(A review) on cavity quantum electrodynamics from a quantum measurement perspective

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2. Quantum many-body physics in tailored reality (PRB, PRE)

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Radiation sources for atom-EM field interaction

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- UV, laser, X, syncrotron: external driving field

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Light-matter interaction in resonator

Coupled equations of motion for the system's degree of freedom

Controlled degrees of freedom in CQED

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- Maxwell–Bloch-equations (semiclassical), Heisenberg–Langevin-equations (quantum)

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- Maxwell–Lorentz–Bloch-equations, external degrees of freedom, translational motion of atoms

QED effects on the atomic structure

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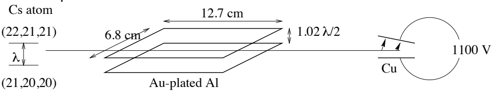
MIT setup



λ=0.45 mm

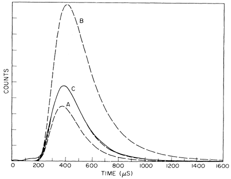


FIG. 1. Time-of-flight signals for various spontaneous decay rates. Mean time of flight $=1.5/A_0$, where A_0 is the free-space radiative decay rate. Curve A, calculated signal, enhanced decay rate $A' = \frac{3}{2}A_0$. Curve B, calculated signal, no radiative decay. Curve C, free space, $A' = A_0$; calculated (dashed line), measured (solid line).

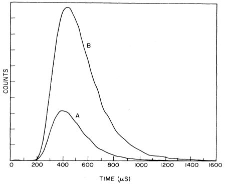
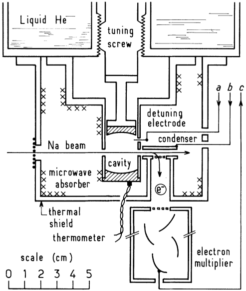


FIG. 2. Inhibited spontaneous emission. Time-of-flight data for inhibited spontaneous emission ($\lambda/2d > 1$, curve B) and enhanced spontaneous emission ($\lambda/2d < 1$, curve A) were taken simultaneously by modulation of the wavelength with an applied electric field.

ENS setup (S. Haroche & J.M. Raimond)

25× enhancement, single atoms



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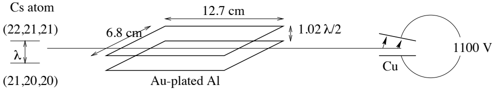

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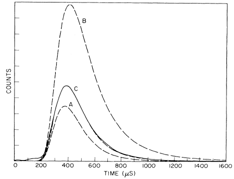

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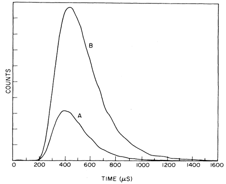

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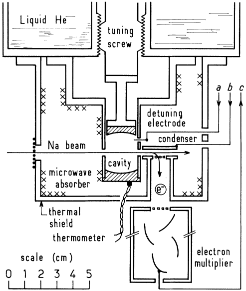
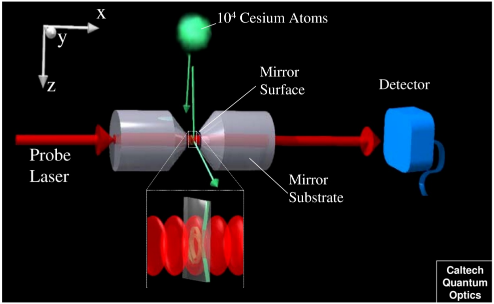


4. optical experiments 1987, M. S. Feld, reduced spont. em (-0.5%) + level shift

Generic experimental scheme



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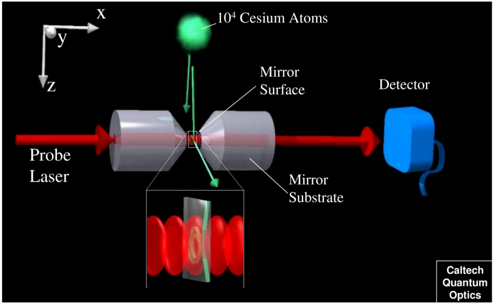

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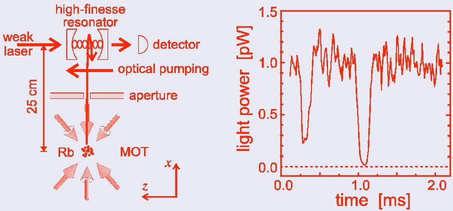

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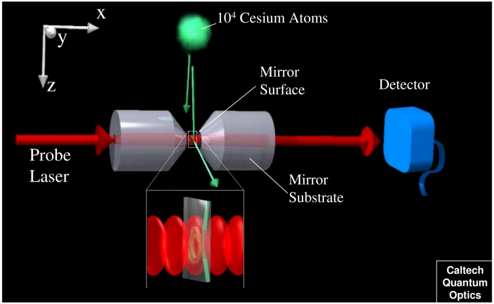

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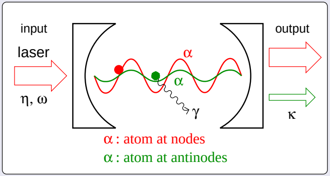

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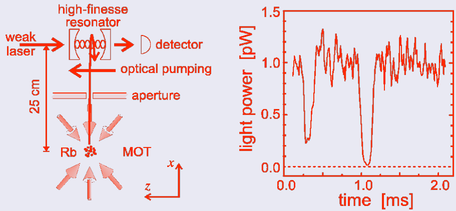

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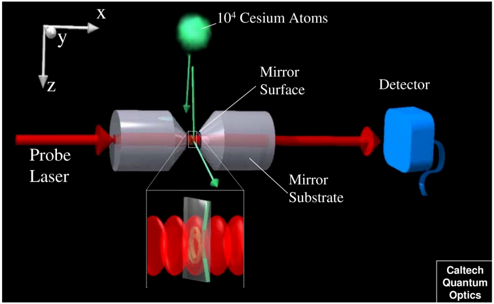

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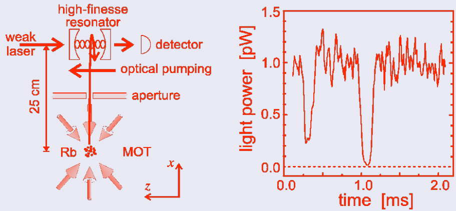

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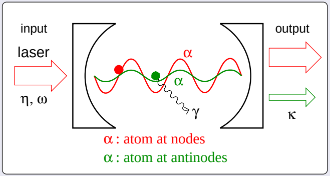
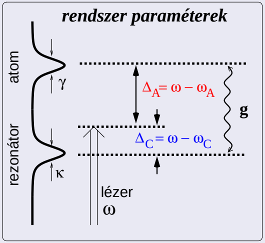
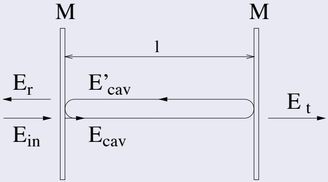


atom-photon molecule



Single-mode radiation field

1D toy model



$$M = \begin{pmatrix} \sqrt{R}e^{i\theta} & i\sqrt{T} \\ i\sqrt{T} & \sqrt{R}e^{-i\theta} \end{pmatrix}$$

$$R + T = 1$$

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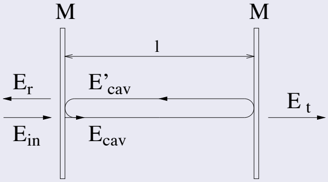

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From mode density to single mode

$$|E_{\text{cav}}|^2 = \frac{T^{-1}}{1 + \left(\frac{\mathcal{F}}{\pi}\right)^2 \sin^2\frac{\phi}{2}} |E_{\text{in}}|^2$$

Airy-function

$\mathcal{F}$inesse: $\mathcal{F} = \frac{\pi\sqrt{1-T}}{T} \gg 1$

$$n_{\text{cav}} = |\alpha|^2 = \frac{2\kappa j_{\text{in}}}{(\omega - \omega_C)^2 + \kappa^2}$$

Lorentzian

$j_{\text{in}} = \epsilon_0 |E_{\text{in}}|^2 cA/\hbar\omega$, $\kappa = Tc/2l$,
$\eta = \sqrt{2\kappa j_{\text{in}}}$

$$\dot{\alpha} = i(\omega - \omega_C)\alpha - \kappa\alpha + \eta$$

Mode (density) can be mimicked by a damped-driven harmonic oscillator

Basic model

Open quantum system

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Generic 1D model for single atom and single cavity mode

$$H = \frac{p^2}{2M} - \hbar\Delta_C a^\dagger a - \hbar\Delta_A \sigma^+\sigma^- - i\hbar g f(\hat{x})(\sigma^+ a - a^\dagger \sigma^-) - i\hbar\eta(a - a^\dagger)$$

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- many dimensions
- many modes
- many atoms

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Cooperativity

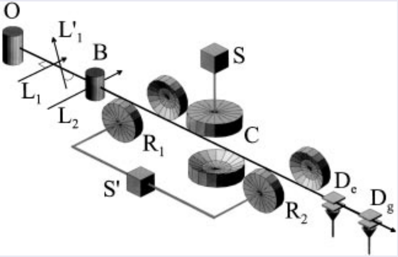
$$\frac{g^2}{\kappa\gamma} = \frac{6\mathcal{F}}{k^2 w^2} = 2\pi\mathcal{F}\frac{\sigma_A}{\mathcal{A}}$$

σ_A radiative cross section

$\mathcal{A}$ Gaussian mode c. s.

