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Proton scattering on carbon nuclei in bichromatic laser field at moderate energies

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ABSTRACT

We present the general theory for proton nuclei scattering in a bichromatic laser field. As a physical example we consider proton collision on ^{12}C at 49 MeV/amu moderate energies in the field of a titan sapphire laser with its second harmonic.

1. Introduction

Optical laser field intensities exceed the 10^{22} W/cm^2 limit nowadays, where radiation effects dominate the electron dynamics. In such a strong field non-linear laser-matter interaction in atoms, molecules and plasmas can be investigated both theoretically and experimentally. Two of such high-field effects are high harmonic generations, or plasma-based laser-electron acceleration. These field intensities open a path to high field quantum electrodynamics phenomena like vacuum-polarization effects of pair production [1]. In most of the presented studies the dynamics of the participating electrons are investigated. Numerous surveys on laser assisted electron collisions are available such as [2]. However, there are only few nuclear photo-excitation investigations done where some low-lying first excited states of medium or heavy elements are populated with the help of X-ray free electron laser pulses [3]. Nuclear excitation by atomic electron re-scattering in a laser field was investigated by Kornev [4] and concerning nuclear quantum optics see [5]. Recently nuclear collisions in laser-created plasmas and laser-assisted recollisions have been studied [6–9] in the frame of classical mechanics, based on trajectory descriptions. Our present study deals with the quantum mechanical description based on the inelastic diffraction of de Broglie wave of the proton.

To our knowledge there are no publications available where laser assisted proton or nucleus collisions with nuclei were investigated in strong laser fields.

For the projectile target interaction we consider the global optical Woods–Saxon (WS) [10] model potential. This formalism has been a very successful method to study the single particle spectra of nuclei in the last half century. Detailed description can be found in any basic nuclear physics textbooks like Greiner and Mahrun [11].

The nuclear physics community recently evaluated the closed analytic form of the Fourier transformed WS interaction [12] which is an important point concerning our present considerations.

We incorporate these results into a first Born approximation scattering cross section formula where the initial and final proton wave functions are Volkov waves and the induced photon emission and absorption processes are taken into account up to arbitrary orders.

The general theory of laser-assisted collision in bichromatic fields were worked out by Varró and Ehlotzky [13–15]. An overview about the field can be found in [16]. On the present example for laser-assisted scattering in a bichromatic field, we wish to demonstrate on the proton scattering that in case of two frequencies whose ratio is a natural number (here ν and 2ν) the multiphoton transition amplitudes (say for an energy change $4 \times \nu$ and $2 \times (2\nu)$) interfere in such a way, that this interference crucially depends on the relative phase of the components.

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A positron impact ionization of atomic hydrogen in bichromatic field was investigated by Jun [17] by theoretical mean which is one of the latest result is this field. In another recent paper [18] the question of strong-field ionization with two-color circularly polarized laser fields were discussed.

In the following study we extend our former description of proton-nuclei collision in monochromatic laser fields to bichromatic ones [19]. As examples we consider proton ^{12}C collisions at 49 MeV (where the numerical parameters of the WS potential is well-known and tested [20]) in the presence of IR laser fields.

2. Theory

Now we summarize our non-relativistic quantum mechanical description. The laser field is handled in the classical way via the minimal coupling. The laser beam is taken to be linearly polarized and the dipole approximation is used. If the dimensionless intensity parameter (or the normalized vector potential) $a_0 = 8.55 \cdot 10^{-10}\sqrt{I(\frac{W}{cm^2})}\lambda(\mu m)$ of the laser field is smaller than unity the non-relativistic description in dipole approximation is valid. For 800 nm laser wavelength this means a critical intensity of $I = 2.13 \cdot 10^{18}$ W/cm^2 for electrons. In case of protons a_0 is replaced by $a_p = ((m_p/m_e)^{-1})a_0$, where the proton to electron mass ratio is $(m_p/m_e) = 1836$. Accordingly, for 800 nm wavelength the critical intensity for protons is $I_{crit} = 3.91 \cdot 10^{21}$ W/cm^2.

Additionally, we consider a moderate proton kinetic energy, not so much above the Coulomb barrier and neglect the exchange term between the proton projectile and the target carbon protons. This proton exchange effect could be included in the presented model with the help of Woods–Saxon potentials of non-local type [21] but not in the scope of the present study.

