Relativistic Nuclear Collisions in 3-Fluid Hydrodynamics

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in collaboration with

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1-FLUID IDEAL HYDRODYNAMICS

baryon current conservation

$$\boxed{\partial_\mu \, J^\mu(x) = 0}$$

the energy–momentum conservation of the fluids

$$\boxed{\partial_\mu \, T^{\mu\nu}(x) = 0}$$

with

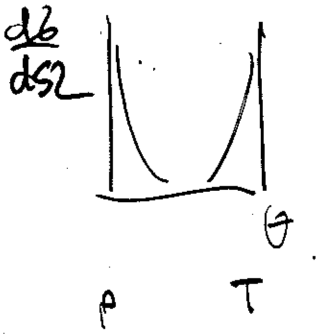
$$J^\mu = n u^\mu$$

$$T^{\mu\nu} = (\varepsilon + P) \, u^\mu \, u^\nu - g^{\mu\nu} P$$

Main hydro assumption $\boxed{\text{Local equilibrium at every point } x}$

Basic assumption for 3(2)-fluid hydrodynamics $f(x,p) = \sum_i^3 f_i(x,p)$

$$f_b(x,p) = f_p(x,p) + f_t(x,p)$$



$f_m(x,p)$

•Three-fluid hydrodynamic model

$$\boxed{\partial_\mu J^\mu_\alpha(x) = 0}$$

baryonic current $J^\mu_\alpha = n_\alpha u^\mu_\alpha, \qquad J^\mu = J^\mu_p + J^\mu_t$
(α = target, projectile; f = created mesonic fireball)
energy-momentum tensor $T^{\mu\nu}_\alpha = (\varepsilon_\alpha + P_\alpha)\, u^\mu_\alpha\, u^\nu_\alpha - g^{\mu\nu} P_\alpha$

$$\boxed{\begin{aligned}
\partial_\mu T^{\mu\nu}_p(x) &= -F^\nu_p(x) + F^\nu_{fp}(x)\\
\partial_\mu T^{\mu\nu}_t(x) &= -F^\nu_t(x) + F^\nu_{ft}(x)\\
\partial_\mu T^{\mu\nu}_f(x) &= \int d^4x\; \delta^4(x - x' - \tau V_f(x'))\; [F^\nu_p(x') + F^\nu_t(x')]\\
&\quad -F^\nu_{fp}(x) - F^\nu_{ft}(x)
\end{aligned}}$$

τ is the formation time. The coupling (friction) term

$$F^\nu_\alpha = \underline{\rho_p\, \rho_t} \left[(u^\nu_\alpha - u^\nu_{\bar\alpha})\, D_P + \left(u^\nu_p + u^\nu_t \right) D_E \right], \qquad \boxed{\times\, \xi^2(s_{pt})}$$

α =p and t, $\bar{\text{p}}$ =t and $\bar{\text{t}}$ =p with the sum of scalar densities of all particles $\underline{\rho_\alpha} = \sum_{i\in\alpha} m_i \int d^4q\; f_i(q)$ and

$$D_{P/E} = m_N\; V^{pt}_{rel}\; \sigma_{P/E}(s_{pt}),$$

where $s_{pt} = m_N^2 \left(u^\nu_p + u^\nu_t \right)^2$ is the mean invariant energy squared. For the mesonic fireball

$$F^\nu_{f\alpha}(x) \simeq V^{f\alpha}_{rel}\; \sigma^{N\pi\to R}_{tot}(s_{f\alpha})\; \frac{T^{(eq)0\nu}_f}{u^0_f}\; \rho_\alpha\,.$$

$$\boxed{\begin{array}{r} n_\alpha, T_\alpha, P_\alpha, u^\mu_\alpha \\ T_f, P_f, u^\mu_f \end{array}} \Rightarrow \boxed{\text{14 Hydro Eqs.}} + (EoS_t = EoS_p = EoS_f)$$

HADRONIC EOS (GASEOS)

Energy density:

$$\varepsilon(n_B,T) = \underbrace{\varepsilon_{gas}(n_B,T)}_{\text{gas of free hadrons}} + \underbrace{W(n_B)}_{\text{mean field}}$$

Pressure:

$$P(n_B,T) = \underbrace{P_{gas}(n_B,T)}_{\text{gas of free hadrons}} \underbrace{+ n_B \frac{dW(n_B)}{dn_B} - W}_{\text{mean field}}$$

$$W(n_B) = n_B m_N \left[-b \left(\frac{n_B}{n_0} \right) + c \left(\frac{n_B}{n_0} \right)^{\gamma+1} \right]$$

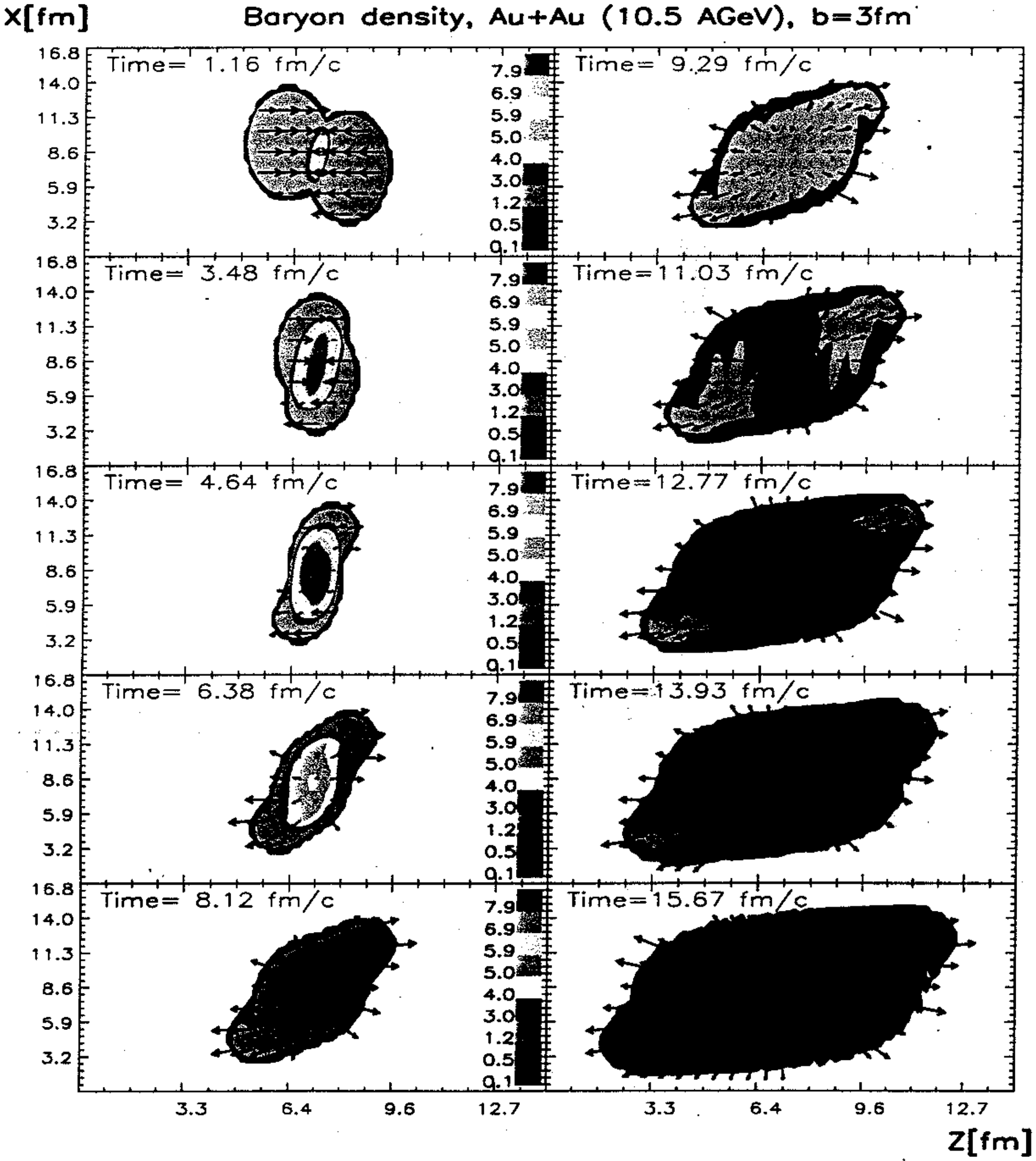
$W(n_B)$ saturates the cold nuclear matter at $n_0 = 0.15$ fm^{-3} and $\varepsilon(n_0, T=0)/n_0 - m_N = 16$ MeV, and provides incompressibility of nuclear matter $K = 235$ MeV.