CQED in the microwave regime

LKB ENS experiments, Paris



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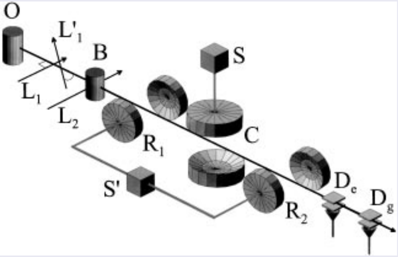
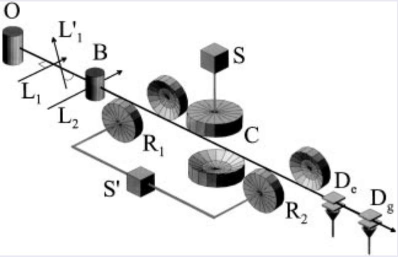


atoms

Rb, circular Rydberg states (n=51, l=50, m=50)
$|e\rangle \leftrightarrows |g\rangle$ 51.1 GHz
d=1256 a. u.
v=100–400 m/s , full path L=20 cm
efficient state-selective ionization

CQED in the microwave regime

LKB ENS experiments, Paris



atoms

Rb, circular Rydberg states (n=51, l=50, m=50)
$|e\rangle \leftrightarrows |g\rangle$ 51.1 GHz
d=1256 a. u.
v=100–400 m/s , full path L=20 cm
efficient state-selective ionization

cavity

Ni supraconducting mirrors
Fabry-Pérot resonator, TEM_{900}
l= 2.76 cm, w=6 mm, V_{eff}=770 mm^3
interaction time τ=1-10 ms
T=0.6 K

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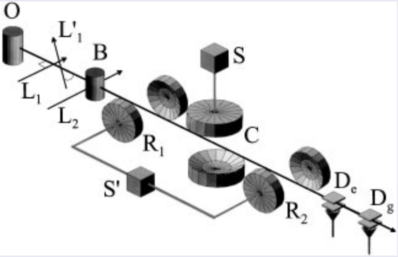

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$g = 2\pi \times 25/\mathrm{ms} \gg \gamma \approx 0.03/\mathrm{ms}$, $\kappa \approx 1$– $0.008/\mathrm{ms}$,

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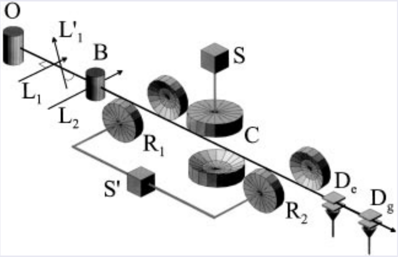

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CQED parameters

$g = 2\pi \times 25/\text{ms} \gg \gamma \approx 0.03/\text{ms}$, $\kappa \approx 1$– 0.008/ms,

Reversible, non-perturbative dynamics (small dissipation)

Pure Hamiltonian dynamics

Jaynes-Cummings Hamiltonian

$$H = -\hbar\Delta_C a^\dagger a - \hbar\Delta_A \sigma^+\sigma^- - i\hbar g\,(\sigma^+ a - a^\dagger \sigma^-)$$

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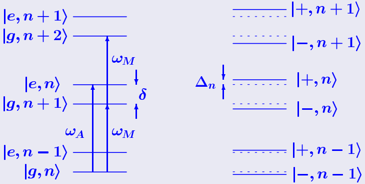
$$E_{|\pm,n\rangle} = (n+1)\hbar\omega_M \pm \hbar\sqrt{g^2(n+1) + \frac{\delta^2}{4}}$$



Pure Hamiltonian dynamics

Quantized Rabi oscillation

$|\psi(0)\rangle = |e\rangle \sum c_n |n\rangle$

$$P_e(t) = 1 - \sum_n |c_n|^2 \frac{4g^2(n+1)}{4g^2(n+1)+\delta^2} \sin^2\left(\sqrt{g^2(n+1)+\delta^2/4}\, t\right)$$

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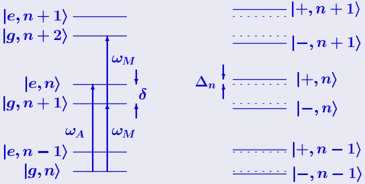

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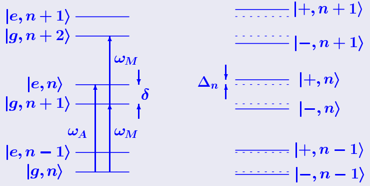


Brune et al. PRL (1996)

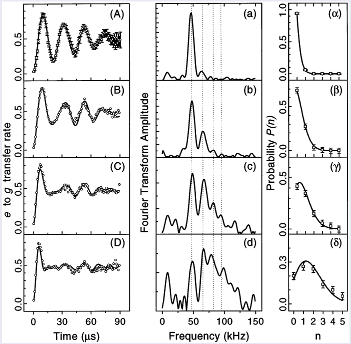
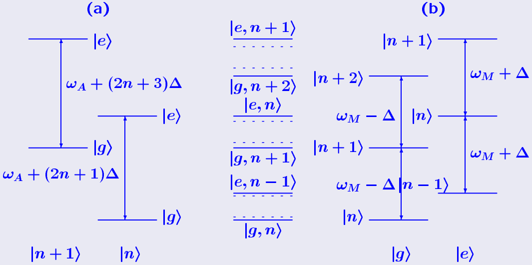


FIG. 2. (A), (B), (C), and (D): Rabi nutation signal representing $P_{e,g}(t)$, for fields with increasing amplitudes. (A) No injected field and 0.06(±0.01) thermal photon on average; (B), (C), and (D) coherent fields with 0.40(±0.02), 0.85(±0.04), and 1.77(±0.15) photons on average. The points are experimental [errors bars in (A) only for clarity]; the solid lines correspond to theoretical fits (see text). (a), (b), (c), (d) Corresponding Fourier transforms. Frequencies $\nu = 47$ kHz, $\nu\sqrt{2}$, $\nu\sqrt{3}$, and 2ν are indicated by vertical dotted lines. Vertical scales are proportional to 4, 3, 1.5, and 1 from (a) to (d). (α), (β), (γ), (δ) Corresponding photon number distribution inferred from experimental signals (points). Solid lines show the theoretical thermal (α) or coherent [(β), (γ), (δ)] distributions which best fit the data.

Non-resonant interaction

Large detuning

$\delta^2 >> g^2(n+1)$



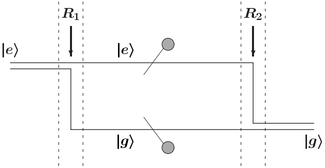
$$\hat{\mathcal{H}}_{\text{int}} \approx \hbar\Delta \left(a^\dagger a \,|g\rangle\langle g| - (a^\dagger a + 1)\,|e\rangle\langle e|\right)$$

$$\Delta = g^2/\delta$$

accumulated phase shift $\frac{g^2 t_{\text{int}}}{\delta} \geq 2\pi$

Quantum measurement models

$$\left.\begin{array}{l}|e\rangle|\alpha\rangle \to |e\rangle\left|\alpha e^{i\phi}\right\rangle \\ |g\rangle|\alpha\rangle \to |g\rangle\left|\alpha e^{-i\phi}\right\rangle\end{array}\right\} \phi = \frac{g^2}{\delta} t_{int}$$



$$|e\rangle \to \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle)$$
$$|g\rangle \to \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$$

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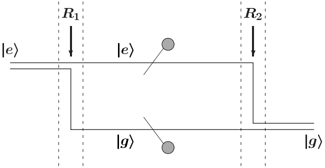

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Ramsey interference signal

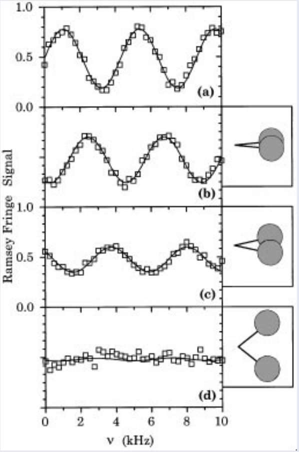

FIG. 3. $P_g^{(1)}(\nu)$ signal exhibiting Ramsey fringes: (a) C empty, $\delta/2\pi = 712$ kHz; (b)–(d) C stores a coherent field with $|\alpha| = \sqrt{9.5} = 3.1$, $\delta/2\pi = 712, 347$, and 104 kHz, respectively. Points are experimental and curves are sinusoidal fits. Insets show the phase space representation of the field components left in C.