The following calculation is similar to the monochromatic case which was published earlier [19].

To describe the non-relativistic scattering process of a proton on a nucleus in a spherically symmetric external field the following Schrödinger equation has to be solved

$$\left[\frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 + U(\mathbf{r})\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}, \tag{1}$$

where $\hat{\mathbf{p}} = -i\hbar\partial/\partial\mathbf{r}$ is the momentum operator of the proton, and $U(\mathbf{r})$ represents the scattering potential of the nucleon.

Our results for the scattering is not affected by the choice of the gauge, because the possible gauge factor drops out from the scattering amplitude. This is clearly seen if we perform a gauge transformation by using a generating function $\mathbf{r}\cdot\mathbf{A}(t)$ in the exponent of the wave function (in the dipole approximation used throughout the paper). This exponential expression naturally comes in if we start with a Hamiltonian in the $\mathbf{r}\cdot\mathbf{E}(t)$-gauge. The Volkov states in the $\mathbf{r}\cdot\mathbf{E}(t)$-gauge contains the mentioned exponent $\mathbf{r}\cdot\mathbf{A}(t)$, which does depend on the kinematics of the scattering, and it drops out from the transition amplitudes.

Let's consider the following external laser field in the form of

$$\mathbf{A}(t) = \boldsymbol{\epsilon}_1(\mathcal{E}_1/\omega)cos(\omega t) + \boldsymbol{\epsilon}_m(\mathcal{E}_m/m\omega)cos(m\omega t + \tilde{\varphi}) \tag{2}$$

where $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_m$ are the two independent polarization vectors. We take the same linear polarization for both fields from now on $\mathcal{E}_1$ and $\mathcal{E}_m$ are the two electric field strengths and $\tilde{\varphi}$ is the relative phase between them. In experiments the value of m is fixed to 2, 3 which corresponds to the second and third harmonics. The maximal, experimentally achievable $\mathcal{E}_2/\mathcal{E}_1$ ratio is about 10 percent created on non-linear medium, for the third harmonic the ratio is even less. However, with weakening of the main beam with the fundamental frequency any kind of $\mathcal{E}_{2,3}/\mathcal{E}_1$ ratio is available in realistic experiments. Fig. 1 shows the scattering geometry, where the $\mathbf{p}_i$ and $\mathbf{p}_f$ are the initial and final proton momenta, θ is the scattering angle of the proton, the laser is linearly polarized in the x–z plane, and the propagation of the laser field is parallel to the z axis. The carbon nucleus is fixed, because it has a much large weight (the scattering is studied in relative coordinates).

The ponderomotive potential for an extremely large-intensity laser field could, of course, modify the overall trajectories both of the target and the projectile. However, in the scattering process we are discussing, the quantum–mechanical diffraction is not affected by the ponderomotive potential, which is practically homogeneous at the collision place. Moreover, no channel-closing or -opening takes place in this process, in contrast to the phenomenon of "peak-suppression" in multiphoton ionization processes. Thus, we think that the role of the ponderomotive potential is likely very marginal concerning the basic features of the process under study.

Without the external scattering potential $U(\mathbf{r})$ the particular solution of (1) can be immediately written down as non-relativistic Volkov states $\varphi_p(\mathbf{r},t)$ which exactly incorporate the interaction with the laser field,

$$\varphi_p(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^{3/2}}exp\left[\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r} - \int_{t_0}^{t}dt'\frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}(t')\right)^2\right] \tag{3}$$

Volkov states, which are modulated de Broglie waves, parametrized by momenta $\mathbf{p}$ and form an orthonormal and complete set,

$$\int d^3r\varphi_p^*(\mathbf{r},t)\varphi_{p'}(\mathbf{r},t) = \delta_3(\mathbf{p}-\mathbf{p}')$$
$$\int d^3p\varphi_p(\mathbf{r},t)\varphi_p^*(\mathbf{r}',t) = \delta_3(\mathbf{r}-\mathbf{r}'). \tag{4}$$

To solve the original problem of Eq. (1) we write the exact wave function as a superposition of an incoming Volkov state and a correction term, which vanishes at the beginning of the interaction (in the remote past $t_0 \to -\infty$). The correction term can also be expressed in terms of the Volkov states, since these form a complete set (see the equation of (4)),

$$\Psi(\mathbf{r},t) = \varphi_{p_i}(\mathbf{r},t) + \int d^3p a_p(t)\varphi_p(\mathbf{r},t), \quad a_p(t_0) = 0. \tag{5}$$