To preserve causality at high n_B

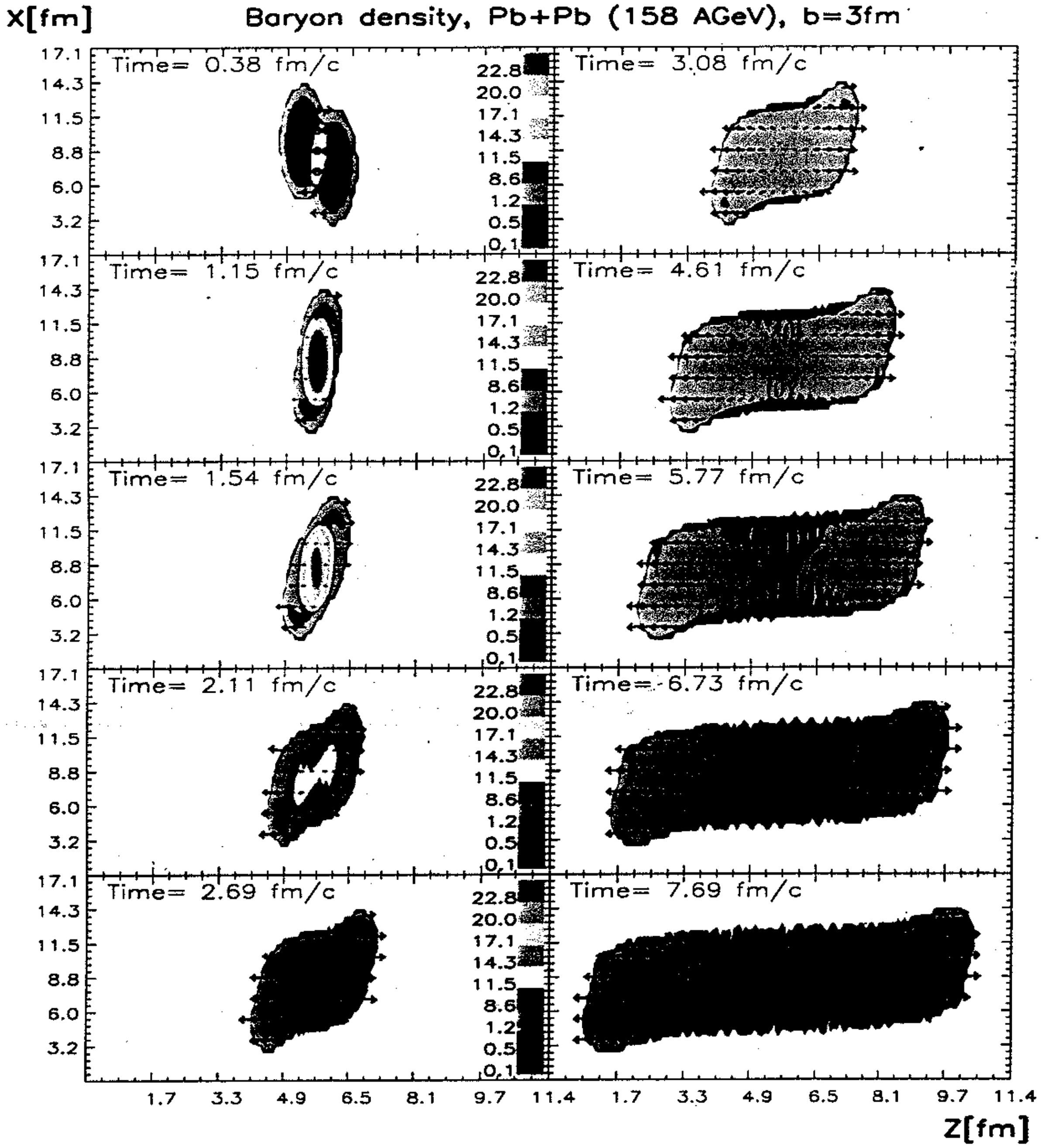
$$\varepsilon(n_B, T=0) = n_0 m_N \left[A \left(\frac{n_B}{n_0} \right)^2 + C + B \left(\frac{n_B}{n_0} \right)^{-1} \right], \quad n_B > n_c \approx 6n_0$$

Parameters are determined on the condition that $\varepsilon(n_B, T=0)$ and its two first derivatives are continues at n_c.