Quantum measurement models

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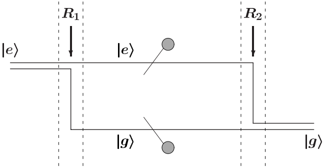

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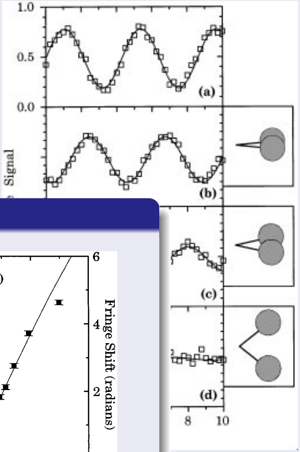
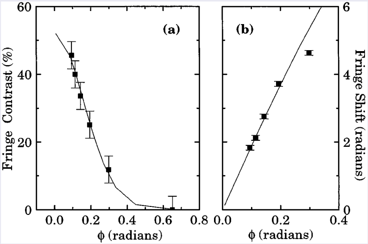


g Ramsey fringes: (a) C) C stores a coherent field 712, 347, and 104 kHz, re- al and curves are sinusoidal representation of the field

Fringe contrast



[G. 4. Fringes contrast (a) and shift (b) versus ϕ, for a ›herent field with $|\alpha| = 3.1$ (points: experiment; line: theory).

Quantum measurement models

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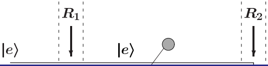


Observing decoherence

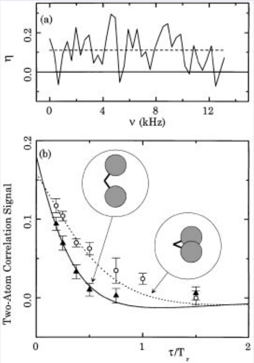
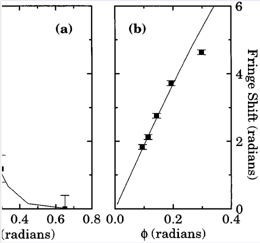


FIG. 5. (a) Two-atom correlation signal η versus ν for $n = 3.3$, $\delta/2\pi = 70$ kHz, and $\tau = 40$ μs. (b) ν-averaged η values versus τ/T_r for $\delta/2\pi = 170$ kHz (circles) and $\delta/2\pi = 70$ kHz (triangles). Dashed and solid lines are theoretical. Insets: pictorial representations of corresponding field components separated by 2ϕ.

ast



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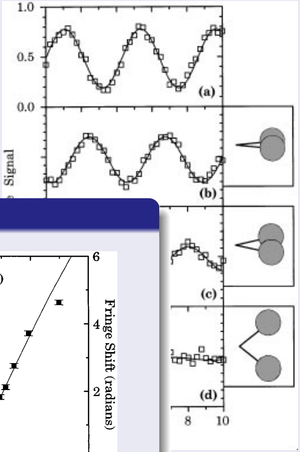

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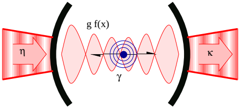
Approach 1. Minimal model



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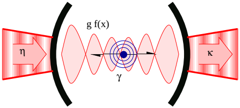

polarization

$$\dot{\sigma}^- = (i\Delta_A - \gamma)\sigma^- + 2gf(\hat{x})\sigma_z a + \xi_-$$

Adiabatic elimination of the internal atomic dynamics

$$\sigma^- \approx -\frac{i\Delta_A + \gamma}{\Delta_A^2 + \gamma^2} gf(\hat{x})a$$

noise neglected
saturation is low: $\sigma_z = -1/2$

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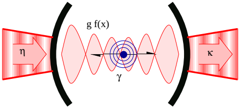

Parameters

$$U_0 = -\frac{\omega_C}{V}\chi' = \frac{g^2\Delta_A}{\Delta_A^2+\gamma^2}\,, \quad \Gamma_0 = -\frac{\omega_C}{V}\chi'' = \gamma\frac{g^2}{\Delta_A^2+\gamma^2}$$

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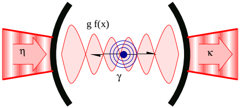

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Effective von Neumann equation

$$H = \frac{\hat{p}^2}{2M} - \hbar\Delta_C a^\dagger a + \hbar U_0 f^2(\hat{x})a^\dagger a - i\hbar\eta(a - a^\dagger)$$

$$\mathcal{L}\rho = -\kappa\left(a^\dagger a\rho + \rho a^\dagger a - 2a\rho a^\dagger\right) - \Gamma_0\Big(f^2(\hat{x})a^\dagger a\rho + \rho f^2(\hat{x})a^\dagger - 2\int_{-1}^{1} du N(u) a f(\hat{x}) e^{-iu\hat{x}} \rho e^{iu\hat{x}} a^\dagger f(\hat{x}) du\Big)$$

This minimal model is ‘exact’ for a linearly polarizable particle

Approach 1a. Brute force quantum solution

On a grid of 128 points
Fock space up to $|20\rangle$

$\Big\}$ Monte Carlo wavefunction method works quite well

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On a grid of 128 points, Fock space up to $|20\rangle$ } Monte Carlo wavefunction method works quite well

But

- one dimensional motion
- single atom
- low photon number

Approach 1b. Semiclassical approximation

Joint atom–field Wigner function

$$\chi(\sigma,\tau,\xi,\xi^*) = Tr\left[\hat{\rho}\exp\left\{\xi\hat{a}^\dagger - \xi^*\hat{a} + i/\hbar(\sigma\hat{x}+\tau\hat{p})\right\}\right]$$

$$W(x,p,\alpha,\alpha^*) = \frac{1}{\pi(2\pi\hbar)^2}\int \chi(\sigma,\tau,\xi,\xi^*)\exp\{-(\xi\alpha^* - \xi^*\alpha + i/\hbar(\sigma x + \tau p))\}$$

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$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}\rho \longrightarrow$$

$$\frac{d}{dt}W(x,p,\alpha,\alpha^*) = \mathrm{PDE}\left(\alpha,\frac{\partial}{\partial\alpha},x,p,\frac{\partial}{\partial x},\frac{\partial}{\partial p}\right)W(x,p,\alpha,\alpha^*)$$

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Rules

$$a\rho \rightarrow \left(\alpha + \frac{1}{2}\frac{\partial}{\partial\alpha^*}\right)W(\alpha)$$

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similar rules for $\hat{x}\rho$, $\rho\hat{x}$, ...

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Approximations

- 'semiclassical' motion : $\Delta p \gg \hbar k$ (cold atoms)
- semiclassical field: $|\alpha|^2 \gg \frac{\partial^2}{\partial\alpha\partial\alpha^*}$
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Equivalent Langevin equations

$$\dot x = \frac{p}{M}$$

$$\dot p = -\hbar U_0\left(|\alpha|^2 - \frac{1}{2}\right)\nabla f^2(x) + \xi_p$$

$$\dot\alpha = \eta - i\left(U_0 f^2(x) - \Delta_C\right)\alpha - \left(\kappa + \Gamma_0 f^2(x)\right)\alpha + \xi_\alpha$$

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$$\dot\alpha = \eta - i\left(U_0 f^2(x) - \Delta_C\right)\alpha - \left(\kappa + \Gamma_0 f^2(x)\right)\alpha + \xi_\alpha$$

≡ classical equations + quantum noise

Approximations

- ‘semiclassical’ motion : $\Delta p \gg \hbar k$ (cold atoms)
- semiclassical field: $|\alpha|^2 \gg \frac{\partial^2}{\partial\alpha\partial\alpha^*}$
  for coherent state $\frac{\partial^2}{\partial\alpha\partial\alpha^*} \sim 1$
- truncate at second order ∼ match the closest classical probabilistic process

