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Effect of Nonuniform Magnetic Field on Ferrofluid Flow and Heat Transfer

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Abstract. Our aim is to investigate the flow and heat transfer between a heated viscous incompressible ferrofluid and a wall in the presence of a spatially varying field. Similarity transformation is applied to convert the governing nonlinear boundary-layer equations into coupled nonlinear ordinary differential equations. These equations are numerically solved using a commercial software. The effects of governing parameters corresponding to miscellaneous physical conditions are analyzed. Numerical results are represented for distributions of velocity and temperature, further more are obtained for the dimensionless wall skin friction and heat-transfer coefficients. In special case, two bifurcate solutions have been achieved and one of the two solutions compares well with previous studies.

INTRODUCTION

In the recent decades, nanofluids are intensively investigated by various researchers due to their wide applicability in industry. These suspensions are prepared with various metals or non-metals and the base fluid. Ferrofluids are stable colloidal suspensions of non-magnetic carrier liquid containing very fine magnetized particles(see [1]). Nanofluids can be used in technological processes e.g., in heat exchanger, vehicle cooling, nuclear reactor, cooling of electronic devices.

When magnetizable materials are subjected to an external magnetizing field **H**, the magnetic dipoles or line currents in the material will align and create a magnetization **M**.

Problem of magnetohydrodynamic (MHD) flow near infinite plate has been studied intensively by a number of researchers (see, e.g., [2], [3], [4], [5]). In recent years Neuringer and Rosensweig [6] developed a model, where the effect of magnetic body force was considered under the assumption that the magnetization vector **M** is parallel to the magnetic field vector **H**.

Andersson [2] extended the so-called Crane's problem by studying the influence of the magnetic field, due to a magnetic dipole, on a shear driven motion, on a flow over a stretching sheet of a viscous non-conducting ferrofluid. It has been shown that the effect of the magnetic field is a slowing of the fluid movement compared to the hydrodynamic case. Neuringer [7] has examined numerically the dynamic response of ferrofluids to the application of non-uniform magnetic fields with studying the effect of magnetic field on two cases, the two-dimensional stagnation point flow of a heated ferrofluid against a cold wall and the two-dimensional parallel flow of a heated ferrofluid along a wall with linearly decreasing surface temperature.

Our goal is to re-investigate the two-dimensional parallel flow of a heated ferrofluid along a wall with varying surface temperature of power-law type and the behaviour of ferrofluids in magnetic field using similarity analysis. The similarity method is applied for the governing equations to transform partial differential equations to nonlinear ordinary differential equations. Numerical solutions are obtained with higher derivative method. The heat transfer, velocity and temperature distribution in the boundary layer are provided. The behaviour of the velocity and thermal distribution is presented. The effects of the parameters involved in the boundary value problem are graphically illustrated.

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Mathematical formulation

Consider a steady two-dimensional flow of an incompressible, viscous and electrically non-conducting ferromagnetic fluid over a flat sheet in the horizontal direction.

The dipole of the magnet is placed at a distance a from the surface, in such a way its center lies on y-axis. The magnetic field ($\mathbf{H}$) due to the magnetic dipole is directed towards positive x-direction. The ferrofluid influences by the dipole of the permanent magnet whose scalar potential is

$$\phi(x,y) = -\frac{I_0}{2\pi}\left(\tan^{-1}\frac{y+a}{x} + \tan^{-1}\frac{y-a}{x}\right), \tag{1}$$

where I_0 denotes the dipole moment per unit length and a is the distance of the line current from the leading edge. The wall temperature is a decreasing function of x and is given by $T_w = T_c + Ax^{m+1}$, where T_c denotes the Curie temperature, A and m are real constants.

The negative gradient of the magnetic scalar potential ϕ equals to the applied magnetic field, i.e. $\mathbf{H} = -\nabla\phi$.

In paper [6] it was showed that the existence of spatially varying fields is required in ferrohydrodynamic interactions. We shall have the following assumptions for the exposition of ferrohydrodynamic interaction, that the fluid temperature must be less than Curie temperature, and the applied magnetic field is inhomogeneous.

Then, the dynamic response of ferrofluids to the application of non-uniform magnetic fields follows from the fact that the force per unit volume on a piece of magnetized material of magnetization $\mathbf{M}$ (i.e. dipole moment per unit volume) in the field of magnetic intensity $\mathbf{H}$ is given by the form $\mu_0 M\nabla H$, where $H = \sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2}$, μ_0 denotes the free space permeability and M represents the magnitude of $\mathbf{M}$. Since $(\partial\phi/\partial x)_{y=0} = 0$ and $\left(\partial^2\phi/\partial y^2\right)_{y=0} = 0$ at the wall, then $[\nabla H]_y$ vanishes.