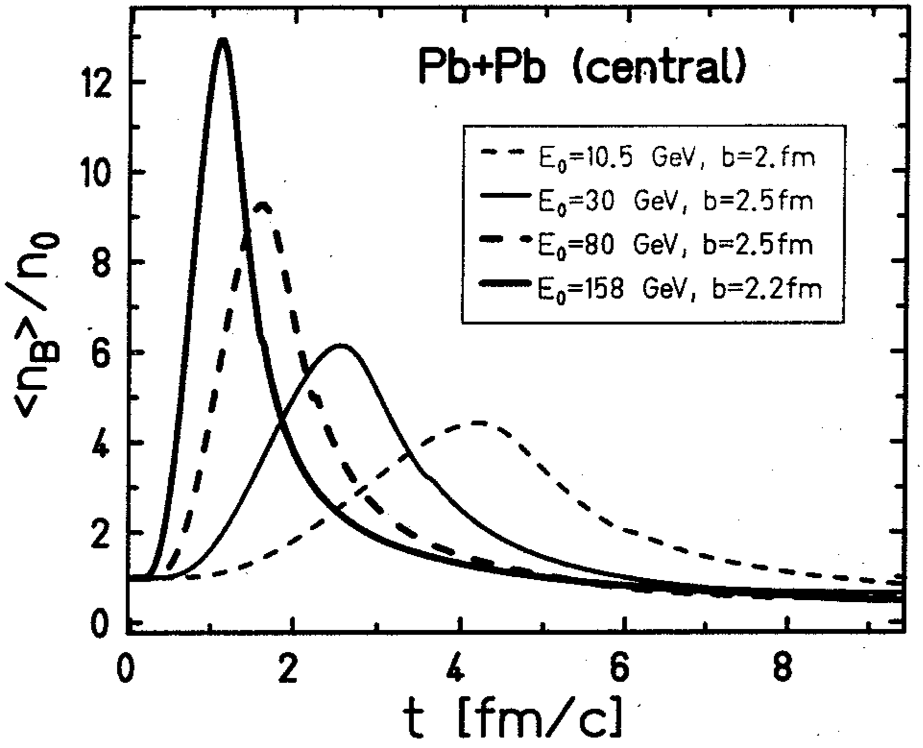
GLOBAL DYNAMICS

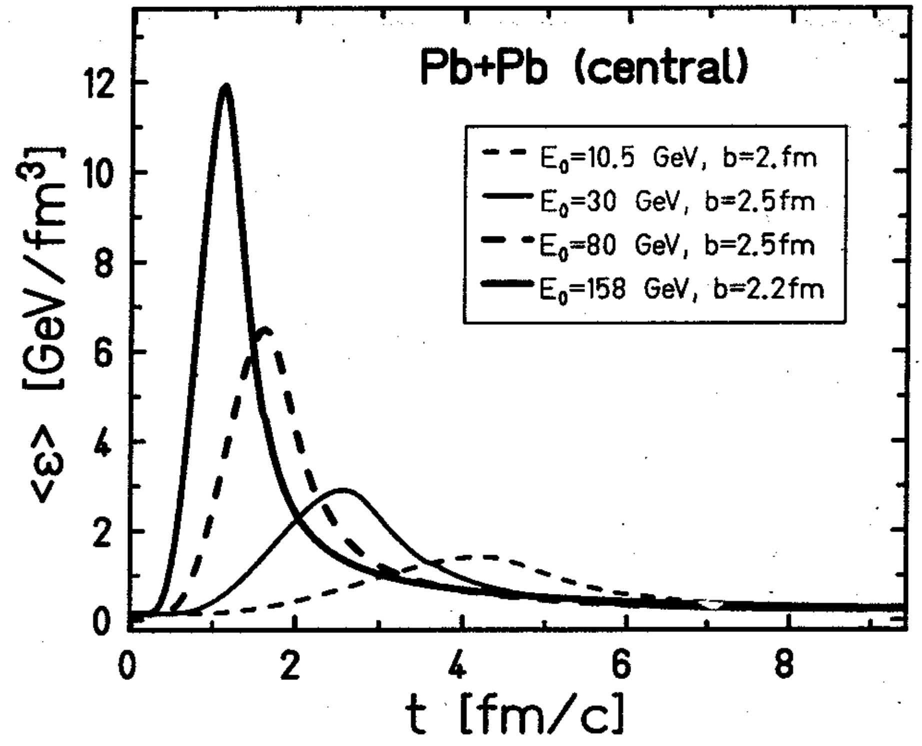


GLOBAL DYNAMICS

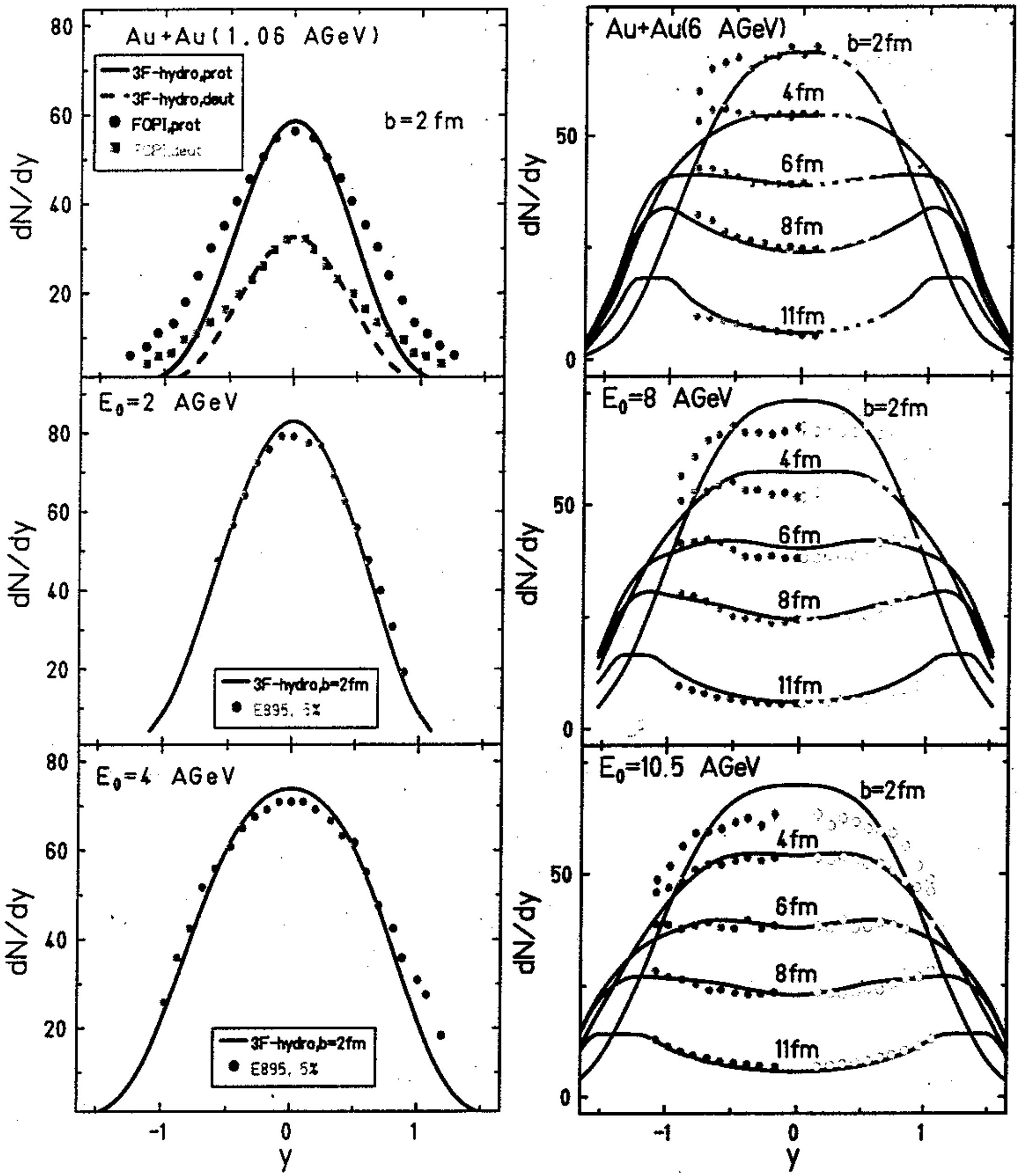


Evolution of Thermodynamic Quantities





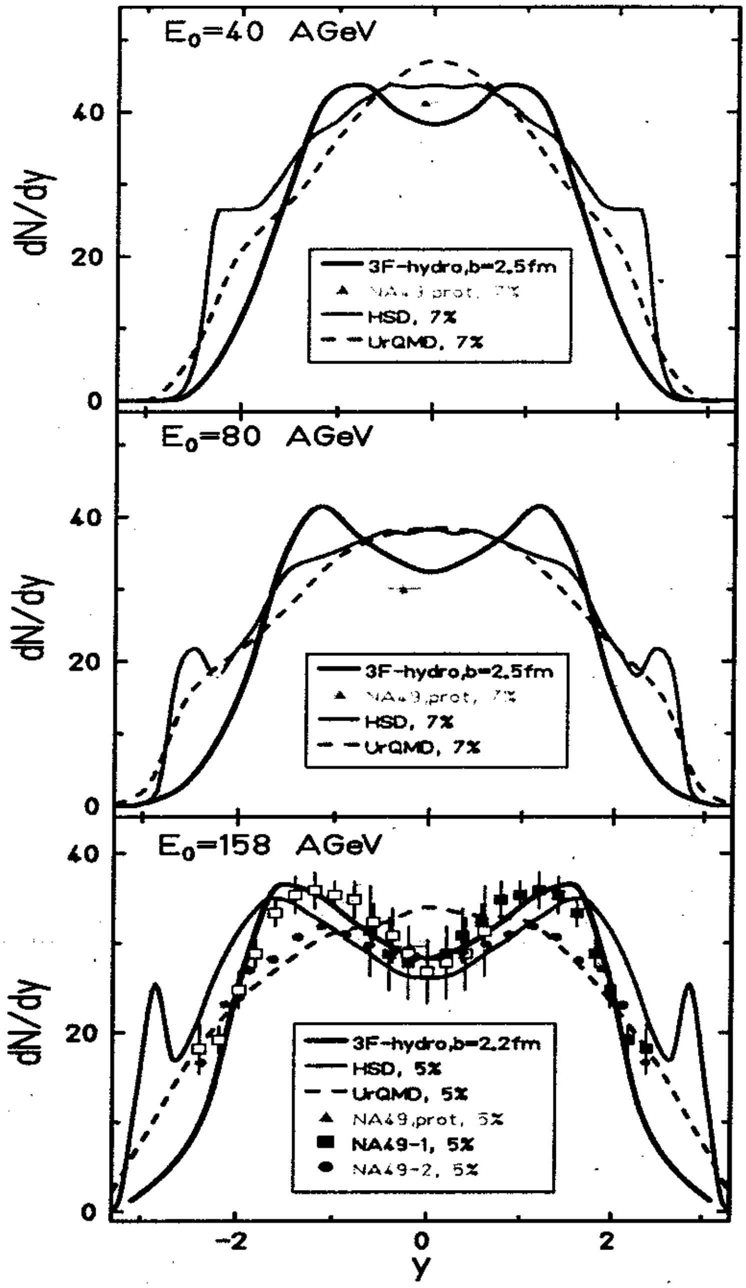
SIS&AGS DATA
PROTON RAPIDITY DISTRIBUTIONS



3-Fluids: gasEoS

FOPI: N.Herrmann,Nucl.Phys. A610 (1996) 49c [Au(1.06 GeV/nucl.)+Au]
E895: Phys. Rev. **C68** (2003) 054905 [Au(2 and 4 GeV/nucl.)+Au]
E917: Phys. Rev. Lett. **86** (2001) 1970 [Au(6, 8 and 10.5 GeV/nucl.)+Au]