Approach 1b. Semiclassical approximation

Joint atom–field Wigner function

$$\chi(\sigma,\tau,\xi,\xi^*) = Tr\left[\hat\rho \exp\left\{\xi\hat a^\dagger - \xi^*\hat a + i/\hbar(\sigma\hat x + \tau\hat p)\right\}\right]$$

$$W(x,p,\alpha,\alpha^*) = \frac{1}{\pi(2\pi\hbar)^2}\int \chi(\sigma,\tau,\xi,\xi^*)\exp\{-(\xi\alpha^* - \xi^*\alpha + i/\hbar(\sigma x + \tau p))\}$$

$$\dot\rho = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}\rho \longrightarrow$$

$$\frac{d}{dt}W(x,p,\alpha,\alpha^*) = \mathrm{PDE}\left(\alpha, \frac{\partial}{\partial\alpha}, x, p, \frac{\partial}{\partial x}, \frac{\partial}{\partial p}\right) W(x,p,\alpha,\alpha^*)$$

Rules

$$a\rho \to \left(\alpha + \frac{1}{2}\frac{\partial}{\partial\alpha^*}\right)W(\alpha)$$

$$a^\dagger\rho \to \left(\alpha^* - \frac{1}{2}\frac{\partial}{\partial\alpha}\right)W(\alpha)$$

$$\rho a \to \left(\alpha - \frac{1}{2}\frac{\partial}{\partial\alpha^*}\right)W(\alpha)$$

$$\rho a^\dagger \to \left(\alpha^* + \frac{1}{2}\frac{\partial}{\partial\alpha}\right)W(\alpha)$$

similar rules for $\hat x\rho$, $\rho\hat x$, ...

Equivalent Langevin equations

$$\dot x = \frac{p}{M}$$

$$\dot p = -\hbar U_0\left(|\alpha|^2 - \frac{1}{2}\right)\nabla f^2(x) + \xi_p$$

$$\dot\alpha = \eta - i\left(U_0 f^2(x) - \Delta_C\right)\alpha - \left(\kappa + \Gamma_0 f^2(x)\right)\alpha + \xi_\alpha$$

$\Longrightarrow$ force depends not only on the position but also on the velocity

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  for coherent state $\frac{\partial^2}{\partial\alpha\partial\alpha^*} \sim 1$
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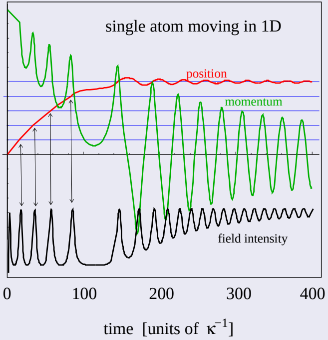
Correlated dynamics of the atom and the field mode

Simulation of noiseless motion

$$\left.\begin{aligned} \Delta_C &= -4\kappa \\ U_0 &= -3\kappa \end{aligned}\right\} \quad |\Delta_C - U_0| = \kappa$$

$$\eta = 1.5\kappa$$

$$\gamma = 0.1\kappa \qquad \text{(negligible)}$$



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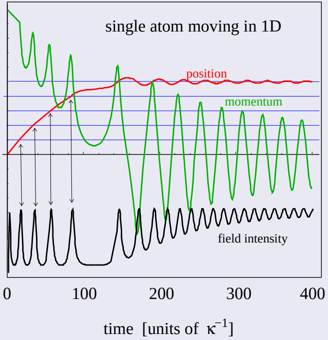
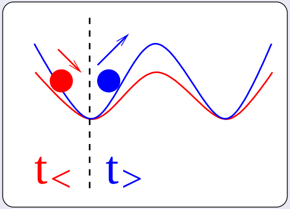


Sisyphus interpretation

Cooling can be attributed to the time lag with which the field adapts itself to the momentary position of the atom.



Diffusion matrix

$$d\xi_{\parallel} = \frac{\alpha_r}{|\alpha|} d\xi_r + \frac{\alpha_i}{|\alpha|} d\xi_i \text{ (amplitude noise)}, \quad d\xi_{\perp} = -\frac{\alpha_i}{|\alpha|} d\xi_r + \frac{\alpha_r}{|\alpha|} d\xi_i \text{ (phase noise)}$$

$$\mathbf{D}\, dt = \left\langle \begin{pmatrix} d\xi_{\parallel} \\ d\xi_{\perp} \\ d\xi_p \end{pmatrix} \left(d\xi_{\parallel},\, d\xi_{\perp},\, d\xi_p \right) \right\rangle = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_1 & d_3 \\ 0 & d_3 & d_2 \end{pmatrix} dt$$

$$\begin{aligned} d_1 &= \frac{1}{2}\left(\kappa + \Gamma_0 f^2(x)\right) \\ d_2 &= 2\Gamma_0 \left(|\alpha|^2 - \frac{1}{2}\right)\left((\hbar \nabla f(x))^2 + \hbar^2 k^2 \bar{u^2} f^2(x)\right) \\ d_3 &= \Gamma_0 |\alpha| \hbar f(x) \nabla f(x) \end{aligned}$$

Domokos, Horak, Ritsch, J. Phys. B 2001

Quantum vs. semiclassical solution

parameters

$\Delta_A = -20\gamma$
$\Delta_C = U_0 = -0.312\gamma$
$g = 2.5\gamma$

Vukics, Janszky, Domokos, J. Phys. B38, 1453 (2005)

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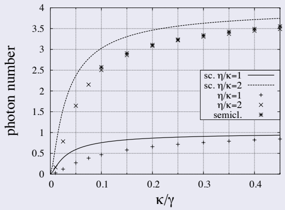

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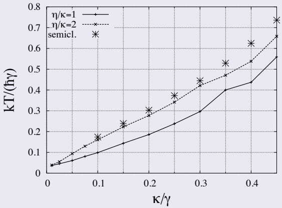
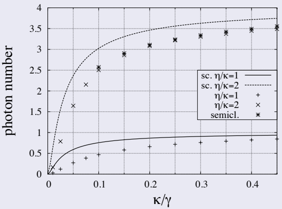


general cavcool result:

$$k_B T \approx \hbar\kappa$$

photon number



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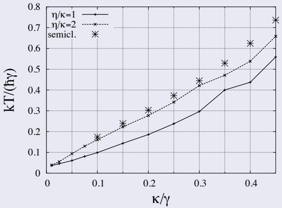

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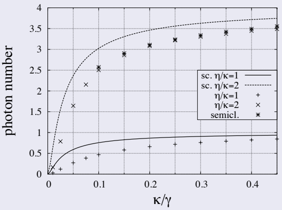
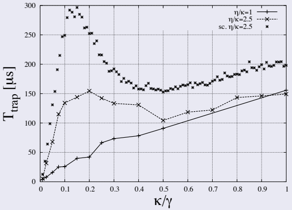


trapping time



Cavity cooling

Idea of cavity cooling

In the strongly coupled dynamics of a moving dipole and the cavity field every available dissipation channel is shared by the components.

Cooling by photon loss κ

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Promises

- temperature not limited by γ

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Cavity cooling

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Cooling by photon loss κ

Promises

- temperature not limited by γ
- cooling molecules
- exempt from spontaneous rescattering
  $\Rightarrow$ cooling ensembles

Approach II. 'Analytic' model

semiclassical motion (slow)

Langevin–equation

$$\dot{x} = p/m$$
$$\dot{p} = f + \beta p/m + \Xi$$
$$\text{where} \quad \langle \Xi(t_1)\Xi(t_2)\rangle = D\delta(t_1 - t_2)$$

Aim: determine the parameters from the internal dynamics

Horak, Hechenblaikner, Gheri, Ritsch, prl 1997; pra 1998

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Force operator

$$\hat{F} = \dot{p} = -\frac{i}{\hbar}[p, H]$$
$$= -ig\,\frac{\partial f(x)}{\partial x}(\sigma^\dagger a - a^\dagger \sigma)$$

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Aim: determine the parameters from the internal dynamics

Internal dynamics (fast)

x is a parameter

$$\dot{a} = (i\Delta_C - \kappa_n)a + g(x)\sigma + \eta + \xi$$
$$\dot{\sigma} = (i\Delta_A - \gamma)\sigma + 2g(x)\sigma_z a + \zeta$$
$$\dot{\sigma}_z = -g(x)\left(\sigma^\dagger a + a^\dagger \sigma\right) - 2\gamma(\sigma^z + 1/2) + \zeta^z$$

Force operator

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linearisation

$$\sigma_z a \approx -\frac{1}{2}a$$

- for $\langle \sigma_z \rangle \approx -\frac{1}{2}$, or
- for subspace $\{|g,0\rangle, |g,1\rangle, |e,0\rangle\}$

