Dynamics of few-body Coulomb systems at low energies

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Atomic physics with antiprotons

i. possibility of long-lived states in antiprotonic lithium (J. Révai, V. Belyaev)

ii. formation probability of different antiprotonic helium states in the capture reaction

$$He + \bar{p} \rightarrow (He^{+}\bar{p})_{Jv} + e^{-}$$

(J.Révai, N. Shevchenko)

iii. antihydrogen formation in the reaction

$$(e^{+}e^{-}) + \bar{p} \rightarrow \bar{H} + e^{-}$$

(J. Révai, V. Belyaev, A. Os'kin)

i.

The study of the long-lived states in antiprotonic helium is (was) a great experimental and theoretical success.
Question: is there a possibility of such states in the next simplest atomic system, the antiprotonic lithium: $(Li^{3+} - 2e^- - \bar{p})$

Two basic causes of metastability:

a) the collisional de-excitation of these states is suppressed
b) certain states can emit only large orbital momentum ($l \geq 4$) Auger-electrons

We checked the possibility of a) for the case of $(Li^{3+} - 2e^- - \bar{p})$,
calculating the spectrum of this 4-body system.

Two-step adiabatic approximation:
– two electron ground state energy $\varepsilon(R)$ in the field of the lithium nucleus and antiproton, fixed at a distance R;
– heavy particle relative motion bound states in the potential

$$V(R) = \frac{J(J+1)}{2MR^2} - \frac{3}{R} + \varepsilon(R)$$

The two-electron wave function (energy) was approximated by a product (sum) of two single-electron two-center wave functions (energies) with ef-fective charges to simulate the neglected electron-electron interaction. The effective charges were chosen to reproduce the experimental binding energies in the two limiting cases $R \to 0$ and $R \to \infty$:

$$\varepsilon(0) = E_{gs}(He) \;\; ; \;\; \varepsilon(\infty) = E_{gs}(Li^{+})$$

Conclusions:

- the density of states in antiprotonic lithium is much higher, than in antiprotonic helium;
- many possible candidates for long-lived states according to criterium a).

However:

no information on the collisional stability and on formation probabilities, thus the experimental observability of long lived states in Li is still open

"Search for long lived states in antiprotonic lithium"
J. Révai and V.B. Belyaev
Phys. Rev. A **67**(2003)032507

ii.

A less studied aspect of long-lived states of antiprotonic helium is the for-mation probability of different *(Jv)* states in the capture reaction

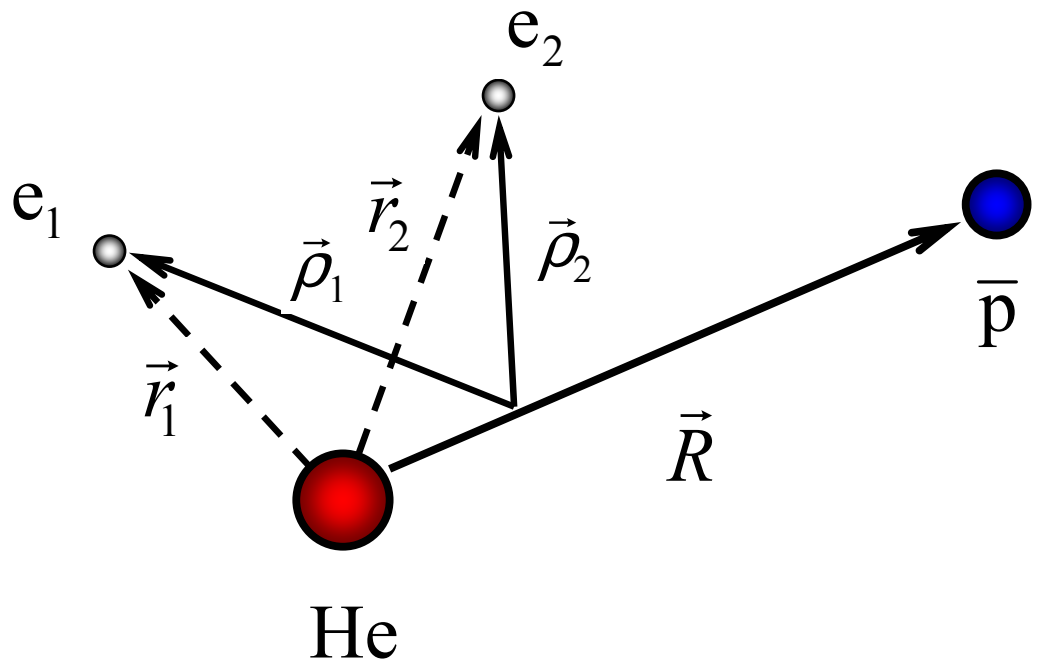
$$He + \overline{p} \rightarrow (He^{+}\overline{p})_{Jv} + e^{-}$$

