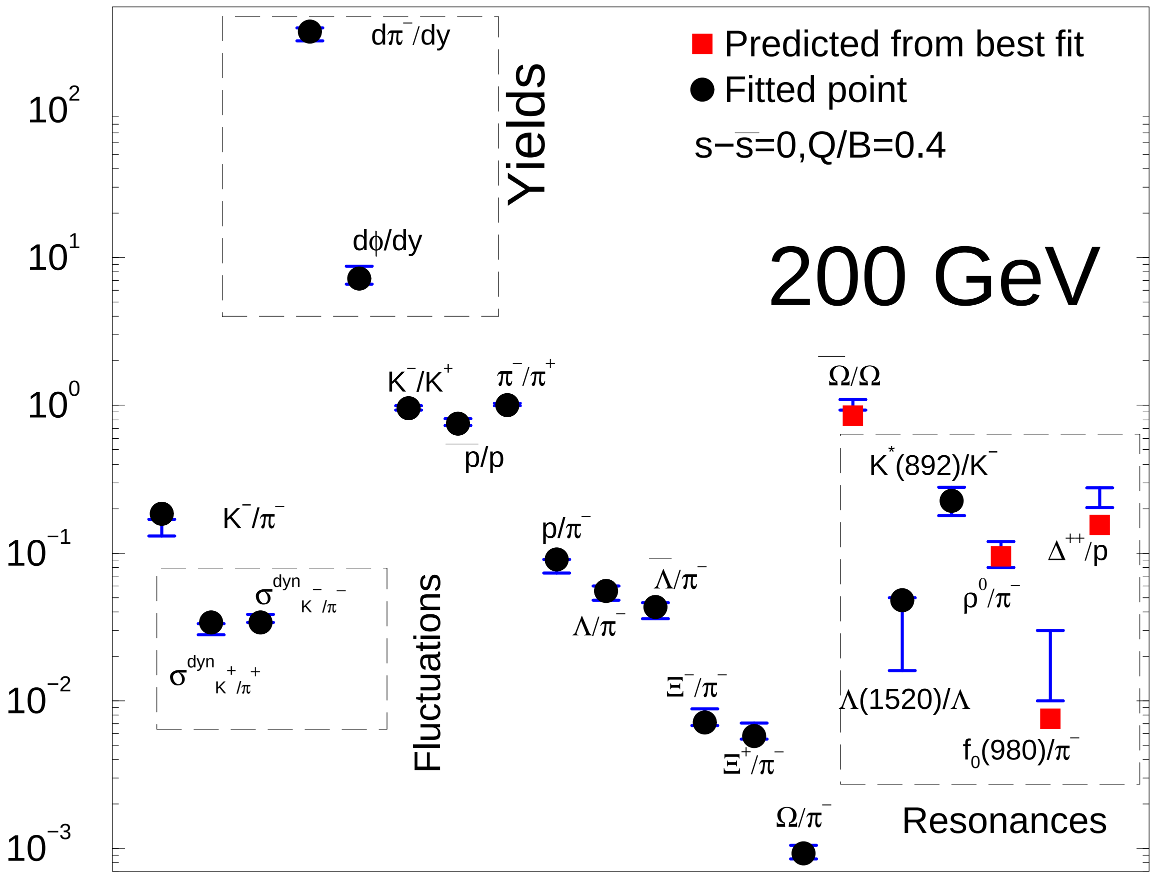
Thus, σ_{dyn} robust against detector acceptance but needs "volume" to be described.
No diagrams (3 parameters at least) but can use it in fit, including yields at same centrality as σ_{dyn}.
large acceptance $\rightarrow$ fit both σ_{dyn} and σ



$\gamma_q > 1$ fits both yields and fluctuations.

Equilibrium does not!

But see details on my poster!!!



$\gamma_q > 1$ fits yields, (most) ratios and fluctuations.

Equilibrium does not!.

Some resonances not well-described. But see details on my poster!!!

Instead of a conclusion I

- We hope that some kind of statistical production applies
  Statistical production $\rightarrow$ Experimental Study of QCD thermodynamics

- all statistical models are falsifiable.
  Data presented at this conference could well make or break some or all of them

Instead of a conclusion II:Many publically available codes calculating observables for statistical models now available

SHARE (Torrieri, Rafelski,Steinke, Florkowski, Broniowski) Yields, fluctuations and bulk quantities. Chemical non-equilibrium. Advanced fitting tools (statistical significance, parameter sensitivity, stability against data points etc.)

Thermus (Cleymans,Redlich,Wheaton) Yields within Canonical and Grand-canonican ensembles

Therminator (Florkowski, Broniowski) Spectra with statistical model and flow, Montecarlo Canonical and Grand-canonican ensembles

Coming soon (Becattini, Ferroni) Microcanonical ensemble yields and fluctuations (In development)

If you're an experimentalist, test 'em vs. your data