It is clear that the unknown expansion coefficients $a_p(t)$ describe the non-trivial transition symbolized as $\mathbf{p}_i \to \mathbf{p}$, from a Volkov state of momentum $\mathbf{p}_i$ to another Volkov state with momentum $\mathbf{p}$. If we take the projection of Ψ into some Volkov state $\varphi_p(t)$ we get

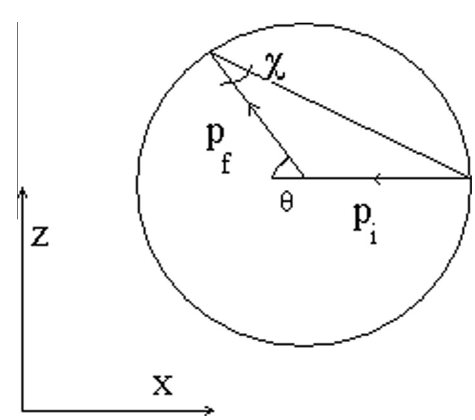

Fig. 1. The geometry of the scattering process. The ^{12}C nucleus is in the center of the circle, $\mathbf{p}_i$ and $\mathbf{p}_f$ stand for the initial and final scattered proton momenta, θ is the proton scattering angle, laser pulse propagates parallel to the x axis and linearly polarized in the x–z plane. The χ angle is needed for the laser-proton momentum transfer.

$$\int d^3r\varphi_p^*(\mathbf{r},t)\Psi(\mathbf{r},t) = \delta_3(\mathbf{p}-\mathbf{p}_i) + a_p(t). \quad (6)$$

By inserting Ψ of Eq. (5) into the complete Schrödinger Eq. (1), we receive the following integro-differential equation for the coefficients $a_p(t)$,

$$i\hbar\dot{a}_{p'}(t) = \int d^3r\varphi_{p'}^*(\mathbf{r},t')U(\mathbf{r})\varphi_{p_i}(\mathbf{r},t') + \int d^3pa_p(t) \times \int d^3\varphi_{p'}^*(\mathbf{r},t')U(\mathbf{r})\varphi_p(\mathbf{r},t'), \quad (7)$$

where the scalar product was taken with $\varphi_{p'}(t)$ on both sides of the resulting equation and the orthogonality property of the Volkov sates was taken after all (see the first Eq. of (4)). Owing to the initial condition $a_p(t_0) = 0$, displayed already in Eq. (4) the formal solution of (6) can be written down as

$$a_{p'}(t) = -\frac{i}{\hbar}\int_{t_0}^t dt' \int d^3r\varphi_{p'}^*(\mathbf{r},t')U(\mathbf{r})\varphi_{p_i}(\mathbf{r},t') - \frac{i}{\hbar}\int_{t_0}^t dt' \times \int d^3pa_p(t') \times \int d^3r\varphi_{p'}^*(\mathbf{r},t')U(\mathbf{r})\varphi_p(\mathbf{r},t'). \quad (8)$$

In the spirit of the iteration procedure used in scattering theory the $(k+1)$th iterate of $a_p(t)$ is expressed by the kth iterate on the right hand side in (8) like

$$a_p^{(k+1)}(t) = -\frac{i}{\hbar}\int_{t_0}^t dt' \int d^3r\varphi_{p'}^*(\mathbf{r},t')U(\mathbf{r})\varphi_{p_i}(\mathbf{r},t') - \frac{i}{\hbar}\int_{t_0}^t dt' \times \int d^3pa_p^{(k)}(t') \times \int d^3r\varphi_{p'}^*(\mathbf{r},t')U(\mathbf{r})\varphi_p(\mathbf{r},t'). \quad (9)$$

In the first Born approximation (where the transition amplitude is linear in the scattering potential $U(\mathbf{r})$) we receive the transition amplitude in the following form

$$T_{fi} = \lim_{t\to\infty}\lim_{t_0\to-\infty} a_{p_f}^{(1)}(t) = -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt' \int d^3r\varphi_{p_f}^*(\mathbf{r},t')U(\mathbf{r})\varphi_{p_i}(\mathbf{r},t'). \quad (10)$$