Friction

expansion

$$x \to x(t) \approx x + vt$$

$$\frac{d}{dt} \to \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}$$

$$a_{\mathrm{ss}}(x, v) = a^{(0)}(x) + va^{(1)}(x) + \mathcal{O}(v^2)$$

$$\sigma_{\mathrm{ss}}(x, v) = \sigma^{(0)}(x) + v\sigma^{(1)}(x) + \mathcal{O}(v^2)$$

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Quantum Bloch–equations to linear order in velocity

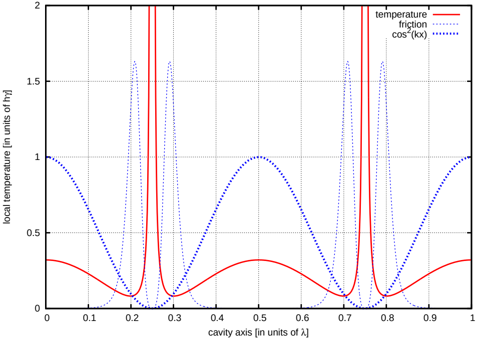
$$\frac{\partial}{\partial x}a^{(0)} = (i\Delta_C - \kappa_n)a^{(1)} + g(x)\sigma^{(1)}$$
$$\frac{\partial}{\partial x}\sigma^{(0)} = (i\Delta_A - \gamma)\sigma^{(1)} - g(x)a^{(1)}$$

linear friction coefficient (analytical)

$$\beta = \quad -ig\,\tfrac{\partial f(x)}{\partial x}\left(\sigma^{(0)\dagger}a^{(1)} - a^{(1)\dagger}\sigma^{(0)}\right) \quad \text{non-adiabatic field}$$
$$\quad -ig\,\tfrac{\partial f(x)}{\partial x}\left(\sigma^{(1)\dagger}a^{(0)} - a^{(0)\dagger}\sigma^{(1)}\right) \quad \text{non-adiabatic atom}$$

Local friction coefficient

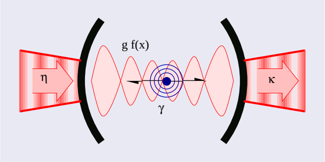
$\Delta_C = 0$, $\Delta_A = 10\gamma$, $g = 4\gamma$, $\kappa = \gamma/6$



need for averaging
What is the distribution?

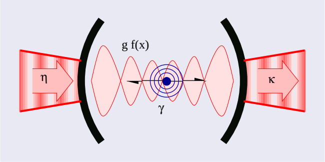
Far off resonance trap (FORT)

optical lattice potential



Far off resonance trap (FORT)

optical lattice potential



Limit of large detuning

- spontaneous photon scattering rate $2\gamma P_e \propto \Omega^2/\Delta_A^2$
- optical potential depth $U \propto \Omega^2/\Delta_A$
- Friction and diffusion are slow $\rightarrow$ almost conservative potential

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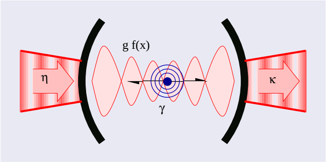

equilibrium

$$k_B T_{\text{Doppler}} = \frac{\hbar\gamma}{2}\left(\frac{\Delta_A}{\gamma} + \frac{\gamma}{\Delta_A}\right)$$

$$k_B T_{\text{FORT}} = \hbar\Delta_A/2 \gg U$$

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Far off-resonance trap in a cavity

free space

$$k_B T_{\mathrm{FORT}} = \hbar \Delta_A / 2$$

switching on cavity

$$\Omega^2 = g^2 \langle a^\dagger a \rangle \propto \frac{N_{phot}}{\mathcal{V}}$$

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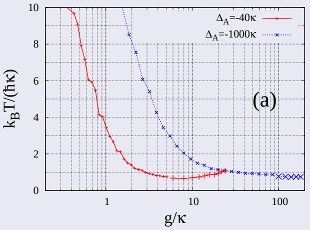
temperature in a cavity



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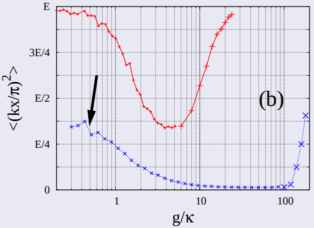
localisation



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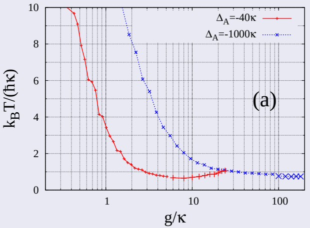

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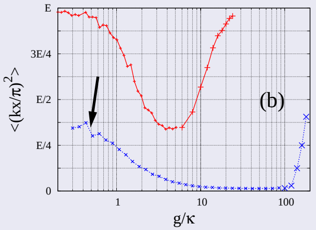
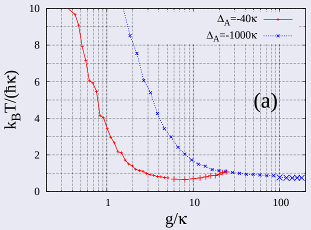
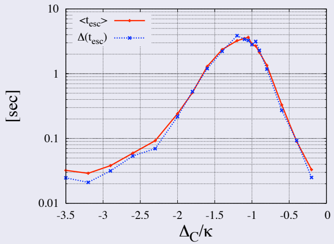


temperature in a cavity

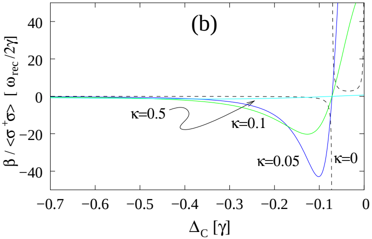


capture time



Far off-resonance trap in a cavity

$$\frac{\beta}{2\gamma P_e} = \frac{\hbar k^2}{2m\gamma} 4\sin^2(kx) \frac{2g^2(\Delta_C - U_0\cos^2(kx))(\kappa + \Gamma_0\cos^2(kx))}{\left((\Delta_C - U_0\cos^2(kx))^2 + (\kappa + \Gamma_0\cos^2(kx))^2\right)^2}$$



molecool: spontaneous scattering events (rate of $2\gamma P_e$) are likely to lead out from the space $\Rightarrow$ large cooperativity is needed

Vukics, Domokos, pra 2005; K. Murr et al. pra, 2007
P. Domokos, A. Vukics, and H. Ritsch, Phys. Rev. Lett. **92**, 103601, 2004.

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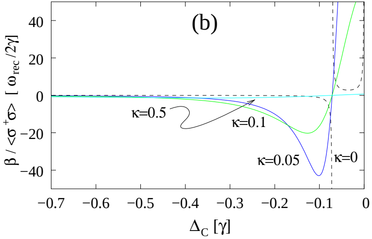

Optimum detuning

$$\Delta_C \approx -\kappa - \Gamma_0 + U_0 \approx -\kappa + U_0$$

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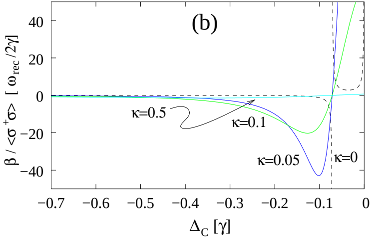

Friction is independent of detuning Δ_A

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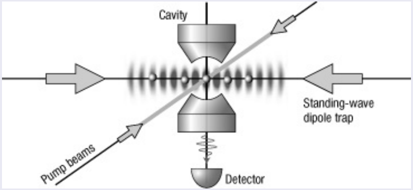
Capturing single atoms for long times

optical transport



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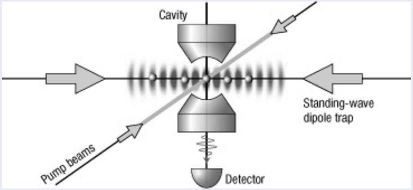


forces

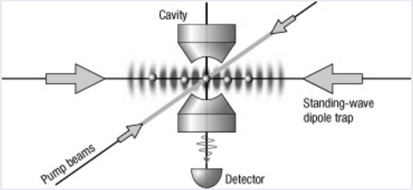
$$\mathcal{H} = ... + (\Delta_A + V \cos kz)\sigma^\dagger \sigma$$

van Enk et al. pra 2001
K. Murr et al, pra 2006

S. Nussmann, et al (Garching, A. Kuhn, G. Rempe), prl, nature phys, 2006

Capturing single atoms for long times

optical transport



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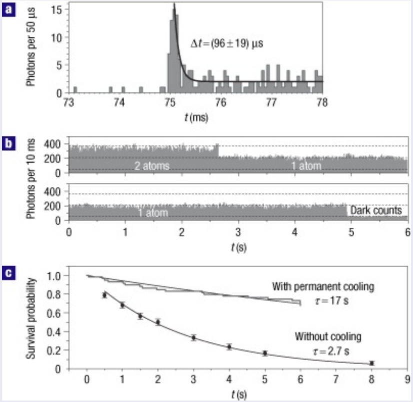