SPS Data
$(p - \bar{p})$ Rapidity Distributions



Pb + Pb

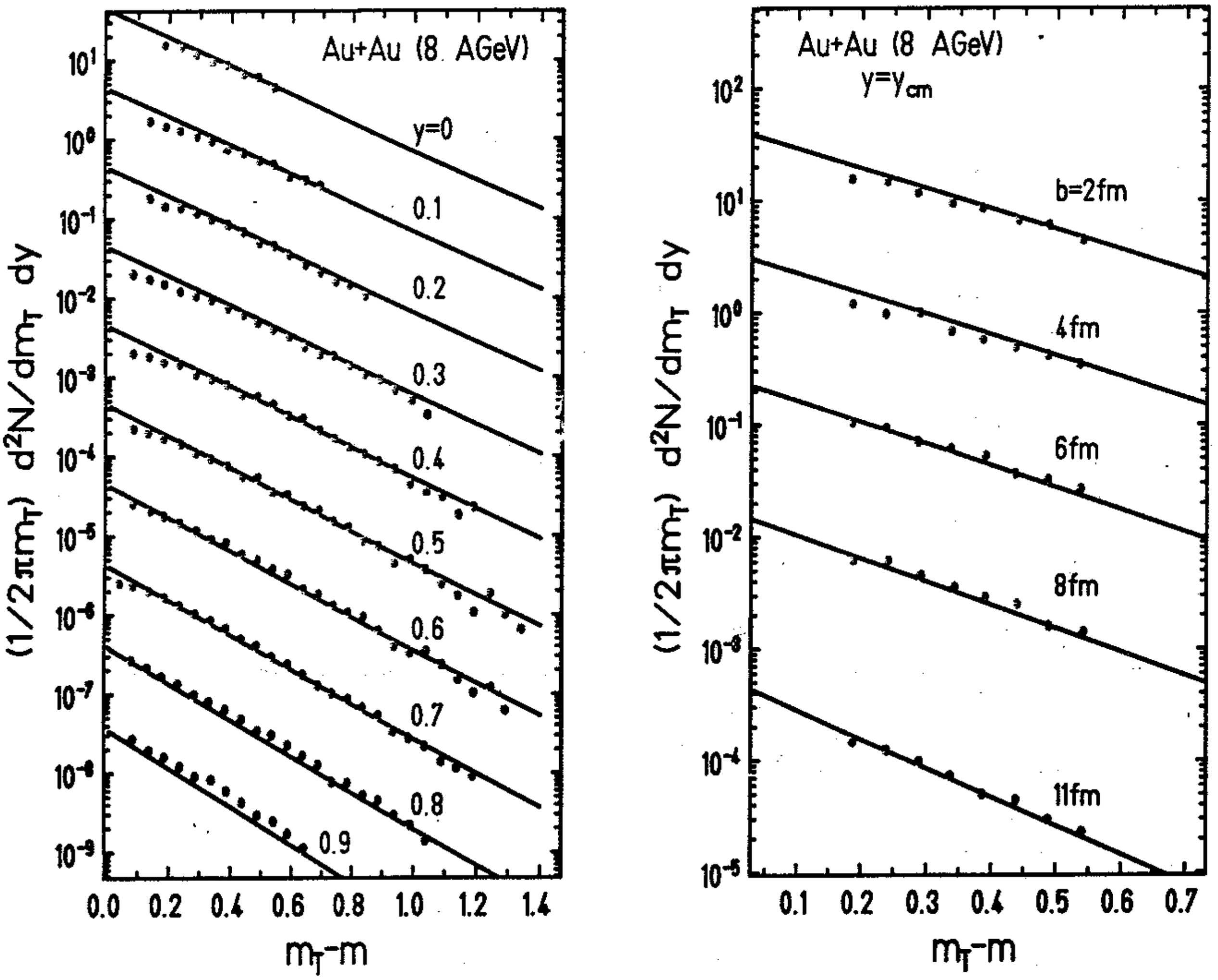
3-Fluids: gasEoS

b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV are experimental estimates.

NA49 (prot.): Phys. Rev. C69 (2004) 024902
NA49–1: Phys. Rev. Lett. 82 (1999) 2471
NA49–2(preliminary): Nucl. Phys. A661 (1999) 362c
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker, Phys. Rev. C67 (2003) 014904

AGS DATA: PROTON p_T SPECTRA

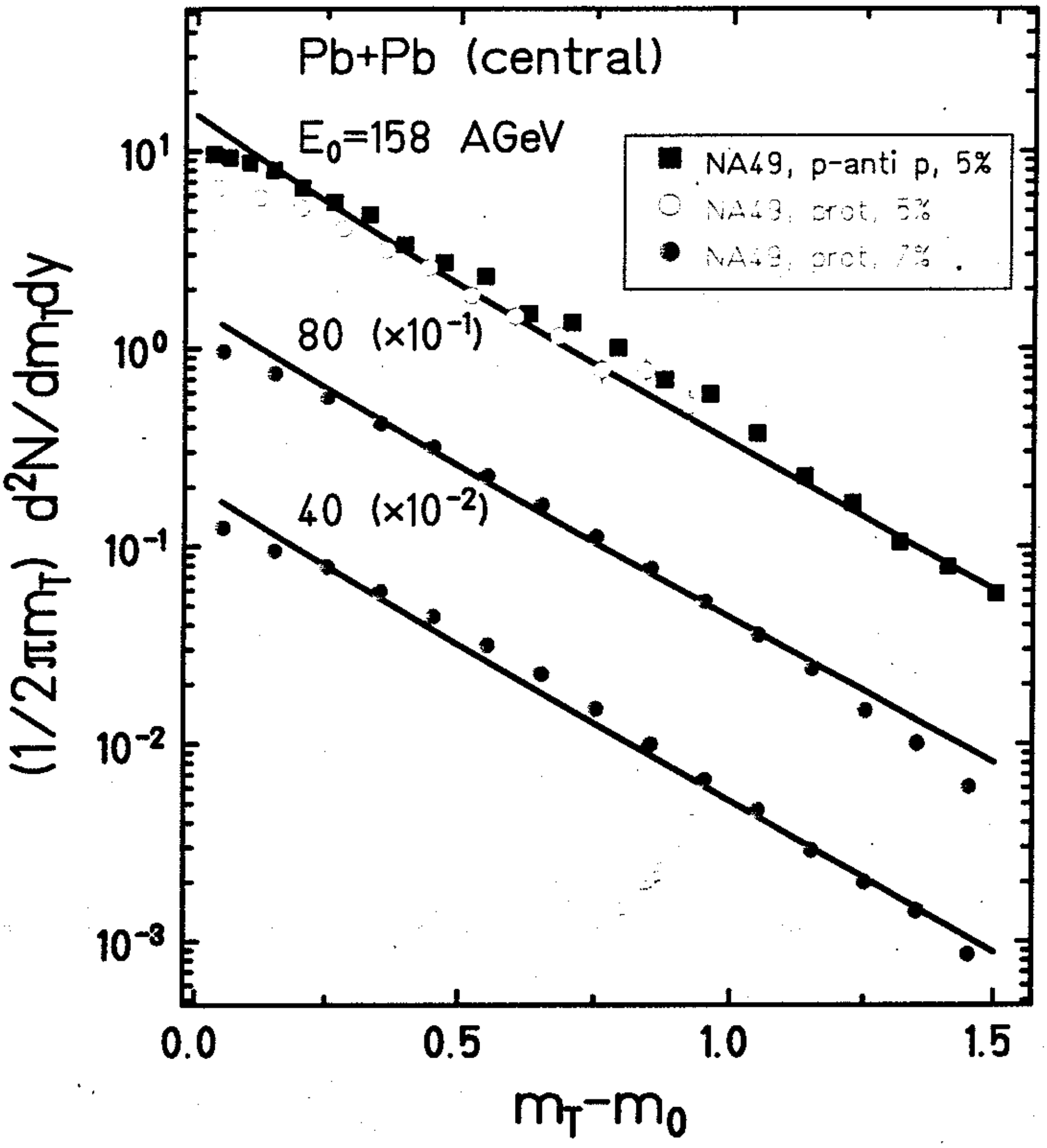
b = 2 fm



3-Fluids: gasEoS

E917: Phys. Rev. Lett. **86** (2001) 1970 [Au(6, 8 and 10.5 GeV/nucl.)+Au]