Only classical and semiclassical estimates were available for the cross sections.
Our aim: full, though approximate, quantum mechanical calculation. Full in the sense, that all degrees of freedom and all relevant quantum numbers are treated explicitely.

$$\sigma_{Jv} \sim \left| \left\langle \Phi^{f}_{Jv,\boldsymbol{K}_f} \mid V_f \mid \Psi^{i}_{He,\boldsymbol{K}_i} \right\rangle \right|^2$$

To be able to speak about the functions involved, let's define our coordinate system

The set of coordinates



$\vec{r}_i$ - vectors from helium to the *i*-th electron;

$\vec{\rho}_i$ - Jacobian vectors connecting the electrons with the center of mass of the *He* - antiproton system;

$\vec{R}$ - vector between *He* and antiproton.

$$\Psi^{i}_{He,\boldsymbol{K}_i} \Leftarrow \Phi^{i}_{He,\boldsymbol{K}_i} = \Phi_{He}(\boldsymbol{r}_1,\boldsymbol{r}_2)(2\pi)^{-3/2} e^{i\boldsymbol{K}_i\boldsymbol{R}}$$

$$\Phi^{f}_{Jv,\boldsymbol{K}_f} = \Phi^{BO}_{Jv}(\boldsymbol{\rho}_1,\boldsymbol{R})(2\pi)^{-3/2} e^{i\boldsymbol{K}_f\boldsymbol{\rho}_2}$$

All functions are four-body wave functions, depend on 3 vector variables.

$\Psi^{i}_{He,\boldsymbol{K}_i}$ can not be calculated exactly, approximations are needed

First attempt: Born approximation $\Psi^{i}_{He,\boldsymbol{K}_i} \approx \Phi^{i}_{He,\boldsymbol{K}_i}$

We calculated the σ_{Jv} cross sections for a wide range of (Jv) and hoped that due to the realistic final state functions at least the relative magnitude of the cross sections will contain useful information.

Two basic drawbacks of the Born approximation:

• the antiproton “feels” an interaction from the He atom, its wave function has to be distorted from a plane wave;

• the He electrons also “feel” the approaching antiproton, the polarization of their wave functions has to be taken into account.

Slow collision — adiabatic approach: $\Psi^{i}_{He,\boldsymbol{K_i}} \approx \Phi_{He}(\boldsymbol{r_1},\boldsymbol{r}_2;R)\chi^{+}_{\boldsymbol{K}_i}(\boldsymbol{R})$

$\Phi_{He}(\boldsymbol{r_1},\boldsymbol{r}_2;R)$ — wave function of the He atom in the presence of the $\bar{p}$ at a fixed distance R from it with the corresponding energy $\varepsilon(R)$

$\chi^{+}_{\boldsymbol{K}_i}(\boldsymbol{R})$ — scattering wave function of the antiproton in the potential

$$V(R) = \frac{J(J+1)}{2MR^2} - \frac{2}{R} + \varepsilon(R)$$

$\Phi_{He}(\boldsymbol{r_1},\boldsymbol{r}_2;R)$ was approximated — similarly to the Li case — by a product of two single-particle two-center functions with effective charges fitted to

$$\varepsilon(0) = E_{gs}(H^-) \quad ; \quad \varepsilon(\infty) = E_{gs}(He)$$

Again, the full transition matrix element was calculated to yield the $\sigma_{Jv}(E)$ for a smaller range of (*Jv*) since the calculation time is significantly increased compared to the Born case.

We think, we have reliable cross sections now, however, since the energy distribution of the slowed down antiprotons is unknown, the experimentally observed population numbers can not be deduced directly.

