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Localization of nonlocal potentials by a Taylor expansion method

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Abstract. The Taylor expansion procedure introduced by Apagyi and Scheid (1983 *J. Phys. G: Nucl. Phys.* **9** 73) for localization of nonlocal potentials has been re-examined and improved to include third-order corrections. The method has been applied to the real part of the nonlocal potential describing n–^{40}Ca scattering. Comparison with exact results and investigation of phase shifts show that inclusion of second-order correction terms may be sufficient to obtain a quick assessment of the local space effect of the underlying nonlocal potential.

1. Introduction

It is well known that nonlocal potentials arise in the collisions of composite quantum mechanical systems. These nonlocalities are due to contributions of inelastic and reaction channels and the Pauli principle. Although the resulting coupled integro-differential equations can be solved by standard numerical techniques, the accurate solution of the collision problem is tedious and time-consuming even in the simplest case. Therefore, it is reasonable to investigate and develop localization procedures which make the potentials local, thus enabling a quick assessment of the local space effect of the underlying nonlocal potential.

A localization procedure was carried out by Perey and Buck [1]. Bencze and Zimányi, and Buttle and Goldfarb outlined a similar localization method [2]. These investigations led to the introduction of the so-called Perey–Buck factor which proved useful in several DWBA codes to account for the exchange nonlocality in nucleon transfer reactions. Fiedeldey and co-workers [3] applied a semi-classical WKB method for the localization of nonlocal potentials. Peierls and Vin Mau [4] used a Wigner transform technique for treating the nonlocality. Localization of nonlocal potentials arising in a microscopic treatment of nuclear reactions has been carried out in [5].

In this paper we reconsider and improve the Taylor expansion localization method introduced by Apagyi and Scheid [6]. This method is quite simple and predicts an energy and angular momentum dependence of the localized potentials. Although the approximate local potentials belonging to the different orders of the expansion are formally not phase-equivalent with the nonlocal potential, they may still provide accurate phase shifts if enough terms are retained in the procedure. In the original formulation [6], the expansion up to second order ($\lambda_{\max} = 2$) was considered and here we also include the third-order correction terms ($\lambda_{\max} = 3$). If the sequence of the approximate local potentials corresponding to different

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orders of the Taylor expansion shows a convergence, then these local potentials can describe the scattering effect of the original nonlocal potential, giving similar phase shifts.

As a simplification of the original method [6] we apply the Taylor expansion technique to the radial wavefunction $f_l(r) = rR_l(r)$ instead of $R_l(r)$. In doing so, much simpler expressions can be obtained than those given in [6]. By retaining terms up to the third order in the Taylor expansion, the equations can be cast into a Schrödinger equation with an effective local potential. The localized potentials so obtained depend explicitly on the scattering energy E and the angular momentum quantum number l. An analytical formulation of the method has been elaborated elsewhere [7] for the case of a nonlocal model potential of the form $K_l(r, r') = (rr')^l \exp(-ar - a'r')$. This analytical model has served as a test case of the numerical code for localizing more realistic nonlocal potentials.

The method can be applied to nonlocal kernels $K_l(r, r')$ which are peaked around the main diagonal in the r, r' space. This condition is usually fulfilled in nuclear physics where Gaussian-type effective interactions determine the potentials. As an application of the procedure, we consider the real part of the nonlocal potential arising in the scattering of neutrons by the nucleus ^{40}Ca. This potential of Frahn–Lemmer-form usually serves as the archetype of different localization procedures [3–5] so that comparison of results with others is easily possible.

Section 2 contains the theoretical background of the localization procedure. In section 3 the theory is applied to a nonlocal model by calculating the localized potentials in various orders of the Taylor expansion and the phase shifts generated by them. This paper ends with a short summary.