SPS DATA: PROTON p_T SPECTRA



Pb + Pb

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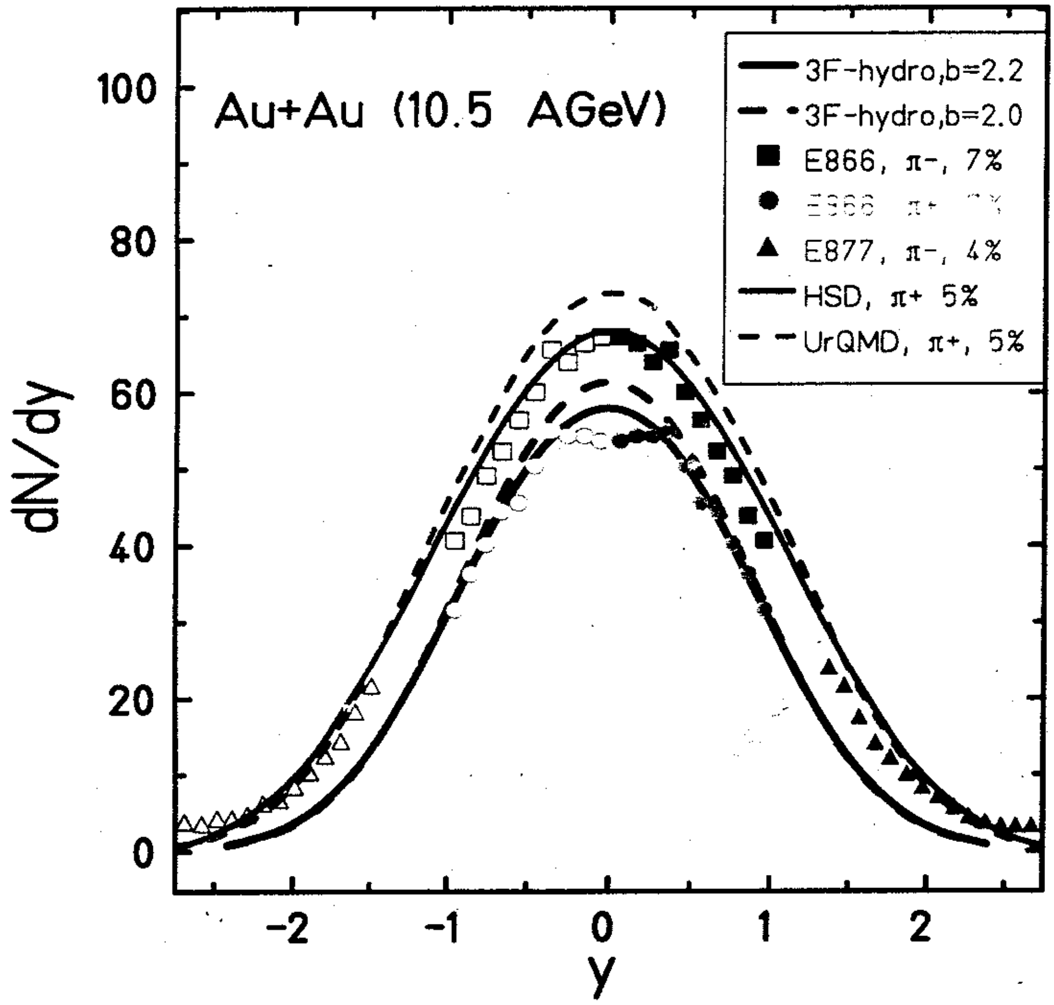
b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV are experimental estimates.

NA49: Phys. Rev. Lett. **82** (1999) 2471
NA49: Nucl. Phys. A715 (2003) 166c

AGS DATA
PION RAPIDITY DISTRIBUTIONS

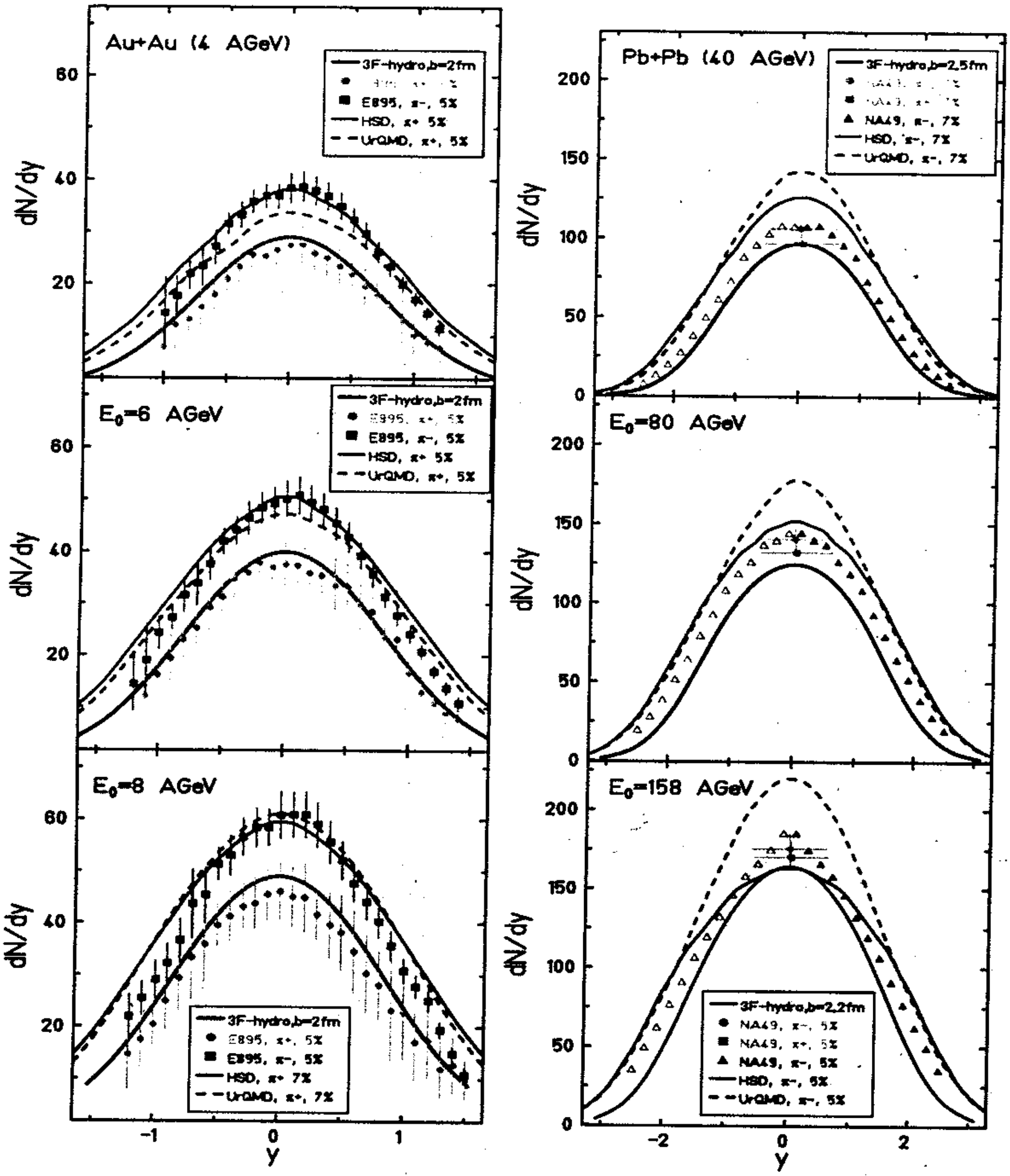


3-Fluids: gasEoS is too hard

b = 2.0 fm for 7%σ and b = 1.5 fm for 4%σ are experimental estimates.