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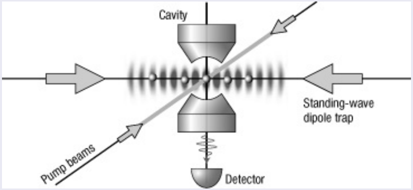
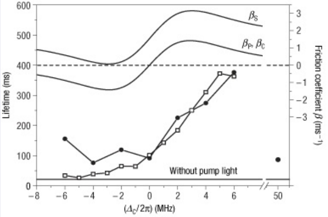


cavity cooling



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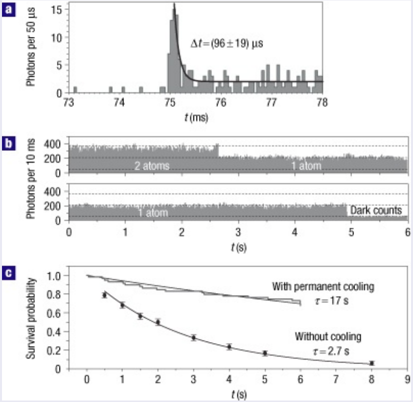

Many-body physics with cold and ultracold atoms

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   - Feschbach resonance: tuning from weak coupling to strongly correlated matter
   - specific: atoms interacting through the EM radiation field
   - Contrast to collisions, ion crystal, dipolar gas: global, long-range coupling

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   - mean-field model

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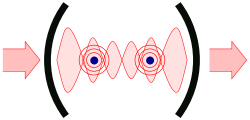

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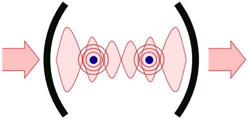
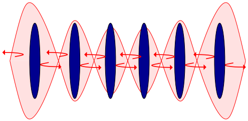
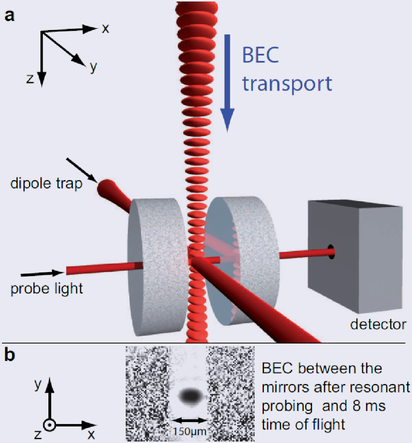


3. Collective effects in free space: opto-mechanical coupling in an optical lattice
   - Bragg-mirror regime
   - collective excitations
     $\Rightarrow$ density waves
   - dynamical instability



Many-body physics of atoms in optical resonators

BEC in a cavity



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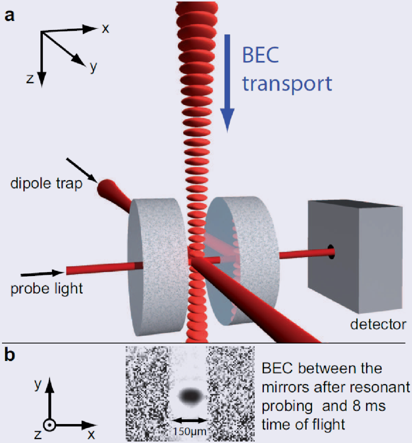

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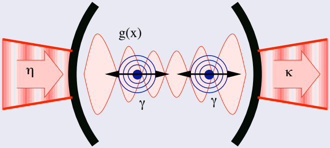

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- long-range
- global coupling (Kuramoto model)

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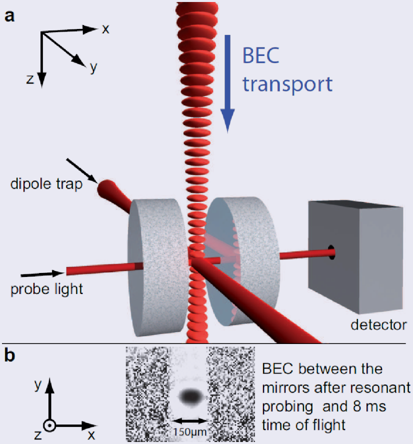

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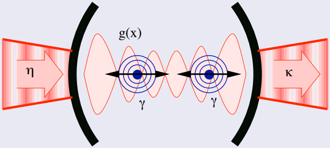

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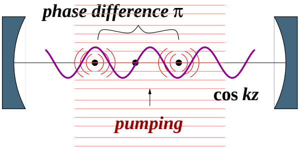
Experiments

- Esslinger (ETH, Zürich), Stamper-Kurn (Berkeley)
- Hemmerich (Hamburg), Zimmermann (Tübingen)

Scattering into the cavity



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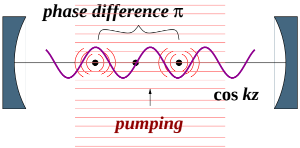

atom-atom coupling by interference

$|x_1 - x_2| = (2n+1)\,\lambda/2 \rightarrow$ destructive interference
$\rightarrow |\alpha|^2 = 0$

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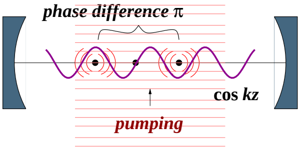

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Spatial self-organization of atom clouds

P. Domokos, H. Ritsch, PRL 89, 253003 (2002), Black, Chan, Vuletic, PRL 91, 203001 (2003)

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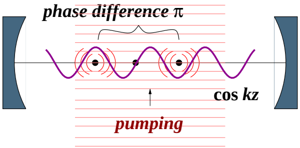

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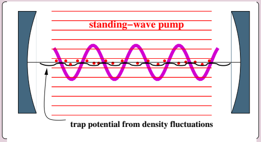

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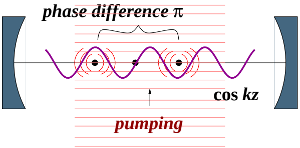

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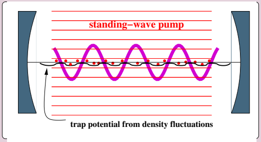

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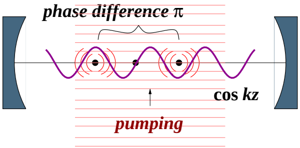

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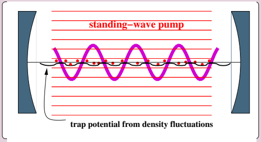

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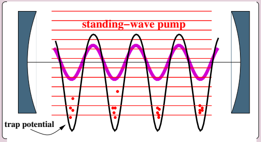
crystalline order



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Quantized atom field in a single-mode resonator

One-dimensional toy model for coupled matter and light fields

$$H = -\Delta_C \, \hat{a}^\dagger \hat{a} + i\eta \, (\hat{a}^\dagger - \hat{a}) + \int \hat{\Psi}^\dagger(x) \Big[-\frac{\hbar}{2\,m} \frac{d^2}{dx^2} + N g_c \hat{\Psi}^\dagger(x) \hat{\Psi}(x)$$

$$+ U_0 \, \hat{a}^\dagger \hat{a} \cos^2(kx) + i\eta_t \cos kx (\hat{a}^\dagger - \hat{a}) \Big] \hat{\Psi}(x) dx,$$

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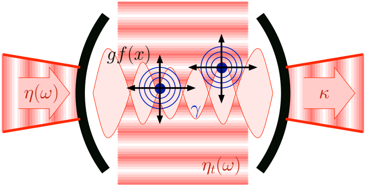

scattering processes (four-wave mixing)

1. absorption and induced emission of cavity photons
2. absorption of a pump photon and emission into the cavity

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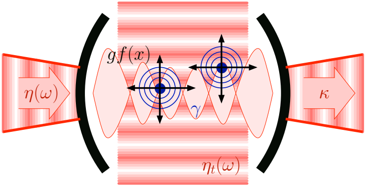

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dissipation and noise

$$\frac{d}{dt}\hat{a} = -\frac{i}{\hbar}[\hat{a}, H] - \kappa\hat{a} + \hat{\xi} \qquad \langle\hat{\xi}(t)\hat{\xi}^\dagger(t')\rangle = \kappa\delta(t - t')\;.$$