By taking the explicit form of the Volkov states (3) with the bichromatic vector potential (2) we note that the A^2 term drops out from the transition matrix element (10), and T_{fi} becomes

$$T_{fi} = \sum_{n=-\infty}^{\infty} T_{fi}^{(n)},$$
$$T_{fi}^{(n)} = -2\pi i\delta\left(\frac{p_f^2 - p_i^2}{2m} + n\hbar\omega\right)\frac{U(\mathbf{q})}{(2\pi\hbar)^3}C_n(a,b;\tilde{\varphi}), \quad (11)$$

where the Dirac delta stands for energy conservation, $U(\mathbf{q})$ is the Fourier transformed interaction potential. The main difference is to the monochromatic field $C_n(a,b;\varphi)$ which is the generalized Bessel function with the form of

$$C_n(a,b;\tilde{\varphi}) = \sum_{\lambda=-\infty}^{\infty} J_{n-m\lambda}(a)J_\lambda(b)e^{-i\lambda\tilde{\varphi}} \quad (12)$$

for the second or third harmonics if $m = 2,3$. The generalized phase-dependent Bessel functions can be obtained by expanding its generating function into a Fourier series viz.

$$e^{\{i[asin(\omega t)+bsin(m\omega+\tilde{\varphi})]\}} = \sum_{n=-\infty}^{\infty} e^{in\omega t}C_n(a,b;\tilde{\varphi}). \quad (13)$$

Here the various different Fourier components of the expansions of the two exponential on the left-hand side into ordinary Bessel functions J_λ yielding the same final harmonic frequency $n\omega$ may be considered in this classical problem as the infinite number of different phase-dependent "*reaction channels*" contributing to the matrix element of (11). Note that this is a generalization of the Jacobi–Anger formula which was used in the monochromatic case

$$e^{iasin(\omega t)} = \sum_{\lambda=-\infty}^{\infty} J_\lambda(a)e^{i\lambda\omega t}. \quad (14)$$

With the well-known symmetry property of the Bessel functions of the first kind $J_{-\lambda}(a) = (-1)^\lambda J_\lambda(a)$ we can easily verify the following symmetry relations for the second and third harmonics (12)

$$C_{-n}(a,b_2;\tilde{\varphi}) = (-1)^n C_n^*(a,b_2;\tilde{\varphi}-\pi),$$
$$C_{-n}(a,b_3;\tilde{\varphi}) = (-1)^n C_n^*(a,b_3;\tilde{\varphi}). \quad (15)$$

The $U(\mathbf{q})$ is the Fourier transformed of the scattering potential with the momentum transfer of $\mathbf{q} \equiv \mathbf{p}_i - \mathbf{p}_f$ where $\mathbf{p}_i$ is the initial and $\mathbf{p}_f$ is the final proton momentum. The absolute value is $q = \sqrt{p_i^2 + p_f^2 - 2p_ip_fcos(\theta_{p_i,p_f})}$. In our case, for 49 MeV energy protons absorbing optical photons the following approximation is valid $q \approx 2p_i|sin(\theta/2)|$. As we mentioned earlier the polarization vector of the laser pulses are parallel with the initial proton momentum $\epsilon\|\mathbf{p}_i$, then $\boldsymbol{\epsilon}(\mathbf{p}_i - \mathbf{p}_f) = p_i(1-cos(\theta))$.

The Dirac delta describes photon absorptions ($n < 0$) and emissions ($n > 0$) with energy conservation. The arguments of the two Bessel functions are the following

$$a = \frac{m_e}{m_p}a_0\boldsymbol{\epsilon}(\mathbf{p}_i - \mathbf{p}_f)c/\hbar\omega \quad (16)$$

$$b_m = \frac{m_e}{m_p}a_m\boldsymbol{\epsilon}_m(\mathbf{p}_i - \mathbf{p}_f)c/\hbar m\omega_0 \quad (17)$$

where $a_m = \frac{eE_m}{m_ecm\omega} = 10^{-9}\sqrt{I}_m/E_{ph}$. where the laser energy $\hbar\omega$ is measured in eV, the proton energy E_p in MeV and the laser intensity I in W/cm^2.