E895: Phys. Rev. C68 (2003) 054905
E877: Phys. Rev. C62 (2000) 024901
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker,
Phys. Rev. **C67** (2003) 014904

AGS&SPS DATA
PION RAPIDITY DISTRIBUTIONS



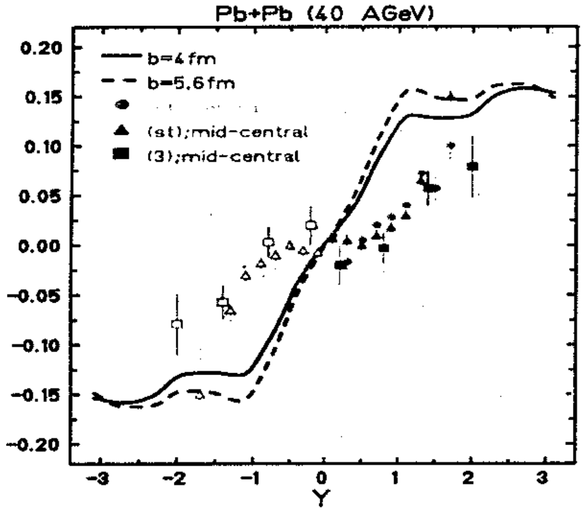
3-Fluids: gasEoS is too hard

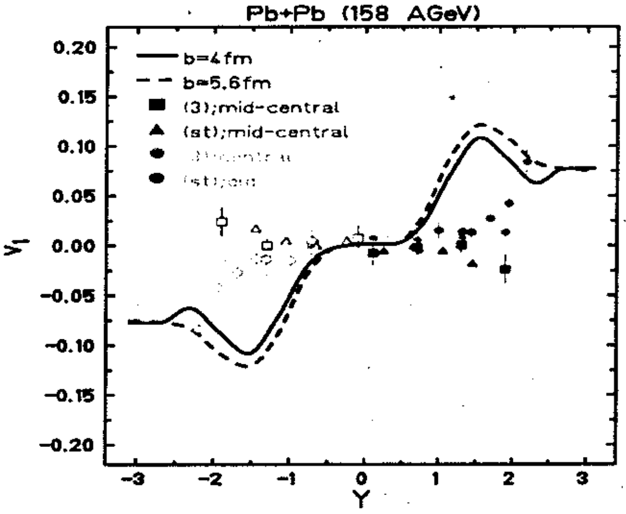
b = 2.0 fm for 4, 6 and 8 AGeV, b = 2.2 fm for 158 AGeV, and b = 2.5 fm for 40 and 80 AGeV, are experimental estimates.

NA49: Phys. Rev. C66 (2002) 054902
E895: Phys. Rev. C68 (2003) 054905
Models: H. Weber, E.L. Bratkovskaya, W. Cassing and H. Stöcker, Phys. Rev. **C67** (2003) 014904

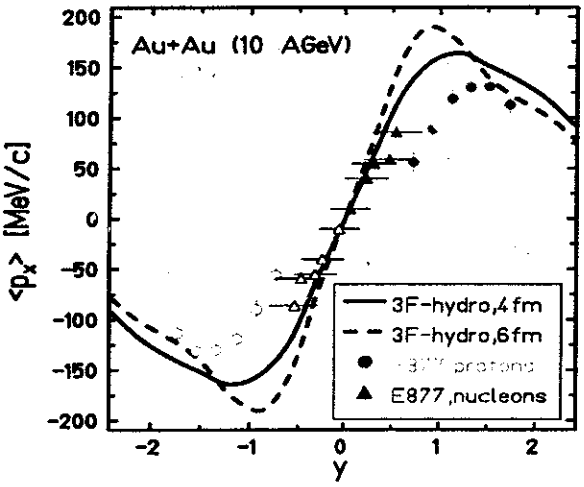
SPS/AGS DATA: NUCLEON v_1/p_x FLOW

$$v_1(y) = \int d^2p_T \frac{p_x}{p_T}\frac{dN}{d^3p} \Big/ \int d^3p \frac{dN}{d^3p}$$





b = 3.7 fm for central coll., and b ≈ 7 fm for midcentral coll.
are experimental estimates.
(st): standard method; (3): 3-particle correlation method
NA49: Phys. Rev. Lett. **80** (1998) 4136
NA49: Phys. Rev. C68 (2003) 034903