Mean-field approach

Separation of mean field and quantum fluctuations

$$\hat{a}(t) = \alpha(t) + \delta\hat{a}(t) \qquad \hat{\Psi}(x,t) = \sqrt{N}\varphi(x,t) + \delta\hat{\Psi}(x,t)$$

Szirmai, Nagy, Domokos, PRL 102, 080401 (2009), **M** is non-normal $\longrightarrow$ excess noise

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Gross–Pitaevskii-type equation

$$i\frac{\partial}{\partial t}\alpha = \Big\{-\Delta_C + NU_0\langle\cos^2(kx)\rangle - i\kappa\Big\}\alpha + N\eta_t\langle\cos(kx)\rangle + \eta$$

$$i\frac{\partial}{\partial t}\varphi(x,t) = \Big\{-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} + |\alpha(t)|^2 U_0\cos^2(kx) + 2Re\{\alpha(t)\}\eta_t\cos(kx) + Ng_c|\varphi(x,t)|^2\Big\}\varphi(x,t)$$

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Linearized quantum fluctuations

$$\frac{\partial}{\partial t}\vec{\hat{R}} = -i\mathbf{M}\vec{\hat{R}} + \vec{\hat{\xi}}, \qquad \begin{cases} \vec{\hat{R}} \equiv [\delta\hat{a}, \delta\hat{a}^\dagger, \delta\hat{\Psi}(x), \delta\hat{\Psi}^\dagger(x)] \\ \mathbf{M} = \mathbf{M}(\alpha_0, \varphi_0(x), \mu) \\ \vec{\hat{\xi}} = [\hat{\xi}, \hat{\xi}^\dagger, 0, 0] \end{cases}$$

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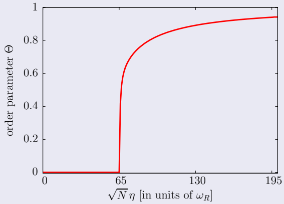
Self-organization of a BEC in a cavity

order parameter

$$\Theta = \langle \varphi | \cos kx | \varphi \rangle$$



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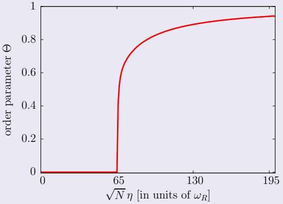
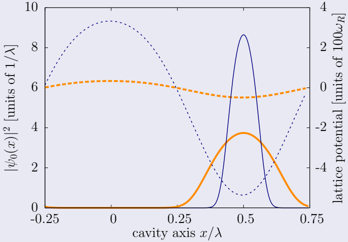


steady-states



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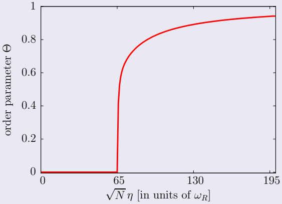

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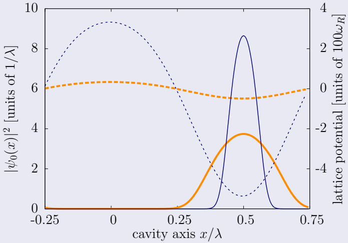


threshold

$$\sqrt{N}\eta_c = \sqrt{\frac{\delta_C^2 + \kappa^2}{2|\delta_C|}} \sqrt{\omega_R + 2Ng_c}$$

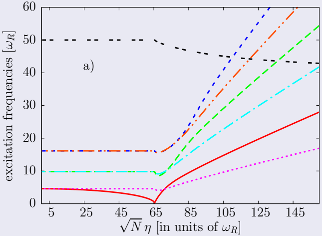
$$\delta_C = \Delta_C - NU_0/2 \qquad \omega_R = \frac{\hbar k^2}{2m}$$

temperature ↔ kinetic energy + collision

Nagy, Szirmai, Domokos, Eur. Phys. J. D 48, 127 (2008)

Spectrum of fluctuations

frequencies



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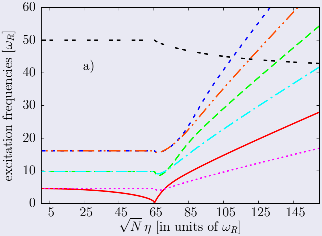
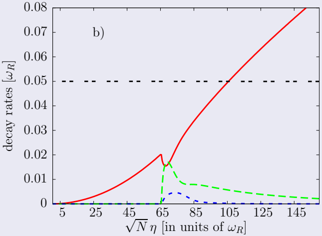


damping (cavity cooling)



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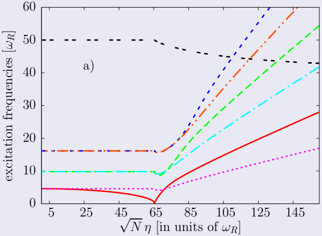

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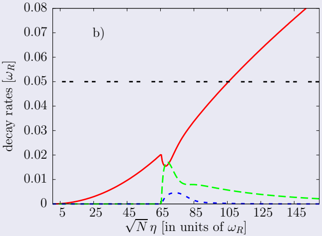
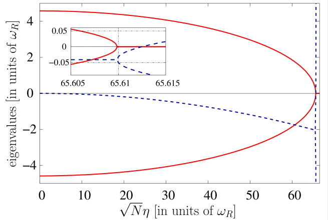


critical "point"

2 modes, $T = 0$, analytical



Quantum phase transition of the Dicke model

1. Reduce the size of the Hilbert space, to the subspace sufficient to describe the self-organization

Nagy, Konya, Szirmai, Domokos, PRL 104, 130401 (2010)

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2. Study quantum statistical properties

$$\hat{\Psi}(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}} c_1 \cos kx \qquad \left[c_i, c_i^\dagger\right] = 1 \quad i = 0, 1$$

Number of particles: $c_0^\dagger c_0 + c_1^\dagger c_1 = N \quad$ fixed

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Two-mode H: analogy with the Dicke Hamiltonian

$$H/\hbar = -\delta_C\, a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x / \sqrt{N} + u a^\dagger a \left(\frac{1}{2} + \hat{S}_z / N\right)$$

$$\omega_R = \hbar k^2/m$$

$$\left.\begin{aligned} \delta_C &= \Delta_C - 2u \\ u &= N U_0/4 \\ y &= \sqrt{2N}\eta_t \end{aligned}\right\} \text{tunable}$$

Nagy, Konya, Szirmai, Domokos, PRL 104, 130401 (2010)

Quantum phase transition of the Dicke model

1. Reduce the size of the Hilbert space, to the subspace sufficient to describe the self-organization
2. Study quantum statistical properties

$$\hat{\Psi}(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}} c_1 \cos kx \qquad \left[c_i, c_i^\dagger\right] = 1 \quad i = 0, 1$$

Number of particles: $c_0^\dagger c_0 + c_1^\dagger c_1 = N$ fixed

Spin representation

$$\hat{S}_x = \tfrac{1}{2}(c_1^\dagger c_0 + c_0^\dagger c_1) \quad \hat{S}_y = \tfrac{1}{2i}(c_1^\dagger c_0 - c_0^\dagger c_1) \quad \hat{S}_z = \tfrac{1}{2}(c_1^\dagger c_1 - c_0^\dagger c_0)$$

Two-mode H: analogy with the Dicke Hamiltonian

$$H/\hbar = -\delta_C\, a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x/\sqrt{N} + u a^\dagger a \left(\tfrac{1}{2} + \hat{S}_z/N\right)$$

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Threshold

$$y_{\mathrm{crit}} = \sqrt{-\delta_C \omega_R}$$

c.f. $\kappa = 0$ before

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Quantum statistical properties of the ground state

Holstein-Primakoff representation

$S_- = \sqrt{N - b^\dagger b}\, b,\ S_+ = b^\dagger\ \sqrt{N - b^\dagger b},\ S_z = b^\dagger b - N/2\,,\qquad b$ boson for $N \to \infty$

$$H/\hbar = -\delta_C\, a^\dagger a + \omega_R b^\dagger b + u a^\dagger a b^\dagger b/N + \frac{i}{2} y (a^\dagger - a) \left(b^\dagger\ \sqrt{1 - \frac{b^\dagger b}{N}} + \sqrt{1 - \frac{b^\dagger b}{N}}\, b \right)$$