Collecting the constants together the very final formulas for the arguments are

$$a = 10^{-4}\sqrt{I}[1-cos(\theta)], \quad b_m = \frac{10^{-4}\sqrt{I}_m[1-cos(\theta)]}{m}. \quad (18)$$

(Note, that this formula is valid for any kind of external laser field. For a 49 MeV proton projectile even the 10 keV X-ray laser has a negligible energy.) The final differential cross section formula for the laser associated collision with simultaneous nth-order photon absorption and emission processes in a bichromatic field of a frequency plus it's mth-order high-harmonic is

$$\frac{d\sigma^{(n,m)}}{d\Omega} = \frac{p_f}{p_i}\frac{d\sigma_B}{d\Omega}\left|\sum_{\lambda=-\infty}^{\infty} J_{n-m\lambda}(a)J_\lambda(b_m)e^{-i\lambda\tilde{\varphi}}\right|^2 \quad (19)$$

where $\frac{d\sigma_B}{d\Omega}$ is the usual Born cross section for the scattering on the potential as was mentioned above. At the low energy limit, where the photon energy is much below the kinetic energy of the scattered particle the Born cross section can be extracted from the sum. The expression Eq. (19) was first calculated by [13–15].

If we investigate $d\sigma^{n,m}/d\sigma_B = |C_n(a,b_m,\tilde{\varphi})|^2$, we can study the modification of the cross sections due to the interaction with the laser fields. This is the main goal of the present study.

We note that the structure of the transition matrix elements of the scattering process is the same in case of two circularly polarized waves. In this case the A^2 terms also drop out from the transition matrix elements, and we arrive again with the generalized Bessel functions $C_n(a,b,\varphi)$, where now $a \sim |\epsilon_1 \cdot (p_i - p_f)|$ and $b \sim |\boldsymbol{\epsilon}_2 \cdot (p_i - p_f)|$, where $\boldsymbol{\epsilon}_{1,2}$ are the complex polarization vectors of the circularly polarized components. The difference is that the relative phase is modified by the difference of the arguments of the complex products $\boldsymbol{\epsilon}_1 \cdot (p_i - p_f)$ and $\boldsymbol{\epsilon}_2 \cdot (p_i - p_f)$ which depends on the kinematics of the scattering. In the case of co-propagating fields this modification is zero, so in this sense the study of the scattering in the presence of circularly polarized fields does not bring additional new features. The functional dependence on the relative phase of the assisting bichromatic components essentially remains the same.

We must say some words about the central scattering potential $U(r)$ which is the sum of the Coulomb potential of a uniform charged sphere [22] and a short range optical [10] potential

$$U(r) = V_c(r) + V_{ws}(r) + i[W(r) + W_s(r)] + V_{ls}(r)\mathbf{l}\cdot\sigma \quad (20)$$

where the Coulomb term is

$$V_c = \frac{Z_p Z_t e^2}{2R_0}\left(3 - \frac{r^2}{R_c^2}\right) \quad r < R_c$$
$$V_c = \frac{Z_p Z_t e^2}{r} \quad r \geqslant R_c \quad (21)$$

where $R_c = r_0 A_t^{1/3}$ is the target radius calculated from the mass number of the target with $r_0 = 1.25$ fm. Z_p, Z_t are the charge of the projectile and the target and e is the elementary charge. This kind of regularized Coulomb potential helps us to avoid singular cross sections and is routinely used in nuclear physics.

The short range nuclear part is given via

$$\begin{aligned}
V_{ws}(r) &= -V_r f_{ws}(r, R_0, a_0)\\
W(r) &= -V_v f_{ws}(r, R_s, a_s)\\
W_s(r) &= -W_s(-4a_s) f'_{ws}(r, R_s, a_s)\\
V_{ls}(r) &= -(V_{so} + iW_{so})(-2) g_{ws}(r, R_{so}, a_{so})\\
f_{ws}(r, R, a) &= \frac{1}{1 + exp(\frac{r-R}{a})}\\
f'_{ws}(r, R, a) &= \frac{d}{dr} f_{ws}(r, R, a)\\
g_{ws}(r, R, a) &= f'_{ws}(r, R, a)/r.
\end{aligned} \quad (22)$$