In the boundary layer for regions close to the wall when distances from the leading edge large compared to the distances of the line sources from the plate, i.e. $x \gg a$, then one gets

$$[\nabla H]_x = -\frac{I_0}{\pi}\,\frac{1}{x^2}. \tag{2}$$

From the above said consideration of the flow analysis, the governing equations (conservation of mass, momentum and energy) of the boundary layer flow are formed according to the assumptions of [7] and the variation of magnetization M is the linear function of temperature as reported by $M = K(T_c - T)$, where K is the pyromagnetic coefficient and T_c denotes the Curie temperature proposed by [8].

Then, the governing equations are described as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{I_o\mu_0 K}{\pi\rho}(T_c - T)\frac{1}{x^2} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{4}$$

$$c\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

where the x and y axes are taken parallel and perpendicular to the plate, u and v are the parallel and normal velocity components to the plate, respectively, μ_0 means the permeability of the vacuum, c is the thermal heat capacity, k is the thermal conductivity, ν is the kinematic viscosity and ρ denotes the density of the ambient fluid, which will be assumed constant. Equations (3)-(5) are considered under the boundary conditions at the surface ($y = 0$) with

$$u(x,0) = 0, \qquad v(x,0) = 0, \qquad T(x,0) = T_w, \tag{6}$$

where $T_w = T_c + Ax^{m+1}$ and as y leaves the boundary layer ($y \to \infty$) with

$$u(x,y) \to u_\infty, \qquad T(x,y) \to T_\infty \tag{7}$$

where $T_\infty = T_c$, and u_∞ is the exterior streaming speed which is assumed throughout the paper to be $u_\infty = U_\infty x^m$ ($U_\infty = const.$). Parameter m is relating to the power law exponent. The parameter $m = 0$ refers to a linear temperature profile and constant exterior streaming speed. In case of $m = 1$, the temperature profile is quadratic and the streaming speed is linear. The value of $m = -1$ corresponds to no temperature variation on the surface.

Introducing the stream function ψ and using the following transformations, the structure of (4)–(7) allows us to look for similarity solutions of a class of solutions ψ and T in the form (see [9])

$$\psi(x,y) = C_1 x^b f(\eta), \qquad T = T_c + A x^{m+1}\Theta(\eta), \qquad \eta = C_2 x^d y, \tag{8}$$

where b and d satisfy the scaling relation $b + d = m$ and for positive coefficients C_1 and C_2 the relation $C_1/C_2 = \nu$ are fulfilled. The real numbers b, d are such that $b - d = 1$ and $C_1 C_2 = U_\infty$, i.e. $b = (m+1)/2$, $d = (m-1)/2$, $C_1 = \sqrt{\nu U_\infty}$, $C_2 = \sqrt{U_\infty/\nu}$.

By taking into account (8), equations (4) and (5) and conditions (6) and (7) lead to the following system of coupled ordinary differential equations

$$\frac{d^3 f}{d\eta^3} - m\left(\frac{df}{d\eta}\right)^2 + \frac{m+1}{2}\, f\, \frac{df}{d\eta} - \beta\,\Theta = 0, \tag{9}$$

$$\frac{d^2\Theta}{d\eta^2} + (m+1)\ \mathrm{Pr}\left(\frac{1}{2}\, f\, \frac{d\Theta}{d\eta} - \Theta\,\frac{df}{d\eta}\right) = 0. \tag{10}$$

The boundary conditions reduces to the following equations subjected to the boundary conditions

$$f(0) = 0, \qquad \frac{d}{d\eta} f(0) = \lambda, \qquad \Theta(0) = 1, \tag{11}$$

$$\frac{d}{d\eta} f(\eta) = 1, \qquad \Theta(\eta) = 0 \qquad as\ \eta \to \infty, \tag{12}$$

where $\mathrm{Pr} = c\nu/k$ is the Prandtl number and $\beta = I_0\mu_0 K A/(\pi\rho U_\infty^2)$ is the ferromagnetic parameter.

