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Heat and Mass Transfer of Nanofluids Containing Metallic Nanoparticles

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Abstract. In this study will be investigated nanofluids, which contain metallic nanoparticles. A special type of these fluids is so-called ferrofluids. They can change their physical properties in magnetic fields and their nanoparticles can enhance the base fluid's thermal properties. The dynamic response of ferrofluids to the application of non-uniform magnetic fields will be studied for ferrofluids. The steady two-dimensional flow of an incompressible, viscous, and electrically nonconducting nanofluid over a linearly stretching flat sheet in the horizontal direction is considered when the wall temperature is a decreasing function of the distance from the leading edge. Magnetic nanoparticles are incorporated with water as a base fluid. The standard boundary layer theory is applied, the similarity variables are used to convert the system of partial differential equations to a system of ordinary differential equations. The numerical solutions are examined, and the influence of various governing factors is studied on the heat and mass transfer. The skin friction coefficient, the heat transfer rate, and the Sherwood number at the wall are compared for the different additives.

INTRODUCTION

The study of flow over a stretching sheet has generated a great deal of as a result of its applications in many engineering areas, industrial processes, and various industries. Applications include extrusion of plastics, fiberglass production, condensation processes of metal sheets in refrigerators, and glass and polymer industry. Sakiadis [1] started the examination of boundary layer flow over constantly moving solid surfaces at constant speed, while examination of the flow field above a stretching flat surface can be dated back to Crane's work [2]. Gupta and Gupta [3] analyzed the heat and mass transfer on a stretching film corresponding to the boundary layer similarity solution with suction and injection. Flow heat and mass transfer characteristics were analyzed in a saturated porous medium by Vajravelu [4] with frictional heating over an impermeable stretching sheet and internal heat generation or absorption. The boundary layer past a flat sheet stretched with a velocity proportional to the distance along the sheet was introduced by Danberg and Fansler [5]. According to experiments, the stretching velocity of the plastic film drawing is proportional to the distance from the slot [6]. Then classical model was then generalized in different directions [7-20].

Nowadays, more and more work is dealing with the question of how and to what extent the heat transfer characteristics of conventional liquids (oil, water, glycol etc.) used in technical practice can be increased. The addition of nanoparticles to the base fluid showed good results [21], [22]. The addition of even small amount of nano-sized particles provided an improvement of the thermal conductivity. Choi [21] showed that mixing 1% volume fraction nanoparticles to the base fluid has doubled the thermal conductivity of the working fluid. The influence of the nanoparticles on the fluid flow due to a stretching sheet is analyzed in papers [7-10].

Ferrofluids are nanofluids of ferromagnetic particles. Some of them are applied to reduce the surface friction. In the magnetic field the ferrofluid becomes strongly magnetic and a significant increase in heat transfer can be observed. The fluid flow properties can be controlled by the strength of the magnetic field [12, 13, 17-19]. Due to

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the practical relevance, the attention of researchers has been attracted to the effect of the magnetic field on the boundary layer. Heat transfer in magnetohydrodynamic (MHD) nanofluid flow past a stretching sheet was studied by Ibrahim [15].

The MHD flow of an Oldroyd-B fluid over stretching surface was analyzed in [10]. Authors obtained numerical results applying the similarity method in [17-20].

The objective of this paper is investigating the heat and mass transfer of ferrofluids along a linearly decreasing temperature wall in a non-uniform magnetic field. We shall apply similarity transformation to model the physical phenomena and solve it numerically. The effect of varying physical parameters (shear stress, heat and mass transfer at the wall) will be shown in figures and a table.

MODELLING OF THE PROBLEM

We consider the heat and mass transfer of three types of ferrofluids with different volume fractions. Steady twodimensional flow past an impermeable, linearly stretching sheet is analyzed.

The horizontal sheet is at y = 0 and the wall temperature is assumed to be variable $T_w = T_C - \frac{T_C x}{l}$, where l denotes the length of the plate. The stretching velocity is $u_w = U_w/\sqrt{x}$ and for the concentration C zero flux condition is considered as $C_B T \nabla C + \frac{C_T C \nabla T}{T} = 0$. The ambient fluid temperature attained at infinite distance from the surface is T_∞ . The ambient fluid velocity is $u_\infty = U_\infty/\sqrt{x}$ and C tends constant value C_∞ . The magnetic field is to the result of two-line currents, perpendicular to flow plane and equidistant from the leading edge (Fig. 1). It is assumed that the fluid is electrically non-conducting, and the magnetization direction of a fluid element is always in the direction of the local magnetic field.

The boundary layer equations for a two-dimensional and incompressible flow express the conservation of mass, continuity, momentum, and energy. In our analysis we assume [13] that the applied field is of sufficient strength to saturate the ferrofluid everywhere inside the boundary layer. the variation of magnetization of the field is described by $M = K(T_c - T)$, where K is the pyromagnetic coefficient and T_c is the Curie temperature as proposed in [13]. We neglect the induced field resulting from the induced magnetization compared to the applied field. Thermal heat capacity C_p , thermal conductivity k, and viscosity ν are considered independent of temperature in the temperature range investigated here.