The constants V_r, W_v, V_{so} and W_{so} are the strength parameters, and $a_{0,s,so}, R_{0,s,so}$ are the diffuseness and the radius parameters given for large number of nuclei. The f function is called the shape function of the interaction. As we will see at moderate collisions energies the complex terms become zero. According to the work of [12] the complete analytic form of the Fourier transform of the WS potential can be calculated on the complex plain with contour integration using the residuum theorem. For exhaustive details see [12]. The Fourier transformed second term of Eq. (20) reads

$$\begin{aligned}
V_{ws}(q) = \frac{V_r}{\pi^2}\Bigg\{ & \frac{\pi a_0 e^{-\pi a_0 q}}{q(1 - e^{-2\pi a_0 q})^2}\left[R_0(1 - e^{-2\pi a_0 q})cos(qR_0)\right.\\
& \left. -\pi a_0 (1 + e^{-2\pi a_0 q}) sin(qR_0)\right]\\
& -a_0^3 e^{-\frac{R_0}{a_0}}\left[\frac{1}{(1 + a_0^2 q^2)^2} - \frac{2e^{-\frac{R_0}{a_0}}}{(4 + a_0^2 q^2)^2}\right]\Bigg\}.
\end{aligned} \quad (23)$$

The surface term $W_s(r)$ (fourth term in Eq. (20)) gives the following formula in the momentum space:

$$\begin{aligned}
W_s(q) = & -4a_s \frac{W_s}{\pi^2}\Bigg\{\frac{\pi a_s e^{-\pi a_s q}}{(1 - e^{-2\pi a_s q})^2}\left[\pi a_s (1 + e^{-2\pi a_s q})\right.\\
& \left. -\frac{1}{q}(1 - e^{-2\pi a_s q})cos(qR_s) + R_s(1 - e^{-2\pi a_s q}) sin(qR_s)\right]\\
& + a_s^2 e^{-\frac{R_s}{a_s}}\left[\frac{1}{(1 + a_s^2 q^2)^2} - \frac{4e^{-\frac{R_s}{a_s}}}{(4 + a_s^2 q^2)^2}\right]\Bigg\}.
\end{aligned} \quad (24)$$

The last term in Eq. (20), the transformed spin–orbit coupling term leads to

$$\begin{aligned}
V_{ls}(q) = & -\frac{a_{so}}{\pi^2}(V_{so}\\
& + iW_{so})\left\{\frac{2\pi e^{-\pi a_{so} q}}{1 - e^{-2\pi a_{so} q}} sin(qR_{so}) + e^{\frac{-R_{so}}{a_{so}}}\left(\frac{1}{1 + a_{so}^2 q^2} - \frac{2e^{-R_{so}/a_{so}}}{4 + a_{so}^2 q^2}\right)\right\}
\end{aligned} \quad (25)$$

where $\mathbf{q}$ is the momentum transfer as defined above. The low energy transfer approximation formula $q \approx 2p_i|sin(\theta/2)|$ is valid. The Fourier transform of the charged sphere Coulomb field is also far from being trivial

$$\begin{aligned}
V_c(q) = & \frac{Z_p Z_t e^2}{2^{\frac{5}{6}}\sqrt{\pi} q^3}\left(-2\cdot 3^{\frac{1}{3}} q \cos[2^{\frac{2}{3}} 3^{\frac{1}{3}} q] + 2^{\frac{1}{3}}(1 + 2\cdot 2^{\frac{1}{3}} 3^{\frac{2}{3}} q^2)\sin[2^{\frac{2}{3}} 3^{\frac{1}{3}} q]\right)\\
& + 3Z_p Z_t e^2 \sqrt{\frac{2}{\pi}}\left(\frac{i\pi|q|}{2q} - \mathrm{Ci}[2^{\frac{2}{3}} 3^{\frac{1}{3}} q] + \log(q) - \log|q| - i\mathrm{Si}[2^{\frac{2}{3}} 3^{\frac{1}{3}} q]\right)
\end{aligned} \quad (26)$$

where Ci and Si are the cosine integral and the sine integral functions, respectively. For details see [23].

3. Results

We applied the non-perturbative method, outlined in the previous section to 49 MeV proton – ^{12}C scattering. The parameters of the Woods-Saxon potential are the following the three potential strength V_R, W_s, V_{so} are 31.31, 5.98, 2.79 MeV, the three diffuseness a_0, a_s, a_{so} are 0.68, 0.586, 0.22 fm and the three radius parameters R_0, R_s, R_{so} are 1.276, 0.89, 0.716 fm, respectively.

The shape of the Fourier transformed angular differential cross section functions of the various Woods–Saxon potential terms for 49 MeV elastic proton – ^{12}C scattering Eqs. (23)–(25) are presented and analyzed in our former study [19] in details. Our calculated total cross section of the elastic scattering is 201 mbarn which is consistent with the data of [20].