2. Theory

In [6] the following Schrödinger equation with nonlocal kernel K_l and spherical symmetric direct potential $V_D(r)$ was investigated:

$$\left(-\frac{\hbar^2}{2\mu}\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}r + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} + V_D(r) - E\right) R_l(r) + \int_0^\infty K_l(r, r')R_l(r')r'^2\,\mathrm{d}r' = 0. \tag{1a}$$

However, it is more useful to introduce the usual transformation $R_l(r) = f_l(r)/r$ and rewrite (1a) as the Schrödinger equation for the radial wavefunction $f_l(r)$,

$$\left(-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} + V_D(r) - E\right) f_l(r) + \int_0^\infty \tilde{K}_l(r, r')\,f_l(r')\,\mathrm{d}r' = 0, \tag{1b}$$

where the radial nonlocal kernel $\tilde{K}_l$ given by

$$\tilde{K}_l(r, r') = rK_l(r, r')r' = \tilde{K}_l^\dagger(r, r')$$

is assumed as Hermitian.

The known boundary conditions for the radial wavefunctions are

$$f_l(0) = 0, \qquad \text{and} \qquad f_l(r \to \infty) = \mathrm{A}_l \sin(kr + \delta_l), \tag{1c}$$

with δ_l and A_l being the phase shift and normalization constant, respectively, in the lth partial wave. In the following we use a Taylor expansion for the radial wavefunction $f_l(r')$ valid in the vicinity of r:

$$f_l(r') = \sum_{\lambda=0}^{\infty} \frac{(r'-r)^\lambda}{\lambda!}\partial_r^\lambda f_l(r). \tag{2}$$

In order to apply this expansion we consider general matrix elements of the nonlocal term in (1b)

$$\int_0^\infty \int_0^\infty g_l^*(r)\tfrac{1}{2}(\tilde{K}_l(r, r') + \tilde{K}_l^\dagger(r, r'))\,f_l(r')\,\mathrm{d}r\,\mathrm{d}r'. \tag{3}$$

These matrix elements are Hermitian. They can be written with Taylor expansions for $g_l(r)$ and $f_l(r)$ as follows:

$$\int_0^\infty g_l^*(r) W_\lambda^{(l)}(r) f_l(r)\,\mathrm{d}r$$

with

$$W_\lambda^{(l)}(r) = \frac{1}{2\lambda!}(U_\lambda^{(l)}(r)\partial_r^\lambda + (-1)^\lambda \partial_r^\lambda U_\lambda^{(l)*}(r)) = W_\lambda^{(l)\dagger}(r), \tag{4}$$

where the moments of the radial kernel are defined by

$$U_\lambda^{(l)} = \int_0^\infty \tilde{K}_l(r, r')(r' - r)^\lambda\,\mathrm{d}r'. \tag{5}$$

Therefore, we are allowed to replace the nonlocal kernel in (1*b*) by a local Hermitian potential:

$$\int_0^\infty \tilde{K}_l(r, r') f_l(r')\,\mathrm{d}r' \Rightarrow \sum_{\lambda=0}^\infty W_\lambda^{(l)}(r) f_l(r). \tag{6}$$

Let us relate, in (2), the quantities $(r - r')$ and ∂_r to the measure Δ of the nonlocality and the local wavenumber $k(r)$, respectively. Then one can assess that the Taylor expansion procedure is useful if $\Delta \times k$ is small, i.e. in cases of low energies and/or narrow nonlocalities.

In [6] the Taylor expansion has been performed explicitly up to the second order ($\lambda_{\max} = 2$) only. However, (6) allows one to go beyond the second-order approximation and to also include terms arising from the third-order expansion with $\lambda_{\max} = 3$. The terms containing the bare third-order derivative occurring in the form $g(r)\partial_r^3$, where $g(r)$ is an arbitrary function of r, drop out if $U_\lambda^{(l)} = U_\lambda^{(l)*}$. This condition is fulfilled for all real-valued nonlocal kernels $K_l(r, r') = K_l^*(r, r')$. The bare first-order derivative $g(r)\partial_r$ occurring in the second- and third-order terms ($W_2^{(l)}$ and $W_3^{(l)}$, see later) will be transformed out as shown below.

In this way, we obtain a scheme where the equation with terms resulting from the expansion up to third order ($\lambda_{\max} = 3$) in the Taylor expansion can be cast into a Schrödinger equation with effective (localized) potentials in which the contributions from the various orders of λ can explicitly be studied and visualized.