We introduce the magnetic scalar potential ϕ for which the magnetic intensity $H = -\nabla \phi$ by

$$\phi(x,y) = -\frac{I_0}{2\pi} \Big(tan^{-1} \frac{y+a}{x} + tan^{-1} \frac{y-a}{x} \Big),$$

where I_0 is the dipole moment per unit length and a denotes the distance of the line current from the leading edge.

In the regions near the wall, when distances from the leading edge are large compared to those of the line sources from the plate $(x \gg a)$, then for the magnetic field one gets $[\nabla H]_x = -\frac{I_0}{\pi} \frac{1}{x^2}$.

The governing equation for this problem can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{I_0\mu_0K}{\pi\rho}(T_C - T)\frac{1}{x^2} + v\frac{\partial^2 u}{\partial y^2} + u_\infty\frac{\partial u_\infty}{\partial x}$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_C} \left(\frac{\partial T}{\partial y} \right)^2 \right],\tag{3}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D_B \frac{\partial^2 c}{\partial y^2} + \frac{D_T}{T_C} \frac{\partial^2 T}{\partial y^2},\tag{4}$$

where u and v are the parallel and normal velocity components to the plate and $\alpha = k/(C_p)$, μ_0 the permeability of the vacuum.

Introducing the stream function ψ satisfies the continuity equation (1) as $u = \partial \psi / \partial y$ and $v = -\partial \psi / \partial y$. In order to transform the system of equations (1)-(4) to a system of ordinary differential equations, the similarity transformation is applied in the form

$$\eta = \left(\frac{U_w}{v_x}\right)^{1/4} y, \ \psi = (U_w v x)^{1/2} f(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad g(\eta) = \frac{C - C_\infty}{C_w - C_\infty}.$$

Employing similarity transformation equations, (2)-(4) are reformulated in the nondimensional form

$$f''' + \frac{1}{4}ff'' + \frac{1}{2}(f')^2 + \beta\theta - \frac{1}{2}A^2 = 0,$$
(5)

$$\theta^{\prime\prime} + Pr\left[\frac{1}{4}f\theta^{\prime} + N_bg^{\prime}\theta^{\prime} + N_t(1+g)\theta^{\prime 2}\right] = 0, \tag{6}$$

$$Scfg' + g'' + \frac{\Delta T}{T_{\infty}}g'\theta' + \frac{N_t}{N_b} \left[-\frac{\Delta T}{T_{\infty}}(1+g)\theta'^2 + g'\theta' + (1+g)\theta'' \right] = 0$$
⁽⁷⁾

with ferromagnetic parameter $\beta = \frac{I_0 \mu_0 K}{\pi}$, $A = \frac{U_{\infty}}{U_w}$, $\Delta T = T_w - T_c$, $Sc = \frac{\mu}{\rho C_B T_{\infty}}$, Prandtl number $Pr = \frac{\nu}{\alpha}$, and the corresponding boundary conditions are

$$f(0) = 0, \ f'(0) = 1, \ \lim_{\eta \to \infty} f'(\eta) = A, \tag{8}$$

$$\theta(0) = 1, \lim_{\eta \to \infty} \theta(\eta) = 0, \tag{9}$$

$$g'(0) + \frac{N_T}{N_B} (1 + g(0))\theta'(0) = 0, \quad \lim_{\eta \to \infty} g(\eta) = 0.$$
(10)

The skin friction coefficient C_f , the local Nusselt number Nu_x , and the local Sherwood number Sh_x can be written as follows

where the wall shear stress:

the wall shear flux:

the wall mass flux:

$$\begin{array}{l} \frac{\tau_w}{\rho U_w^2}, \quad N u_x = \frac{x \, q_w}{k(T_w - T_C)}, \quad S h_x = \frac{x \, h_m}{D_b \left(C_w - C_{nf} \right)}, \\ \tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \\ q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \\ h_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}. \end{array}$$

Applying the nondimensional quantities one can define the formulas

 $C_f =$

 $-f''(0) = Re^{1/2}C_f, \ -\theta'(0) = Re^{-1/2}Nu_x, \ -g'(0) = Re^{-1/2}Sh_x,$ where *Re* is the Reynolds number.

DISCUSSION

The reduced equations (5)-(7) with boundary conditions (8)-(10) are solved numerically with the bvp4c method using MATLAB software considering various values of parameters involved. The asymptotic boundary condition is approximated by $\eta_{max} = 10$. The step size is set at $\Delta \eta = 0.01$ and the criterion of convergence at 10-9.

We analyzed the effect of parameters on the temperature, concentration, and velocity. Equations (5)-(7) are nonlinear coupled differential equations. Figures 2-8 demonstrate the non-dimensional velocity profiles at different parameter values while keeping the values of the other parameters fixed, and particularly A = 0.01. We observed that the decrease in ΔT , when T_w is decreasing, f''(0) shows a decrease.

Numerical results for the non-dimensional concentration equation (7) for the nanoparticles are also obtained. The concentration distribution and boundary layer thickness decrease with an increase in the parameters Pr, Nb, Nt, β, A, Sc but increase with increasing ΔT .





CONCLUSION

The heat and mass transfer characteristics in a non-uniform magnetic field for ferrofluid due to a stretching sheet have been studied numerically. The influence of ferromagnetic parameter β , Prandtl number Pr and Schmidt number Sc, Brownian motion parameter Nb and thermophoresis parameter Nt, temperature difference ΔT and velocity ratio A was investigated.

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