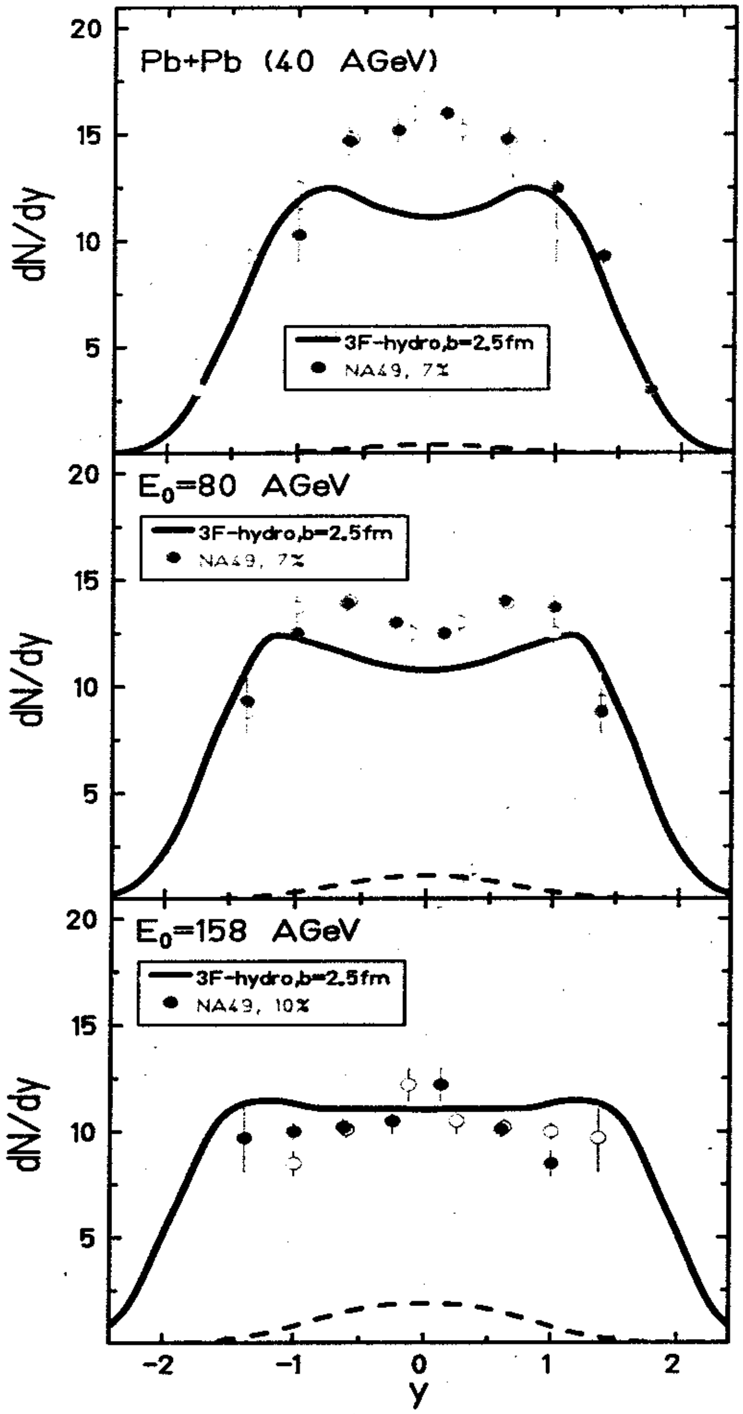


$$\langle p_x\rangle(y) = \frac{\int d^2p_T p_x(dN/d^3p)}{\int d^3p(dN/d^3p)}$$

E877: Phys. Rev. **C56** (1997) 3254

3-Fluids: **gasEoS** is too hard

SPS DATA: $\Lambda + \Sigma^0$ RAPIDITY DISTRIBUTIONS PRELIMINARY

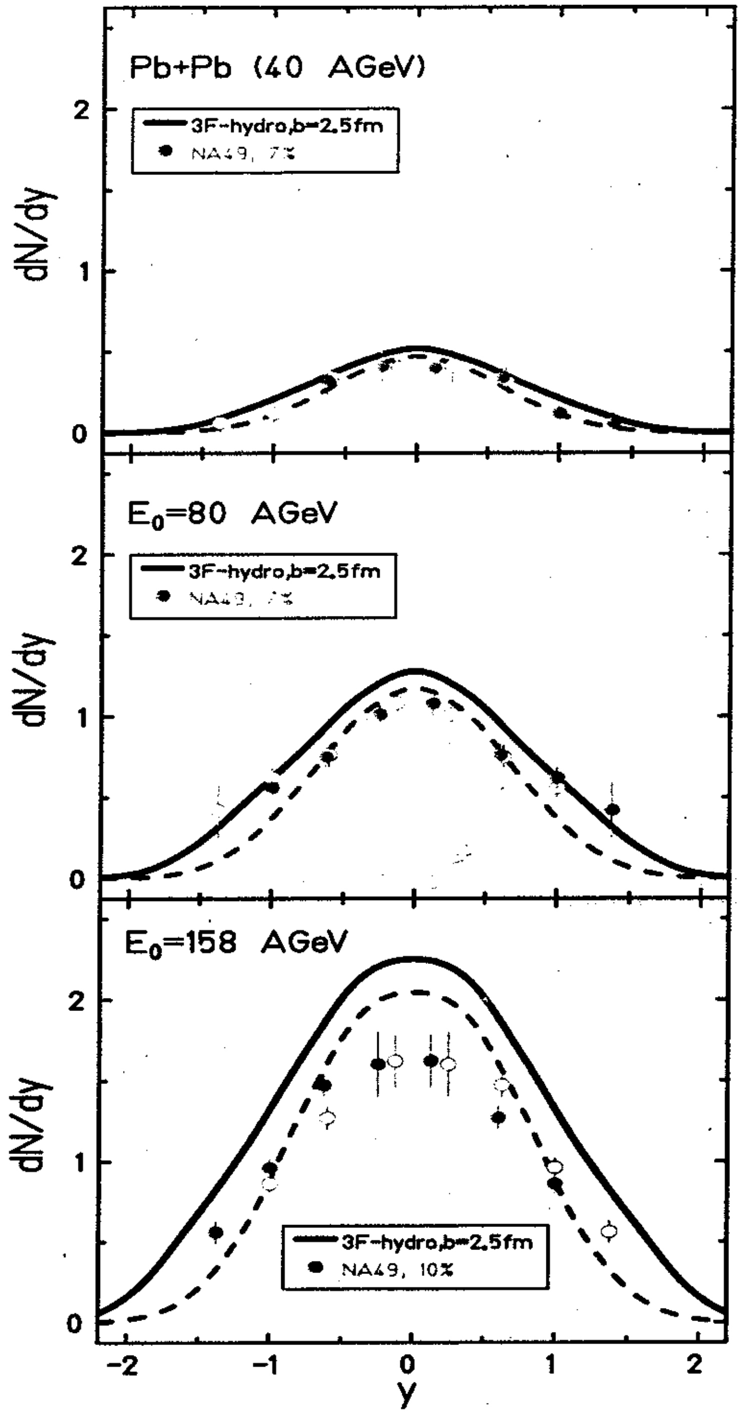


3-Fluids: gasEoS

dashed line = contribution from the fireball fluid

NA49: nucl-ex/0311024

SPS Data: $\bar{\Lambda}+\bar{\Sigma}^0$ Rapidity Distributions Preliminary



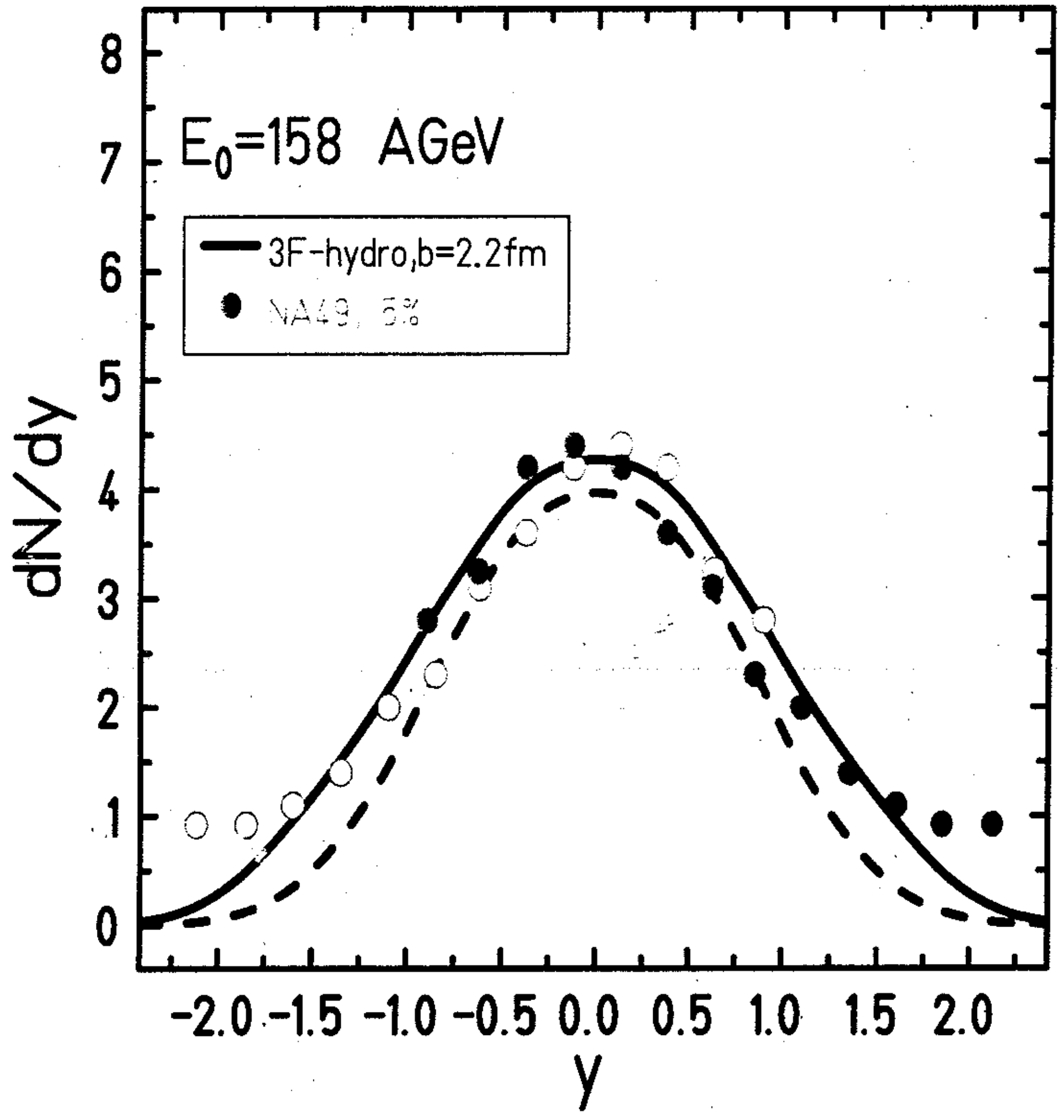
3-Fluids: gasEoS

dashed line = contribution from the fireball fluid

NA49: nucl-ex/0311024

SPS DATA: $\bar{p}$ RAPIDITY DISTRIBUTIONS PRELIMINARY

($\bar{\Lambda} \rightarrow \bar{N} + \pi$ is excluded)