Going up to third order in the Taylor expansion (putting $\lambda_{\max} = 3$ in (2)), one has to deal with the following Hermitian potential operators:

$$W_0^{(l)}(r) = U_0^{(l)}(r), \tag{7}$$

$$W_1^{(l)}(r) = -\tfrac{1}{2}U_1^{(l)'}(r), \tag{8}$$

$$W_2^{(l)}(r) = \tfrac{1}{4}U_2^{(l)''}(r) + \tfrac{1}{2}U_2^{(l)'}(r)\partial_r + \tfrac{1}{2}U_2^{(l)}(r)\partial_r^2, \tag{9}$$

$$W_3^{(l)}(r) = -\tfrac{1}{12}U_3^{(l)'''}(r) - \tfrac{3}{12}U_3^{(l)''}(r)\partial_r - \tfrac{3}{12}U_3^{(l)'}(r)\partial_r^2, \tag{10}$$

where primes denote derivatives with respect to the relative distance r. It is a simple matter to prove that the potential operators given by (7)–(10) are indeed Hermitian.

By using these formulae in (1*b*) together with (6), the radial Schrödinger equation becomes

$$A_l(r)\frac{\mathrm{d}^2 f_l(r)}{\mathrm{d}r^2} + B_l(r)\frac{\mathrm{d}f_l(r)}{\mathrm{d}r} + (C_l(r) - E) f_l(r) = 0, \tag{11}$$

where $f_l(r)$ is now an approximation of the true wavefunction appearing in (1*b*).

The functions $A_l(r)$, $B_l(r)$, and $C_l(r)$ are defined as follows:

$$A_l = -\frac{\hbar^2}{2\mu} + \frac{1}{2}U_2^{(l)} - \frac{3}{12}U_3^{(l)'} \equiv -\frac{\hbar^2}{2\tilde{M}_l(r)}, \tag{12}$$

$$B_l = \tfrac{1}{2}U_2^{(l)'} - \tfrac{3}{12}U_3^{(l)''} = A_l', \tag{13}$$

$$C_l = \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} + V_D(r) + U_0^{(l)} - \frac{1}{2}U_1^{(l)'} + \frac{1}{4}U_2^{(l)''} - \frac{1}{12}U_3^{(l)'''}. \tag{14}$$

In the next step we transform (11) into a usual Schrödinger equation with a constant mass μ and without a first-order derivative. This can be achieved by introducing a transformation $T_l(r)$ with the following definition:

$$T_l(r) f_l(r) = \tilde{f}_l(r), \qquad T_l(r \to \infty) = 1. \tag{15a}$$

The condition of disappearance of the first derivative $\tilde{f}_l'$ from (11) leads to a differential equation for the transformation function $T_l(r)$:

$$T_l'/T_l = B_l/(2A_l). \tag{15b}$$

Finally, we obtain the Schrödinger equation in the required form:

$$-\frac{\hbar^2}{2\mu}\tilde{f}_l'' + \left(\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} + \tilde{V}_l - E\right)\tilde{f}_l = 0 \tag{16}$$

with the 'localized' potential

$$\tilde{V}_l = E - \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} - \frac{\hbar^2}{2\mu}\left[\frac{C_l - E}{A_l} - \frac{1}{4}\left(\frac{B_l}{A_l}\right)^2 - \frac{1}{2}\frac{B_l'A_l - B_lA_l'}{A_l^2}\right] \tag{17}$$

which is correct up to third order of the Taylor expansion.

The effective potential (17) depends explicitly on the scattering energy E and the angular momentum l in the interaction domain of the scattered particles. At large relative distances where the effect of the nonlocality is negligible and the moments $U_\lambda^{(l)}$ of the nonlocal kernel $\tilde{K}_l(r, r')$ become zero, the effective potential approaches the direct potential $V_D(r)$ contained in the function $C_l(r)$ (see (14)).

The function $A_l(r)$ is related to the effective mass $\tilde{M}_l(r) = -\hbar^2/\big(2A_l(r)\big)$ whose dependence on r and l originates entirely from the nonlocality via the second- and third-order moments, $U_2^{(l)}$ and $U_3^{(l)'}$, respectively. In the regions where $A_l(r) = 0$ occurs (in the case of broad nonlocality), the effective mass becomes infinite; in such cases an investigation of higher-order corrections is indispensable. The function $B_l(r)$ multiplying the first derivative of the approximate scattering wavefunction in (11) is related to the momentum. The function $C_l(r)$ is related to the centrifugal potential and the direct interaction $V_D(r)$. It is observed that both functions A_l and B_l acquire constant values in the lower approximations ($\lambda_{\max} = 0, 1$); in those cases the nonlocal effects enter via the function $C_l(r)$.