The components of the non-dimensional velocity $\mathbf{v} = (u, v, 0)$ can be expressed by

$$u = U_\infty\, x^m\, \frac{df(\eta)}{d\eta}, \tag{13}$$

$$v = -\sqrt{\nu U_\infty}\, x^{(m-1)/2}\left(\frac{m+1}{2}\, f(\eta) + \frac{m-1}{2}\frac{df(\eta)}{d\eta}\,\eta\right). \tag{14}$$

The physical quantities that specify the surface drag and heat transfer rate can be derived. Mathematically these quantities are interpreted in the following form

$$\tau_{y=0} = \nu\rho\left(\frac{\partial u}{\partial y}\right)_{y=0} = \rho U_\infty\sqrt{\nu U_\infty}\, x^{\frac{3m-1}{2}}\,\frac{d^2}{d\eta^2} f(0), \tag{15}$$

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -kA\sqrt{\frac{U_\infty}{\nu}}\, x^{\frac{3m+1}{2}}\,\frac{d}{d\eta}\Theta(0), \tag{16}$$

where $(d^2 f/d\eta^2)(0)$ denotes the skin friction coefficient and $(d\Theta/d\eta)(0)$ stands for the heat transfer coefficient.

Numerical Results

The system of equations (9)–(10) with the corresponding conditions (11)–(12), is interpreted numerically using BVP solution technique built in Maple. During our investigations, the velocity and temperature changes in the boundary layer are examined and the effects of the parameters on the solution are illustrated on the figures.

The boundary value problems can have the situation that either no solution or multiple solutions exist even for the simple set of differential equations (see [10] and [11]). The Prandtl number $Pr = 10$ is fixed as a typical value of kerosine based ferrofluid. The obtained solutions of velocity and thermal distribution can be seen on Figs. 1-2. The ferromagnetic parameter β highlights the effect of the external magnetic field. The variation of λ is shown on Figs. 1-2. It is can also be noticed that the boundary layer thickness is different for different values of β and parameter m. The thermal boundary-layer thickness is smaller than the corresponding velocity boundary-layer thickness.

Conclusions

This paper presents similarity solution of the boundary layer flow and heat transfer over a cold wall of a ferrofluid flow in the presence of spatially varying magnetic field. By means of similarity transformation, the governing mathematical equations are reduced into ordinary differential equations which are then solved numerically using BVP solution technique. The effects of some governing parameters namely ferromagnetic parameter β, Prandtl number and power law parameter on the flow, and heat transfer characteristics are graphically presented and discussed.

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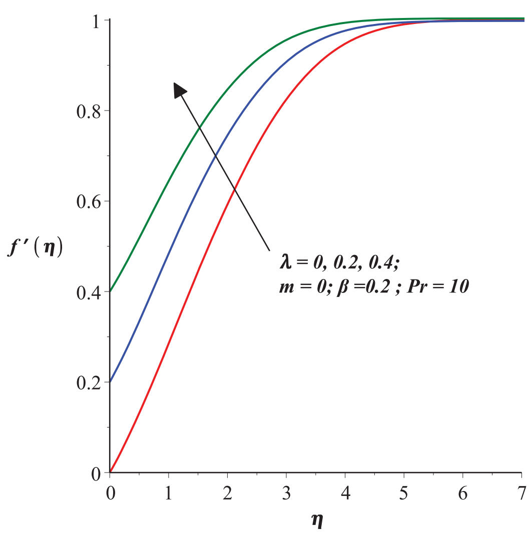

FIGURE 1. The velocity distribution

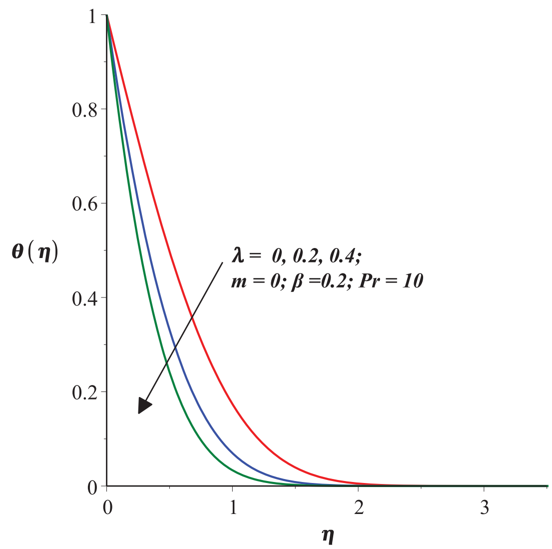

FIGURE 2. The temperature distribution

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