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quadratic Hamiltonian

$$H/\hbar = E_0 - (\delta_C - u\beta_0^2) a^\dagger a + \frac{M_x + M_y}{2} b^\dagger b + \frac{M_x - M_y}{4}\left(b^{\dagger 2} + b^2\right) + i\frac{M_c}{2}(a^\dagger - a)(b^\dagger + b)$$

meanfield

$$\beta_0^2 = \frac{\delta_C}{u}\left(1 - \sqrt{1 - \frac{u}{\delta_C}\frac{y^2 - y_{\mathrm{crit}}^2}{y^2 - \frac{u}{\delta_C} y_{\mathrm{crit}}^2}}\right),$$

$$M_x = \omega_R - y\alpha_0\beta_0 \frac{3 - 2\beta_0^2}{\left(1 - \beta_0^2\right)^{3/2}}$$

$$M_y = \omega_R - y\alpha_0\beta_0 \frac{1}{\left(1 - \beta_0^2\right)^{1/2}}$$

$$M_c = 2u\alpha_0\beta_0 + y\frac{1 - 2\beta_0^2}{\left(1 - \beta_0^2\right)^{1/2}}$$

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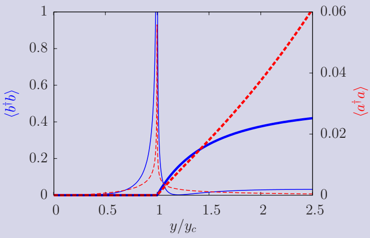
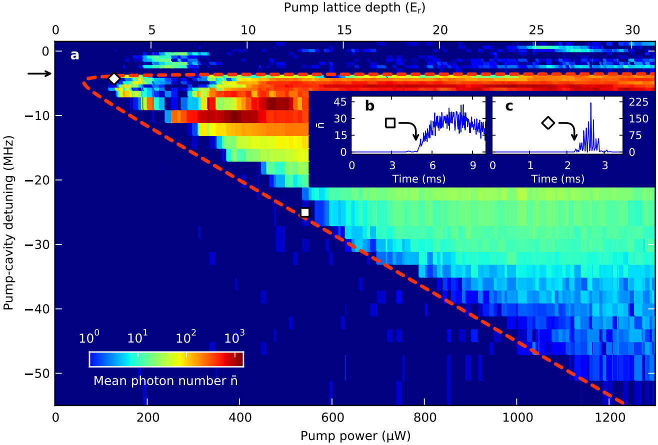
second-order correlations



Nagy, Konya, Szirmai, Domokos, PRL 104, 130401 (2010)

Experimental mapping of the phase diagram



Baumann, Guerlin, Brennecke, Esslinger, Nature 464, 1301 (2010)

Photon measurement induced back action

The ground state is fragile due to the irreversible loss of photons (=measurement) ⇒ quantum noise analysis

Szirmai, Nagy, Domokos, PRL 102, 080401 (2009)

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Normal mode decomposition

- left and right eigenvectors of M → $(\vec{l}^{(k)}, \vec{r}^{(l)}) = \delta_{k,l}$
- normal modes $\hat{\rho}_k = (\vec{l}^{(k)}, \hat{\vec{R}})$
- $\frac{\partial}{\partial t}\hat{\rho}_k = -i\omega_k\hat{\rho}_k + \hat{Q}_k$
- projected noise $\hat{Q}_k \equiv (\vec{l}^{(k)}, \hat{\vec{\xi}})$

1.) quasi-mode excitation $\frac{\delta}{\delta t}\langle\rho_+^\dagger\rho_+ + \rho_-^\dagger\rho_-\rangle$
2.) measureably excitation $\delta N(t) = \left\langle a^\dagger a + b^\dagger b\right\rangle$

$$\frac{\delta N(t)}{\delta t} \approx 2\kappa \sum_{k,l} l_1^{(k)*} l_2^{(l)*} \left(r_2^{(k)} r_1^{(l)} + r_4^{(k)} r_3^{(l)}\right) \Theta\left(\delta t^{-1} - |\omega_k + \omega_l|\right)$$

Szirmai, Nagy, Domokos, PRL 102, 080401 (2009)
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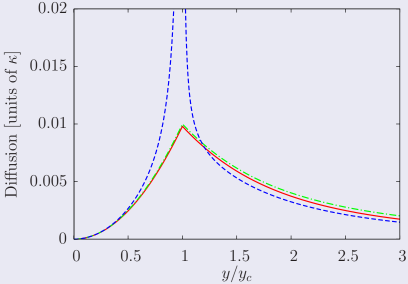
Depletion rate

coarse graining: $|\delta_C|^{-1} \ll \delta t \ll \omega_R^{-1}$

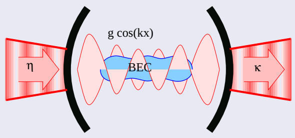


From coarse-grained effective quantum master eq.

$$\frac{\delta N(t)}{\delta t} = \kappa \frac{M_c^2}{\delta_C^2 + \kappa^2} \approx \frac{\kappa\omega_R}{|\delta_C|}$$

Open system dynamics away from equilibrium

BEC in a driven cavity



Nagy, Domokos, Vukics, Ritsch EPJD 55, 659 (2009)

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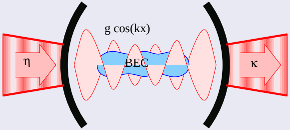

Microscopic model

$$H = -\Delta_C\, a^\dagger a + i\eta(a^\dagger - a) + \int \Psi^\dagger(x)\left[-\frac{1}{2\hbar m}\frac{d^2}{dx^2} + U_0\, a^\dagger a \cos^2(kx)\right]\Psi(x)dx,$$

$$\dot{\rho} = \frac{1}{i\hbar}[H,\rho] - \kappa\left(a^\dagger a\rho + \rho a^\dagger a - 2a\rho a^\dagger\right)$$

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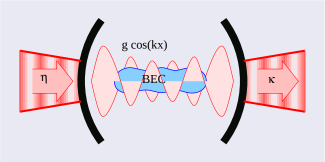

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Reduced Hilbert-space

$$\Psi(x) = c_0 + \sqrt{2}\, c_2 \cos 2kx$$

$$X = \frac{1}{\sqrt{2}}(c_2^\dagger + c_2) \quad P = \frac{i}{\sqrt{2}}(c_2^\dagger - c_2)$$

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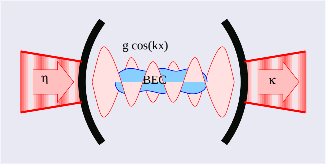

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Analogy to opto-mechanics

$$H = -\tilde\Delta_C\, a^\dagger a + i\eta\,(a^\dagger - a) + 2\omega_R\,(X^2 + P^2) + u\, a^\dagger a\, X\,.$$

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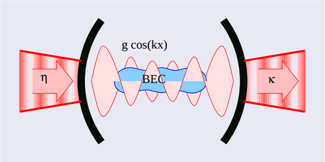

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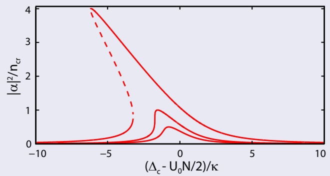
Optical bistability



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Environment filtered through the cavity field

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Reduced master equation

diffusion friction

$$\dot{\rho} = \frac{1}{i\hbar}[H_{\text{eff}}, \rho] - [d(X), [d(X), \rho]] - \frac{i}{2}[g(X), \{P, \rho\}]$$

$$a(t) = \frac{\eta}{\kappa - i\delta} + \int_0^t e^{(i\delta-\kappa)(t-t')}\xi(t')dt' , \quad \delta \equiv \delta(X) = \tilde{\Delta}_C - uX , \quad \langle \xi(t)\,\xi^\dagger(t') \rangle = 2\kappa\,\delta(t-t') ,$$

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Coefficients

$$H_{\text{eff}} = 4\omega_R\,\frac{1}{2}(X^2 + Y^2) + \frac{\eta^2}{\kappa}\arctan\left(\frac{uX - \tilde{\Delta}_C}{\kappa}\right)$$

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Lindblad?

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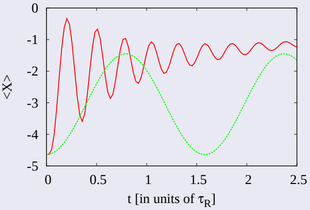
Tunneling oscillations



Method I. Non-adiabatic effects

$$\alpha(X,t) = \alpha_0(X) + \frac{1}{2}\{Y, \alpha_1(X)\}$$

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Hierarchy in powers of Y

$$\alpha_0(X) = \frac{\eta}{-i\delta(X) + \kappa}\;.$$

$$\alpha_1(X) = \frac{4\omega_R}{i\delta - \kappa}\,\frac{\partial \alpha_0(X)}{\partial X} = i\,\frac{4\omega_R u \eta}{(\kappa - i\delta(X))^3}.$$

Summary

Many-body effects in the motion of atoms in a cavity

- global coupling
- non-equilibrium phase transitions
- experimental realization of Dicke-type phase transition
- driven-damped system, controlled dissipation channel
- Open question: stationary state of the system