One may generalize the above method to coupled channels along the lines of [6]. In this case the functions $\tilde{V}_l$, A_l, B_l, and C_l are matrices depending on the channel quantum numbers. Formula (17) for the local potential matrix coincides with the corresponding equation (37) of [6], the only exception is the different definition of the moments $U_\lambda^{(l)}$.

3. Application and discussion

3.1. Frahn–Lemmer-type nonlocality for n–^{40}Ca scattering

The radial kernel of the Frahn–Lemmer-type nonlocality [3, 5, 8] is given by (all lengths measured in fm):

$$\tilde{K}_l(r, r') = 4\pi r r' V(R) H_l(r, r'), \qquad \left(R = \frac{r + r'}{2}\right), \tag{18a}$$

with the form factor

$$V(R) = \frac{-71}{1 + \mathrm{e}^{(R-4.17)/0.65}}\ \text{MeV}, \tag{18b}$$

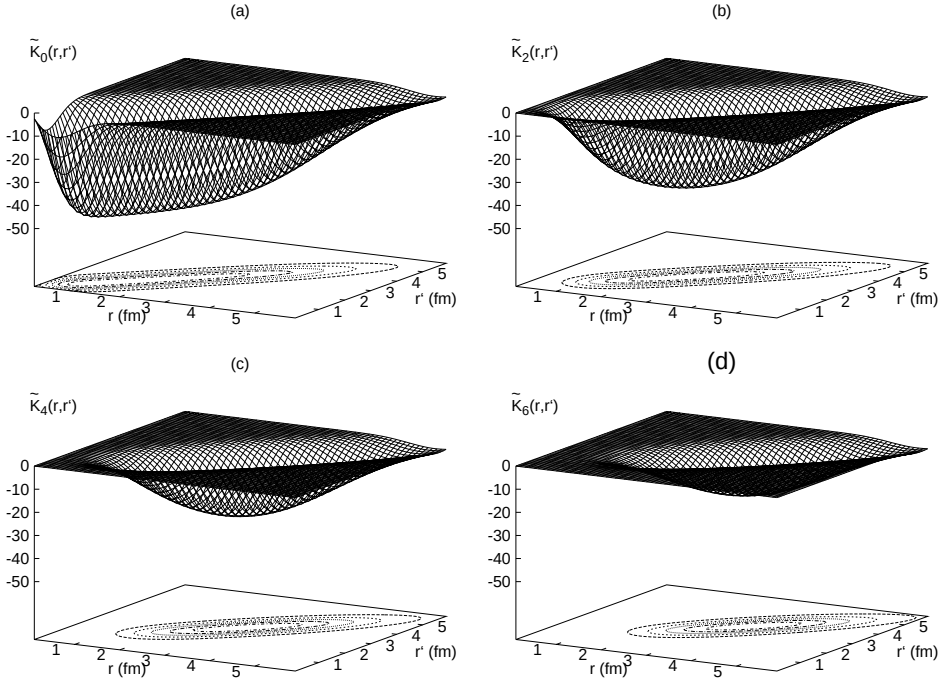

Figure 1. Radial nonlocal kernels $\tilde{K}_l(r, r')$ of Frahn–Lemmer-type in units of MeV fm^{-1} for scattering of neutrons by ^{40}Ca; (*a*) $l = 0$, (*b*) $l = 2$, (*c*) $l = 4$, (*d*) $l = 6$.

and the nonlocality factor

$$H_l(r, r') = \frac{1}{(\pi\gamma^2)^{3/2}} \mathrm{e}^{-(r^2+r'^2)/\gamma^2} i_l(2rr'/\gamma^2), \qquad (\gamma = 0.85 \text{ fm}^2). \tag{18c}$$

The modified Bessel functions i_l originate from the angular integration [9].