3-Fluids: gasEoS

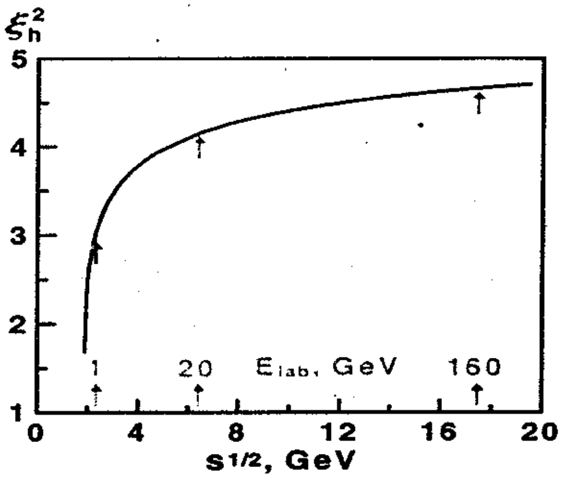
dashed line = contribution from the fireball fluid

NA49: Nucl. Phys. **A661** (1999) 45c.

SUMMARY

- All global observables, considered up to now (!!!), are reasonably reproduced with a simple hadronic EoS, provided the friction is enhanced as follows

This enhancement reproduces the observable Stopping Power.



- Is it reasonable enhancement in view of model uncertainties? (medium effects, multiparticle collisions, poor knowledge of various σ)

◦ Mixed quark/hadron phase formation ⇒ at $T \sim T_c$ the scattering length for $q-\bar{q}$ (quasi-)mesons and gluons goes through ∞ ? (at RHIC the enhancement factor $10-10^2$ is needed for partonic σ !; E.Shuryak and I.Zahed, hep-ph/0307276; "sticky moalesses" : G.E.Brown, C.-H.Lee, M.Rho, hep-ph/0402207)

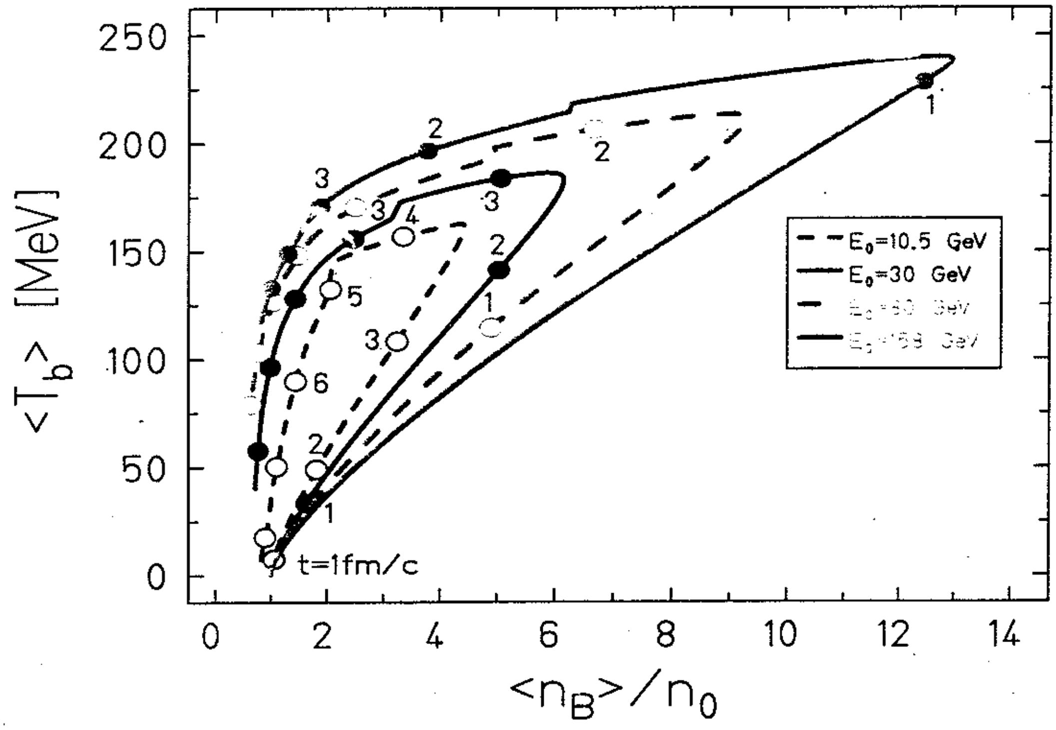
- Different EoS (with different order of phase transition) should be probed

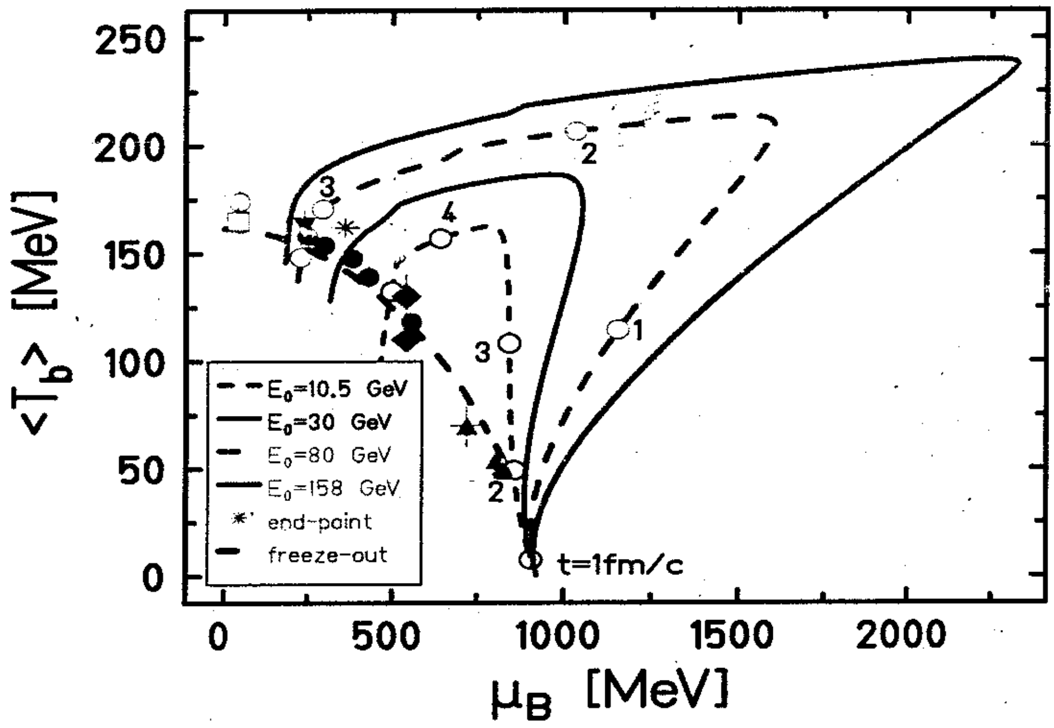
- Observable Stopping Power ⇒ there are certain windows of incident energies, where a matter with desired properties is most efficiently produced, e.g.

◦ 15 GeV/nucl. $< E_{lab} <$ 80 GeV/nucl. is preferable for production of thermalized baryonic matter with $n_B > 8n_0$

PHASE DIAGRAMS

central Pb+Pb collisions





Critical end-point: Z.Fodor, S.D.Katz, hep-lat/0402006.

- T.S. Biro, A.A. Shanenko and V.D. Toneev, *Towards Thermodynamical Consistency of Quasiparticle Picture,* Yad. Fiz. **66** (2003) 982; [nucl-th/0102027].
- A.A. Shanenko, T.S. Biro and V.D. Toneev, *Quasiparticles and Thermodynamical Consistency,* Heavy Ion Collisions **18** (2003) 91.
- A.S.Parvan and T.S. Biro, *Extensive Renyi statistics from non-extensive entropy,* hep-ph/0407131.