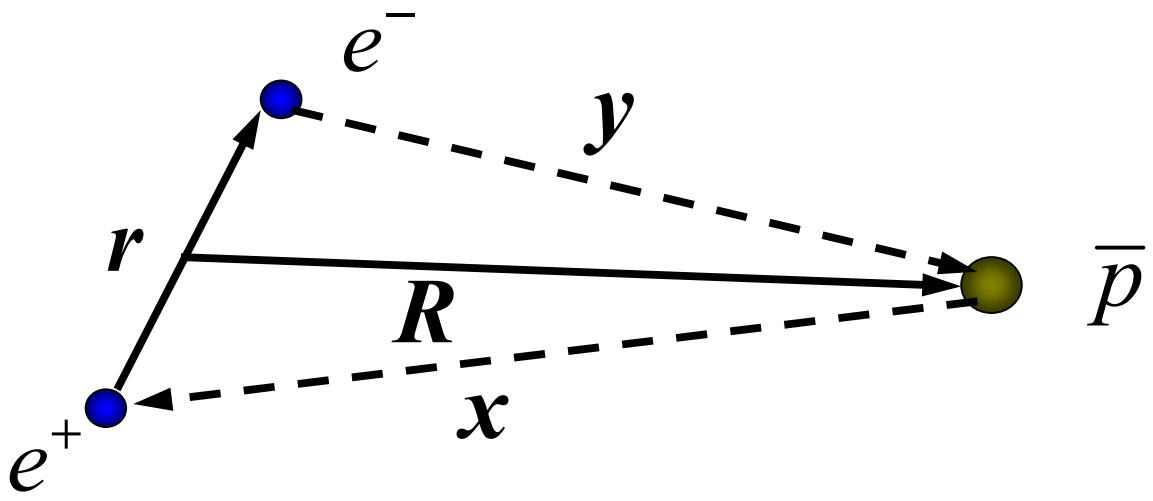
"Primary population of antiprotonic helium states"
J. Révai, N. V. Shevchenko
arXive, physics/0310153, and talks at
Workshop on computational physics dedicated to the memory of S. P. Merkuriev, St. Petersburg, Russia, 24-27 August, 2003
19th European conference on few-body problems in physics, Groningen, The Netherlands, 23-27 August, 2004

iii.

Description of antihydrogen formation in the reaction

$$(e^+e^-)_{2s} + \bar{p} \rightarrow (\bar{H})_{2s} + e^-$$

Rearrangement collision in a 3-body Coulomb system — exact description extremely difficult.
Approximate solution — close coupling:



$$H = H_0 - \frac{1}{r} - \frac{1}{x} + \frac{1}{y}; \quad H_0 = -\Delta_{\boldsymbol{r}} - \frac{1}{4}\Delta_{\boldsymbol{R}} = -\frac{1}{2}\Delta_{\boldsymbol{x}} - \frac{1}{2}\Delta_{\boldsymbol{y}}$$

$$\Psi \sim \sum_i \varphi_i(r) u_i(R) + \sum_j \phi_j(x) U_j(y)$$

$$(-\Delta_{\boldsymbol{r}} - \frac{1}{r} - E_i)\varphi_i(r) = 0; \; (-\frac{1}{2}\Delta_{\boldsymbol{x}} - \frac{1}{x} - E_j)\phi_j(x) = 0$$

Integro-differential equations for $u_i(R)$ and $U_j(y)$

Disandvantages:

• due to the exponential decay of bound state functions, the $u_i(R)$ and $U_j(y)$ "feel" only short range interactions. In the Coulomb case the polarization potentials are important.

• the bound state functions are "frozen", their disrtortion by the interaction with the projectile is only taken into account by higher order terms in the expansion.

Modified close coupling:

$$\Psi \sim \sum_i \tilde{\varphi}_i(r;R)\tilde{u}_i(R) + \sum_j \tilde{\phi}_j(x;y)\tilde{U}_j(y)$$

$$\left[-\Delta_r - \frac{1}{r} - \underbrace{\frac{1}{|\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}|}}_{x} + \underbrace{\frac{1}{|\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}|}}_{y} - \tilde{E}_i(R)\right]\tilde{\varphi}_i(r;R) = 0$$

$$\left[-\frac{1}{2}\Delta_x - \frac{1}{x} + \frac{1}{y} - \underbrace{\frac{1}{|\boldsymbol{x}+\boldsymbol{y}|}}_{r} - \tilde{E}_j(y)\right]\tilde{\phi}_j(x;y) = 0$$

$\tilde{\varphi}_i(r;R)$ and $\tilde{\phi}_j(x;y)$ are special “three-center” functions, which take into account the polarization of bound states on one hand, and through their eigenvalues $\tilde{E}_i(R)$ and $\tilde{E}_j(y)$ on the other, provide long range polarization potentials for the scattering functions $\tilde{u}_i(R)$ and $\tilde{U}_j(y)$.

In spite of the considerable amount of preparatory work on this subject, no real results can be presented, since one of the participants, (A. Os'kin) stopped working on the integro-differential equation solving part of the problem. The idea seems to be attractive, we plan to continue working on it.