This potential fits the low-energy data of neutron scattering on a ^{40}Ca nucleus. The model has been used in [5] to compare localized potentials with exact ones and in [8] to reconstruct the nonlocality from the phase shifts. The kernel (18*a*) is visualized in figures 1(*a*)–(*d*) for $l = 0, 2, 4, 6$, respectively. The three-dimensional plots clearly show that the nonlocality is concentrated along the diagonal $r \sim r'$ and its magnitude becomes smaller as l increases.

The functions $A_l(r)$, $B_l(r)$, and $C_l(r)$ which play a decisive role in the construction of the localized potentials by the Taylor expansion method are exhibited in figures 2(*a*)–(*c*) for $l = 0, 2, 4, 6$ in the third-order approximation ($\lambda_{\max} = 3$). It can be seen in figure 2(*a*) that the function $A_l(r)$ admits only negative values. This ensures that the effective mass arising due to nonlocalities via the second- and third-order moments $U_2^{(l)}$ and $U_3^{(l)}$ remains positive on the whole axis of r. The nonlocal effect which decreases the effective mass (increases $|A_l|$) becomes smaller for higher angular momenta l. As figure 2(*b*) shows, B_l is generally small compared with A_l indicating that the change of the effective mass is not significant in the domain of the nonlocality of Frahn–Lemmer-type. Outside the nonlocality range, the function $C_l(r)$ is equal to the sum of the centrifugal barrier and the direct potential (see (14)). In the present model, the local potential is set to zero: $V_D(r) = 0$. Therefore, C_0 (full curve in figure 2(*c*)) directly shows the local contributions of the nonlocal kernel via the various moments up to $\lambda_{\max} = 3$. The steep up-rising behaviour of $C_0(r)$ as $r \to 0$ results from the derivatives of the various moments $U_1^{(0)'}$, $U_2^{(0)''}$, $U_3^{(0)'''}$ (see (14)). For $l \neq 0$, C_l is dominated by the centrifugal potential.