Fig. 2 shows the angular dependence of the differential cross section for the elastic, monochromatic and bichromatic fields at 10^{12} W/cm^2 laser intensities. In our former study we found that at higher laser intensities the laser assisted cross sections are many magnitudes below the elastic Born cross sections. This can be understood from the mathematical properties of the Bessel functions. The maximum (with respect to the index n) of the Bessel function is around $n_{max} \approx z$ where z is the argument. If the intensity is $I = 10^{16}$ W/cm^2 the argument of (18) is about 10^4. This also means that for a satisfactory convergence to calculate the general-

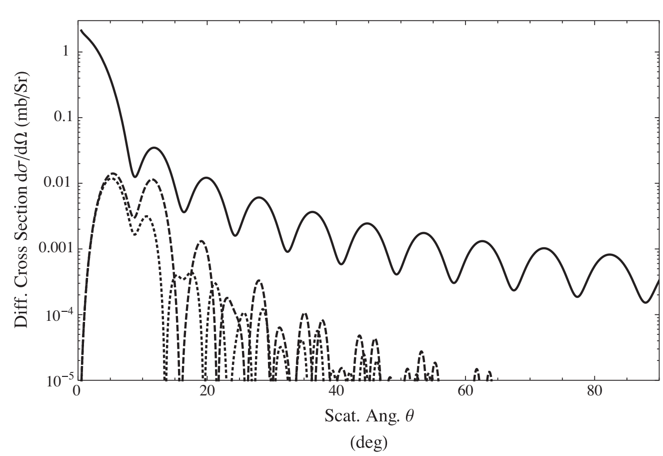

Fig. 2. The calculated angular differential cross sections for $I = 10^{12}$ W/cm^2 laser field intensity. The thick solid line is the $n = 0$ elastic Born cross section, the dashed line is for the monochromatic case for one photon absorption, and the doted line is for bichromatic fields at single photon absorption Eq. (19). The intensity ratio is $I/I_2 = 2$ with $\tilde{\varphi} = 0$ relative phase difference.

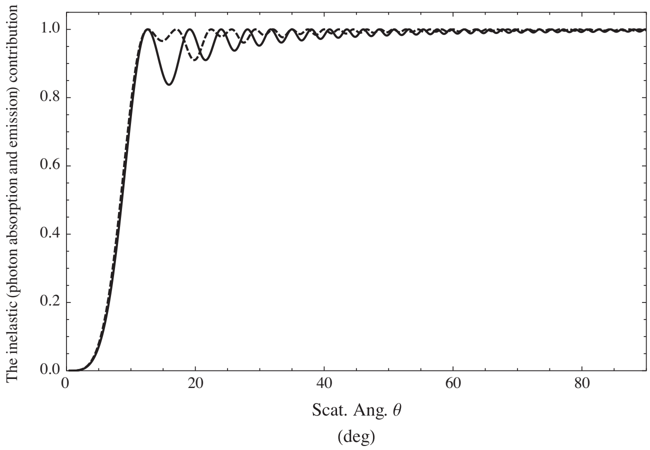

Fig. 3. The total inelastic contributions of the laser assisted scattering. Solid line is for the monochromatic and the dashed line is for the bichromatic field. Parameters are the same as in Fig. 2.

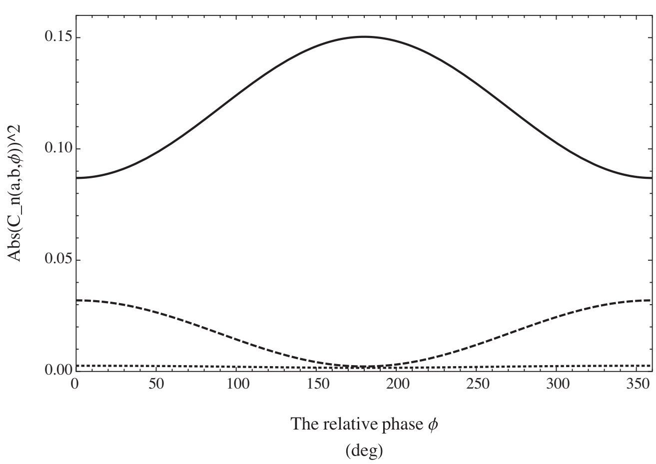

Fig. 4. Shows the dependence on the relative phase between the fundamental and the second harmonics for $I = 10^{12}$ W/cm² intensities. The solid line is for one photon the dashed line is for two photon and the dotted line is for three photon absorptions, respectively.

ized Bessel functions the value of the sum $\lambda = 1000$ should be. For such large values of the argument ($|n| \ll |z|$) the following asymptotic expansion can approximate [24] (formula 8.451)

$$J_{\pm n}(z) \approx \sqrt{\frac{2}{\pi z}} cos(z \mp n\pi/2 - \pi/4). \quad (27)$$

For small arguments however, ($|z| \ll |n|$) the power expansion is valid $J_n(z) \approx (z/2)^n$ [24] (formula 8.440). As we mentioned earlier instead of the complete angular differential cross section it is enough to investigate the properties of $d\sigma_{Total}/d\sigma_{Born} = J_n^2(z)$ for monochromatic or $d\sigma_{Total}/d\sigma_{Born} = |C_n(a, b_m, \tilde{\varphi})|^2$ for bichromatic laser assisted scattering to know how the differential cross sections behave at parameter changes. Is is known that $\sum_{\lambda=-\infty}^{+\infty} J_\lambda^2(z) = 1$ or similarly $\sum_{\lambda=-\infty}^{+\infty} |C_\lambda(a, b_m, \tilde{\varphi})|^2 = 1$. Therefore the total inelastic contributions (where any photons are absorbed or emitted) are the following: $1 - J_0^2(z)$ for the monochromatic case and $1 - |C_0(a, b_m, \tilde{\varphi})|^2$ for bichromatic case. Fig. 3 presents such results for $I = 10^{12}$ W/cm² intensities. Note, that the remarkable contributions are at small scattering angles.

The role of the relative phase between the two corresponding frequencies $\tilde{\varphi}$, as a coherent control parameter is also instructive

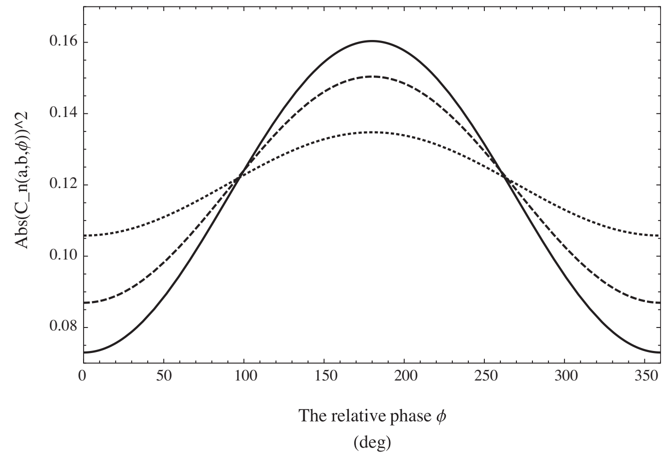

Fig. 5. The role of relative intensity of the two frequencies I/I_2 for single photon absorption as the function of the relative phase $\tilde{\varphi}$. The solid line refers to the numerical value $I/I_2 = 1$, the dashed line to 2 and the dotted line to 10 intensity ratios, respectively.

to examine. First, we may fix the scattering angle to a fix degree say $\theta = 7^\circ$ and investigate how the various phases change the absolute value of the C_n. Fig. 4 presents the relative phase dependence of $|C_n(a, b_2, \tilde{\varphi})|$ for one, two and three photon absorptions. The major contribution is at the single photon absorption which meets our physical intuition.

Fig. 5 presents the role of the intensity ratios I/I_2 at the above fixed range for various relative phases $\tilde{\varphi}$. The larger the intensity of the second harmonic the larger the role of the relative phase.

4. Summary

We presented a formalism which gives an analytic angular differential cross section for the proton-nucleon scattering on a Woods–Saxon optical potential in a bichromatic laser field where the *n*th-order photon absorption and emission is taken into account simultaneously in a non-perturbative way. As a physically relevant example we investigated the proton – ^{12}C collision system at moderate 49 MeV proton energies in the field of an optical Ti: sapphire system with wavelength of 800 nm and its second harmonic. The role of the relative phase between the fields are investigated. We hope that our study will perhaps give an impetus and bring the nuclear and laser physics community together to perform such experiments in the ELI [25] or X-FEL [26] facilities which will be available within a couple of years.

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