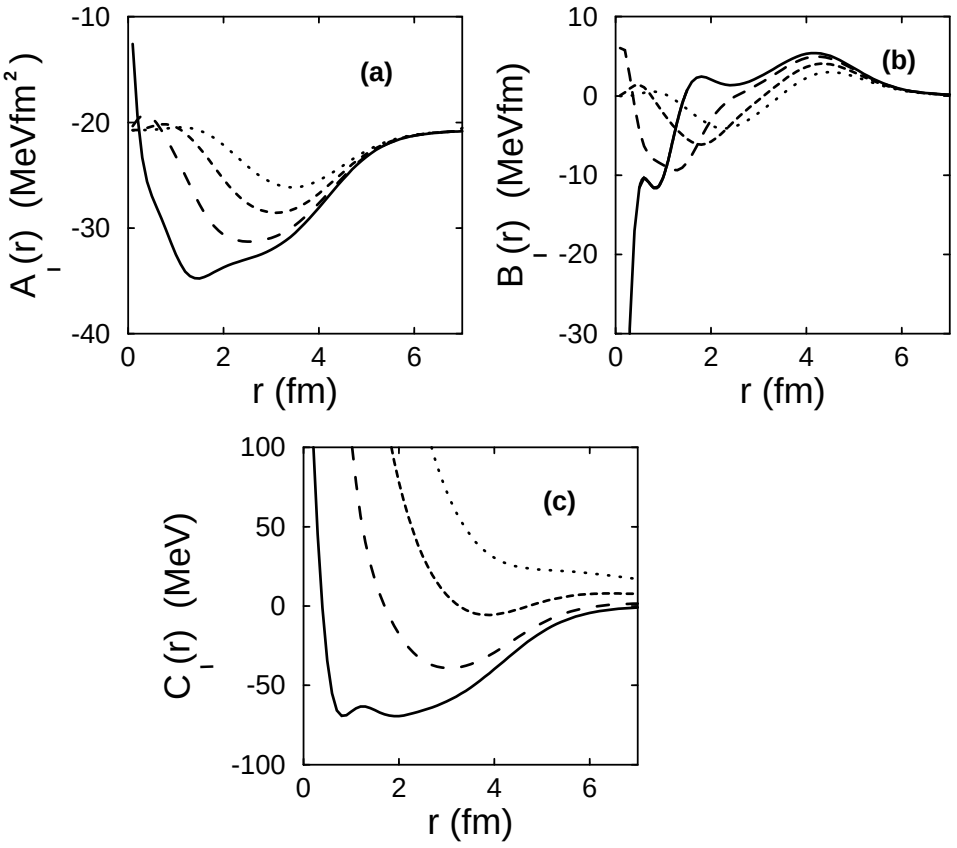

Figure 2. The functions $A_l(r)$, $B_l(r)$, and $C_l(r)$ involved in the Taylor expansion up to $\lambda_{\max} = 3$ for localization of the Frahn–Lemmer-type potential of (18); full curves: $l = 0$, long-dashed curves: $l = 2$, dashed curves: $l = 4$, dotted curves: $l = 6$.

3.2. Localized potentials

Figure 3 shows the localized (effective) potentials $\tilde{V}_l(r)$ at $l = 0, 2, 4, 6$ and $E = 30$ MeV, obtained by the Taylor expansion method using (17) for the Frahn–Lemmer-type nonlocality given by (18). Each part of figure 3 exhibits at least four curves corresponding to the four possible orders of the approximation in the Taylor expansion up to $\lambda_{\max} = 0, 1, 2$, or 3, respectively. The dot-dashed curves in figures 3(*a*)–(*c*) also contain the exact local potentials which have been taken from [5]. These phase equivalent potentials can be obtained by manipulating with the Wronskian of the exact solutions $f_l(r)$. (For details see [5].)

From figure 3 it is apparent that the localized potentials belonging to the two lower approximations ($\lambda_{\max} = 0, 1$; dotted and short-dashed curves) and to the two higher approximations ($\lambda_{\max} = 2, 3$; long-dashed and full curves), show a different behaviour. This can be explained by the structure of the functions $A_l(r)$, $B_l(r)$, and $C_l(r)$. According to (12)–(14) we obtain $A_l = \text{const} \neq 0$, $B_l = 0$, and $C_l = \hbar^2 l(l+1)/2\mu r^2 + V_D + U_0^{(l)} - U_1^{(l)'}/2$ in the lower approximations ($\lambda_{\max} = 0, 1$) so that the localized potential (17) simplifies to $\tilde{V}_l = V_D + U_0^{(l)} - U_1^{(l)'}/2$. The first nontrivial corrections come from the second-order expansion ($\lambda_{\max} = 2$) modifying the mass of the scattered particle as a function of r due to the nonlocal interaction. This modification may be quite large and dramatic, since $A_l(r)$

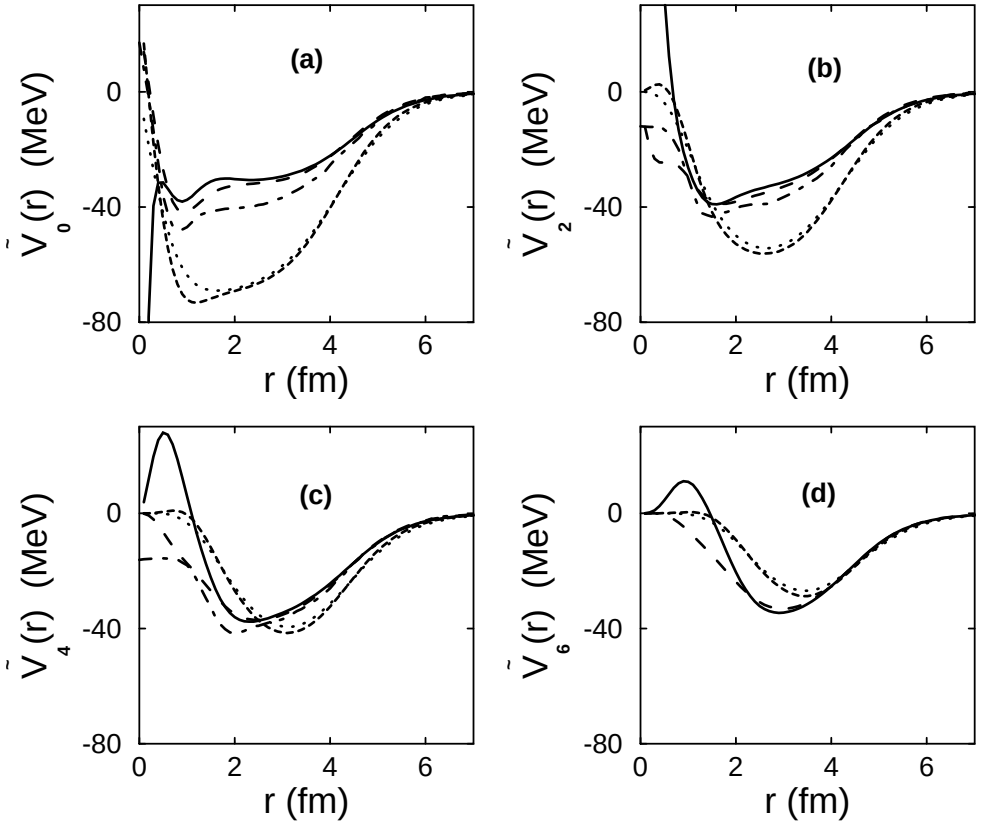

Figure 3. Localized potentials $\tilde{V}_l(r)$ obtained for the Frahn–Lemmer nonlocal potentials in various orders λ of the Taylor expansion at $E = 30$ MeV; dotted curves: $\lambda = 0$, short-dashed curves: $\lambda = 1$, long-dashed curves: $\lambda = 2$, full curves: $\lambda = 3$, dot-dashed curves: exact (Wronskian) results as given by [5]. (*a*) $l = 0$, (*b*) $l = 2$, (*c*) $l = 4$, (*d*) $l = 6$.

appears in the denominator in (17). If $A(r)$ becomes very small compared with $B_l(r)$ and $C_l(r)$, then the localized potential is much enhanced. The first-order term $\lambda_{\max} = 3$ in (12) does not contribute much to the mass because $\frac{1}{2}U_2^{(l)} \gg \frac{3}{12}U_3^{(l)'}$. Going up to higher order in $\lambda_{\max}$, higher derivatives with respect to r cannot be eliminated with any kind of transformation to obtain a second-order Schrödinger equation. Their contributions should be small if the Taylor expansion (6) of the nonlocal kernel term is rapidly converging with λ. We observe in figures 3(*a*)–(*c*) that localized potentials in the second- and third-order approximation lie much closer to the exact (Wronskian) ones. Notice that the Taylor expansion localization method, in the second- and third-order aproximation, also begins to mimic the 'kink' of the exact potential around $r \sim 1$ fm in the s-wave channel (see figure 3(*a*)).

In order that the Taylor expansion procedure be practical, i.e. the second-order approximation be a good one, the localized potentials in the second and third approximations ($\lambda_{\max} = 2$ and 3) should not differ too greatly from each other. That is precisely what we observe in each part of figure 3, apart from the very interior region of $0 \leqslant r \leqslant 0.5$–$1.5$ fm. From the difference between the second- and third-order potentials we expect the sequence of phase shifts generated by the localized potentials of different order λ of approximation to show a faster convergence to the exact values in the higher partial waves l than in the lower ones.

Table 1. Tangents of several phase shifts generated by the localized potentials in various orders λ of approximation at three energies. Values in lines marked by $\lambda = \infty$ refer to exact values of $\tan\delta_l$ generated by the nonlocal potential given by (18).

E (MeV)	λ	$\tan\delta_0$	$\tan\delta_1$	$\tan\delta_2$		
1	0	4.666	−0.208	0.166		
	1	7.433	−0.191	0.141		
	2	−0.776	−0.463	0.037		
	3	−0.774	−0.490	0.035		
	∞	−0.624	−0.436	0.040		
E (MeV)	λ	$\tan\delta_0$	$\tan\delta_1$	$\tan\delta_3$	$\tan\delta_5$	
10	0	−1.188	−8.352	0.068	0.075	
	1	−0.978	−5.720	0.067	0.063	
	2	1.236	1.184	−0.196	0.067	
	3	1.274	1.082	−0.205	0.067	
	∞	1.790	1.373	−0.182	0.068	
E (MeV)	λ	$\tan\delta_0$	$\tan\delta_2$	$\tan\delta_4$	$\tan\delta_6$	$\tan\delta_8$
30	0	1.206	0.142	−1.098	0.766	0.048
	1	1.424	0.201	−1.017	0.690	0.040
	2	−0.700	−0.760	−2.077	0.644	0.042
	3	−0.656	−0.819	−2.117	0.655	0.043
	∞	−0.374	−0.619	−1.951	0.655	0.043

Let us discuss the transformation function $T_l(r) = \exp(-\int_r^\infty B_l(r)/(2A_l(r))\,\mathrm{d}r)$ obtained from (15*b*). In the outer region $r > 1.5 - 3$, $B_l(r)$ is positive and $A_l(r)$ negative as shown in figure 2. Therefore, $T_l(r)$ increases over 1.0 for decreasing values of r up to a maximum at $B_l(r) = 0$. In the interior region $T_l(r)$ is decreasing with decreasing r.

In table 1 the tangents of phase shifts generated by the various localized potentials are listed at three scattering energies and at several l values, together with the exact values listed in the lines indicated by $\lambda = \infty$. From table 1 we see that it is necessary to go up to second order in the Taylor approximation since the values of $\tan\delta_l$ belonging to the zeroth- and first-order approximations ($\lambda = 0, 1$) differ greatly, sometimes by an order of magnitude and even in sign, from the exact values. We also observe, as a justification of the earlier analysis in connection with figure 3, that the convergence is better in the higher partial waves. In the s-wave the convergence gets better as the energy becomes lower. This is also expected as remarked above. It is characteristic for the Taylor expansion method that it proceeds with the power of $\Delta \times k$, where Δ denotes the range of the nonlocality and $k(r)$ denotes the local wavenumber. This wavenumber takes the effects of the potential including the centrifugal part into account. Therefore, the convergence of the method is faster for larger l-values, because then the local wavenumber is smaller. As the energy gets smaller, the succesive terms in the Taylor expansion give even smaller contributions so that the convergence gets better.

Finally, we note that performing calculations of phase shifts without using the symmetrized form of $W_\lambda^{(l)}(r)$ given by (4), but using the localized potential operator $W_\lambda^{(l)}$ in a non-Hermitian form

$$\tilde{W}_\lambda^{(l)}(r) = \frac{1}{\lambda!} U_\lambda^{(l)}(r)\partial_r^\lambda, \tag{19}$$

gives practically the same results as shown in table 1. Additionally, the underlying localized potentials are almost the same. However, in more complicated cases such as of coupled channel reactions or of more sophisticated nonlocal potentials, the use of $W_\lambda^{(l)}(r)$ of (4) is the correct

procedure. In cases of narrow nonlocalities, we may use (19) instead of (4) at the price of not being able to assess the effect of the third-order corrections. In that case one obtains simpler expressions for the functions A_l, B_l, and C_l, whereas (17) for the localized potentials remains the same.

4. Summary

The localization procedure with a Taylor expansion of [6] is modified, improved and investigated. The modification is connected with the use of the transformed radial wavefunction f_l in the Taylor expansion instead of the full radial wavefunction $R_l = f_l/r$ considered in [6]. An improvement of the old procedure is the introduction of the third-order terms into the localization procedure. The inclusion of third-order terms allows one to draw conclusions about the convergence of the localization procedure since there is a marked difference between the lower-order ($\lambda_{\max} = 0, 1$) and higher-order ($\lambda_{\max} = 2, 3$) approximations.

The modified and improved procedure is applied, for the first time, to the case of nonlocal potentials of Frahn–Lemmer-type. We found that the approximation up to $\lambda_{\max} = 3$ should be calculated in order to draw conclusions about the convergence of the method and the effect of the underlying nonlocality. The Taylor expansion localization procedure can be used for nonlocal potentials where the nonlocality is restricted along the main diagonal of the (r, r')-space. In the case of broader nonlocalities, for example in the case of a nonlocality originating from the exchange of nucleons between clusters, higher orders of the Taylor expansion are needed [7]. However, this procedure leads to higher than second-order differential equations whose solution and interpretation in the usual terms of local potentials might be difficult.

In summary, the investigations above suggest that the Taylor expansion localization technique may be useful in cases when the nonlocality is concentrated along the main diagonal $r \sim r'$, the local wavenumber is less or comparable with the inverse of the range of the nonlocality, and/or higher angular momentum channels than s-waves are of interest.

Acknowledgments

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